Controle Adaptativo - Lista 3 - 2023

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1) Consider the signal

$$y \,=\, A\cos\left(\omega t \,+\, \varphi\right), \quad (*)$$

which is broadcast with a known frequency ω but an unknown phase φ and an unknown amplitude A. Assuming that the signal y is available for measurement, our objective is to estimate A and φ using the knowledge of ω and measurements of y. Using the identity

$$A\cos\left(\omega t \,+\, \varphi\right) \,=\, A\cos\varphi\cos\omega t \,-\, A\sin\varphi\sin\omega t,$$

we obtain

$$y \,=\, \theta^{*T}\phi,$$

with

$$\theta^* \,=\, \begin{bmatrix} A_1 & A_2 \end{bmatrix}^T$$

and

$$\phi(t) \,=\, \begin{bmatrix} \cos\omega t & -\sin\omega t \end{bmatrix}^T,$$

where

$$A_1 \,=\, A\cos\varphi \qquad \text{and} \qquad A_2 \,=\, A\sin\varphi.$$

Now, θ^* can be estimated as $\theta \,=\, \begin{bmatrix} \hat{A}_1 & \hat{A}_2 \end{bmatrix}^T$, using

$$\left\{ \begin{array}{ccc} \dot{\theta} & = & \Gamma\epsilon\phi \\ \epsilon & = & \frac{y-\hat{y}}{m_s^2} \,=\, \frac{y-\theta^T\phi}{m_s^2} \end{array} \right.$$

Since $\phi \in \mathcal{L}_\infty$, we can take the normalizing signal $m_s \,=\, 1$. Once, $\theta^* \,=\, \begin{bmatrix} \hat{A}_1 & \hat{A}_2 \end{bmatrix}^T$, we can obtain the estimates of A and φ as

$$\hat{A} \,=\, \sqrt{\hat{A}_1^2 \,+\, \hat{A}_2^2}$$

and

$$\hat{\varphi} \,=\, \cos^{-1}\left(\frac{\hat{A}_1}{\hat{A}}\right).$$

Now, assume that $A = 3$, $\varphi = 0.436\,rad$ for $0 \leq t \leq 10\,sec$, $A = 5$, $\varphi = 0.611\,rad$ for $t > 10\,sec$, and $\omega = 5\,rad/sec$. Let us choose the adaptive gain Γ as the identity matrix. The initial values of the estimates are taken to be $\hat{A}_1(0) = 0$ and $\hat{A}_2(0) = \frac{\pi}{2}$.

2) Consider the second-order stable system

$$\dot{x} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u,$$

where x and u are available for measurement, $u \in \mathcal{L}_\infty$ and a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2 are unknown parameters. Design an online estimator to estimate the unknown parameters. Simulate your scheme using $a_{11} = -5$, $a_{12} = 0.5$, $a_{21} = 0.7$, $a_{22} = -3$, $b_1 = 1$, $b_2 = 2.2$, and $u = 10sin(2t)$. Repeat the simulation when $u = 10sin(2t) + 7sin(3.6t)$. Comment on your results.