

Задача 1.11. Поведение динамической системы описывается уравнением

$$0,002\ddot{y}(t) + 0,21\dot{y}(t) + 4y(t) = 2e^{-5t}.$$

Определить собственное движение системы $y_c(t)$, если $y(0) = y_0$; $\dot{y}(0) = 0$.

$$\frac{K^2}{500} + \frac{21K}{100} + 4 = 0$$

$$\frac{(K+25)(K+80)}{500} = 0$$

$$K_1 = -25 \quad K_2 = -80$$

$$\bar{y} = \frac{C}{e^{25t}} + \frac{C_1}{e^{80t}}$$

$$y = \frac{A}{e^{5t}}$$

$$y^* = -\frac{5A}{e^{5t}}$$

$$y^* = \frac{25A}{e^{5t}}$$

$$\frac{3A}{e^{5t}} = \frac{2}{e^{5t}}$$

$$3A = 2 \Rightarrow A = \frac{2}{3}$$

$$y^* = \frac{2}{3e^{5t}}$$

$$y = y^* + \bar{y}$$

$$y = \frac{2}{3e^{5t}} + \frac{C}{e^{25t}} + \frac{C_1}{e^{80t}}$$

$$\begin{cases} y(0) = y_0 \\ y'(0) = 0 \end{cases} \Rightarrow \begin{cases} \frac{2}{3} + C + C_1 = \frac{2}{3} \\ -\frac{10}{3} - 25C - 80C_1 = 0 \end{cases}$$

$$y' = -\frac{10e^{-5t}}{3} - 25Ce^{-25t} - 80C_1e^{-80t}$$

$$- \begin{cases} C + C_1 = 0 \\ 25C + 80C_1 = -\frac{10}{3} \end{cases} \Rightarrow C_1 = -C$$

$$24C + 79C_1 = -\frac{10}{3}$$

$$24C - 79C = -\frac{10}{3}$$

$$55C = \frac{10}{3}$$

$$C = \frac{10}{165}$$

$$C_1 = -\frac{10}{165}$$