Mathematical modeling

Airline hub study

Nicholas Wong

(1464763)

Background:

Airport hubs are used by airlines to concentrate passenger traffic and flight operations to a particular airport. They also serve as a transfer point for passengers going to the final destination. Airlines operate flights from non-hub cities to hub cities or through hub cities if the final destination is not a hub city. It is important for an airline to have a hub so that they can provide more flights which suit their passengers' needs and to maximize their company's revenue and profit. It is also crucial to consider the flow and the population of the city. In other words, a hub can be described as "a big airport with a lot of direct flight".

There are various operating costs and expenses airlines have to consider, such as the maintenance, fuel cost, hardware (e.g type of aeroplane) and labor cost (e.g salaries). Also, airlines have to pay extra by having an airport hub, since they will need more employees, facilities which will increase extra costs and reduce profit margin. Therefore, it is crucial to consider the efficiency of the flight network and minimize the airport hubs, to maximize the profit for the airline.

In this study project, the ultimate goal is to find an optimum number of airline hubs which would minimize the operating cost for the airline. In particular, Short-Hop airline. The airline will provide transportation between nine cities. All of the nine cities could be used for optimum hub locations. This report will introduce a simple model for solving this optimization problem.

Assumptions:

The airline has given 2 constraints and I have made several assumptions for this study. The first rule is that all cities should be within 200 miles of a hub city and the number of hub cities should be as small as possible. The assumptions I have made are:

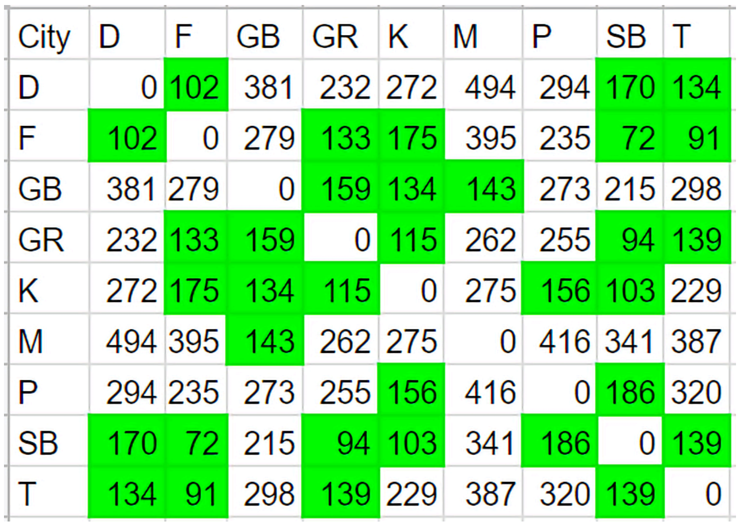
- Assume all aeroplane have the same efficiency level
- The cost of a flight from one city to another city is determined **only** by the distance between the cities/airport
- 1 mile = $10
- No loop

Data:

Code	Name
D	Dayton
F	Fort Wayne
GB	Green Bay
GR	Grand Rapids
K	Kenosha
M	Marquette
P	Peoria
SB	South Bent
T	Toledo

Table 1. The nine airports

The distance matrix from each city to another



City	D	F	GB	GR	K	M	P	SB	T
D	0	102	381	232	272	494	294	170	134
F	102	0	279	133	175	395	235	72	91
GB	381	279	0	159	134	143	273	215	298
GR	232	133	159	0	115	262	255	94	139
K	272	175	134	115	0	275	156	103	229
M	494	395	143	262	275	0	416	341	387
P	294	235	273	255	156	416	0	186	320
SB	170	72	215	94	103	341	186	0	139
T	134	91	298	139	229	387	320	139	0

Figure 1.1

Nine of the cities are placed into Table 1, indicated with the corresponding code names. It is easier to work with the simplified names. The distances between cities are put into the matrix figure 1.1, since this study will use the distance between city i and city j to evaluate the cost for traveling. The green highlighted entries are the selective ‘minimized’ distance, the distance between i and j which are less than 200 miles, qualifying the constraints given by the airline. This method is essentially making every airport a hub, then slowly eliminating each airport based on cost and efficiency. Notice that the matrix figure 1.1 is symmetric, because the airports are pairwise. Also, the diagonal of the matrix is zero since there are no flights going to the same city.

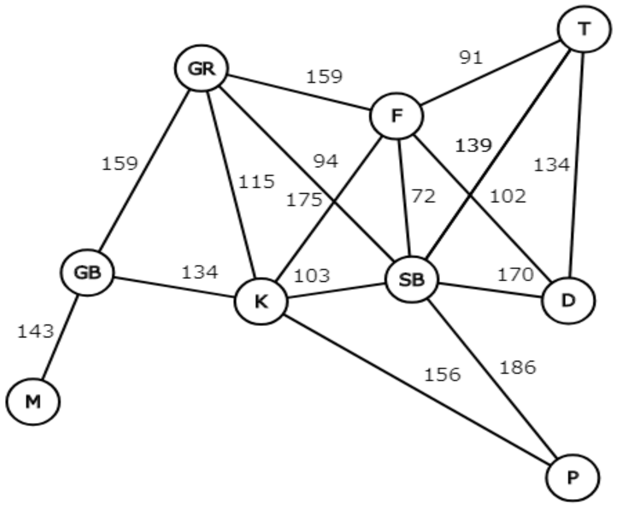


Figure 1.2

Figure 1.2 represents the minimized routes connecting the airports, the distances are the edges. They are all less than 200 miles. Each of the nine airports is a vertex and if there is only one hub, then that particular vertex ensures that every flight goes through the hub first before going to its final destination. In this case, it is not possible to have **only** 1 hub to satisfy the constraints given by the company (less than 200 miles). As clearly shown, M is the only isolated airport. When M is the final destination or a starting point, reaching or leaving M must pass through a point which is GB. However, even though there are possible routes for flight to travel from GB to airport M, GR and K. There are no direct routes to travel from GB to other airports (have to satisfy constraints less than 200 miles), which are F, SB, D, P and T and vice versa since it is pairwise. **Which means there has to be at least one more hub somewhere, in order to operate the airlines correctly.**

GB is one of the ideal hubs to fly to airport M, K and GR, through simple minimization, each flight costs approximately $1430, $1340 and $1590 respectively.

e.g) Cost for M → GB: $143\ miles\ \times \frac{\$10}{1\ mile} = \$1430$

To find out which airport is the best as a second hub, this study will use Dijkstra's algorithm. Finding the shortest paths between vertices in a graph and minimizing the number of hubs- figure 1.2.

Simple Model Formulation:

$d_{ij} = distance\ from\ city\ i\ to\ j$ ... (1)

$x_{ij} = \ fly\ directly\ from\ \ i\ to\ j$ ... (2)

$x_{ii} = 0, \forall i\ not\ flying\ to\ the\ same\ city$ ... (3)

$min\ H\ =\ minimum\ airport\ hubs$

$initial\ boundary := \ 1\ \leq min\ H\ \leq 9$

$new\ boundary\ with\ a\ least\ 2\ hub := \ 1 + 1\ \leq min\ H\ \leq 9 \Rightarrow 2 \leq min\ H \leq 9$

$c\ = cost$ ... (4)

$c\ = d_{ij},\ cost\ of\ direct\ flight$ ... (5)

$if\ d_{ij} \neq x_{ij}$

$c\ = min \sum d_{ij} = \ cost\ of\ flight\ \ , if\ flying\ through\ 1\ or\ 2\ hub\ or\ more\ than\ 1\ city$ ... (6)

Now, all the ideas and assumptions are put into a mathematical manner. This is a simple model, the first term is the measurement of distance. The second term indicates a direct flight from i to j. Third term constraints flight cannot fly to the same airport. Fifth term is the cost of a direct flight from i to j. If it is not a direct flight from i to j, then flight will pass via one or two hubs or serverals airports before arriving at the final destination, the cost is the total of minimized and shortest distance. By using the idea of Dijkstra's algorithm, perform the algorithm on each airport. The airport that repeated the most in the algorithm is the best solution, the boundary for min hub is $2 \le min\ H \le 7$. Airport M and P are eliminated as options because it is simply economically inefficient, with 1 path and 2 path respectively.

Calculation:

$$2 \le min\ H \le 9, \text{elimination of M and P}$$

$$2 \le min\ H \le 9 - 1_{\ M} - 1_{\ P} \Rightarrow 2 \le min\ H \le 7$$

As a result, boundary becomes $2 \le min\ H \le 7$.

Algorithms:

Pairwise

Repeated airport highlighted in different color

Choosing M and P as starting point

From M → T :

- M → GB → K→ SB→ T (shorter, the Dijkstra's solution)
- M → GB → K→ SB→ F→ T

From M → D:

- M→GB→K→ SB→ D

From M→ F:

- M→GB→ K→ SB→ F (same distance)
- M→GB→ K→F (same distance, the Dijkstra's solution)

From M→ GR

- M→ GB→ GR (shorter, Dijkstra's solution)
- M→ GB→ K→ GR

From P→ T

- P→ SB→ T

From P→ D

- P→ SB→ D

From P→ M

- P→ K→ GB→ M

From P→ GR

- P→ K→ GR (shorter, Dijkstra's solution)
- P→ SB→ GR

The sequences above are performed by Dijkstra's algorithm, choosing the shortest pathway given at a vertex. I have also included an alternative path in some particular routes, which is not labeled as 'Dijkstra's solution'; the alternative pathway is not performed by Dijkstra's algorithm but only by connecting the vertex with a different route. By choosing a longer route like from M to T, the method can ensure the sub-routes like GB → K→ SB→ T or K→ SB→ T are also the shortest and considered.

$$GB \rightarrow K \rightarrow SB \text{ and } K \rightarrow SB \rightarrow T \subseteq M \rightarrow T$$

It is clear that regardless of which direction the flight is flying, there are 3 major airports the aeroplane must pass through. They are airport GB, K and SB.
Therefore, setting the 3 airports GB, K and SB as the maximum airport hubs, the new boundary becomes $2 \leq min\ H \leq 3$. Yet, we are sure that GB has to be a hub, because it is the only airport connecting airport M.

Consider the of airport K and SB, using the route P→ GR:

- P→ K→ GR = 156 + 115 = 271
- P→ SB→ GR = 186 + 94 = 280

Clearly, by using a route through K, it is cheaper to operate, it saves $90.

$$(280 \ - \ 271\ )miles \times \frac{\$10}{1\,mile} = \$90$$

However, SB is still a better airport as a hub. SB is a far more efficient airport, it has 6 pairwise direct routes, and connects major routes. Essentially, SB and K could be a hub but to minimize the cost and number of hubs, choosing the more effective one is preferred. Therefore, the boundary becomes $2 \leq min\, H \ \leq 2$

Similar to Squeeze theorem, as a result, $min\, H \ = 2$

And using this method, not only can we find the minimum numbers of hubs but also the explicit solution of most efficient hubs available. Note that Dijkstra's algorithm gives the shortest path, but not the best solution, as it does not account for efficient factors.

Conclusion

Using the distance factor and analysis, the model computed the optimum number of hubs and the optimum location for the airline, for the purpose of saving cost and efficiency. If the airline were to make 2 hubs, it would be airport GB and SB. The need of opening the third hub is very low, as GB and SB dominate all major lines already. It would be pointless to create another hub next to the domain area, like K. The result relied heavily on the distance as the cost, since there is insufficient data to calculate otherwise. Also, Dijkstra's algorithm only finds the shortest path for a graph but it ignores a lot of realistic factors, particularly efficiency and expenses. The use of computer programming is also not included, brute force calculation was used since it is simple with only 9 vertices. If the studies have more airports to begin with, coding must be implemented. Otherwise, it would take forever to find the shortest path via the algorithm. Nevertheless, the model seemed too simple and could be improved a lot more with more data and numbers. Afterall, distance alone was not the only factor that contributed to the cost of operating and finding the optimum hub. For future improvement to the current model, this study will need a lot more data and guidelines from the airline. A more realistic way to model will require the population of each city, frequency and flow of flight in each airport, capacity of the airport, discount rate for delay cost, landing fees, and many more factors have to be taken into account.

Given the limited data and constraints, this is the most realistic approach to minimize the cost and to find the optimum number of hubs, it also gives us explicitly which two airports are ideal. Therefore, the solution is unique as long as the airline set the minimum of hub to be $min\,H = 2$. There is no general method to solve such a problem, it all depends on the assumptions, constraints and most importantly the data. However, note that the tool Dijkstra's algorithm is infamous for solving problems involving shortest distance and path within a graph and network.

Acknowledgement:

Prof. Hassan Safouhi (from University of Alberta)
Mr. Kenneth Chau (from Western University)

Appendix:

Note: No coding was used in this study. If there is the necessity for codes, below is the pseudocode.

```
dist[0] ← 0        %distance to source vertex is zero
for all v ∈ $V - \{s\}$
        dist[v] ← ∞ %set all other distances to infinity
S ← ϕ %S set of visited vertices is empty at start
W ← $V$ % W, contains all vertices
While W≠ ϕ %while W is not empty
```

```
 u ← mindist(W, dist) %select element of W with the minimum distance
        S ← S ∪{u} %add u to list of visited vertices
        for all v∈neighbors[u]
                    if dist[v] > dist[u] + g(u,v)  %shortest path located
                    then
                    d[v] ← d[u] +g(u,v) %new value of shortest path
return dist
```