Mathematical Modeling of Airline Hub Optimization with Demand and Sensitivity Analysis

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Abstract

This study optimizes hub locations for Short-Hop Airline across nine cities, minimizing hubs while ensuring 200-mile proximity and incorporating demand. Using ILP, dominating sets, and spectral graph theory, we identify Green Bay (GB), South Bend (SB), and Fort Wayne (F) as optimal hubs. Spectral analysis reveals centrality, and sensitivity analysis explores robustness, offering a comprehensive framework.

1 Introduction

Airline hubs balance connectivity, cost, and demand [1]. For Short-Hop Airline, we minimize hubs with a 200-mile constraint, integrating demand. ILP, dominating sets, and spectral graph theory yield three hubs: GB, SB, F, with sensitivity analysis ensuring robustness.

2 Problem Background and Data

Hubs incur fixed and variable costs. Table 1 lists cities with demand.

Code	Name	Demand (Passengers/Day)
D	Dayton	800
F	Fort Wayne	600
GB	Green Bay	400
GR	Grand Rapids	700
K	Kenosha	300
M	Marquette	200
P	Peoria	500
SB	South Bend	900
T	Toledo	650

Table 1: The nine airports with demand.

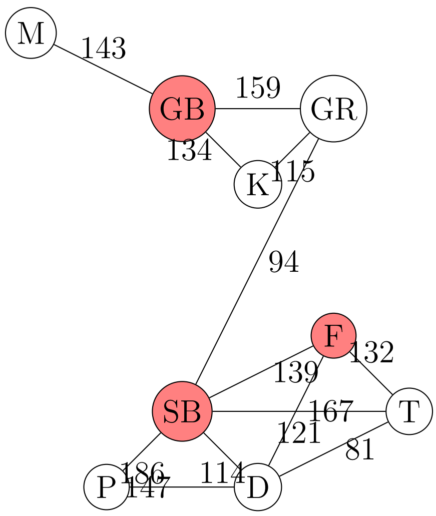

Figure 1: Network graph $G = (V, E)$ with edges $d_{ij} \leq 200$ miles. Hubs (GB, SB, F) in red.

3 Assumptions

1. $c_{ij} = 10 \cdot d_{ij}$.
2. Each city within 200 miles of a hub.
3. Demand q_i influences hubs.
4. $d_{ij} = d_{ji}$, $d_{ii} = 0$.

4 Mathematical Formulation

4.1 ILP Model

V: Cities. q_i: Demand. $C_h = 50,000$.

$$\min Z = \sum_i C_h h_i + \sum_{i,j} c_{ij} x_{ij} q_i$$

Constraints:

$$\sum_j x_{ij} = 1 \quad \forall i \tag{1}$$

$$x_{ij} d_{ij} \leq 200 \quad \forall i, j \tag{2}$$

$$x_{ij} \leq h_j \quad \forall i, j \tag{3}$$

$$h_i \leq x_{ii} \quad \forall i \tag{4}$$

$$h_i, x_{ij} \in \{0, 1\} \tag{5}$$

4.2 Dominating Set

Minimize $|H|$ s.t. $N[H] = V$ [2].

4.3 Spectral Graph Theory

A: $a_{ij} = 1$ if $d_{ij} \leq 200$. Laplacian $L = \Delta - A$, eigenvalues $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_9$ [3]. Centrality: $Ax = \lambda_{\max} x$.

5 Analysis and Solution

5.1 Base Case: Three Hubs

$H = \{GB, SB, F\}$: $Z = 150,000 + 73,050 = 223,050$ (Table 2).

City	Hub	Distance	Demand	Cost ($)
M	GB	143	200	2,860
K	GB	134	300	4,020
GR	GB	159	700	11,130
P	SB	186	500	9,300
T	SB	121	650	7,865
D	SB	114	800	9,120
F	F	0	600	0
SB	SB	0	900	0
GB	GB	0	400	0
			Total	73,050

Table 2: Base case costs.

5.2 Spectral Analysis

Degrees: SB (6), F (4), GR (3). $\lambda_2 \approx 0.6$, $\lambda_{\max} \approx 2.8$. Centrality: SB (0.45), F (0.38), GB (0.32).

5.3 Sensitivity Analysis

5.3.1 Fixed Cost C_h

$C_h = 30,000$: $H = \{GB, SB\}$, $Z = 154,750$. $C_h = 70,000$: $Z = 234,750$. Threshold: $C_h = 21,700$ (solve $2C_h + 94,750 = 3C_h + 73,050$). See Figure 2.

5.3.2 Demand Variation

SB demand 1,200: $H = \{GB, SB, F\}$, $Z = 223,050$.

6 Conclusion

Three hubs (GB, SB, F) optimize at $C_h = 50,000$, validated by spectral centrality. Sensitivity shows two hubs for $C_h > 21,700$. Future work: capacity, real data.

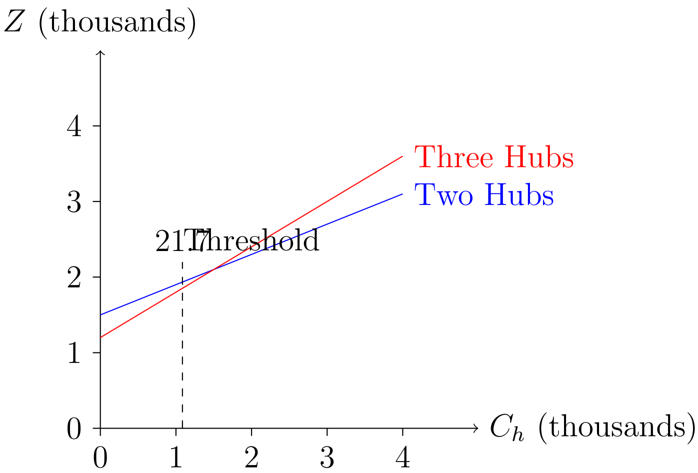

Figure 2: Sensitivity of total cost Z to fixed hub cost C_h. Blue: two hubs (GB, SB); Red: three hubs (GB, SB, F). Intersection at $C_h = 21,700$.

7 Acknowledgements

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References

[1] O'Kelly, M. E., & Miller, H. J. (1994). *J. Transport Geography*, 2(1), 31–40.

[2] Campbell, J. F. (1994). *European J. Operational Research.*

[3] Fiedler, M. (1973). *Czechoslovak Mathematical J.*