Decision Analysis

Decision Analysis Sequence

This is the introduction (conclusion) to my decision analysis sequence. It covers (much more quickly and less completely) what you would expect to see in a semester-long course on decision making. The posts are:

1. **Uncertainty**: the basics of treating uncertainties as probabilities and doing Bayesian math.
2. **5 Axioms of Decision Making**: the five steps / assumptions that form the foundation of careful decision-making.
3. **Compressing Reality to Math**: how to take a sticky, complicated situation and condense it down to something a calculator can solve, without feeling like you've left something important out.
4. **Measures, Risk, Death, and War**: how to deal with many similar prospects (utilities), risks of death, and adversaries.
5. **Value of Information: Four Examples**: how to value information-gathering activity, like tests or waiting, and incorporate it into your decision-making process.

I'd like to welcome any comments about the sequence here. What parts did I do well? What parts need work? What parts would you like to see expanded (or removed)?

One of the difficulties in posting about a topic like this is that it's foundational: basic, but important to get right. The idea of an expected utility calculation is not new (although the approach I take here may be novel for many of you) and, like I say in the VoI post, there's often more benefit in applying the process to examples than repeatedly talking about the process. The case studies I have access to, though, are not ones I can publish online, and I don't think I can construct an example that would work as well as a real one. Do people have problems they would like me to analyze with this framework as examples?

5 Axioms of Decision Making

This is part of a sequence on decision analysis; the first post is a primer on Uncertainty.

Decision analysis has two main parts: abstracting a real situation to math, and then cranking through the math to get an answer. We started by talking a bit about how probabilities work, and I'll finish up the inner math in this post. We're working from the inside out because it's easier to understand the shell once you understand the kernel. I'll provide an example of prospects and deals to demonstrate the math, but first we should talk about axioms. In order to be comfortable with using this method, there are five axioms1 you have to agree with, and if you agree with those axioms, then this method flows naturally. They are: **Probability, Order, Equivalence, Substitution,** and **Choice.**

Probability

You must be willing to assign a probability to quantify any uncertainty important to your decision. You must have consistent probabilities.

Order

You must be willing to order outcomes without any cycles. This can be called transitivity of preferences: if you prefer A to B, and B to C, you must prefer A to C.

Equivalence

If you prefer A to B to C, then there must exist a *p* where you are indifferent between a deal where you receive B with certainty and a deal where you receive A with probability *p* and C otherwise.

Substitution

You must be willing to substitute an uncertain deal for a certain deal or vice versa if you are indifferent between them by the previous rule. Also called "do you really mean it?"

Choice

If you have a choice between two deals, both of which offer A or C, and you prefer A to C, then you must pick the deal with the higher probability of A.

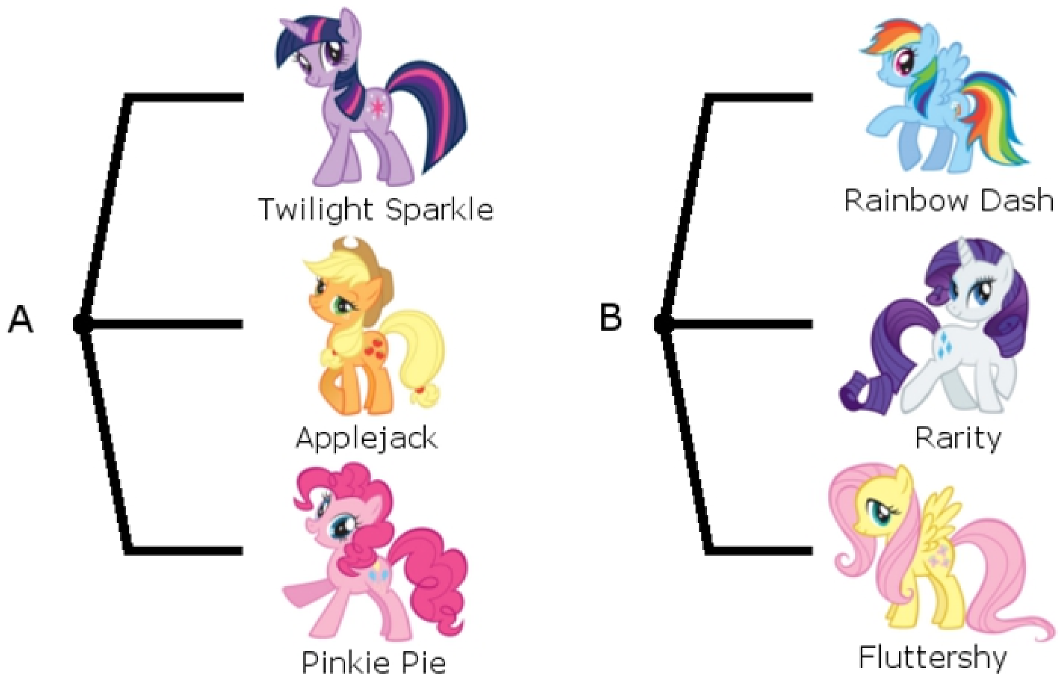
These five axioms correspond to five actions you'll take in solving a decision problem. You assign **probabilities**, then you **order** outcomes, then you determine **equivalence** so you can **substitute** complicated deals for simple deals, until you're finally left with one obvious **choice**.

You might be uncomfortable with some of these axioms. You might say that your preferences genuinely cycle, or you're not willing to assign numbers to uncertain events, or you want there to be an additional value for certainty beyond the prospects involved. I can only respond that these axioms are prescriptive, not descriptive: you will be better off if you behave this way, but you must choose to.

Let's look at an example:

My Little Decision

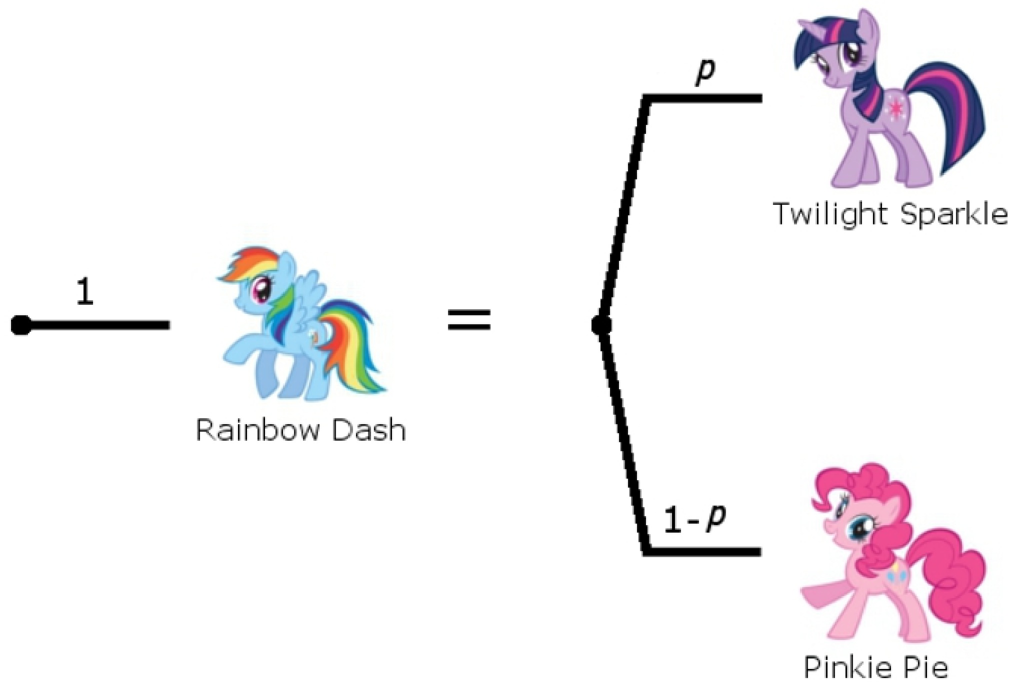
Suppose I enter a lottery for MLP toys. I can choose from two kinds of tickets: an A ticket has a 1/3 chance of giving me a Twilight Sparkle, a 1/3 chance of giving me an Applejack, and a 1/3 chance of giving me a Pinkie Pie. A B ticket has a 1/3 chance of giving me a Rainbow Dash, a 1/3 chance of giving me a Rarity, and a 1/3 chance of giving me a Fluttershy. There are two deals for me to choose between- the A ticket and the B ticket- and six prospects, which I'll abbreviate to TS, AJ, PP, RD, R, and FS.



(Typically, decision nodes are represented as squares, and work just like uncertainty nodes, and so A would be above B with a decision node pointing to both. I've displayed them side by side because I suspect it looks better for small decisions.)

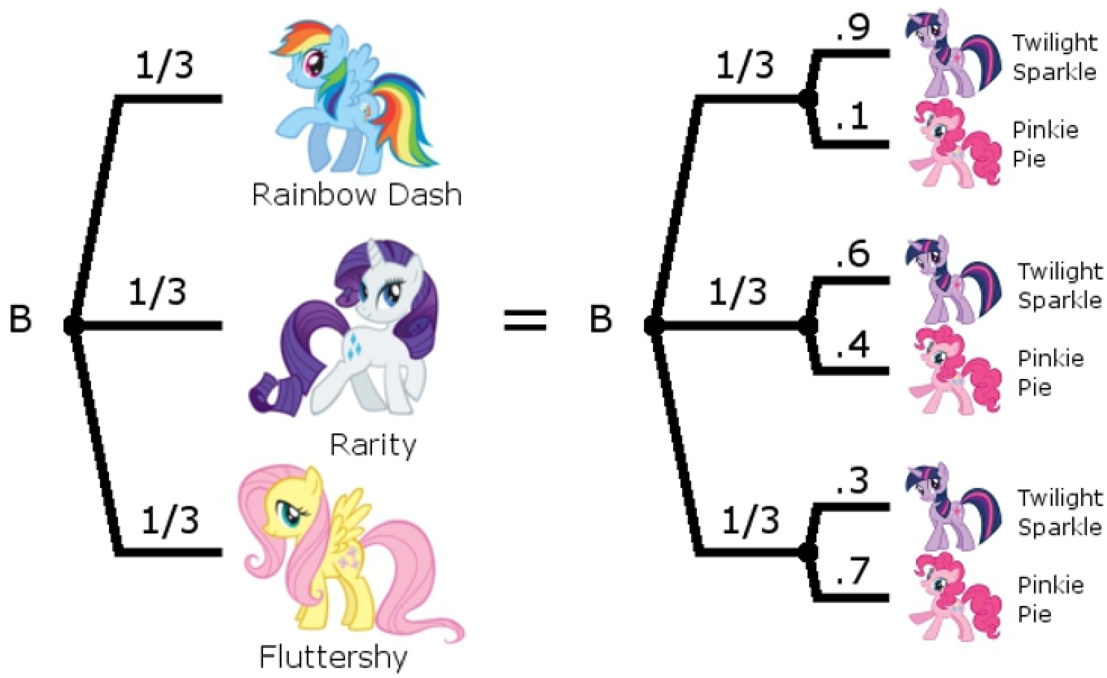
The first axiom- probability- is already taken care of for us, because our model of the world is already specified. We are rarely that lucky in the real world. The second axiom- order- is where we need to put in work. I need to come up with a preference ordering. I think about it and come up with the ordering TS > RD > R = AJ > FS > PP. Preferences are *personal*- beyond requiring internal consistency, we shouldn't require or expect that everyone will think Twilight Sparkle is the best pony. Preferences are also a source of uncertainty if prospects satisfy multiple different desires, as you may not be sure about your indifference tradeoffs between those desires. Even when prospects have only one *measure*, that is, they're all expressed in the same unit (say, dollars), you could be uncertain about your risk sensitivity, which shows up in preference probabilities but deserves a post of its own.

Now we move to axiom 3: I have an ordering, but that's not enough to solve this problem. I need a preference scoring to represent how much I prefer one prospect to another. I might prefer cake to chicken and chicken to death, but the second preference is far stronger than the first! To determine my scoring I need to imagine deals and assign indifference probabilities. There are a lot of ways to do this, but let's jump straight to the most sensible one: compare every prospect to a deal between the best and worst prospect.2

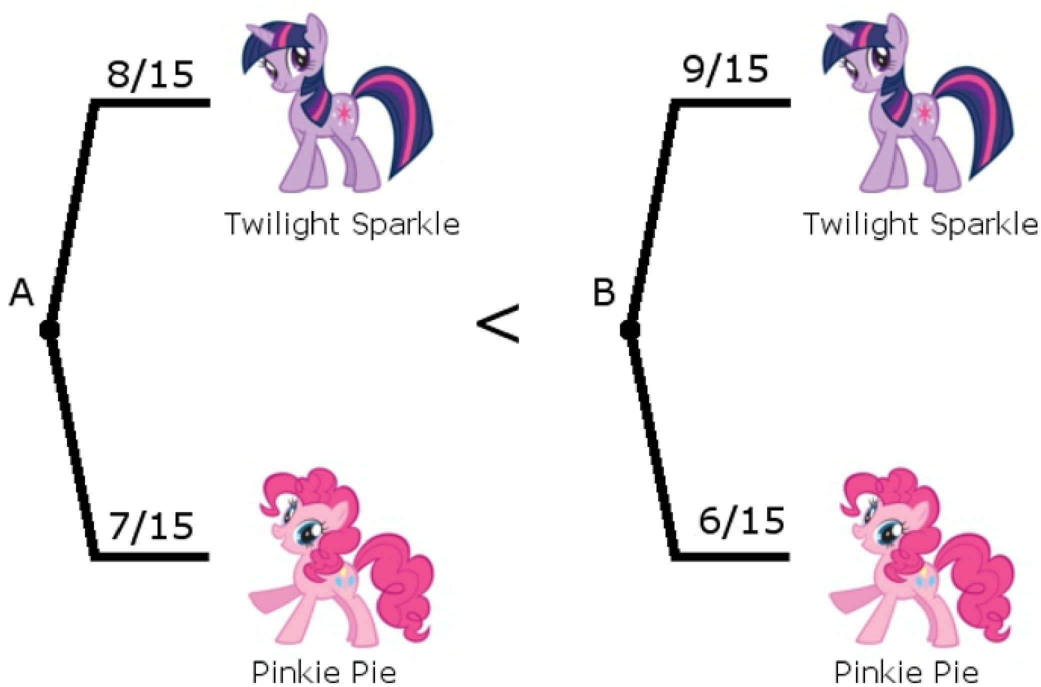


I need to assign a *preference* probability *p* such that I'm indifferent between the two deals presented: either RD with certainty, or a chance at TS (and PP if I don't get it). I think about it and settle on .9: I like RD close to how much I like TS.3 This indifference needs to be two-way: I need to be indifferent about trading a ticket for that deal for a RD, and I need to be indifferent about trading a RD for that deal.4 I repeat this process with the rest, and decide .6 for R and AJ and .3 for FS. It's useful to check and make sure that all the relationships I elicited before hold- I prefer R and AJ the same, and the ordering is all correct. I don't need to do this process for TS or PP, as *p* is trivially 1 or 0 in that case.

Now that I have a preference scoring, I can move to axiom 4. I start by making things more complicated- I take all of the prospects that weren't TS or PP and turn them into deals of {*p* TS, 1-*p* PP}. (Pictured is just the expansion of the right tree; try expanding the tree for A. It's much easier.)



Then, using axiom 1 again, I rearrange this tree. The A tree (not shown) and B tree now have only two prospects, and I've expressed the probabilities of those prospects in a complicated way that I know how to simplify.



And we have one last axiom to apply: choice. Deal *B* has a higher chance of the better prospect, and so I pick it. Note that that's the case even though my actual chance of receiving TS with deal B is 0%- this is just how I'm representing my preferences, and this computation is telling me that my probability-weighted preference for deal B is higher than my probability-weighted preference for deal A. Not only do I know that I should choose deal B, but I know how much better deal B is for me than deal A.5

This was a toy example, but the beauty of this method is that all calculations are local. That means we can apply this method to a problem of arbitrary size without changes. Once we have probabilities and preferences for the possible outcomes, we can propagate those from the back of the tree through every node (decision or uncertainty) until we know what to do everywhere. Of course, whether the method will have a runtime shorter than the age of the universe depends on the size of your problem. You could use this to decide which chess moves to play against an opponent whose strategy you can guess from the board configuration, but I don't recommend it.6 Typical real-world problems you would use this for are too large to solve with intuition but small enough that a computer (or you working carefully) can solve it exactly if you give it the right input.

Next we start the meat of decision analysis: reducing the real world to math.

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1. These axioms are Ronald Howard's 5 Rules of Actional Thought.

2. Another method you might consider is comparing a prospect to its neighbors; RD in terms of TS and R, R in terms of RD and FS, FS in terms of R and PP. You could then unpack those into the preference probability

3. Assigning these probabilities is tough, especially if you aren't comfortable with probabilities. Some people find it helpful to use a probability wheel, where they can *see* what 60% looks like, and adjust the wheel until it matches what they feel. See also 1001 PredictionBook Nights and This is what 5% feels like.

4. In actual practice, deals often come with friction and people tend to be attached to what they have beyond the amount that they want it. It's important to make sure that you're actually coming up with an indifference value, *not* the worst deal you would be willing to make, and flipping the deal around and making sure you feel the same way is a good way to check.

5. If you find yourself disagreeing with the results of your analysis, double check your math and make sure you agree with all of your elicited preferences. An unintuitive answer can be a sign of an error in your inputs or your calculations, but if you don't find either make sure you're not trying to start with the bottom line.

6. There are supposedly 10120 possible games of chess, and this method would evaluate all of them. Even with computation-saving implementation tricks, you're better off with another algorithm.

Compressing Reality to Math

This is part of a sequence on decision analysis and follows 5 Axioms of Decision-Making, which explains how to turn a well-formed problem into a solution. Here we discuss turning reality into a well-formed problem. There are three basic actions I'd like to introduce, and then work through some examples.

Scope

The first thing you have to decide with a problem is, well, what the problem is. Suppose you're contemplating remodeling your kitchen, and the contractor you're looking at offers marble or granite countertops. While deciding whether you want marble or granite, you stop and wonder- is this really the contractor that you should be using? Actually, should you even be remodeling your kitchen? Maybe you should to move to a better city first. But if you're already thinking about moving, you might even want to emigrate to another country.

At this point the contractor awkwardly coughs and asks whether you'd like marble or granite.

Decisions take effort to solve, especially if you're trying to carefully avoid bias. It helps to partition the world and deal with local problems- you can figure out which countertops you want without first figuring out what country you want to live in. It's also important to keep in mind lost purposes- if you're going to move to a new city, remodeling your kitchen is probably a mistake, even after you already called a contractor. Be open to going up a level, but not paralyzed by the possibility, which is a careful balancing act. Spending time periodically going up levels and reevaluating your decisions and directions can help, as well as having a philosophy of life.

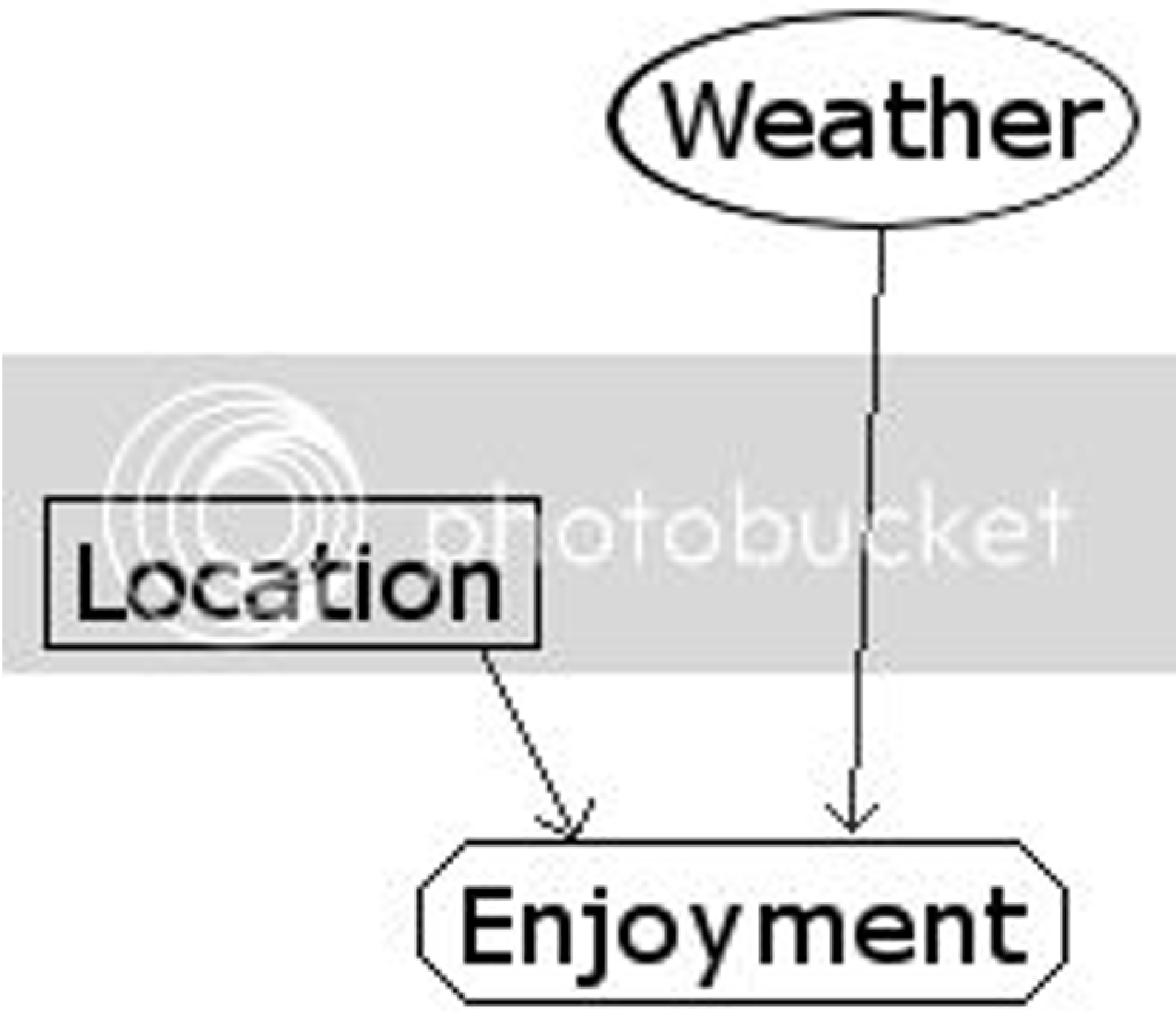
Model

Now that you've got a first draft of what your problem entails, how does that corner of the world work? What are the key decisions and the key uncertainties? A tool that can be of great help here is an influence diagram, which is a directed acyclic graph[1] which represents the uncertainties, decisions, and values inherent in a problem. While sketching out your model, do you become more or less comfortable with the scope of the decision? If you're less comfortable, move up (or down) a level and remodel.

Elicit

Now that you have an influence diagram, you need to populate it with numbers. What (conditional) probabilities do you assign to uncertainty nodes? What preferences do you assign to possible outcomes? Are there any other uncertainty nodes you could add to clarify your calculations? (For example, when making a decision based on a medical test, you may want to add a "underlying reality" node that influences the test results but that you can't see, to make it easier to elicit probabilities.)

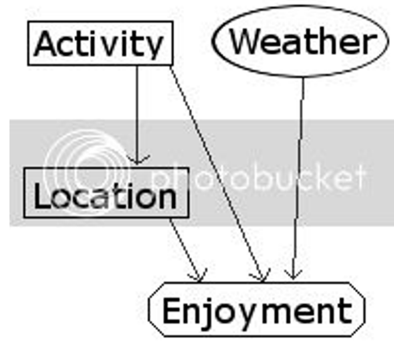
Outing with a Friend



A friend calls me up: "Let's do something this weekend." I agree, and ponder my options. We typically play Go, and I can either host at my apartment or we could meet at a local park. If it rains, the park will not be a fun place to be; if it's sunny, though, the park is much nicer than my apartment. I check the weather forecast for Saturday and am not impressed: 50% chance of rain.[2] I check calibration data and see that for a 3 day forecast, a 50% prediction is well calibrated, so I'll just use that number.[3]

If we play Go, we're locked in to whatever location we choose, because moving the board would be a giant hassle.[4] But we could just talk- it's been a while since we caught up. I don't think I would enjoy that as much as playing Go, but it would give us location flexibility- if it's nice out we'll go to the park, and if it starts raining we can just walk back to my apartment.

Note that I just added a decision to the problem, and so I update my diagram accordingly. With influence diagrams, you have a choice of how detailed to be- I could have made the activity decision point to two branches, one in which I see the weather then pick my location, and another where I pick my location and then see the weather. I chose a more streamlined version, in which I choose between playing Go or walking wherever the rain isn't (where the choice of location isn't part of my optimization problem, since I consider it obvious.



Coming up with creative options like this is one of the major benefits of careful decision-making. When you evaluate every part of the decision and make explicit your dependencies, you can see holes in your model or places where you can make yourself better off by recasting the problem. Simply getting everything on paper in a visual format can do wonders to help clarify and solve problems.

At this point, I'm happy with my model- I could come up with other options, but this is probably enough for a problem of this size.[5] I've got six outcomes- either (we walk, play go (inside, outside)) and it (rains, doesn't rain). I decide that I would most prefer playing go outside and it doesn't rain, and would least prefer playing go outside and it rains.[6] Ordering the rest is somewhat difficult, but I come up with the following matrix:

	Go at Park	Go at Home	Walk
Rains	0	.9	.7
Doesn't	1	.5	.8

If we walk, I won't enjoy it quite as much as playing Go, but if we play Go in my apartment and it's sunny I'll regret that we didn't play outside. Note that all of the preferences probabilities are fairly high- that's because having a game interrupted by rain is much worse than all of the other outcomes. Calculating the preference value of each deal is easy, since the probability of rain is independent of my decision and .5: I decide that playing Go at Park is the worst option with an effective .5 chance of the best outcome, Go at Home is better with an effective .7 chance of the best outcome, and Walk is best with an effective .75 chance of the best outcome.

Note that working through everything worked out better for us than doing scenario planning. If I knew it would rain, I would choose Go at Home; if I knew it wouldn't rain, I would choose Go at Park. Walk is dominated by both of those in the case of certainty, but its lower variance means it wins out when I'm very uncertain about whether it will rain or not.[7]

What's going up a level here? Evaluating whether or not I want to do something with this friend this weekend (and another level might be evaluating whether or not I want them as a friend, and so on). When evaluating the prospects, I might want to compare them to whatever I was planning before my friend called to make sure this is actually a better plan. It could be the chance of rain is so high it makes this plan worse than whatever alternatives I had before I knew it was likely to rain.

Managing a Store

Suppose I manage a store that sells a variety of products. I decide that I want to maximize profits over the course of a year, plus some estimate of intangibles (like customer satisfaction). I have a number of year-long commitments (the lease on my space, employment contracts for full-time employees, etc.) already made, which I'll consider beyond the scope of the problem. I then have a number of decisions I have to make month-to-month[8] (how many seasonal employees to hire, what products to order, whether to change wages or prices, what hours the store should be open), and then decisions I have to make day-to-day (which employees to schedule, where to place items in the store, where to place employees in the store, how to spend my time).

I look at the day-to-day decisions and decide that I'm happy modeling those as policies rather than individual decisions- I don't need to have mapped those out now, but I do need to put my January orders in next week. Which policies I adopt might be relevant to my decision, though, and so I still want to model them on that level.

Well, what about my uncertainties? Employee morale seems like one that'll vary month-to-month or day-to-day, though I'm comfortable modeling it month-to-month, as that's when I would change the employee composition or wages. Customer satisfaction is an uncertainty that seems like it would be worth tracking, and so is customer demand- how the prices I set will influence sales. And I should model demand by item- or maybe just by category of items. Labor costs, inventory costs, and revenue are all nodes that I could stick in as uncertainty nodes (even though they might just deterministically calculate those based on their input nodes).

You can imagine that the influence diagram I'm sketching is starting to get massive- and I'm just considering this year! I also need to think carefully about how I put the dependencies in this network- should I have customer satisfaction point to customer demand, or the other way around? For uncertainty nodes it doesn't matter much (as we know how to flip them), but may make elicitation easier or harder. For decision nodes, order is critical, as it represents the order the decisions have to be made in.

But even though the problem is getting massive, I can still use this methodology to solve it. This'll make it easier for me to keep everything in mind- because I won't need to keep everything in *mind*. I can contemplate the dependencies and uncertainties of the system one piece at a time, record the results, and then integrate them together. Like with the previous example, it's easy to include the results of other people's calculations in your decision problem as a node, meaning that this can be extended to team decisions as well- maybe my marketer does all the elicitation for the demand nodes, and my assistant manager does all the elicitation for the employee nodes, and then I can combine them into one decision-making network.

Newcomb's Problem

Newcomb's Problem is a thought experiment designed to highlight the difference between different decision theories that has come up a lot on Less Wrong. AnnaSalamon wrote an article (that even includes influence diagrams that are much prettier than mine) analyzing Newcomb's Problem, in which she presents three ways to interpret the problem. I won't repeat her analysis, but will make several observations:

1. Problem statements, and real life, are often ambiguous or uncertain. Navigating that ambiguity and uncertainty about what problem you're actually facing is a major component of making decisions. It's also a major place for bias to creep in: if you aren't careful about defining your problems, they will be defined in careless ways, which can impose real and large costs in worse solutions.
2. It's easy to construct a thought experiment with contradictory premises, and not notice if you keep math and pictures out of it. Draw pictures, show the math. It makes normal problems easier, and helps you notice when a problem boils down to "could an unstoppable force move an immovable object?",[9] and then you can move on.
3. If you're not quite sure which of several interpretations is true, you can model that explicitly. Put an uncertainty node at the top which points to the model where your behavior and Omega's decision are independent and the model where your behavior determines Omega's behavior. Elicit a probability, or calculate what the probability would need to be for you to make one decision and then compare that to how uncertain you are.

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1. This means that there are a bunch of nodes connected by arrows (directed edges), and that there are no cycles (arrows that point in a loop). As a consequence, there are some nodes with no arrows pointing to them and one node with no arrows leaving it (the value node). It's also worth mentioning that influence diagrams are a special case of Bayesian Networks.

2. This is the actual prediction for me as of 1:30 PM on 12/14/2011. Link here, though that link should become worthless by tomorrow. Also note that I'm assuming that rain will only happen after we start the game of Go / decide where to play, or that the switching costs will be large.

3. The actual percentage in the data they had collected by 2008 was ~53%, but as it was within the 'well calibrated' region I should use the number at face value.

4. If it started raining while we were playing outside, we would probably just stop and do something else rather than playing through the rain, but neither of those are attractive prospects.

5. I strongly recommend satisficing, in the AI sense of including the costs of optimizing in your optimization process, rather than maximization. Opportunity costs are real. For a problem with, say, millions of dollars on the line, you'll probably want to spend quite a bit of time trying to come up with other options.

6. You may have noticed a trend that the best and worst outcome are often paired together. This is more likely in constructed problems, but is a feature of many difficult real-world problems.

7. We can do something called sensitivity analysis to see how sensitive this result is; if it's very sensitive to a value we elicited, we might go back and see if we can narrow the uncertainty on that value. If it's not very sensitive, then we don't need to worry much about those uncertainties.

8. This is an artificial constraint- really, you could change any of those policies any day, or even at any time during that day. It's often helpful to chunk continuous periods into a few discrete ones, but only to the degree that your bins carve reality at the joints.

9. The unstoppable force being Omega's prediction ability, and the immovable object being causality only propagating forward in time. Two-boxers answer "unmovable object," one-boxers answer "unstoppable force."

Measures, Risk, Death, and War

This is the fourth post of a sequence on decision analysis, preceded by Compressing Reality to Math. It touches on a wide variety of topics which didn't seem to work well as posts of their own, either because they were too short or too long.

Measures over Prospects

So far, we've looked at distinct prospects: different toys, different activities, different life experiences. Those are difficult to compare, and we might actually be unsure about the ordering of some of them. Would I prefer playing Go or chatting more? It takes a bit of effort and imagination to say.

Oftentimes, though, we face prospects that are measured in the same units. Would you prefer having $10 to having $4? There's no effort or imagination necessary: the answer is yes.[1] Facing wildly different prospects- a vacation to the Bahamas, a new computer, a raise at work- it can be helpful to try and reduce them to common units, so that preferences are easy to calculate. This is especially true if the prospects are fungible: you could sell your new computer at some time cost to receive a dollar amount, or could buy one from a store at some dollar cost. It doesn't make sense to value winning a computer higher than the cost to buy one (or gain from selling one, if that manages to be higher), even if you value it much more highly than its cost.[2]

As always, adding uncertainty makes things interesting: would you prefer having {.5 $10, .5 $0} or {1 $4}?[3] The answer depends on your circumstances: if lunch costs $3 and you get $10 worth of value out of eating lunch (the first time), then the certain deal is probably better. If these are your marginal investment dollars, though, a 20% expected return is probably worth jumping on.

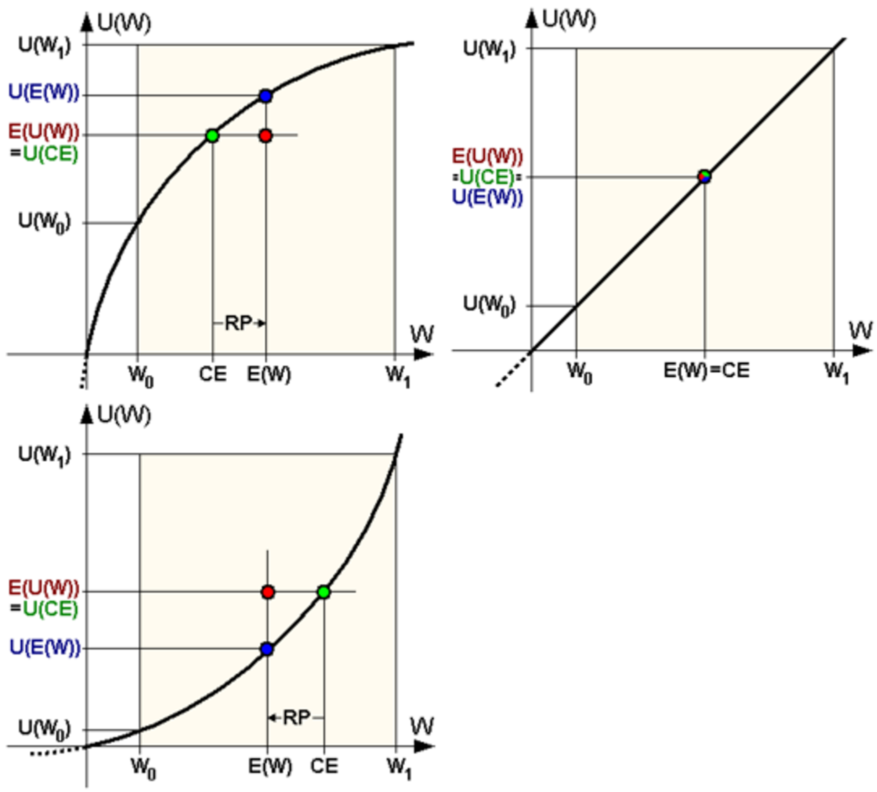
When dealing with a complicated problem with lots of dollar prospects, we could express each one as the certain equivalent of a deal between the highest and lowest dollar prospects. If we aren't great at eliciting preferences, though, we might end up with weird results we don't really agree with, and adding a new dollar amount requires eliciting a new preference probability.

An alternative is to come up with a function that maps the prospects to preference probabilities. The function can be fit with only a few elicited parameters, and then just evaluated for every prospect, making large problems and adding new prospects easy.

As you've probably guessed, that function is called a utility function.[4] I haven't brought it up before now because it's not necessary,[5] though a useful computational trick, and it's dangerous to think of utilities as measurable numbers out there, rather than expressions of an individual's preferences.

Risk Aversion

The primary information encoded by a utility function over one type of prospect is risk sensitivity. We can divide risk sensitivity into risk-averse, risk-neutral, and risk-loving (also called risk-affine)- basically, whether the utility function is concave, flat, or convex.



Averse, Neutral, and Loving. Images from wikipedia.

In the risk-averse case, you essentially take some function of the variance off of the expected value of each deal. A risk averse person might rather have 9±1 than 10±10. Notice that which one they prefer depends on how curved their utility function is, i.e. how much penalty they charge risk. In the risk neutral case, variance is simply unimportant- you only decide based on expected values. In the risk-loving case, you *add* some function of the variance to the expected value- a risk affine person might prefer 9±10 to 10±1.

Globally risk-loving people are hard to come by, although it's easy to imagine a utility function that's locally risk-loving (especially one that's risk-loving up to a certain point). True risk neutrality is also hard to come by- typically, if you multiply the scale by 10 enough times someone becomes risk-averse. Local risk neutrality, though, is the norm- zoom in on any utility function close enough and it'll be roughly flat.

So the utility functions we'll look at will be concave. Log is a common choice, but is sometimes awkward in that log(0) is negative infinity, and log(infinity) is also infinity- it's unbounded both above and below. Exponential is better behaved- 1-exp(0)=0 and 1-exp(infinity)=1, and so it's bounded both above and below. It also follows what's called the Delta property: if we add a constant amount to every prospect, our behavior doesn't change.[6] The irrelevance of 'money in the bank' is sometimes sensible, but sometimes not- if we re-examine the earlier deal of ({$10, .5; $0, .5} or {$4, 1}), and add $3 to replace it with ({$13, .5; $3, .5} or {$7, 1}), the investor will just up his price by $3, whereas the lunch-buyer might switch from the second choice to the first. Thinking about the delta property- as well as risk premiums (how much would you pay to narrow an outcome uncertainty?) helps determine whether you should use linear, log, or exponential utility functions.

Micromorts

The methodology we've discussed seems like it might have trouble comparing things of wildly different value. Suppose I like reading in the park more than reading in my home, but getting to the park requires traveling, and also suppose that traveling includes some non-zero chance of death. If I had a categorical preference that ranked the continuation of my life first, I would never choose to go to the park.

But that seems far too cautious. If the difference in enjoyment were large enough - say the choice was between attending my daughter's wedding in the park and reading at home - it seems like I should accept the chance of death and travel. But perhaps that is too bold- if it were almost certain that I would die along the way, I suspect it would be wiser to not go, and others would agree with my assessment. That is, if we adopt categorical preferences (no amount of B could compensate for a reduction in A), we can construct realistic scenarios where we would make regrettable decisions.

That suggests what we need to do is make a measured tradeoff. If I have a slight preference for living at the park to living at home, and a massive preference for living at home to dying along the way, then in order to go to the park I need it to be almost certain I will arrive alive, but there is some chance of death small enough that I would be willing to accept it.

How small? The first 'small probability' that comes to mind is 1%, but that would be far, far too large. That's about 600 times riskier than skydiving. I don't expect my mind to process smaller numbers very effectively. When I think of 1 in 10,000 and 1 in 100,000, does the first feel ten times bigger?

Like we just discussed, the way to deal with this sort of elicitation trouble is to turn to utility functions. Howard outlines an approach in a 1984 paper which has some sensible features. Given an exponential utility function, there is some maximum probability of death one will accept money for- and in the example they give it's about 10%, though that number will obvious vary from person to person.[7] Anything riskier, and you couldn't be paid enough to accept.

Conveniently, though, there is a large "safety region" where prices are linear with chance of death. That is, the price of an incremental risk doesn't change until the risks get rather severe. To make this easier to handle, consider a one millionth chance of dying: a micromort. That's a fairly convenient unit, as many risky behaviors have easily imagined scales at one micromort. For example, walking 17 miles is one micromort; and so going to the park a two miles away and coming back represents a 2.5e-7 chance of dying. (You can calculate your baseline chance of dying here, though it should be noted by 'baseline' they mean 'average' rather than 'without doing anything.')

How should we value that incremental amount? Well, it depends on what utility function you want to use, and what you assume about your life. Optimistic singularitarians, for example, should need far more money to accept a chance of dying than others, because they expect their lives to be longer and better than traditional analysis would suggest, but pessimistic singularitarians should need far less money to accept a chance of dying than others, because they expect their lives to be shorter or worse than traditional analysis would suggest.[8] The EPA suggests $8.24 for Americans (in 2011 dollars), but this number should vary based on age, sex, risk attitude, wealth, and other factors. Common values seem to range from $2 to $50; when I ran my numbers a while back I got about $10. If we take the EPA number, it looks like walking to the park will cost me about $2. If I would rather be at the park and $2 poorer than if I were at home, then I should walk over there, even though it brings me a bit closer to death. When considering risky activities like skydiving, I just just adjust the price upwards and decide if I would still want to do it if it cost that much extra, but was safe. (For skydiving, each jump costs about $144 using the EPA micromort value.)

Adversarial Decision Making

So far, we've mostly discussed decision-making under uncertainty by focusing on natural uncertainties- you're not sure if it'll rain or not, you're not sure if you'll win the lottery or not, you're not sure if you'll get involved in an accident on the way to the park or not. That's not the full picture, though: many important decisions include an adversary. Adversaries represent a special kind of uncertainty, because they react to your decisions, have their own uncertainties, and often actively want to make you worse off, rather than just not caring about your preferences.

Game Theory

Game Theory behaves a lot like the methods we've described before. Take a real situation, turn it into an action-payoff matrix, and find equilibria and mixed strategies. It's a large, rich field and I'm not going to describe how it works in detail, as there are other resources for that.

One of the pitfalls with Game Theory, though, is that it requires some strong assumptions about how your opponent makes decisions. Can you really be sure your opponent will play the game-theoretically correct strategy, or that you've determined their payoff matrix correctly?

For example, consider a game of rock-paper-scissors. Game Theory suggests a mixed strategy of throwing each possibility with 1/3 probability. When playing against Bart Simpson, you can do better. Even when playing against a normal person, there are biases you can take advantage of.

As another example, consider that you run a small firm that's considering entering a market dominated by a large firm. After you choose to enter or not, they can choose whether to cut prices or not. You estimate the dollar payoffs are (yours, theirs):

	Enter	Don't
Price War	-1, 8	0, 10
Don't	2, 18	0, 20

You see that, regardless of what you do, they earn more not having a price war, and if they don't go for a price war you would prefer entering to not entering. Indeed, (Enter, Don't) is a Nash Equilibrium. But suppose the scenario instead looked like this:

	Enter	Don't
Price War	-10, 16	0, 18
Don't	2, 18	0, 20

The cells of the matrix all have the same ranking- regardless of whether or not you enter, they earn more by not having a price war. But the difference is much smaller, and the loss to you for entering if they do have a price war is much higher. (This could be because the entire firm will go under if this expansion fails, rather than just losing some money.) Someone might confidently announce that they won't engage in a price war, and so you should enter- but you might want to do a little research first on how strong their preference for dollars are. They might value market share- which isn't included in this payoff matrix- much more highly. That is, the Nash Equilibrium for this matrix (which is still the same cell) might not be the Nash Equilibrium for the real-world scenario.

You can model this uncertainty about your opponent's strategy explicitly: include it as an uncertainty node that leads to several decision nodes, each operating on different preferences. You might decide, say, that you need >83% confidence that they'll behave selfishly rather than vengefully (when it comes to dollars) in the second scenario, but only >33% confidence that they'll behave selfishly rather than vengefully (when it comes to dollars) in the first scenario.

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1. Obviously, this is not true for everything- I might prefer two apples to one apple, but a million apples might be more trouble than they're worth. (Where would I put them?) For dollars, though, more is better in a much more robust way.

2. This assumes that you'll still be able to buy the computer with whatever option you pick. If I decide to receive non-transferable tickets to a show that I enjoy at $1000 instead of a computer that costs $500 but I enjoy at $3000, and don't have the money to buy the computer, I made a mistake. But if I have at least $500 spare, I can consider myself as already having the first computer- the question is what a second one is worth. Ideally, preferences should be calculated over life experiences, not just events- your future life where you got the computer vs. your future life where you got the tickets.

3. That is, option A is $10 50% of the time and $0 the other 50% of the time. Option B is $4 every time.

4. What I described is a special case: a utility function which has a min of 0 and max of 1 on the domain of the problem. General utility functions don't have that constraint.

5. The Von Neumann-Morgenstern axioms and the 5 axioms I discussed earlier are mostly the same, and so if you have someone willing to make decisions the way I'm describing they should also be willing to construct a utility function and compute expected utilities. Indeed, the processes will be difficult to distinguish, besides vocabulary, and so this is more a statement that the word is unnecessary than that the idea is unnecessary.

6. It's so called because Δ is often used to signify a small amount- this is going through and replacing all prospects x_i with $x_i+\Delta$.

7. Incidentally, this is one of the differences between a log utility function and a exponential utility function. Someone with a log utility function would accept an arbitrarily small chance of life with an arbitrarily high wealth- but as wealth is practically bounded (there's only one Earth to own at present) that bounds the maximum chance of death.

8. Interestingly, cryonics doesn't seem to alter this calculation, unless you think freezing technology will rapidly improve over your lifespan. That said, instead of just tracking risk of death you also need to track risk of not being frozen soon enough, meaning cause of death is much more relevant.

Value of Information: Four Examples

Value of Information (VoI) is a concept from decision analysis: how much answering a question allows a decision-maker to improve its decision. Like opportunity cost, it's easy to define but often hard to internalize; and so instead of belaboring the definition let's look at some examples.

Gambling with Biased Coins

Normal coins are approximately fair.[1] Suppose you and your friend want to gamble, and fair coins are boring, so he takes out a quarter and some gum and sticks the gum to the face of the quarter near the edge. He then offers to pay you $24 if the coin lands gum down, so long as you pay him $12 to play the game. Should you take that bet?

First, let's assume risk neutrality for the amount of money you're wagering. Your expected profit is $24*p*-12*,* where *p* is the probability the coin lands gum down. This is a good deal if *p*>.5, but a bad deal if *p*<.5. So... what's *p*? More importantly, how much should you pay to figure out *p*?

A Bayesian reasoner looking at this problem first tries to put a prior on *p*. An easy choice is a uniform distribution between 0 and 1, but there are a lot of reasons to be uncomfortable with that distribution. It might be that the gum will be more likely to be on the bottom- but it also might be more likely to be on the top. The gum might not skew the results very much- or it might skew them massively. You could choose a different prior, but you'd have trouble justifying it because you don't have any solid evidence to update on yet.[2]

If you had a uniform prior and no additional evidence, then the deal as offered is neutral. But before you choose to accept or reject, your friend offers you *another* deal- he'll flip the coin once and let you see the result before you choose to take the $12 deal, but you can't win anything on this first flip. How much should you pay to see one flip?

Start by modeling yourself after you see one flip. It'll either come up gum or no gum, and you'll update and produce a posterior for each case. In the first case, your posterior on *p* is P(*p*)=2p; in the second, P(*p*)=2-2p. Your expected profit for playing in the first case is $4;[3] your expected profit for playing in the second case is **negative** $4. You think there's a half chance it'll land gum side up, and a half chance it'll land gum side down, and if it lands gum side down *you can choose not to play*. There's a half chance you get $4 from seeing the flip, and a half chance you get nothing (because you don't play) from seeing the flip, and so $2 is the **VoI** of seeing one flip of the biased coin, *given your original prior*.

Notice that, even though it'd be impossible to figure out the 'true' chance that the coin will land gum down, you can model how much it would be worth it to you to figure that out. If I were able to tell you *p* directly, then you could choose to gamble only when *p*>.5, and you would earn an average of $3.[4] One coin flip gives you two thirds of the value that perfect information would give you.

Also notice that you need to **change your decision** to get any value out of more information. Suppose that, instead of letting you choose whether or not to gamble, your friend made you decide, flipped two coins, and then paid you if the second coin landed gum down and you paid him. The coin is flipped the same number of times, but you're worse off because you have to decide with less information.

It's also worth noting that multimodal distributions- where there are strong clusters rather than smooth landscapes- tend to have higher VoI. If we knew the biased coin would either always come up heads or always come up tails, and expected each case were equally likely, then seeing one flip is worth $6, because it's a half chance of a guaranteed $12.

Choosing where to invest

Here's an example I came across in my research:

Kleinmuntz and Willis were trying to determine the value of doing detailed anti-terrorism assessments in the state of California for the Department of Homeland Security. There are hundreds of critical infrastructure sites across the state, and it's simply not possible to do a detailed analysis of each site. There are terrorism experts, though, who can quickly provide an estimate of the risk to various sites.

They gave a carefully designed survey to those experts, asking them to rate the relative probability that a site would be attacked (conditioned on an attack occurring) and the probability that an attack would succeed on a scale from 0 to 10, and the scale of fatalities and economic loss on a logarithmic scale from 0 to 7. The experts were comfortable with the survey[5] and able to give meaningful answers.

Now Kleinmutz and Willis were able to take the elicited vulnerability estimates and come up with an estimated score for each facility. This estimated score gave them a prior over detailed scores for each site- if the experts all agreed that a site was a (0, 1, 2, 3), then that still implies a range over actual values. The economic loss resulting from a successful attack (3) could be anywhere from $100 million to $1 billion. (Notice that having a panel of experts gave them a natural way to determine the spread of the prior beyond the range inherent in their answers- where the experts agreed, they could clump the probability mass together, with only a little on answers the experts didn't give, and where the experts disagreed they knew where to spread the probability out over.) They already had, from another source, data on the effectiveness of the risk reductions available at the various sites and the costs of those reductions.

The highest actual consequence elicited was for $6 billion, assuming a value of $6 million per life. The highest VoI of getting a detailed site analysis, though, was only $1.1 *million*. From the definition, this shouldn't be that surprising- VoI is only large when you would be surprised or uncertainty is high. For some sites, it was obvious that DHS should invest in reducing risk; in others, it was obvious that DHS shouldn't invest in reducing risk. The detailed vulnerability analysis would just tell them what they already knew, and so wouldn't provide any value. Some sites were on the edge- it might be worthwhile to reduce risk, it might not. For those sites, a detailed vulnerability analysis would provide value- but because the site was on the edge, the expected value of learning more was necessarily small![6] Remember, for VoI to be positive you have to **change your decision**, and if that doesn't happen there's no VoI.

Distressingly, they went on to consider the case where risk reduction could not be performed without a detailed vulnerability analysis. Then, rather than measuring VoI, they were mostly measuring the value of risk reduction- and the maximum value shot up to $840 million. When Bayesian evidence is good enough, requiring legal evidence can be costly.[7]

Medical Testing

About two years ago, I was sitting at my computer and noticed a black dot on my upper arm. I idly scratched it, and then saw its little legs move.

It was an tick engorged on my blood, which I had probably picked up walking through the woods earlier. I removed it, then looked up online the proper way to remove it. (That's the wrong order, by the way: you need the information before you make your decision for it to be of any use. I didn't do it the proper way, and thus increased my risk of disease transmission.)

Some ticks carry Lyme disease, and so I looked into getting tested. I was surprised to learn that if I didn't present any symptoms by 30 days, the recommendation was against testing. After a moment's reflection, this made sense- tests typically have false positive rates. If I didn't have any symptoms after 30 days, even if I took the test and got a positive result the EV could be higher for no treatment than for treatment. In that case, the VoI of the test would be 0- **regardless of its outcome, I would have made the same decision.** If I saw symptoms, though, then the test would be worthwhile, as it could distinguish Lyme disease from an unrelated rash, headache, or fever. "Waiting for symptoms to appear" was the test with positive VoI, not getting a blood test right away.

One could argue that the blood test could have "peace of mind" value, but that's distinct from VoI. Even beyond that, it's not clear that you would get positive peace of mind on net. Suppose the test has a 2% false positive rate- what happens when you multiply the peace of mind from a true negative by .98, and subtract the costs of dealing with the false positives by .02? That could easily be negative.

(I remain symptom-free; either the tick didn't have Lyme disease, didn't transfer it to me, or my immune system managed to destroy it.)

Choosing a Career

Many careers have significant prerequisites: if you want to be a doctor, you're going to have to go to medical school. People often have to choose where to invest their time with limited knowledge- you can't know what the career prospects will be like when you graduate, how much you'll enjoy your chosen field, and so on. Many people just choose based on accumulated experience- lawyers were high-status and rich before, so they suspect becoming a lawyer now is a good idea.[8]

Reducing that uncertainty can help you make a better decision, and VoI helps decide what ways to reduce uncertainty are effective. But this example also helps show the limits of VoI: VoI is best suited to situations where you've done the background research and are now considering further experiments. With the biased coin, we started off with a uniform prior; with the defensive investments, we started off with estimated risks. Do we have a comparable springboard for careers?

If we do, it'll take some building. There's a lot of different value functions we could build- it probably ought to include stress, income (both starting and lifetime)[9], risk of unemployment, satisfaction, and status. It's not clear how to elicit weights on those, though. There's research on what makes people in general happy, but you might be uncomfortable just using those weights.[10]

There are also hundreds, if not thousands, of career options available. Prior distributions on income are easy to find, but stress is harder to determine. Unemployment risk is hard to predict over a lifetime, especially as it relies on macroeconomic trends that may be hard to predict. (The BLS predicts employment numbers out 10 years from data that's a few years old. It seems unlikely that they're set up to see crashes coming, though.)

Satisfaction is probably the easiest place to start: there are lots of career aptitude tests out there that can take self-reported personality factors and turn that into a list of careers you might be well-suited for. Now you have a manageable decision problem- probably somewhere between six and twenty options to research in depth.

What does that look like from a VoI framework? You've done a first screening which has identified places where more information might *alter your decision*. If you faint at the sight of blood, it doesn't matter how much surgeons make, and so any time spent looking that up is wasted. If you do a quick scoring of the six value components I listed above (after brainstorming for other things relevant to you), just weighting them with those quick values may give you good preliminary results. Only once you know what comparisons are relevant- "what tradeoff between status and unemployment risk am I willing to make?"- would you spend a long time nailing down your weights.

This is also a decision problem that could take a long, long time. (Even after you've selected a career, the option to switch is always present.) It can be useful to keep upper and lower bounds for your estimates and update those along with your estimates- their current values and their changes with the last few pieces of information you found can give you an idea of how much you can expect to get from more research, and so you can finish researching and make a decision at a carefully chosen time, rather than when you get fatigued.

Conclusion

Let's take another look at the definition: how much *answering* a question allows a decision-maker to *improve* its *decision*.

The "answering" is important because we need to consider all possible answers.[11] We're replacing one random variable with two random variables- in the case of the biased coin, it replaced one unknown coin (one flip) with either the lucky coin and the unlucky coin (two flips- one to figure out which coin, one to bet on). When computing

VoI, you can't just consider one possible answer, but all possible answers considering their relative likelihood.[12]

The "improve" is important because VoI isn't about sleeping better at night or covering your ass. If you don't expect to change your decision after receiving this information, or you think that the expected value of the information (the chance you change your decision times the relative value of the decisions) is lower than the cost of the information, just bite the bullet and don't run the test you were considering.

The "decision" is important because this isn't just curiosity. Learning facts is often fun, but for it to fit into VoI some decision has to depend on that fact. When watching televised poker, you know what all the hands are- and while that may alter your enjoyment of the hand, it won't affect how any of the players play. You shouldn't pay much for that information, but the players would pay quite a bit for it.[13]

1. Persi Diaconis predicts most human coin flips are fair to 2 decimals but not 3, and it's possible through training to bias coins you flip. With a machine, you can be precise enough to get the coin to come up the same way every time.

2. There is one thing that isn't coin-related: your friend is offering you this gamble, and probably has information you don't. That suggests the deal favors him- but suppose that you and your friend just thought this up, and so neither of you has more information than the other.

3. Your profit is 24p-12; your distribution on p is P(p)=2p, and so your distribution on profit is $48p^2$-24p integrated from 0 to 1, which is 4.

4. Again, your profit is 24p-12; you have a uniform distribution on what I will tell you about p, but you only care about the section where p>.5. Integrated from .5 to 1, that's 3.

5. Whenever eliciting information from experts, make sure to repeat back to them what you heard and ensure that they agree with it. You might know decision theory, but the reason you're talking to experts is because they know things you don't. Consistency can take a few iterations, and that's to be expected.

6. A common trope in decision analysis is "if a decision is hard, flip a coin." Most people balk at this because it seems arbitrary (and, more importantly, hard to justify to others)- but if a decision is hard, that typically means both options are roughly equally valuable, and so the loss from the coin flip coming up the wrong value is necessarily small.

7. That said, recommendations for policy-makers are hard to make here. Legal evidence is designed to be hard to game; Bayesian evidence isn't, and so Bayesian evidence is only "good enough" if it's not being gamed. Checking your heuristic (i.e. the expert's estimates) to keep it honest can provide significant value. Performing detailed vulnerability analysis on some (how many?) randomly chosen sites for calibration is often a good choice. Beyond that, I can't do much besides point you to psychology to figure out good ways to diagnose and reduce bias.

8. It doesn't appear that this is the case anymore. The supply of lawyers has dramatically increased, and so wages are declining; as well, law is a pretty soul-crushing field from a stress, work-life balance, and satisfaction perspective. If law

looks like the best field for you and you're not in it for the money or status, the advice I hear is to specialize in a niche field that'll put food on the table but stay interesting and tolerably demanding.

9. Both of these capture different information. A job with a high starting salary but no growth prospects might translate into more happiness than a job with a low starting salary but high growth prospects, for example.

10. Most of the happiness/satisfaction literature I've seen has asked people about their attributes and their happiness/satisfaction. That's not a randomized trial, though, and so there could be massive selection effects. If we find that engineers are collectively less happy than waiters, does that mean engineering causes unhappiness, unhappiness causes engineering, that unhappiness and engineering are caused by the same thing, or none of those?

11. Compare this with information theory, where bits are a property of answers, *not* questions. Here, VoI is a property of questions, *not* answers.

12. If you already know the cost of the information, then you can stop computing as soon as you find a positive outcome good enough and likely enough that the VoI so far is higher than the cost.

13. In high-stakes poker games, the VoI can get rather high, and the deceit / reading involved is why poker is a more interesting game than, say, the lottery.