Mini Project 2

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Intro

- Motivation, why # Data Generation
- Description of data generation process # Simulation Design
- Description of simulation design
- Tests used (F and LRT) # Results
- Summary tables and figures
-

  for large n

  ———— - compound - random - autoregres -
- type1 LRT -
- type1 F -
- power LRT -
- power F -
-

  for small n

  ———— - compound - random - autoregres -
- type1 LRT -
- type1 F -
- power LRT -
- power F -

$$\Sigma = \begin{pmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{pmatrix}$$

$$\mathrm{Cov}(Y_i) = Z_i \begin{pmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{pmatrix} Z_i' + \sigma^2 I$$

where

$$Z_i = \begin{pmatrix} 1 & t_{i1} \\ 1 & t_{i2} \\ \vdots & \vdots \\ 1 & t_{in_i} \end{pmatrix}, \quad \text{so that} \quad \mathrm{Cov}(Y_i) = \begin{pmatrix} \sigma_0^2 + 2t_{i1}\sigma_{01} + t_{i1}^2\sigma_1^2 & \cdots \\ \vdots & \ddots \end{pmatrix}$$

$V = \sigma_b^2 11' + \sigma_c^2 xx' + \sigma_e^2 I$.

AR(1): $(V)_{ij} = \sigma^2 \rho^{|i-j|}$.

Compute the Cholesky factorization $V = CC'$, where C is lower-triangular.

Generate independent standard normal vectors $z_i \sim N(0, I_5)$ for each subject.

Induce correlation: $y_i = X_i\beta + Cz_i$, which ensures $\mathrm{Var}(y_i) = C\mathrm{Var}(z_i)C' = CIC' = V$.

$$\mathrm{Cov}(Y_{ij}, Y_{ik}) = \rho\sigma^2, \quad \forall i \neq k$$

$$Y_{ij} = \beta_0 + \beta_1 \, \mathrm{time}_{ij} \times \mathrm{treatment}_i + \epsilon_{ij},$$

where

$$\epsilon_i = \begin{pmatrix} \epsilon_{i1} \\ \epsilon_{i2} \\ \vdots \\ \epsilon_{in_i} \end{pmatrix} \sim N(\mathbf{0}, V),$$

and V represents one of the three covariance structures defined above (Compound Symmetric, Random Coefficients, or Autoregressive).

Each subject's repeated measures are thus correlated according to V, while treatment effects are introduced through the fixed effects β_0 and β_1.