Binary Test

Team Binary

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Suppose we have two particles 1 and 2. Their masses, positions and velocities are m_1 and m_2, $\vec{r}_1$ and $\vec{r}_2$, $\vec{v}_1$ and $\vec{v}_1$. The relative position and velocity are $\vec{r} = \vec{r}_1 - \vec{r}_2$ $\vec{v} = \vec{v}_1 - \vec{v}_2$.

We calculate their reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}. \tag{1}$$

The position of the center of mass is

$$\vec{R}_{\mathrm{C}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}. \tag{2}$$

We choose the coordinate where the center of mass is static. The total energy is

$$E = \frac{1}{2}\mu v^2 - \frac{G m_1 m_2}{r}. \tag{3}$$

Here $v = |\vec{v}|$ and $r = |\vec{r}|$.

The angular momentum is

$$L = \vec{r} \times \mu \vec{v}. \tag{4}$$

The major axis is

$$a = -\frac{G m_1 m_2}{2E}. \tag{5}$$

The period is

$$T^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}. \tag{6}$$

The eccentricity is

$$e = \sqrt{1 + \frac{2EL^2}{\mu (G m_1 m_2)^2}}. \tag{7}$$

The maximum and minimum distance between the two particles are $r_{\max} = a(1+e)$ and $r_{\min} = a(1-e)$.

The initial conditions we choose are as follows: $m_1 = 10^6 M_\odot$,$m_2 = 10^6 M_\odot$; $\vec{r}_1 = (0,0,0)$, $\vec{r}_2 = (10^{-5}\,\mathrm{kpc}, 0, 0)$; $\vec{v}_1 = (0,0,0)$, $\vec{v}_2 = (0, 10^{-6}\,\mathrm{kpc/s}, 0)$.