

Homework 5

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Question 1

Confidence intervals in bayesian inference take into account prior knowledge, so a confidence interval in bayesian inference gives the probability of an unknown parameter lying within an interval given the observed data. Traditional inference on the other hand makes the assumption that the data is a random sample of a larger distribution, so confidence intervals are less meaningful.

Question 2

```
x = c(
  2, 2, 3, 1, 3, 3, 4, 2, 3, 1, 3, 5, 2, 2, 4, 4, 2, 2, 2, 4, 3, 4, 3, 4, 2, 1, 4, 5, 3, 2, 3, 6, 2, 4,
  4, 5, 1, 3, 8, 1, 2, 5, 4, 5, 3, 3, 4, 5, 2, 2, 2, 1, 2, 2, 3, 2, 1, 2, 3, 2, 3, 3, 6, 4, 3, 2, 3, 3,
  2, 2, 3, 7, 4, 3, 5, 4, 5, 1, 3, 3, 6, 7, 0, 3, 4, 2, 2, 4, 5, 2, 2, 3, 5, 2, 3, 1, 0, 4
)
```

The normal distribution is not bounded by 0 like x appears to be (no negative values) and the values are integers, implying some kind of categorical data. The data also has a positive or right skew, meaning that the tail is on the right.

For these reasons, a normal distribution is not an ideal distribution for this data.

Question 3

```
log_likelihood <- function(lambda) {
  sum(dpois(x, lambda = lambda, log = TRUE))
}
log_likelihood(1)
```

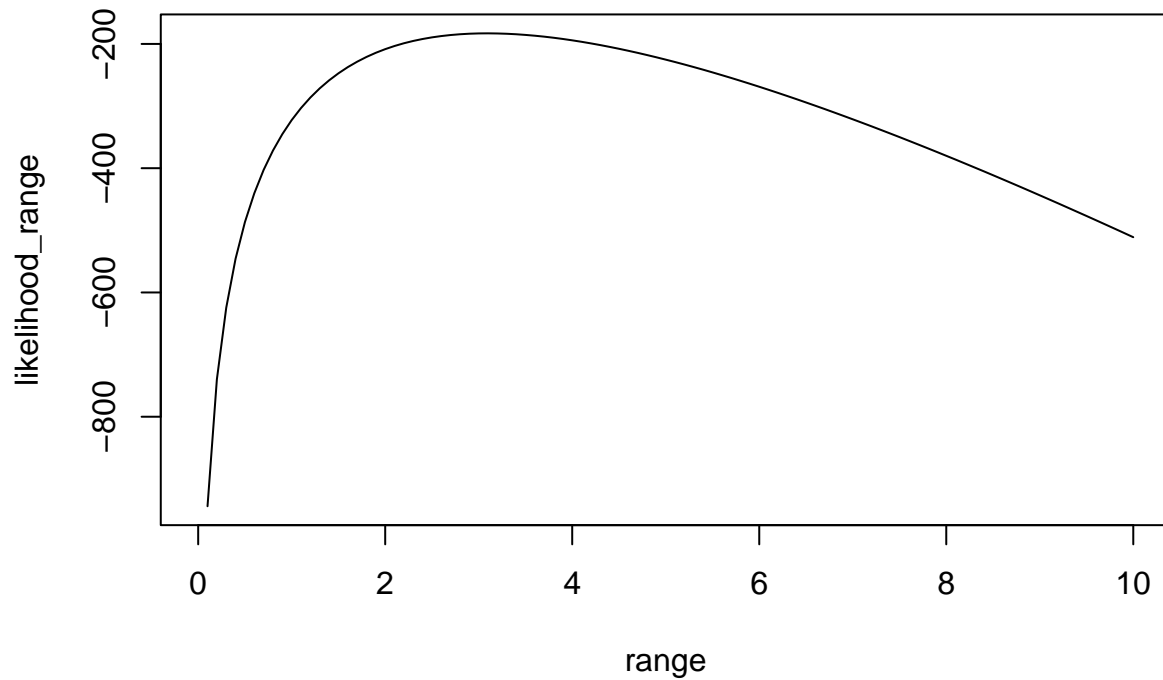
```
## [1] -322.5608
```

The value of the likelihood function at $\lambda = 1$ is -322.5607775.

Plot

```
range <- seq(0, 10, by = 0.1)
likelihood_range <- sapply(range, log_likelihood)

plot(range, likelihood_range, type = "l")
```

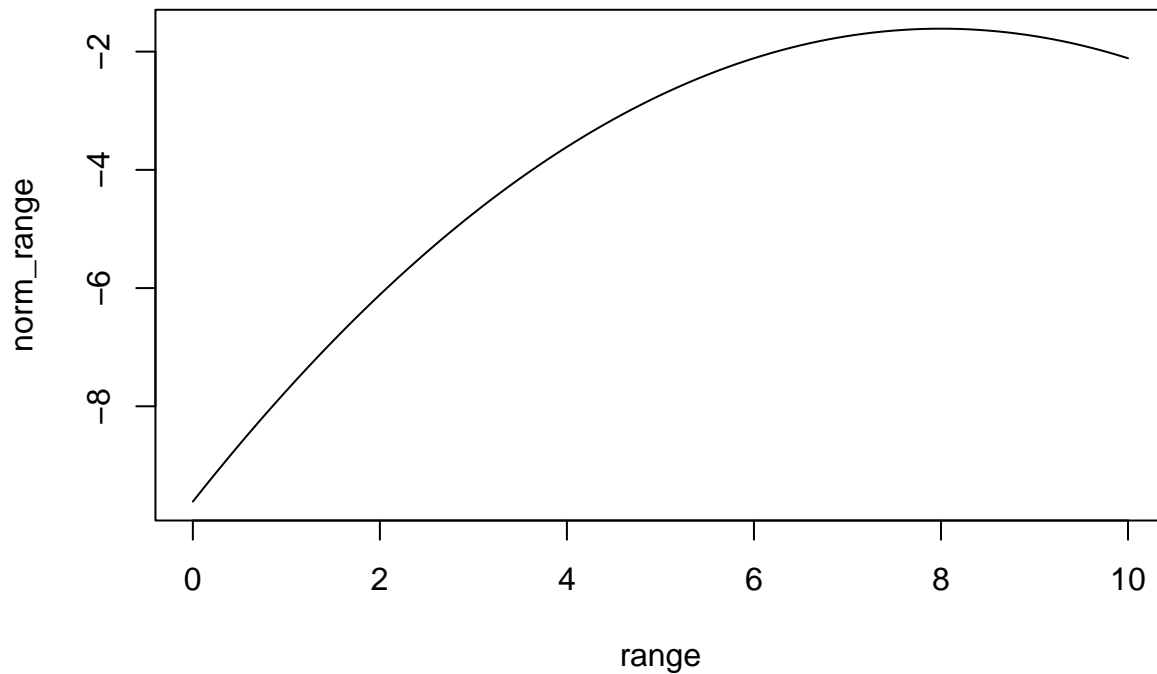


Question 4

Since the interval for λ is bounded by the interval (0, 10), a reasonable choice for the prior distribution is a uniform distribution bounded at 0 and 10. This way, any real lambda is possible within these bounds. Since no other knowledge about λ is available, the uniform distribution is the best (naive) choice.

Question 5

```
norm_prior <- function(lambda) {  
  dnorm(lambda, mean = 8, sd = 2, log = TRUE)  
}  
  
norm_range <- sapply(range, norm_prior)  
plot(range, norm_range, type = "l")
```



Question 6

```
log_posterior <- function(lambda) {
  log_likelihood(lambda) + norm_prior(lambda)
}
```

Question 7

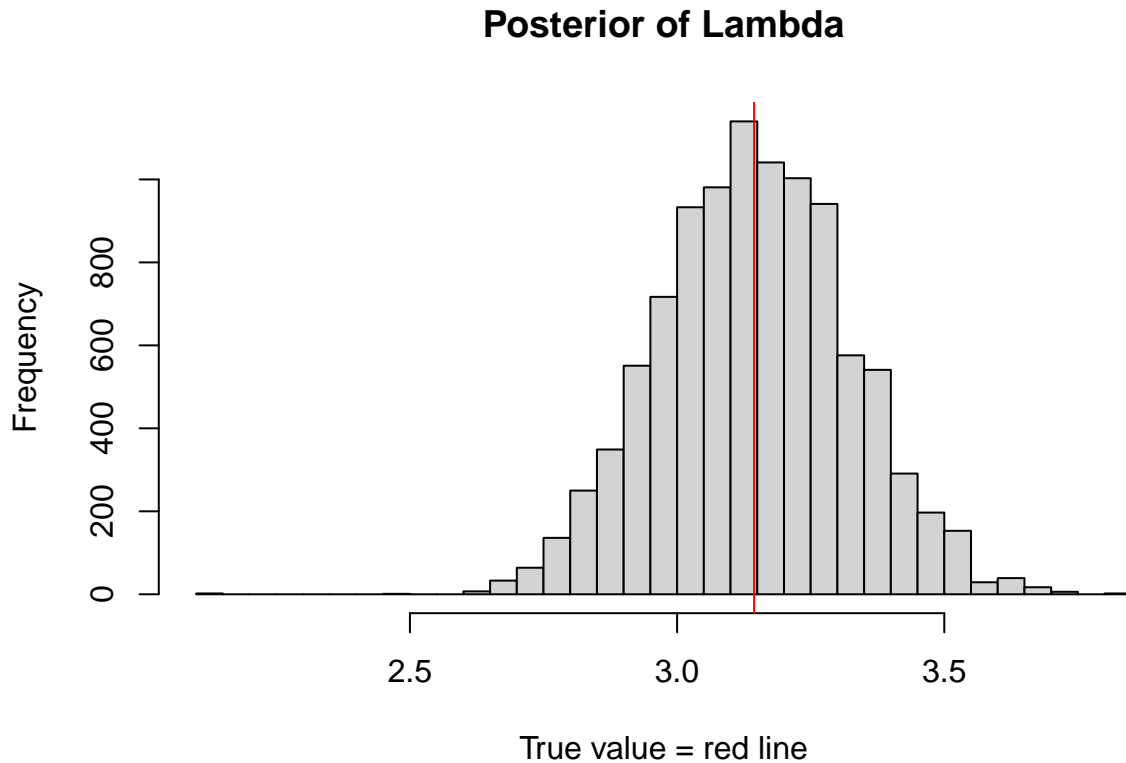
```
proposalFunctionPoiss <- function(theta.old) {
  max(rnorm(1, mean = theta.old, sd = c(1)), 0) # relu?
}

MCMC <- function(theta.start, N) {
  sample <- array(dim = c(N, 1))
  theta.old <- theta.start
  for (i in 1:N) {
    theta.new <- proposalFunctionPoiss(theta.old)
    p <- exp(log_posterior(theta.new) - log_posterior(theta.old))
    if (runif(1) < p) {
      sample[i] <- theta.new
      theta.old <- sample[i]
    } else {
      sample[i] <- theta.old
    }
  }
  return(sample)
}

theta.start <- 1
```

Question 8

```
chain <- MCMC(theta.start, 10000)
# burnin <- 5000
# chain <- chain[1:burnin] didn't seem to have much effect, so I'll leave it off
hist(chain, nclass = 30, , main = "Posterior of Lambda", xlab = "True value = red line")
abline(v = mean(chain), col = "red")
```



```
median(chain)
```

```
## [1] 3.142509
```

```
quantile(chain, prob = c(0.025, 0.975))
```

```
##      2.5%      97.5%
```

```
## 2.802494 3.499042
```

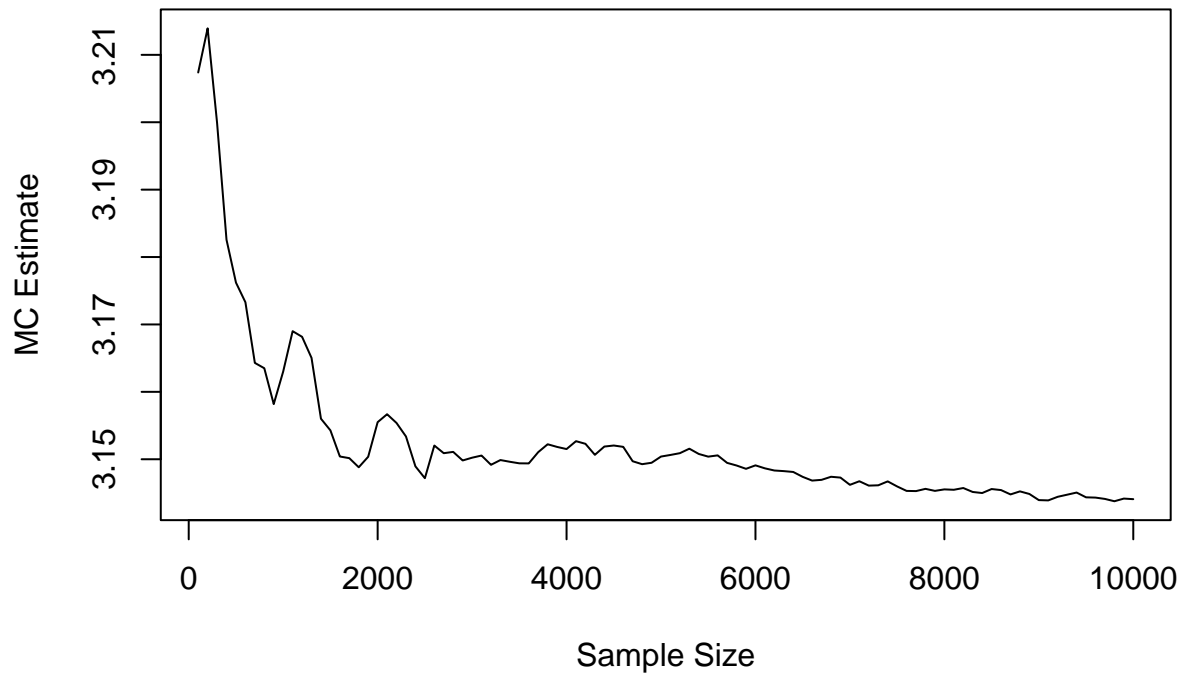
Question 9

```
batchmeans::ess(chain)
```

```
## [1] 1710.666
```

```
batchmeans::estvssamp(chain)
```

Estimate vs Sample Size



The quality of the estimate does not significantly change after 4000 samples, and only changes slightly after the effective samples size.

That being said, the rate of convergence is not particularly high, indicating that the sample may not be the best quality.