

SOAP-RL: Sequential Option Advantage Propagation for Reinforcement Learning in POMDP Environments

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Abstract

This work compares ways of extending Reinforcement Learning algorithms to Partially Observed Markov Decision Processes (POMDPs) with *options*. One view of options is as temporally extended action, which can be realized as a memory that allows the agent to retain historical information beyond the policy’s context window. While option assignment could be handled using heuristics and hand-crafted objectives, learning temporally consistent options and associated sub-policies without explicit supervision is a challenge. Two algorithms, PPOEM and SOAP, are proposed and studied in depth to address this problem. PPOEM applies the forward-backward algorithm (for Hidden Markov Models) to optimize the expected returns for an option-augmented policy. However, this learning approach is unstable during on-policy rollouts. It is also unsuited for learning causal policies without the knowledge of future trajectories, since option assignments are optimized for offline sequences where the entire episode is available. As an alternative approach, SOAP evaluates the policy gradient for an optimal option assignment. It extends the concept of the generalized advantage estimation (GAE) to propagate *option advantages* through time, which is an analytical equivalent to performing temporal back-propagation of option policy gradients. This option policy is only conditional on the history of the agent, not future actions. Evaluated against competing baselines, SOAP exhibited the most robust performance, correctly discovering options for POMDP corridor environments, as well as on standard benchmarks including Atari and MuJoCo, outperforming PPOEM, as well as LSTM and Option-Critic baselines. The open-sourced code is available at <https://github.com/shuishida/SapRL>.

1 Introduction

While deep Reinforcement Learning (RL) has seen rapid advancements in recent years, with numerous real-world applications such as robotics (Gu et al., 2017; Akkaya et al., 2019; Haarnoja et al., 2024), gaming (Van Hasselt et al., 2016; Arulkumaran et al., 2019; Baker et al., 2022), and autonomous vehicles (Kendall et al., 2019; Lu et al., 2023), many algorithms are limited by the amount of observation history they plan on. Developing learnable embodied agents that plan over a wide spatial and temporal horizon has been a longstanding challenge in RL.

With a simple Markovian policy $\pi(a_t|s_t)$, the agent’s ability to make decisions is limited by only having access to the current state as input. Early advances in RL were made on tasks that either adhere to the Markov assumption that the policy and state transitions only depend on the current state, or those can be solved by frame stacking (Mnih et al., 2015) that grants the policy access to a short history. However, many real-world tasks are better modeled as Partially Observable Markov Decision Processes (POMDPs) (Åström, 1965), and necessitate solutions that use working memory. The history of the agent’s trajectory also contains signals to inform the agent to make a more optimal decision. This is due to the reward and next state distribution $p(r_t, s_{t+1}|s_{0:t}, a_{0:t})$ being conditional on the past states and actions, not just on the current state and action.

A common approach of accommodating POMDPs is to learn a latent representation using sequential policies, typically using a Long Short-Term Memory (LSTM) (Hochreiter & Schmidhuber, 1997), Gated Recurrent Unit (GRU) (Cho et al., 2014) or Transformer (Vaswani et al., 2017). This will allow the policy to gain access to signals from the past. Differentiable planners (Tamar et al., 2016; Lee et al., 2018; Ishida & Henriques, 2022) are another line of work that incorporate a learnable working memory into the system. However, these approaches have an inherent trade-off between the duration of history it can retain (defined by the policy’s context window size) and the compute and training data required to learn the policy. This is because the entire history of observations within the context window have to be included in the forward pass at training time to propagate useful gradients back to the sequential policy. Another caveat is that, with larger context windows, the input space is less constrained and it becomes increasingly unlikely that the agent will revisit the same combination of states, which makes learning the policy and value function sample-expensive, and potentially unstable at inference time if the policy distribution has changed during training.

Training RL agents to work with longer working memory is a non-trivial task, especially when the content of the memory is not pre-determined and the agent also has to learn to store information relevant to each task. With the tasks that the RL algorithms are expected to handle becoming increasingly complex (Dulac-Arnold et al., 2021; Milani et al., 2023; Chen et al., 2023), there is a vital need to develop algorithms that learn policies and skills that generalise to dynamic and novel environments. Many real-world tasks are performed over long time horizons, which makes it crucial that the algorithm can be efficiently trained and quickly adapted to changes in the environment.

The aim of this work is to develop an algorithm that (a) can solve problems modeled as POMDP using memory, (b) has a constrained input for the policy and value function so that they are more trainable, (c) only requires the current observation to be forward-passed through a neural network at a time to reduce the Graphical Processing Unit (GPU) memory and computational requirements.

Acquiring transferable skills and composing them to execute plans, even in novel environments, are remarkable human capabilities that are instrumental in performing complex tasks with long-term objectives. Whenever one encounters a novel situation, one can still strategize by applying prior knowledge with a limited budget of additional trial and error. One way of achieving this is by abstracting away the complexity of long-term planning by delegating short-term decisions to a set of specialized low-level policies, while the high-level policy focuses on achieving the ultimate objective by orchestrating these low-level policies.

There has been considerable effort in making RL more generalizable and efficient. Relevant research fields include Hierarchical Reinforcement Learning (HRL) (Vezhnevets et al., 2017; Nachum et al., 2018; Patera et al., 2021; Zhang et al., 2021), skill learning (Pertsch et al., 2020; Nam et al., 2022; Peng et al., 2022; Shi et al., 2023), Meta Reinforcement Learning (Meta-RL) (Wang et al., 2016; Duan et al., 2016; Rakelly et al., 2019; Beck et al., 2023) and the options framework (Sutton et al., 1999; Precup & Sutton, 2000), with a shared focus on learning reusable policies. In particular, this research focuses on the options framework, which extends the RL paradigm with a Hidden Markov Model (HMM) that uses options to execute long-term behavior.

The Option-Critic architecture (Bacon et al., 2017) presents a well-formulated solution for end-to-end option discovery. The authors showed that once the option policies are learned, the Option-Critic agent can quickly adapt when the environment dynamics are changed, whereas other algorithms suffer from the changes in reward distributions.

However, there are challenges with regard to automatically learning options. A common issue is that the agent may converge to a single option that approximates the optimal policy under a Markov assumption. Additionally, learning options from scratch can be sample-inefficient due to the need to learn multiple option policies.

In the following sections, two training objectives are proposed and derived to learn an optimal option assignment.

The first approach, Proximal Policy Optimization via Expectation Maximization (PPOEM), applies Expectation Maximization (EM) to a HMM describing a POMDP for the options framework. The method is an extension of the forward-backward algorithm, also known as the Baum-Welch algorithm (Baum, 1972), applied to options. While this approach has previously been explored (Daniel et al., 2016; Fox et al., 2017; Zhang & Paschalidis, 2020; Giannarino & Paschalidis, 2021), these applications were limited to 1-step Temporal Difference (TD) learning. In addition, the learned options have limited expressivity due to how the option transitions are defined. In contrast, PPOEM augments the forward-backward algorithm with Generalized Advantage Estimate (GAE) (Schulman et al., 2016), which is a temporal generalization of TD learning, and extends the Proximal Policy Optimization (PPO) (Schulman et al., 2017) to work with options. While this approach was shown to be effective in a limited setting of a corridor environment

requiring memory, the performance degraded with longer corridors. It could be hypothesized that this is due to the learning objective being misaligned with the true RL objective, as the approach assumes access to the full trajectory of the agent for the optimal assignment of options, even though the agent only has access to its past trajectory (and not its future) at inference time.

As an alternative approach, Sequential Option Advantage Propagation (SOAP) evaluates and maximizes the policy gradient for an optimal option assignment directly. With this approach, the option policy is only conditional on the history of the agent. The derived objective has a surprising resemblance to the forward-backward algorithm, but showed more robustness when tested in longer corridor environments. The algorithms were also evaluated on the Atari (Bellemare et al., 2013) and MuJoCo (Todorov et al., 2012) benchmarks. Results demonstrated that using SOAP for option learning is more effective and robust than using the standard approach for learning options, proposed by the Option-Critic architecture.

The proposed approach can improve the efficiency of skill discovery in skill-based RL algorithms, allowing them to adapt efficiently to complex novel environments.

2 Background

2.1 Partially Observable Markov Decision Process

POMDP is a special case of an Markov Decision Process (MDP) where the observation available to the agent only contains partial information of the underlying state. In this work, s is used to denote the (partial) state given to the agent, which may or may not contain the full information of the environment (which shall be distinguished from state s as the underlying state \mathfrak{s}).¹ This implies that the “state” transitions are no longer fully Markovian in a POMDP setting, and may be correlated with past observations and actions. Hence, $p(r_t, s_{t+1}|s_{0:t}, a_{0:t})$ describes the full state and reward dynamics in the case of POMDPs, where $s_{0:t}$ is a shorthand for $\{s_t|t_1 \leq t \leq t_2\}$, and similarly with $a_{0:t}$.

2.2 The Options Framework

Options (Sutton et al., 1999; Precup & Sutton, 2000) are temporally extended actions that allow the agent to make high-level decisions in the environment. Each option corresponds to a specialized low-level policy that the agent can use to achieve a specific subtask. In the Options Framework, the inter-option policy $\pi(z_t|s_t)$ and an option termination probability $\varpi(s_{t+1}, z_t)$ govern the transition of options, where z_t is the current option, and are chosen from an n number of discrete options $\{\mathcal{Z}_1, \dots, \mathcal{Z}_n\}$. Options are especially valuable when there are multiple stages in a task that must be taken sequentially (e.g. following a recipe) and the agent must obey different policies given similar observations, depending on the stage of the task.

Earlier works have built upon the Options Framework by either learning optimal option selection over a pre-determined set of options (Peng et al., 2019) or using heuristics for option segmentation (Kulkarni et al., 2016; Nachum et al., 2018), rather than a fully end-to-end approach. While effective, such approaches constrain the agent’s ability to discover useful skills automatically. The Option-Critic architecture (Bacon et al., 2017) proposed end-to-end trainable systems which learn option assignment. It formulates the problem such that inter-option policies and termination conditions are learned jointly in the process of maximizing the expected returns.

2.3 Option-Critic architecture

As mentioned in Section 2.2, the options framework (Sutton et al., 1999; Precup & Sutton, 2000) formalizes the idea of temporally extended actions that allow agents to make high-level decisions. Let there be n discrete options $\{\mathcal{Z}_1, \dots, \mathcal{Z}_n\}$ from which z_t is chosen and assigned at every time step t . Each option corresponds to a specialized sub-policy $\pi_\theta(a_t|s_t, z_t)$ that the agent can use to achieve a specific subtask. At $t = 0$, the agent chooses an option according to its inter-option policy $\pi_\phi(z_t|s_t)$ (policy over options), then follows the option sub-policy until termination, which is dictated by the termination probability function $\varpi_\psi(s_t, z_{t-1})$. Once the option is terminated, a new option z_t is sampled from the inter-option policy and the procedure is repeated.

¹In other literature, o is used to denote the partial observation to distinguish from the underlying state s . While this makes the distinction explicit, many works on standard RL algorithms assume a fully observable MDP for their formulation, leading to conflicting notations.

The Option-Critic architecture (Bacon et al., 2017) learns option assignments end-to-end. It formulates the problem such that the option sub-policies $\pi_\theta(a_t|s_t, z_t)$ and termination function $\varpi_\psi(s_{t+1}, z_t)$ are learned jointly in the process of maximizing the expected returns. The inter-option policy $\pi_\phi(z_t|s_t)$ is an ϵ -greedy policy that takes an argmax z of the option value function $Q_\phi(s, z)$ with $1 - \epsilon$ probability, and uniformly randomly samples options with ϵ probability. In every step of the Option-Critic algorithm, the following updates are performed for a current state s , option z , reward r , episode termination indicator $d \in \{0, 1\}$, next state s' , and discount factor $\gamma \in [0, 1]$:

$$\begin{aligned} \delta &\leftarrow r + \gamma(1-d) \left[(1 - \varpi_\psi(s', z)) Q(s', z) + \varpi_\psi(s', z) \max_z Q_\phi(s', z) \right] - Q_\phi(s, z), \\ Q_\phi(s, z) &\leftarrow Q_\phi(s, z) + \alpha_\phi \delta, \\ \theta &\leftarrow \theta + \alpha_\theta \frac{\partial \log \pi_\theta(a|s, z)}{\partial \theta} [r + \gamma Q_\phi(s', z)], \\ \psi &\leftarrow \psi - \alpha_\psi \frac{\partial \varpi_\psi(s', z)}{\partial \psi} [Q_\phi(s', z) - \max_z Q_\phi(s', z)]. \end{aligned} \quad (1)$$

Here, α_ϕ , α_θ and α_ψ are learning rates for $Q_\phi(s, z)$, $\pi_\theta(a|s, z)$, and $\varpi_\psi(s, z)$, respectively.

Proximal Policy Option-Critic (PPOC) (Klissarov et al., 2017) builds on top of the Option-Critic architecture (Bacon et al., 2017), replacing the ϵ -greedy policy over the option-values with a policy network $\pi_\varphi(z|s)$ parametrized by φ with corresponding learning rate α_φ , substituting the policy gradient algorithm with PPO (Schulman et al., 2017) to optimize the sub-policies $\pi_\theta(a|s, z)$.

A standard PPO's objective is:

$$\mathcal{L}_{\text{PPO}}(\theta) = \mathbb{E}_{s, a \sim \pi} \left[\min \left(\frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)} A_t^{\text{GAE}}, \text{clip} \left(\frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)}, 1 - \epsilon, 1 + \epsilon \right) A_t^{\text{GAE}} \right) \right], \quad (2)$$

where $\frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)}$ is a ratio of probabilities of taking action a at state s with the new policy against that with the old policy, and A_t^{GAE} is the GAE (Schulman et al., 2016) at time step t . GAE provides a robust and low-variance estimate of the advantage function. It can be expressed as a sum of exponentially weighted multi-step TD errors:

$$A_t^{\text{GAE}} = \sum_{t'=t}^T (\gamma \lambda)^{t'-t} \delta_{t'}, \quad (3)$$

where $\delta_t = r_t + \gamma(1 - d_t)V(s_{t+1}) - V(s_t)$ is the TD error at time t , and λ is a hyperparameter that controls the trade-off of bias and variance. Extending the definition of GAE to work with options, the update formula for PPOC can be expressed as:

$$\begin{aligned} A^{\text{GAE}}(s, z) &\leftarrow r + \gamma V(s', z') - V(s, z) + \lambda \gamma(1 - d) A^{\text{GAE}}(s', z'), \\ Q_\phi(s, z) &\leftarrow Q_\phi(s, z) + \alpha_\phi A^{\text{GAE}}(s, z), \\ \theta &\leftarrow \theta + \alpha_\theta \frac{\partial \mathcal{L}_{\text{PPO}}(\theta)}{\partial \theta}, \\ \psi &\leftarrow \psi - \alpha_\psi \frac{\partial \varpi_\psi(s, z)}{\partial \psi} A^{\text{GAE}}(s, z), \\ \varphi &\leftarrow \varphi + \alpha_\varphi \frac{\partial \log \pi_\varphi(z|s)}{\partial \varphi} A^{\text{GAE}}(s, z). \end{aligned} \quad (4)$$

PPOC is used as one of the baselines in this work.

2.4 Expectation Maximization algorithm

The EM algorithm (Dempster et al., 1977) is a well-known method for learning the assignment of latent variables, often used for unsupervised clustering and segmentation. The k -means clustering algorithm (Forgy, 1965) can be considered a special case of EM. The following explanation in this section is a partial summary of Chapter 9 of Bishop's book (Bishop, 2006).

The objective of EM is to find a maximum likelihood solution for models with latent variables. Denoting the set of all observed data as \mathbf{X} , the set of all latent variables as \mathbf{Z} , and the set of all model parameters as Θ , the log-likelihood function is given by:

$$\log p(\mathbf{X}|\Theta) = \log \left\{ \sum_{\mathbf{Z}} p(\mathbf{X}, \mathbf{Z}|\Theta) \right\}. \quad (5)$$

However, evaluating the above summation (or integral for a continuous \mathbf{Z}) over all possible latents is intractable. The EM algorithm is a way to strictly increase the likelihood function by alternating between the E-step that evaluates the expectation of a joint log-likelihood $\log p(\mathbf{X}, \mathbf{Z}|\Theta)$, and the M-step that maximizes this expectation.

In the E-step, the current parameter estimate Θ_{old} (using random initialization in the first iteration, or the most recent updated parameters in subsequent iterations) is used to determine the posterior of the latents $p(\mathbf{Z}|\mathbf{X}, \Theta_{\text{old}})$. The joint log-likelihood is obtained under this prior. The expectation, denoted as $\mathcal{Q}(\Theta; \Theta_{\text{old}})$, is given by:

$$\mathcal{Q}(\Theta; \Theta_{\text{old}}) = \mathbb{E}_{\mathbf{Z} \sim p(\cdot|\mathbf{X}, \Theta_{\text{old}})} [\log p(\mathbf{X}, \mathbf{Z}|\Theta)] = \sum_{\mathbf{Z}} p(\mathbf{Z}|\mathbf{X}, \Theta_{\text{old}}) \log p(\mathbf{X}, \mathbf{Z}|\Theta). \quad (6)$$

In the M-step, an updated parameter estimate Θ_{new} is obtained by maximizing the expectation:

$$\Theta_{\text{new}} = \arg \max_{\Theta} \mathcal{Q}(\Theta, \Theta_{\text{old}}). \quad (7)$$

The E-step and the M-step are performed alternately until a convergence criterion is satisfied. The EM algorithm makes obtaining a maximum likelihood solution tractable (Bishop, 2006).

2.5 Forward-backward algorithm

The EM algorithm can also be applied in an HMM setting for sequential data, resulting in the forward-backward algorithm, also known as the Baum-Welch algorithm (Baum, 1972). Figure 1 shows the graph of the HMM of interest. At every time step $t \in \{0, \dots, T\}$, a latent z_t is chosen out of n number of discrete options $\{\mathcal{Z}_1, \dots, \mathcal{Z}_n\}$, which is an underlying conditioning variable for an observation x_t . In the following derivation, $\{x_t | t_1 \leq t \leq t_2\}$ is denoted with a shorthand $x_{t_1:t_2}$, and similarly for other variables. Chapter 13 of Bishop's book (Bishop, 2006) offers a comprehensive explanation for this algorithm.

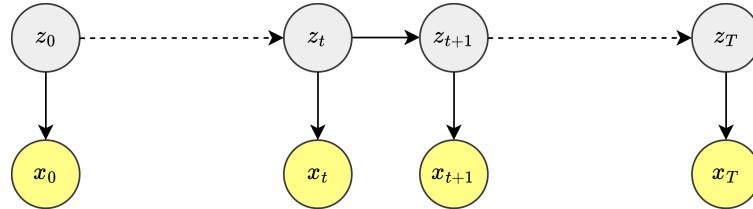


Figure 1: An HMM for sequential data \mathbf{X} of length T , given latent variables \mathbf{Z} .

For this HMM, the joint likelihood function for the observed sequence $\mathbf{X} = \{x_0, \dots, x_T\}$ and latent variables $\mathbf{Z} = \{z_0, \dots, z_T\}$ is given by:

$$p(\mathbf{X}, \mathbf{Z}|\Theta) = p(z_0|\Theta) \prod_{t=0}^T p(x_t|z_t, \Theta) \prod_{t=1}^T p(z_t|z_{t-1}, \Theta). \quad (8)$$

Using the above, EM objective can be simplified as:

$$\begin{aligned}
\mathcal{Q}(\Theta; \Theta_{\text{old}}) &= \sum_{\mathbf{Z}} p(\mathbf{Z}|\mathbf{X}, \Theta_{\text{old}}) \log p(\mathbf{X}, \mathbf{Z}|\Theta) \\
&= \sum_{z_0} p(z_0|\Theta_{\text{old}}) \log p(z_0|\Theta) + \sum_{t=0}^T \sum_{z_t} p(z_t|\mathbf{X}, \Theta_{\text{old}}) \log p(x_t|z_t, \Theta) \\
&\quad + \sum_{t=1}^T \sum_{z_{t-1}, z_t} p(z_{t-1}, z_t|\mathbf{X}, \Theta_{\text{old}}) \log p(x_t, z_t|z_{t-1}, \Theta).
\end{aligned} \tag{9}$$

2.5.1 E-step

In the E-step, $p(z_t|\mathbf{X})$ and $p(z_{t-1}, z_t|\mathbf{X})$ are evaluated. Note that in the following derivation, it is assumed that the probability distributions are conditioned on Θ . Defining $\alpha(z_t) := p(z_t|x_{0:t})$, $\beta(z_t) := \frac{p(x_{t+1:T}|z_t)}{p(x_{t+1:T}|x_{0:t})}$ and normalising constant $c_t := p(x_t|x_{0:t-1})$,

$$p(z_t|\mathbf{X}) = \frac{p(x_{0:T}, z_t)}{p(x_{0:T})} = \frac{p(x_{0:t}, z_t)p(x_{t+1:T}|z_t)}{p(x_{0:t})p(x_{t+1:T}|x_{0:t})} = \alpha(z_t)\beta(z_t), \tag{10}$$

$$\begin{aligned}
p(z_{t-1}, z_t|\mathbf{X}) &= \frac{p(x_{0:T}, z_{t-1}, z_t)}{p(x_{0:T})} = \frac{p(x_{0:t-1}, z_{t-1})p(x_t|z_t)p(z_t|z_{t-1})p(x_{t+1:T}|z_t)}{p(x_{0:t-1})p(x_t|x_{0:t-1})p(x_{t+1:T}|x_{0:t})} \\
&= p(x_t|z_t)p(z_t|z_{t-1})\frac{\alpha(z_t)\beta(z_t)}{c_t}.
\end{aligned} \tag{11}$$

Recursively evaluating $\alpha(z_t)$, $\beta(z_t)$ and c_t ,

$$\begin{aligned}
\alpha(z_t) &= \frac{p(x_{0:t}, z_t)}{p(x_{0:t})} = \frac{p(x_t, z_t|x_{0:t-1})}{p(x_t|x_{0:t-1})} = \frac{\sum_{z_{t-1}} [p(z_{t-1}|x_{0:t-1})p(x_t|z_t)p(z_t|z_{t-1})]}{p(x_t|x_{0:t-1})} \\
&= \frac{p(x_t|z_t)\sum_{z_{t-1}} [\alpha(z_{t-1})p(z_t|z_{t-1})]}{c_t},
\end{aligned} \tag{12}$$

$$\begin{aligned}
\beta(z_t) &= \frac{p(x_{t+1:T}|z_t)}{p(x_{t+1:T}|x_{0:t})} = \frac{\sum_{z_{t+1}} [p(x_{t+2:T}|z_{t+1})p(x_{t+1}|z_{t+1})p(z_{t+1}|z_t)]}{p(x_{t+2:T}|x_{0:t+1})p(x_{t+1}|x_{0:t})} \\
&= \frac{\sum_{z_{t+1}} [\beta(z_{t+1})p(x_{t+1}|z_{t+1})p(z_{t+1}|z_t)]}{c_{t+1}},
\end{aligned} \tag{13}$$

$$c_t = p(x_t|x_{0:t-1}) = \sum_{z_{t-1}, z_t} [p(z_{t-1}|x_{0:t-1})p(x_t|z_t)p(z_t|z_{t-1})] = \sum_{z_{t-1}, z_t} [\alpha(z_{t-1})p(x_t|z_t)p(z_t|z_{t-1})]. \tag{14}$$

Initial conditions are $\alpha(z_0) = \frac{p(x_0|z_0)p(z_0)}{\sum_{z_0} [p(x_0|z_0)p(z_0)]}$, $\beta(z_T) = 1$.

2.5.2 M-step

In the M-step, the parameter set Θ is updated by maximizing $\mathcal{Q}(\Theta; \Theta_{\text{old}})$, which can be rewritten by substituting $p(z_t|\mathbf{X})$ and $p(z_{t-1}, z_t|\mathbf{X})$ in Equation (9) with $\alpha(z)$ and $\beta(z)$ (ignoring the constants) as derived in Section 2.5.1.

2.5.3 Option discovery via the forward-backward algorithm

The idea of applying the forward-backward algorithm to learn option assignments is first introduced in Daniel et al. (2016), and has later been applied in both Imitation Learning (IL) settings (Zhang & Paschalidis, 2020; Giammarino & Paschalidis, 2021) and RL settings (Fox et al., 2017). However, in previous literature, the option policy is decoupled into an option termination probability $\varpi(s_t, z_{t-1})$, and an inter-option policy $\pi(z_t|s_t)$. Due to the inter-option policy being unconditional on the previous option z_{t-1} , the choice of a new option z_t will be uninformed of the previous option z_{t-1} . This may be problematic for learning POMDP tasks as demonstrated in Section 6.1, because if a task consists of a sequence of options, then knowing which one was executed before is important to decide to move on to

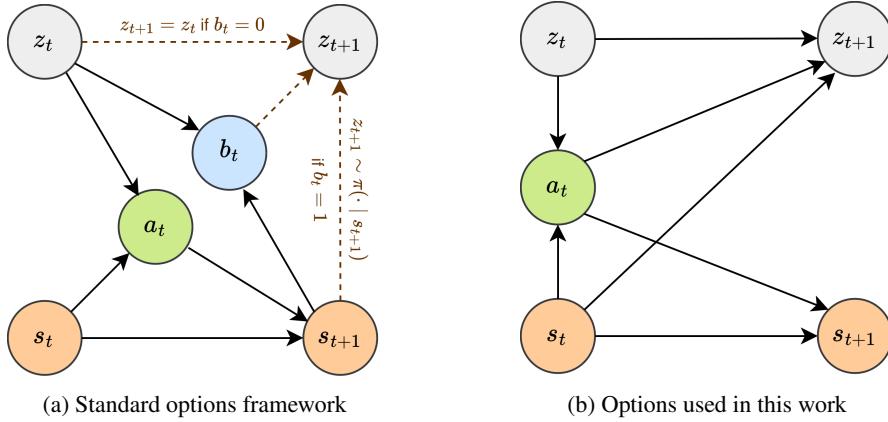


Figure 2: Probabilistic graphical models showing the relationships between options z , actions a and states s at time step t . b_t in the standard options framework denotes a boolean variable that initiates the switching of options when activated. This work adopts a more general formulation compared to the options framework.

the next one. Previous literature also does not address the issues of exponentially diminishing magnitudes that arise from recursively applying the formula. This is known as the scaling factor problem (Bishop, 2006).

This work presents a concise derivation of the forward-backward algorithm applied to an improved version of the options framework. It also addresses the scaling factor problem, by building this factor into the derivation.

3 Option assignment formulation

The aim is to learn a diverse set of options with corresponding policy and value estimates, such that each option is responsible for accomplishing a well-defined subtask, such as reaching a certain state region. At every time step t , the agent chooses an option z_t out of n number of discrete options $\{\mathcal{Z}_1, \dots, \mathcal{Z}_n\}$.

3.1 Option policy and sub-policy

The goal is to learn a sub-policy $\pi_\theta(a|s, z)$ conditional to a latent option variable z , and an option policy $\pi_\psi(z'|s, a, z)$ used to iteratively assign options at each time step, to model the joint option policy

$$p_\Theta(a_t, z_{t+1}|s_t, z_t) = \pi_\theta(a_t|s_t, z_t)\pi_\psi(z_{t+1}|s_t, a_t, z_t). \quad (15)$$

Here, the learnable parameter set of the policy is denoted as $\Theta = \{\theta, \psi\}$.

A comparison of the option policy used in this work and the standard options framework is shown in Figure 2. Unlike the options framework, which further decouples the option policy π_ψ into an option termination probability $\varpi(s_t, z_{t-1})$, and an unconditional inter-option policy $\pi(z_t|s_t)$, in this work the option policy is modeled π_ψ with one network so that the inter-option policy is informed by the previous option z_t upon choosing the next z_{t+1} . A graphical model for the full HMM is shown in Figure 3.

3.2 Evaluating the probability of latents

Let us define an auto-regressive action probability $\alpha_t := p(a_t|s_{0:t}, a_{0:t-1})$, an auto-regressive option forward distribution $\zeta(z_t) := p(z_t|s_{0:t}, a_{0:t-1})$, and an option backward feedback $\beta(z_t) := \frac{p(s_{t:T}, a_{t:T-1}|s_{t-1}, a_{t-1}, z_t)}{p(s_{t:T}, a_{t:T-1}|s_{0:t-1}, a_{0:t-1})}$. Notice that the definitions of action probability α , option forward $\zeta(z_t)$, and option backward $\beta(z_t)$ resemble c_t , $\alpha(z_t)$ and $\beta(z_t)$ defined in Section 2.5, respectively. While it is common practice to denote the forward and backward quantities as α and β in the forward-backward algorithm (also known as the α - β algorithm), here α_t is redefined to denote the action probability (corresponding to the normalizing constant c_t), and $\zeta(z_t)$ for the option forward distribution, to draw attention to the fact that these are probabilities of option z_t and action a_t , respectively.

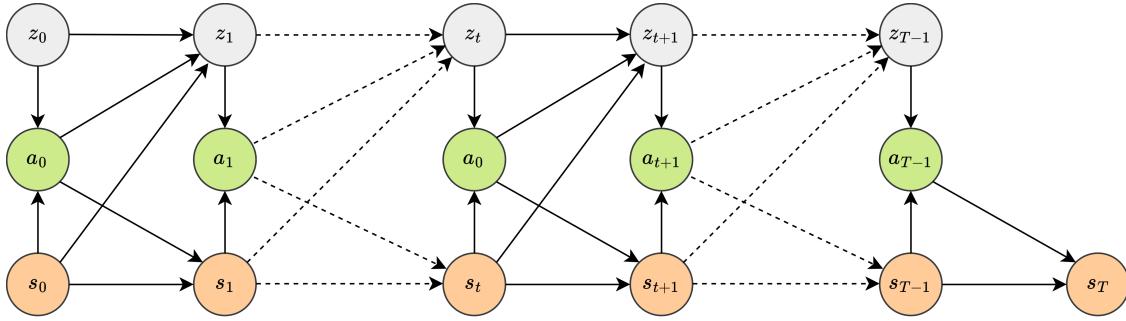


Figure 3: An HMM showing the relationships between options z , actions a and states s .

α_t , $\zeta(z_t)$ and $\beta(z_t)$ can be recursively evaluated as follows:

$$\alpha_t = p(a_t | s_{0:t}, a_{0:t-1}) = \sum_{z_t, z_{t+1}} p(z_t | s_{0:t}, a_{0:t-1}) p_\Theta(a_t, z_{t+1} | s_t, z_t) = \sum_{z_t, z_{t+1}} \zeta(z_t) p_\Theta(a_t, z_{t+1} | s_t, z_t), \quad (16)$$

$$\begin{aligned} \zeta(z_{t+1}) &= \frac{p(z_{t+1}, s_{t+1}, a_t | s_{0:t}, a_{0:t-1})}{p(s_{t+1}, a_t | s_{0:t}, a_{0:t-1})} \\ &= \frac{\sum_{z_t} p(z_t | s_{0:t}, a_{0:t-1}) p_\Theta(a_t, z_{t+1} | s_t, z_t) P(s_{t+1} | s_{0:t}, a_{0:t})}{p(a_t | s_{0:t}, a_{0:t-1}) P(s_{t+1} | s_{0:t}, a_{0:t})} \\ &= \frac{\sum_{z_t} \zeta(z_t) p_\Theta(a_t, z_{t+1} | s_t, z_t)}{\alpha_t}, \end{aligned} \quad (17)$$

$$\begin{aligned} \beta(z_t) &= \frac{p(s_{t:T}, a_{t:T-1} | s_{t-1}, a_{t-1}, z_t)}{p(s_{t:T}, a_{t:T-1} | s_{0:t-1}, a_{0:t-1})} \\ &= \frac{\sum_{z_{t+1}} [p(s_{t+1:T}, a_{t+1:T-1} | s_t, a_t, z_{t+1}) p_\Theta(a_t, z_{t+1} | s_t, z_t) P(s_t | s_{0:t-1}, a_{0:t-1})]}{p(s_{t+1:T}, a_{t+1:T-1} | s_{0:t}, a_{0:t}) p(a_t | s_{0:t}, a_{0:t-1}) P(s_t | s_{0:t-1}, a_{0:t-1})} \\ &= \frac{\sum_{z_{t+1}} [\beta(z_{t+1}) p_\Theta(a_t, z_{t+1} | s_t, z_t)]}{\alpha_t}. \end{aligned} \quad (18)$$

Initial conditions are $\zeta(z_0) = p(z_0) = \frac{1}{n}$ for all possible z_0 , indicating a uniform distribution over the options initially when no observations or actions are available, and $\beta(z_T) = \frac{p(s_T | s_{T-1}, a_{T-1}, z_T)}{p(s_T | s_{0:T-1}, a_{0:T-1})} = 1$.

4 Proximal Policy Optimization via Expectation Maximization

In this section, PPOEM is introduced, an algorithm that extends PPO for option discovery with an EM objective. The expectation of the returns is taken over the joint probability distribution of states, actions and options, sampled by the policy. This objective gives a tractable objective to maximize, which has a close resemblance to the forward-backward algorithm.

4.1 Expected return maximization objective with options

The objective is to maximize the expectation of returns $R(\tau)$ for an agent policy π over a trajectory τ with latent option z_t at each time step t . The definition of a trajectory τ is a set of states, actions and rewards visited by the agent policy in an episode. The objective $J[\pi]$ can be written as:

$$J[\pi_\Theta] = \mathbb{E}_{\tau, Z \sim \pi} [R(\tau)] = \int_{\tau, Z} R(\tau) p(\tau, Z | \Theta). \quad (19)$$

Taking the gradient of the maximization objective,

$$\nabla_\Theta J[\pi_\Theta] = \int_{\tau, Z} R(\tau) \nabla_\Theta p(\tau, Z | \Theta) = \int_{\tau, Z} R(\tau) \frac{\nabla_\Theta p(\tau, Z | \Theta)}{p(\tau, Z | \Theta)} p(\tau, Z | \Theta) = \mathbb{E}_{\tau, Z} [R(\tau) \nabla_\Theta \log p(\tau, Z | \Theta)]. \quad (20)$$

To simplify the derivation, let us focus on the states and actions that appear in the trajectory. The joint likelihood function for the trajectory τ and latent options $Z = \{z_0, \dots, z_{T-1}\}$ is given by:

$$p(\tau, Z|\Theta) = p(s_{0:T}, a_{0:T-1}, z_{0:T}|\Theta) = p(s_0, z_0) \prod_{t=0}^{T-1} [p_\Theta(a_t, z_{t+1}|s_t, z_t) P(s_{t+1}|s_{0:t}, a_{0:t})], \quad (21)$$

Evaluating $\nabla_\Theta \log p(\tau, Z|\Theta)$, the log converts the products into sums, and the terms which are constant with respect to Θ are eliminated upon taking the gradient, leaving

$$\nabla_\Theta \log p(\tau, Z|\Theta) = \sum_{t=0}^{T-1} \nabla_\Theta \log [\pi_\theta(a_t|s_t, z_t) \pi_\psi(z_{t+1}|s_t, a_t, z_t, s_{t+1})]. \quad (22)$$

Substituting Equation (22) into Equation (20) and explicitly evaluating the expectation over the joint option probabilities,

$$\begin{aligned} \nabla_\Theta J[\pi_\Theta] &= \mathbb{E}_{\tau, Z \sim \pi} \left[\sum_{t=0}^{T-1} R(\tau) \nabla_\Theta \log p_\Theta(a_t, z_{t+1}|s_t, z_t) \right] \\ &= \mathbb{E}_{\tau \sim \pi} \int_Z \left[\sum_{t=0}^{T-1} [R(\tau) \nabla_\Theta \log p_\Theta(a_t, z_{t+1}|s_t, z_t)] \right] p(Z|\tau) \\ &= \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{T-1} \sum_{z_t, z_{t+1}} [R(\tau) p(z_t, z_{t+1}|\tau) \nabla_\Theta \log p_\Theta(a_t, z_{t+1}|s_t, z_t)] \right]. \end{aligned} \quad (23)$$

Using the action probability $\alpha_t := p(a_t|s_{0:t}, a_{0:t-1})$, option forward distribution $\zeta(z_t) := p(z_t|s_{0:t}, a_{0:t-1})$, and option backward feedback $\beta(z_t) := \frac{p(s_{t:T}, a_{t:T-1}|s_{t-1}, a_{t-1}, z_t)}{p(s_{t:T}, a_{t:T-1}|s_{0:t-1}, a_{0:t-1})}$ evaluated in Section 3.2, $p(z_t, z_{t+1}|\tau)$ can be evaluated as

$$\begin{aligned} p(z_t, z_{t+1}|\tau) &= \frac{p(s_{0:T}, a_{0:T-1}, z_t, z_{t+1})}{p(s_{0:T}, a_{0:T-1})} \\ &= \frac{p(s_{0:t}, a_{0:t-1}, z_t) p_\Theta(a_t, z_{t+1}|s_t, z_t) p(s_{t+1:T}, a_{t+1:T-1}|s_t, a_t, z_{t+1})}{p(s_{0:t}, a_{0:t-1}) p(a_t|s_{0:t}, a_{0:t-1}) p(s_{t+1:T}, a_{t+1:T-1}|s_{0:t}, a_{0:t})} \\ &= p_\Theta(a_t, z_{t+1}|s_t, z_t) \frac{\zeta(z_t) \beta(z_{t+1})}{\alpha_t}. \end{aligned} \quad (24)$$

Using this, Equation (23) can be evaluated and maximized with gradient descent.

4.1.1 Relationship with Expectation Maximization

The objective derived in Equation (23) closely resembles the objective of the EM algorithm applied to the HMM with options as latent variables. The expectation of the marginal log-likelihood $\mathcal{Q}(\Theta; \Theta_{\text{old}})$, which gives the lower-bound of the marginal log-likelihood $\log p(\tau|\Theta)$, is given by

$$\begin{aligned} \mathcal{Q}(\Theta; \Theta_{\text{old}}) &= \mathbb{E}_{Z \sim p(\cdot|\tau, \Theta_{\text{old}})} [\ln p(\tau, Z|\Theta)] = \mathbb{E}_{\tau \sim \pi} \int_Z p(Z|\tau, \Theta_{\text{old}}) \ln p(\tau, Z|\Theta) dZ \\ &= \mathbb{E}_{\tau \sim \pi} \sum_{t=0}^{T-1} \sum_{z_t, z_{t+1}} [p(z_t, z_{t+1}|\tau, \Theta_{\text{old}}) \log p_\Theta(a_t, z_{t+1}|s_t, z_t)] + \text{const.} \end{aligned} \quad (25)$$

The difference is that the expected return maximization objective in Equation (23) weights the log probabilities of the policy according to the returns, whereas the objective of Equation (25) is to find a parameter set Θ that maximizes the probability that the states and actions that appeared in the trajectory are visited by the joint option policy p_Θ .

4.2 PPO objective with Generalized Advantage Estimation

A standard optimization technique for neural networks using gradient descent can be applied to optimize the policy network. Noticing that the optimization objective in Equation (23) resembles the policy gradient algorithm, the joint option policy can be optimized using the PPO algorithm instead to prevent the updated policy $p_\Theta(a_t, z_{t+1}|s_t, z_t)$ from deviating from the original policy too much.

Several changes have to be made to adapt the training objective to PPO. Firstly, $\nabla \log p_\Theta$ is replaced by $\frac{\nabla p_\Theta}{p_{\Theta_{\text{old}}}}$, its first order approximation, to easily introduce clipping constraints to the policy ratios. Secondly, the return $R(\tau)$ is replaced with the GAE, A_t^{GAE} , as described in Equation (3).

Extending the definition of GAE to work with options,

$$A_t^{\text{GAE}}(z_t, z_{t+1}|\tau) = r_t + \gamma V(s_{t+1}, z_{t+1}) - V(s_t, z_t) + \lambda \gamma (1 - d_t) A_{t+1}^{\text{GAE}}(z_{t+1}|\tau) \quad (26)$$

$$A_t^{\text{GAE}}(z_t|\tau) = \sum_{z_{t+1}} p(z_{t+1}|z_t, \tau) A_t^{\text{GAE}}(z_t, z_{t+1}|\tau). \quad (27)$$

The GAE could be evaluated backwards iteratively, starting from $t = T$ with the initial condition $A_T^{\text{GAE}}(z_{t+1}|\tau) = 0$. The option transition function $p(z_{t+1}|z_t, \tau)$ can be evaluated using $p(z_t, z_{t+1}|\tau)$ (Equation (24)) as:

$$p(z_{t+1}|z_t, \tau) = \frac{p(z_t, z_{t+1}|\tau)}{\sum_{z_{t+1}} p(z_t, z_{t+1}|\tau)}. \quad (28)$$

The target value $V_{\text{target}}(s_t, z_t)$ to regress the estimated value function towards can be defined in terms of the GAE and the current value estimate as:

$$V_{\text{target}}(s_t, z_t) = V^\pi(s_t, z_t) + A_t^{\text{GAE}}(z_t|\tau). \quad (29)$$

5 Sequential Option Advantage Propagation

In the previous section, assignments of the latent option variables Z were determined by maximizing the expected return for complete trajectories. The derived algorithm resembles the forward-backward algorithm closely, and requires the backward pass of $\beta(z_t)$ in order to fully evaluate the option probability $p(Z|\tau)$. During rollouts of the agent policy, however, knowing the optimal assignment of latents $p(z_t|\tau)$ in advance is not possible, since the trajectory is incomplete and the backward pass has not been initiated. Therefore, the policy must rely on the current best estimate of the options given its available past trajectory $\{s_{0:t}, a_{0:t}\}$ during its rollout. This option distribution conditional only on its past is equivalent to the auto-regressive option forward distribution $\zeta(z_t) := p(z_t|s_{0:t}, a_{0:t-1})$.

Since the optimal option assignment can only be achieved in hindsight once the trajectory is complete, this information is not helpful for the agent policy upon making its decisions. A more useful source of information for the agent, therefore, is the current best estimate of the option assignment $\zeta(z_t)$. It is sensible, therefore, to directly optimize for the expected returns evaluated over the option assignments $\zeta(z_t)$ to find an optimal option policy, rather than optimizing the expected returns for an option assignment $p(Z|\tau)$, which can only be known in hindsight.

The following section proposes a new option optimization objective that does not involve the backward pass of the EM algorithm. Instead, the option policy gradient for an optimal forward option assignment is evaluated analytically. This results in a temporal gradient propagation, which corresponds to a backward pass, but with a slightly different outcome. Notably, this improved algorithm, SOAP, applies a normalization of the option advantages in every back-propagation step through time.

As far as the authors are aware, this work is the first to derive the back-propagation of policy gradients in the context of option discovery.

5.1 Policy Gradient objective with options

Let us start by deriving the policy gradient objective assuming options. The maximization objective $J[\pi]$ for the agent can be defined as:

$$J[\pi_\Theta] = \mathbb{E}_{\tau \sim \pi} [R(\tau)] = \int_{\tau} R(\tau) p(\tau|\Theta) d\tau. \quad (30)$$

Taking the gradient of the maximization objective,

$$\nabla_{\Theta} J[\pi_\Theta] = \int_{\tau} R(\tau) \nabla_{\Theta} p(\tau|\Theta) d\tau = \int_{\tau} R(\tau) \frac{\nabla_{\Theta} p(\tau|\Theta)}{p(\tau|\Theta)} p(\tau|\Theta) d\tau = \mathbb{E}_{\tau} [R(\tau) \nabla_{\Theta} \log p(\tau|\Theta)]. \quad (31)$$

So far, the above derivation is the same as the normal policy gradient objective without options. Next, the likelihood for the trajectory τ is given by:

$$p(\tau|\Theta) = p(s_{0:T}, a_{0:T-1}|\Theta) = \rho(s_0) \Pi_{t=0}^{T-1} [p(a_t|s_{0:t}, a_{0:t-1}, \Theta) P(s_{t+1}|s_{0:t}, a_{0:t})]. \quad (32)$$

This is where options become relevant, as the standard formulation assumes that the policy $\pi(a|s)$ is only dependent on the current state without history, and similarly that the state transition environment dynamics $P(s'|s, a)$ is Markovian given the current state and action. In many applications, however, the *states* that are observed do not contain the entire information about the underlying dynamics of the environment², and therefore, conditioning on the history yields a different distribution of future states compared to conditioning on just the current state. To capture this, the policy and state transitions are now denoted to be $p(a_t|s_{0:t}, a_{0:t-1})$ and $P(s_{t+1}|s_{0:t}, a_{0:t})$, respectively. Here, the probabilities are conditional on the historical observations ($s_{0:t}$) and historical actions (e.g. $a_{0:t}$), rather than just the immediate state s_t and action a_t . Note that $p(a_t|s_{0:t}, a_{0:t-1})$ is a quantity α_t that has already been evaluated in Section 3.2.

Evaluating $\nabla_{\Theta} \log p(\tau|\Theta)$, the log converts the products into sums, and the terms that are constant with respect to Θ are eliminated upon taking the gradient, leaving

$$\nabla_{\Theta} \log p(\tau|\Theta) = \sum_{t=0}^{T-1} \nabla_{\Theta} \log p(a_t|s_{0:t}, a_{0:t-1}, \Theta) = \sum_{t=0}^{T-1} \nabla_{\Theta} \log \alpha_t = \sum_{t=0}^{T-1} \frac{\nabla_{\Theta} \alpha_t}{\alpha_t}, \quad (33)$$

where α_t is substituted following its definition in Section 3.2.

Substituting Equation (33) into Equation (31),

$$\nabla_{\Theta} J[\pi_\Theta] = \mathbb{E}_{\tau} [R(\tau) \nabla_{\Theta} \log p(\tau|\Theta)] = \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{T-1} R(\tau) \frac{\nabla_{\Theta} \alpha_t}{\alpha_t} \right]. \quad (34)$$

Similarly to Section 4.2, it is possible to substitute the return $R(\tau)$ with GAE, thereby reducing the variance in the return estimate. Extending the definition of GAE to work with options,

$$A_t^{\text{GAE}}(z_t, z_{t+1}) = r_t + \gamma V(s_{t+1}, z_{t+1}) - V(s_t, z_t) + \lambda \gamma (1 - d_t) A_{t+1}^{\text{GAE}}(z_{t+1}), \quad (35)$$

$$A_t^{\text{GAE}}(z_t) = \sum_{z_{t+1}} p(z_{t+1}|s_t, a_t, z_t) A_t^{\text{GAE}}(z_t, z_{t+1}), \quad (36)$$

$$V_{\text{target}}(s_t, z_t) = V^{\pi}(s_t, z_t) + A_t^{\text{GAE}}(z_t). \quad (37)$$

Notice that, while the definition of these estimates is almost identical to Section 4.2, the advantages are now propagated backwards via the option transition $p(z_{t+1}|s_t, a_t, z_t)$ rather than $p(z_{t+1}|z_t, \tau)$.

²Some literature on POMDP choose to make this explicit by denoting the partial observation available to the agent as observation o , distinguishing from the underlying ground truth state. However, since o can also stand for *options*, and is used in other literature on options, here the input to the agent's policy and value functions is denoted using the conventional s to prevent confusion.

Substituting the GAE into Equation (34),

$$\begin{aligned}\nabla_{\Theta} J[\pi_{\Theta}] &= \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{T-1} \frac{\sum_{z_t} A_t^{\text{GAE}}(z_t) \zeta(z_t)}{\alpha_t} \nabla_{\Theta} \alpha_t \right] \\ &= \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{T-1} \frac{\sum_{z_t} A_t^{\text{GAE}}(z_t) \zeta(z_t)}{\alpha_t} \sum_{z_t, z_{t+1}} [p_{\Theta}(a_t, z_{t+1} | s_t, z_t) \nabla \zeta(z_t) + \zeta(z_t) \nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)] \right].\end{aligned}\quad (38)$$

5.2 Analytic back-propagation of the policy gradient

If a forward pass of the policy can be made in one step over the entire trajectory, a gradient optimization on the objective can be performed directly. However, this would require storing the entire trajectory in GPU memory, which is highly computationally intensive. Instead, this section analytically evaluates the back-propagation of gradients of the objective so that the model can be trained on single time-step rollout samples during training.

Gradient terms appearing in Equation (38) are either $\nabla \zeta(z_t)$ or $\nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)$ for $0 \leq t \leq T - 1$. While $p_{\Theta}(a_t, z_{t+1} | s_t, z_t)$ is approximated by neural networks and can be differentiated directly, $\nabla \zeta(z_{t+1})$ has to be further expanded to evaluate the gradient in recursive form as:

$$\begin{aligned}\nabla \zeta(z_{t+1}) &= \frac{\nabla \sum_{z_t} \zeta(z_t) p_{\Theta}(a_t, z_{t+1} | s_t, z_t)}{\alpha_t} - \zeta(z_{t+1}) \frac{\nabla \alpha_t}{\alpha_t} \\ &= \frac{1}{\alpha_t} \left[\sum_{z_t} \nabla [\zeta(z_t) p_{\Theta}(a_t, z_{t+1} | s_t, z_t)] - \zeta(z_{t+1}) \sum_{z'_t, z'_{t+1}} \nabla [\zeta(z'_t) p_{\Theta}(a_t, z'_{t+1} | s_t, z'_t)] \right].\end{aligned}\quad (39)$$

Using Equation (39), it is possible to rewrite the $\nabla \zeta(z_{t+1})$ terms appearing in Equation (38) in terms of $\nabla \zeta(z_t)$ and $\nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)$. Defining the coefficients of $\nabla \zeta(z_{t+1})$ in Equation (38) as option utility $U(z_{t+1})$,

$$\begin{aligned}\sum_{z_{t+1}} U(z_{t+1}) \nabla \zeta(z_{t+1}) &= \frac{1}{\alpha_t} \sum_{z_t, z_{t+1}} \left[U(z_{t+1}) - \sum_{z'_{t+1}} U(z'_{t+1}) \zeta(z'_{t+1}) \right] \nabla [\zeta(z_t) p_{\Theta}(a_t, z_{t+1} | s_t, z_t)] \\ &= \frac{1}{\alpha_t} \sum_{z_t, z_{t+1}} \left[U(z_{t+1}) - \sum_{z'_{t+1}} U(z'_{t+1}) \zeta(z'_{t+1}) \right] [p_{\Theta}(a_t, z_{t+1} | s_t, z_t) \nabla \zeta(z_t) + \zeta(z_t) \nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)].\end{aligned}\quad (40)$$

Thus, the occurrences of gradients $\nabla \zeta(z_{t+1})$ have been reduced to terms with $\nabla \zeta(z_t)$ and $\nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)$.

Applying this iteratively to Equation (38), starting with $t = T - 1$ in reverse order, Equation (38) could be expressed solely in terms of gradients $\nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)$. Defining the coefficients of $\nabla p_{\Theta}(a_t, z_{t+1} | s_t, z_t)$ as policy gradient weighting $W_t(z_t, z_{t+1})$,

$$\begin{aligned}A_t^{\text{GOA}}(z_{t+1}) &= \sum_{z_t} A_t^{\text{GAE}}(z_t) \zeta(z_t) + (1 - d_t) \left[U(z_{t+1}) - \sum_{z'_{t+1}} U(z'_{t+1}) \zeta(z'_{t+1}) \right], \\ U(z_t) &= \frac{\sum_{z_{t+1}} A_t^{\text{GOA}}(z_{t+1}) p_{\Theta}(a_t, z_{t+1} | s_t, z_t)}{\alpha_t}, \\ W(z_t, z_{t+1}) &= \frac{A_t^{\text{GOA}}(z_{t+1}) \zeta(z_t)}{\alpha_t}.\end{aligned}\quad (41)$$

where $A_t^{\text{GOA}}(z_{t+1})$ is a new quantity derived and introduced in this work as Generalized Option Advantage (GOA), which is a term that appears in evaluating $U(z_t)$ and $W(z_t, z_{t+1})$.

Rewriting the policy gradient objective in Equation (38) with the policy gradient weighting,

$$\nabla_{\Theta} J[\pi_{\Theta}] = \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{T-1} \sum_{z_t, z_{t+1}} \frac{A_t^{\text{GOA}}(z_{t+1}) \zeta(z_t)}{\alpha_t} \nabla_{\Theta} p_{\Theta}(a_t, z_{t+1} | s_t, z_t) \right].\quad (42)$$

5.3 Learning objective for option-specific policies and values

The training objective given in Equation (42) is modified so that it could be optimized with PPO. Unlike in Section 4.2, the training objective is written in terms of ∇p_Θ and not $\nabla \log p_\Theta$. Therefore, the clipping constraints are applied to p_Θ directly, limiting it to the range of $(1 - \epsilon)p_{\Theta_{\text{old}}}$ and $(1 + \epsilon)p_{\Theta_{\text{old}}}$. The resulting PPO objective is:

$$J_\Theta = \mathbb{E}_{s_t, a_t \sim \pi} \sum_{z_t, z_{t+1}} \left[\frac{\zeta(z_t)}{\alpha_t} \min \left(\pi_\Theta(a_t, z_{t+1} | s_t, z_t) A_t^{\text{GOA}}(z_{t+1}), \right. \right. \\ \left. \left. \text{clip} \left(\pi_\Theta(a_t, z_{t+1} | s_t, z_t), (1 - \epsilon)\pi_{\Theta_{\text{old}}}(a_t, z_{t+1} | s_t, z_t), (1 + \epsilon)\pi_{\Theta_{\text{old}}}(a_t, z_{t+1} | s_t, z_t) \right) A_t^{\text{GOA}}(z_{t+1}) \right) \right]. \quad (43)$$

The option-specific value function $V_\phi^\pi(s_t, z_t)$ parameterized by ϕ can be learned by regressing towards the target values $V_{\text{target}}(s_t, z_t)$ evaluated in Equation (37) for each state s_t and option z_t sampled from the policy and option-forward probability, respectively. Defining the objective function for the value regression as J_ϕ ,

$$J_\phi = - \mathbb{E}_{s_t \sim \pi, z_t \sim \zeta} [V_{\text{target}}(s_t, z_t) - V_\phi^\pi(s_t, z_t)]^2. \quad (44)$$

The final training objective is to maximize the following:

$$J_{\text{SOAP}} = J_\Theta + J_\phi. \quad (45)$$

6 Experiments

Experiments were conducted on a variety of RL agents: PPO, PPOC, Proximal Policy Optimization with Long Short-Term Memory (PPO-LSTM), PPOEM (ours), and SOAP (ours). PPO (Schulman et al., 2017) is a baseline without memory, PPOC (Klissarov et al., 2017) implements the Option-Critic algorithm, PPO-LSTM implements a recurrent policy with latent states using an LSTM (Hochreiter & Schmidhuber, 1997), PPOEM is the algorithm developed in the first half of this work that optimizes the expected returns using the forward-backward algorithm, and SOAP is the final algorithm proposed in this work that uses an option advantage derived by analytically evaluating the temporal propagation of the option policy gradients. SOAP mitigates the deficiency of PPOEM that the training objective optimizes the option assignments over a full trajectory which is typically only available in hindsight; SOAP optimizes the option assignments given only the history of the trajectory instead, making the optimization objective better aligned with the task objective.

The aim is to (a) show and compare the option learning capability of the newly developed algorithms, and (b) assess the stability of the algorithms on standard RL environments. All algorithms use PPO as the base policy optimizer, and share the same backbone and hyperparameters, making it a fair comparison. All algorithms use Stable Baselines 3 (Raffin et al., 2021) as a base implementation with the recommended tuned hyperparameters for each environment. In the following experiments, the number of options was set to 4.

6.1 Option learning in corridor environments

A simple environment of a corridor with a fork at the end is designed as a minimalistic and concrete example where making effective use of latent variables to retain information over a sequence is necessary to achieve the agent’s goal.

Figure 4 describes the corridor environment, in which the agent has to determine whether the rewarding cell (colored yellow) is at the top or bottom, based on the color of the cell it has seen at the start (either “blue” or “red”). However, the agent only has access to the color of the current cell, and does not have a bird’s-eye-view of the environment. Hence, the agent must retain the information of the color of the starting cell in memory, whilst discarding all other information irrelevant to the completion of the task. The agent must learn that the information of the color of the starting cell is important to task completion in an unsupervised way, just from the reward signals. This makes the task challenging, as only in hindsight (after reaching the far end of the corridor) is it clear that this information is useful to retain in memory, but if this information was not written in memory in the first place then credit assignment becomes infeasible. The learned options can be interpreted as “move up at the end of the corridor” and “move down at the end”.

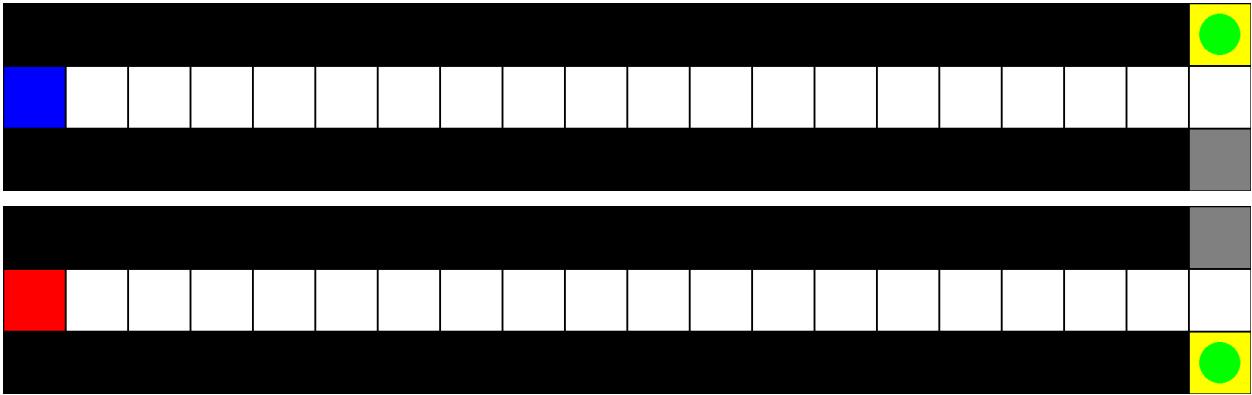


Figure 4: A corridor environment. The above example has a length $L = 20$. The agent represented as a green circle starts at the left end of the corridor, and moves towards the right. When it reaches the right end, the agent can either take an up action or a down action. This will either take the agent to a yellow cell or a grey cell. The yellow cell gives a reward of 1, while the grey cell gives a reward of -1 . All other cells give a reward of 0. The location of a rewarding yellow cell and the penalizing grey cell are determined by the color of the starting cell (either "blue" or "red"), as shown, and this is randomized, each with 50% probability. The agent only has access to the color of the current cell as observation. For simplicity of implementation, the agent's action space is {"up", "down"}, and apart from the fork at the right end, taking either of the actions at each time step will move the agent one cell to the right. The images shown are taken from rollouts of the SOAP agent after training for $100k$ steps. The agent successfully navigated to the rewarding cell in both cases.

The length of the corridor L can be varied to adjust the difficulty of the task. It is increasingly challenging to retain the information of the starting cell color with longer corridors. In theory, this environment can be solved by techniques such as frame stacking, where the entire history of the agent observations is provided to the policy. However, the computational complexity of this approach scales proportionally to corridor length L , which makes this approach unscalable.

Algorithms with options present an alternative solution, where in theory, the options can be used as latent variables to carry the information relevant to the task. In this experiment, PPOC, PPO-LSTM, PPOEM and SOAP are compared against a standard PPO algorithm. The results are shown for corridors with lengths $L = 3$, $L = 10$ and $L = 20$. Due to the increasing level of difficulty of the task, the agents are trained with $8k$, $40k$ and $100k$ time steps of environment interaction, respectively.

The results are shown in Figure 5. As expected, the vanilla PPO agent does not have any memory components so it learned a policy that takes one action deterministically regardless of the color of the first cell. Since the location of the rewarding cell is randomized, this results in an expected return of 0.

With PPOC that implements the Option-Critic architecture, while the options should in theory be able to retain information from the past, it could be observed that the training objective was not sufficient to learn a useful option assignment to complete the task. PPOEM and SOAP, on the other hand, were able to learn to select a different option for a different starting cell color. From Figure 5, it could be seen that the two algorithms had identical performance for a short corridor, but as the corridor length L increased, the performance of PPOEM deteriorated, while SOAP was able to reliably find a correct option assignment, albeit with more training steps.

There are two major differences between PPOC and the proposed algorithms (PPOEM and SOAP) which could be contributing to their significant differences in performance. Firstly, while the option transition function in PPOEM and SOAP are in the form of $\pi_\phi(z_{t+1}|s_t, a_t, z_t)$, which allows the assignment of the new option to be conditional on the current option, the option transition in the Option-Critic architecture is decoupled into an option termination probability $\varpi(s_t, z_{t-1})$, and an unconditional inter-option policy $\pi(z_t|s_t)$. This means that whenever the previous option z_{t-1} is terminated with probability $\varpi(s_t, z_{t-1})$, the choice of the new option z_t will be uninformed of the previous option z_{t-1} , whereas in PPOEM and SOAP the probability of the next option z_{t+1} is conditional on the previous option z_t . Another difference is that, in PPOC a new option is sampled at every time step, but the complete

option forward distribution given the history is not available as a probability distribution. In contrast, in PPOEM and SOAP this is available as $\zeta(z_t) := p(z_t|s_{0:t}, a_{0:t-1})$. Evaluating expectations over distributions gives a more robust estimate of the objective function compared to taking a Monte Carlo estimate of the expectations with the sampled options, which is another explanation of why PPOEM and SOAP were able to learn better option assignments than PPOC.

SOAP’s training objective maximizes the expectation of returns taken over an option probability conditioned only on the agent’s past history, whereas PPOEM’s objective assumes a fully known trajectory to be able to evaluate the option assignment probability. Since option assignments have to be determined online during rollouts, the training objective of SOAP better reflects the task objective. This explains its more reliable performance for longer sequences.

PPO-LSTM achieved competitive performance in a corridor with $L = 3$, demonstrating the capability of latent states to retain past information, but its performance quickly deteriorated for longer corridors. It could be hypothesized that this is because the latent state space of the recurrent policies is not well constrained, unlike options that take discrete values. Learning a correct the value function $V(s, z)$ requires revisiting the same state-latent pair. It is conceivable that with longer sequence lengths during inference time, the latent state will fall within a region that has not been trained well due to compounding noise, leading to an inaccurate estimate of the values and sub-policy.

6.2 Stability of the algorithms on CartPole, LunarLander, Atari, and MuJoCo environments

Experiments were also conducted on standard RL environments to evaluate the stability of the algorithms with options. Results for CartPole-v1 and LunarLander-v2 are shown in Figure 6, and results on 10 Atari environments (Bellemare et al., 2013) and 6 MuJoCo environments (Todorov et al., 2012) are shown in Figure 7 and Figure 8, respectively. There was no significant difference in performances amongst the algorithms for simpler environments like CartPole and LunarLander, with PPO-LSTM slightly outperforming others. For the Atari and MuJoCo environments, however, there was a consistent trend that SOAP achieves similar performances (slightly better in some cases, slightly worse in others) to the vanilla PPO, while PPOEM, PPO-LSTM and PPOC were significantly less stable to train. It could be hypothesized that, similarly to Section 6.1, the policy of PPOC disregarded the information of past options when choosing the next option, which is why the performance was unstable with larger environments. Another point of consideration is that, with N number of options, there are N number of sub-policies to train, which becomes increasingly computationally expensive and requires many visits to the state-option pair in the training data, especially when using a Monte Carlo estimate by sampling the next option as is done in PPOC instead of maintaining a distribution of the option $\zeta(z_t)$ as in PPOEM and SOAP. As for PPO-LSTM, similar reasoning as in Section 6.1 suggests that with complex environments with a variety of trajectories that can be taken through the state space, the latent states that could be visited increases combinatorially, making it challenging to learn a robust sub-policy and value functions.

7 Conclusion

Two competing algorithms, PPOEM and SOAP, are proposed to solve the problem of option discovery and assignments in an unsupervised way. PPOEM implements a training objective of maximizing the expected returns using the EM algorithm, while SOAP analytically evaluates the policy gradient of the option policy to derive an option advantage function that facilitates temporal propagation of the policy gradients. These approaches have an advantage over Option-Critic architecture in that (a) the option distribution is analytically evaluated rather than sampled, and (b) the option transitions are fully conditional on the previous option, allowing historical information to propagate forward in time beyond the temporal window provided as observations.

Experiments in POMDP corridor environments designed to require options showed that SOAP is the most robust way of learning option assignments that adhere to the task objective. SOAP also maintained its performance when solving MDP tasks without the need for options (e.g. Atari with frame-stacking), whereas PPOC, PPO-LSTM and PPOEM were unstable when solving these problems.

8 Future Work

SOAP demonstrated capabilities of learning options in a POMDP environment of corridors, and showed equivalent performances to the baseline PPO agent in other environments. However, even in simple settings, it took the agent

many samples before a correct option assignment was learned. Option discovery is a difficult chicken and an egg problem, since options need to be assigned correctly in order for the rewards to be obtained and passed onto the options, but without the rewards a correct option assignment may not be learned. Furthermore, learning to segment episodes into options in an unsupervised way without any pre-training is an ill-defined problem, since there could be many equally valid solutions. Combining the learning objective of SOAP with methods such as curriculum learning to pre-train diverse sub-policies specialized to different tasks may stabilize training.

In the current formulation of SOAP, the options are discrete variables, and are less expressive compared to latent variables in recurrent policies and transformers. This greatly reduces the memory capacity and could hinder learning in POMDP environments. Further research in extending the derivations of SOAP to work with continuous or multi-discrete variables as latents may lead to making the method scalable to more complex problems.

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A Appendix

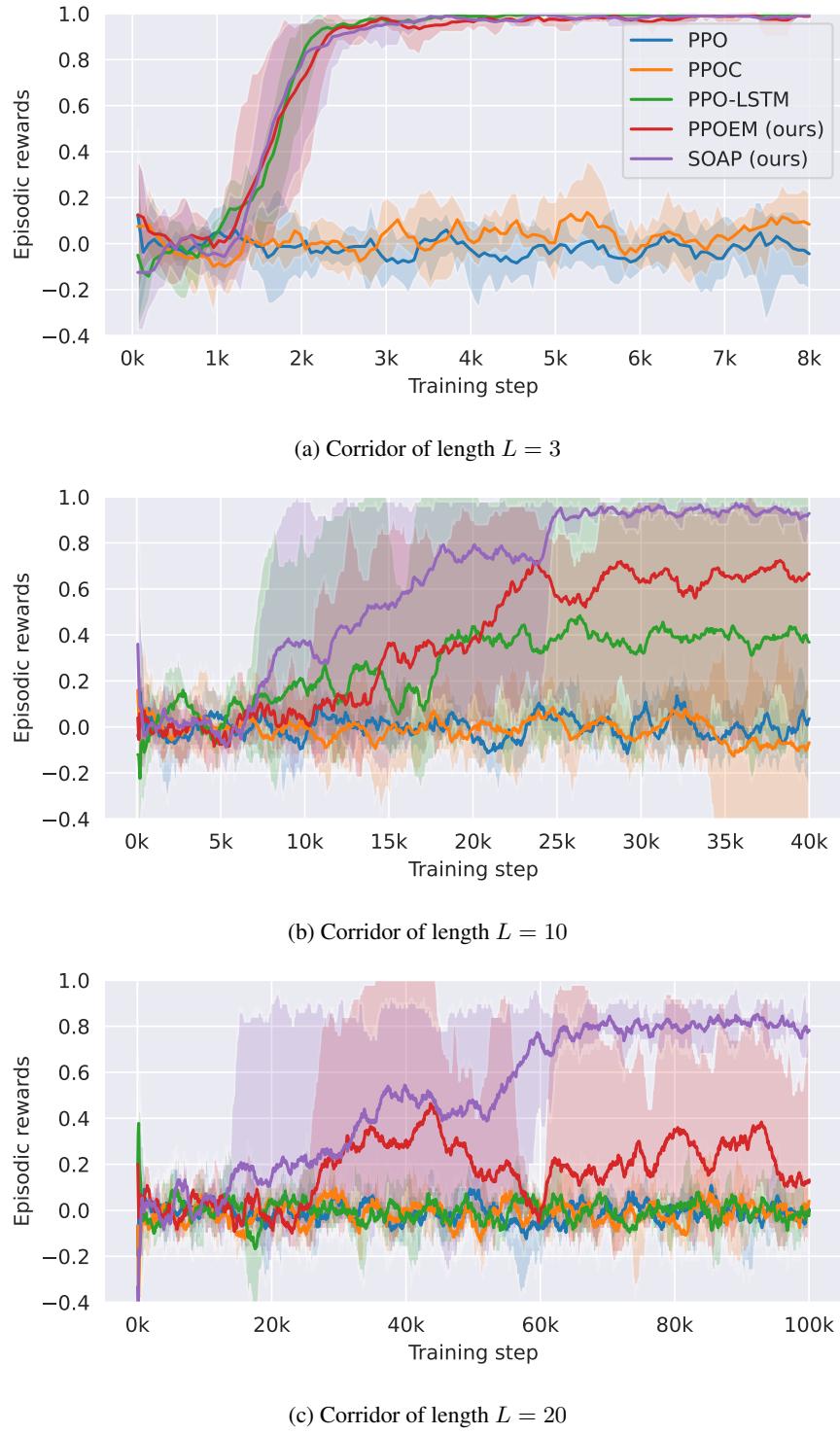


Figure 5: Training curves of RL agents showing the episodic rewards obtained in the corridor environment with varying lengths. The mean (solid line) and the min-max range (colored shadow) for 5 seeds per algorithm are shown.

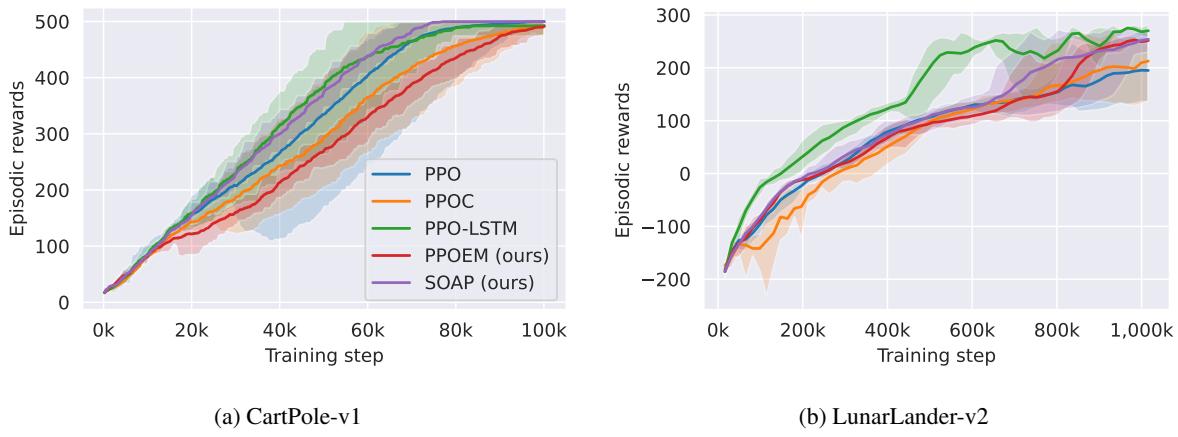


Figure 6: Training curves of RL agents showing the episodic rewards obtained in the CartPole-v1 and LunarLander-v2 environments. The mean (solid line) and the min-max range (colored shadow) for 5 seeds per algorithm are shown.

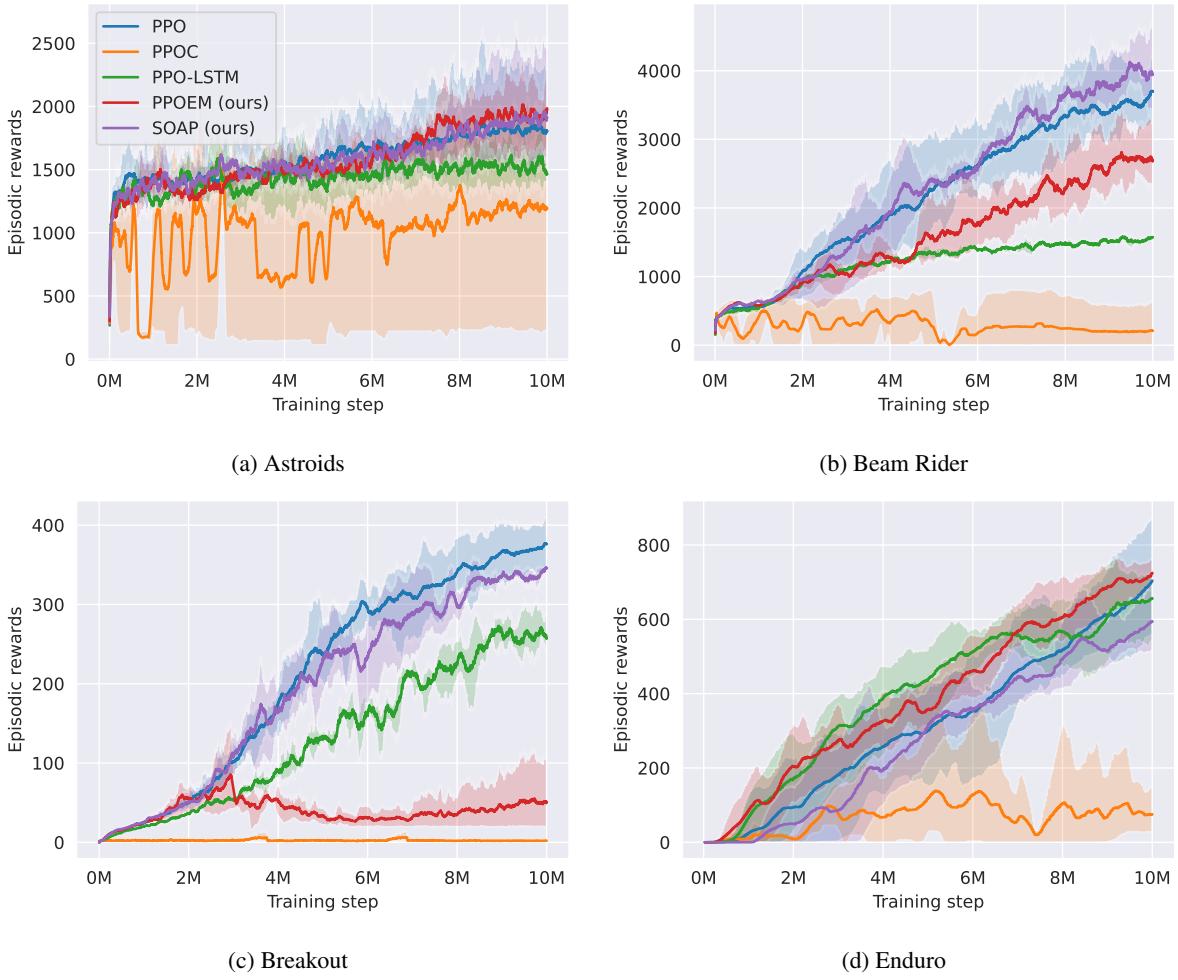
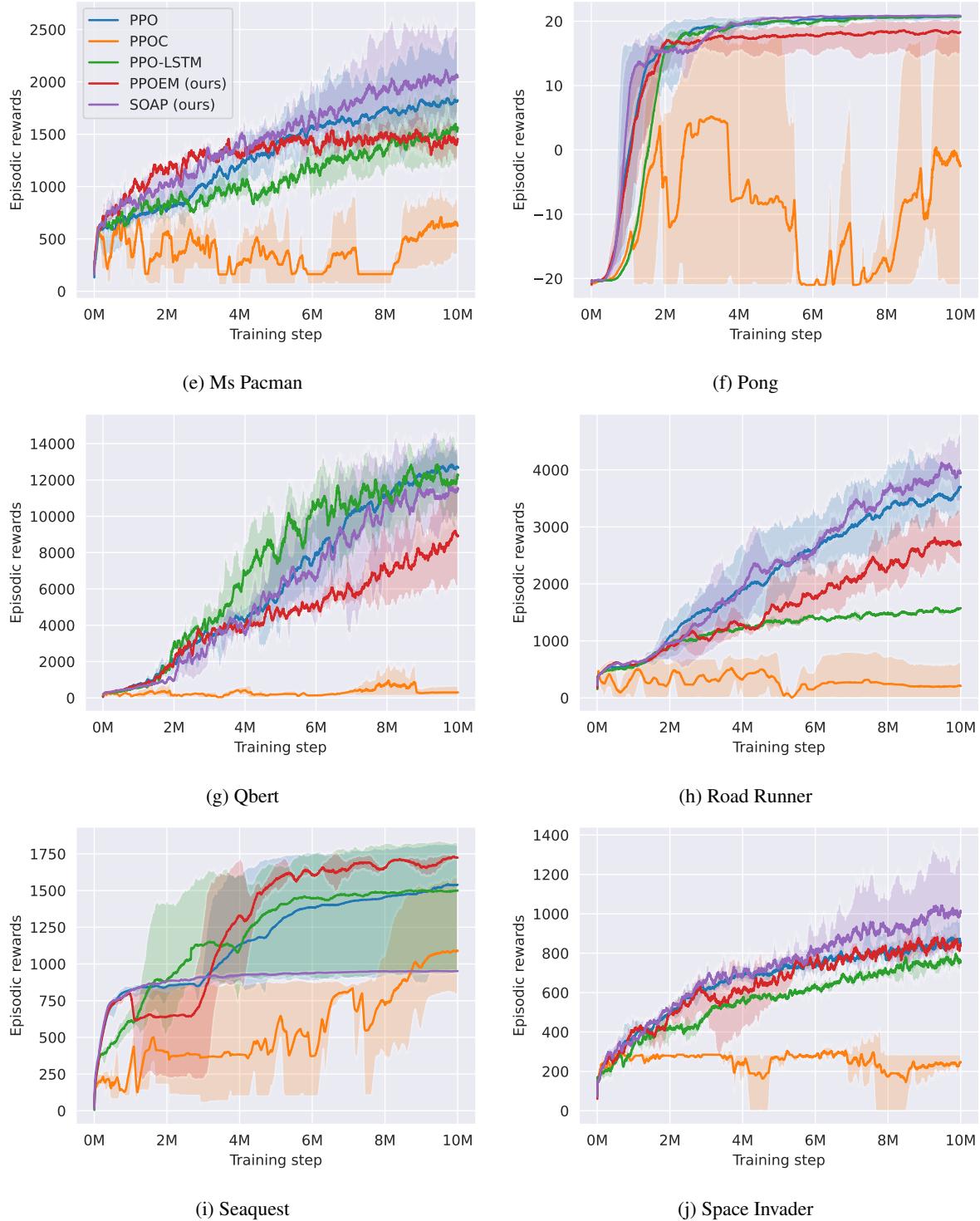


Figure 7: Training curves of RL agents showing the episodic rewards obtained in the Atari environments. The mean (solid line) and the min-max range (colored shadow) for 3 seeds per algorithm are shown. [Spans multiple pages]



[Continued] Training curves of RL agents showing the episodic rewards obtained in the Atari environments. The mean (solid line) and the min-max range (colored shadow) for 3 seeds per algorithm are shown.

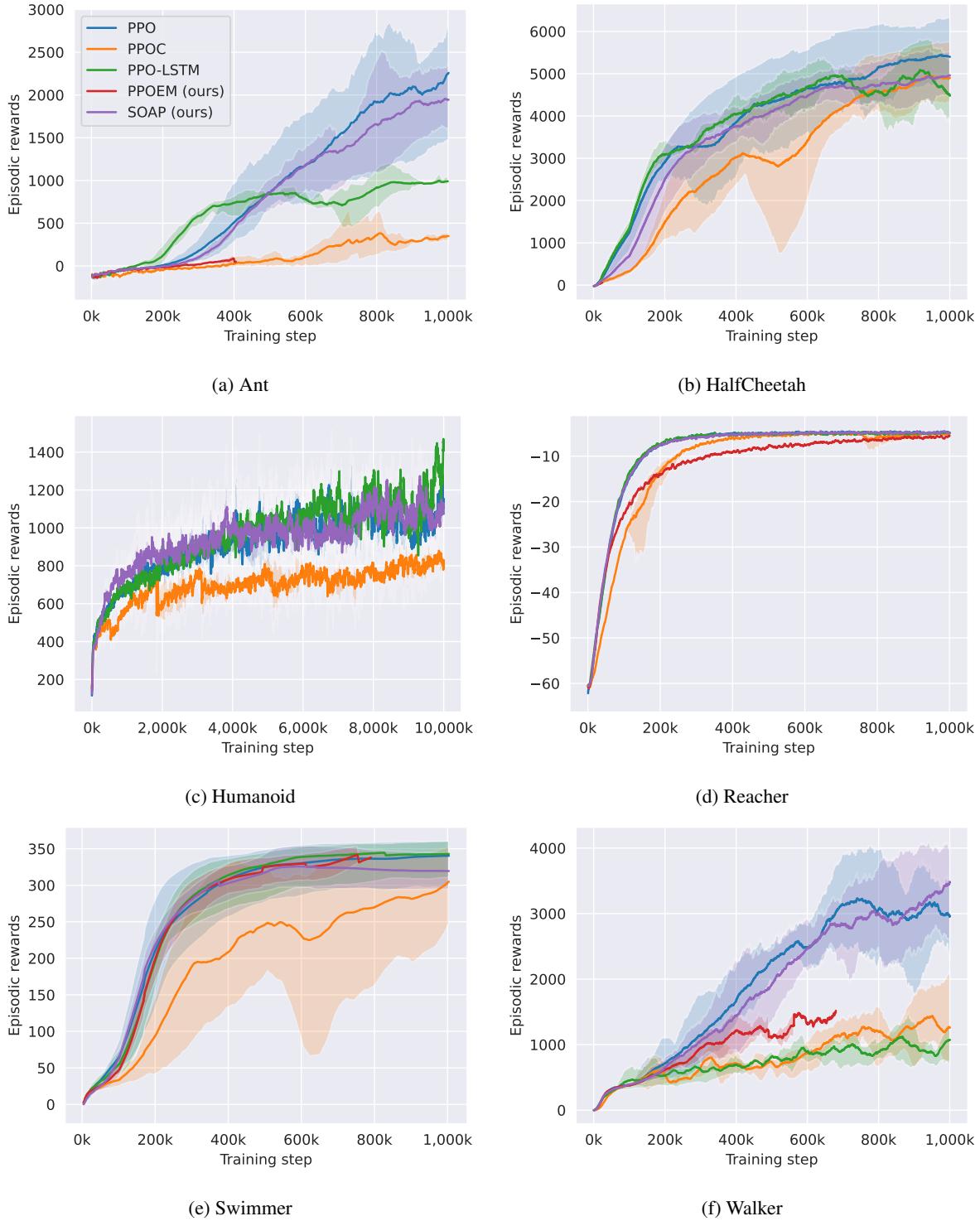


Figure 8: Training curves of RL agents showing the episodic rewards obtained in the MuJoCo environments. The mean (solid line) and the min-max range (colored shadow) for 3 seeds per algorithm are shown. Note that the PPOEM algorithm failed mid-way in some cases due to training instabilities.