1

Monday, January 17, 2022 8:53 PM

Given $u : [0,1] \to \mathbb{R}$ continuous

$$(1+x^2)\,u'' + x u' + x^2 u = 0$$

$$\left.\begin{aligned} u'(1) - 3u(1) &= 0 \\ u(0) &= 1 \end{aligned}\right\} \text{B.C}$$

Find weak form of Problem with

$$\mathcal{S} = \{ w : [0,1] \to \mathbb{R} \text{ smooth} \mid w(0) = 1 \}$$

$$\mathcal{V} = \{ w : [0,1] \to \mathbb{R} \text{ smooth} \mid w(0) = 0 \}$$

Residual equation :

$$(1+x^2)\,u'' + x u' + x^2 u = 0$$

let $v(x) \in \mathcal{V}$ be the test function &

integrate over $[0,1]$

$$\int_0^1 \left[(1+x^2)\,u'' v + x u' v + x^2 u v \right] dx = 0$$

$$\Rightarrow \int_0^1 \left[\underbrace{(1+x^2)\,u'' + 2x u'}_{\left((1+x^2)\,u'\right)'} - x u' + x^2 u \right] v \, dx = 0$$

$\Rightarrow$ Integrate by parts $\left((1+x^2)\,u'\right)' v$

$$(k(x)\,u')' v = (k u')(v)\Big|_0^1 - (k u') v'$$

$$\Rightarrow \int_0^1 \left[-(1+x^2)\,u' v' - x u' v + x^2 u v \right] dx + (1+1^2)\,u'(1)\,v(1) - (1+0^2)\,u'(0)\,v(0) = 0$$

As $u \in \mathcal{S}$ & $v \in \mathcal{V}$ $v(0) = 0$ and from B.C of strong form we have $u'(1) = 3u(1)$

strong form

$$\Rightarrow \int_0^1 \left[-(1+x^2)\,u'v' - x u' v + x^2 u v\right] dx$$

$$+\ 6\,u(1)\,v(1) = 0$$

$\therefore$ The weak form can be expressed as

$$\int_0^1 \left[(1+x^2)\,u'v' + x u' v - x^2 u v\right] dx = \int_0^1 f v\,dx + 6\,u(1)v(1)$$

where f is a zero function i.e

$$f(x) = 0 \quad \forall\, x \in [0,1]$$

$\Rightarrow$ the natural boundary condition is $u'(1) = 3u(1)$

the essential boundary condition is $u(0) = \underline{1}$

From the weak form

$$a(u,v) = \int_0^1 \left[(1+x^2)u'v' + \underline{x u' v} - x^2 u v \right] dx - 6\, u(1)\, v(1)$$

$$a(u+\alpha v, w) = \int_0^1 \left[(1+x^2)(u+\alpha v)' w' + x (u+\alpha v)' w - x^2 (u+\alpha v) v \right] dx - 6\, (u(1)+\alpha v(1)) w(1)$$

$$= \underline{\int_0^1 \left[(1+x^2) w' u' + x w u' - x^2 w u \right] dx - 6\, u(1)\, w(1)}$$

$$+ \alpha \underline{\left[\int_0^1 (1+x^2) w' v' + x w v' - x^2 w v \; dx - 6\, v(1)\, w(1) \right.}$$

$$= \underline{a(u, w)} + \underline{\alpha a(v, w)} \qquad \therefore\; a(u,v) \text{ is bilinear}$$

here $a(u,v)$ is not symmetric as 2^{nd} term

$$\underline{x u' v \neq x v' u}$$

similarly $l(v) = \int_0^1 f v \, dx$

$$l(u+\alpha v) = \int_0^1 f(u+\alpha v)\, dx = \int_0^1 f u \, dx + \alpha \int_0^1 f v \, dx$$

$$= l(u) + \alpha\, l(v)$$

$\therefore$ this is linear form of $l(v) = \int_0^1 f v \, dx$

where f is zero function

PART → a

a) We have subspaces.

$\mathcal{S} = \{ w : [0,1] \to \mathbb{R} \text{ smooth } \mid w(0) = 1 \}$

$\mathcal{V} = \{ w : [0,1] \to \mathbb{R} \text{ smooth } \mid w(0) = 0 \}$

& let us define

$\mathcal{W} = \{ w : [0,1] \to \mathbb{R} \text{ smooth enough} \}$

such that $\mathcal{S} \subset \mathcal{W}$ & $\mathcal{V} \subset \mathcal{W}$ and

$\mathcal{S}$ is affine of $\mathcal{W}$ & $\mathcal{V}$ is same direction of $\mathcal{S}$

Give the span of $\mathcal{W}_h$

$\mathcal{W}_h = \text{span}\{1, x, x^2, x^3\}$

$w_h = 1 \cdot w_0 + x w_1 + x^2 w_2 + x^3 w_3$

To find spaces $\mathcal{V}_h$ & $\mathcal{S}_h$

$\mathcal{V}_h = \mathcal{W}_h \cap \mathcal{V}$ so we need $w_h(0) = 0$

$\Rightarrow w_0 + (0) w_1 + (0)^2 w_2 + (0)^3 w_3 = 0$

$\Rightarrow w_0 = 0$

so, $\mathcal{V}_h = \text{span}(x, x^2, x^3) = \{ w_h \in \mathcal{W}_h \mid w_h(0) = 0 \}$

similarly $\mathcal{S}_h = \mathcal{W}_h \cap \mathcal{S}$ are those in $\mathcal{W}_h$ such that

$w_h(0) = 1$

$w_0 = 1$

$\mathcal{S}_h = \{ w_h \in \mathcal{W}_h \mid w_h(0) = 1 \}$

$= \{ w_h = 1 + c_1 x + c_2 x^2 + c_3 x^3 \mid c_1, c_2, c_3 \in \mathbb{R}^3 \}$

$= \{ w_h = 1 + v_h \mid v_h \in \mathcal{V}_h \}$

$\therefore$ $\mathcal{S}_h$ & $\mathcal{V}_h$ are established

---

b) Part - B

Basis functions

$u_h(x) = \sum_{b=1}^{m} u_b N_b(x)$ & $v_h(x) = \sum_{a=1}^{m} v_a N_a(x)$

where $m = 4$ for $\mathcal{W}_h$, $n = 3$ for $\mathcal{V}_h$ → 1 to 3 forms basis for $\mathcal{V}_h$

$N_1(x) = x$, $N_2(x) = x^2$, $N_3(x) = x^3$ & $N_4(x) = 1$

here we need to find function u_h such that

$u_h = c_1 N_1(x) + c_2(N_2(x)) + c_3 N_3(x) + c_4 N_4(x)$

and the equations we have are

$a(u_h, N_1) = l(N_1)$

$a(u_h, N_2) = l(N_2)$

$$a(u_h, N_3) = l(N_3)$$

× impose boundary condition for $u_h \in S_h$

$$u_4 = 1$$

Now our equation is

$$a(u, v) = \int_0^1 (1+x^2) u' v' + x u' v - x^2 u v \, dx - 6\, u(1) v(1)$$

$$l(v) = \int_0^1 f v \, dx$$

∴ load vector can be represented as

$$F = \begin{bmatrix} l(N_1) \\ l(N_2) \\ l(N_3) \\ u_4 \end{bmatrix} = \begin{bmatrix} \int_0^1 0 \cdot x \, dx \\ \int_0^1 0 \cdot x^2 \, dx \\ \int_0^1 0 \cdot x^3 \, dx \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

stiffness matrix is

$$K = \begin{bmatrix} a(N_1, N_1) & a(N_2, N_1) & a(N_3, N_1) & a(N_4, N_1) \\ a(N_1, N_2) & a(N_2, N_2) & a(N_3, N_2) & a(N_4, N_2) \\ a(N_1, N_3) & a(N_2, N_3) & a(N_3, N_3) & a(N_4, N_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \int_0^1 1 + 2x^2 - x^4 \, dx - 6 & \int_0^1 2x + 4x^3 - x^5 \, dx - 6 & \int_0^1 3x^2 + 6x^3 - x^6 \, dx - 6 & \int_0^1 -x^3 \, dx - 6 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

integration using Matlab

$$K = \begin{bmatrix} -68/15 & -25/6 & -138/35 & -25/4 \\ -53/12 & -379/105 & -25/8 & -31/5 \\ -152/35 & -79/24 & -818/315 & -37/6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴ values of K and F are found

c) Solution to Galerkin Method

$$KU = F$$

$$\therefore U = K^{-1} F$$

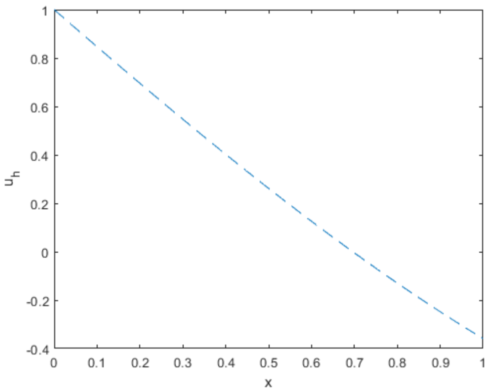
$$KU = F$$

$$U = K^{-1} F$$

where $U \rightarrow$ has coefficient of $u_h(x)$ function

$$U = \begin{bmatrix} -1.543 \\ 0.081 \\ 0.103 \\ 1 \end{bmatrix} \Rightarrow u_h = -1.543x + 0.081x^2 + 0.103x^3 + 1$$

Plot is



We have

$$u_h = 1 - 1.543x + 0.081x^2 + 0.103x^3$$

$$u_h' = -1.543 + 0.162x + 0.309x^2$$

$$u_h(1) = -1.072$$

$$u_h'(1) = -6.359 \Rightarrow 3u_h'(1) = -1.077$$

∴ It is not exactly equal as we made some numerical approximation to 3rd decimal place.

$$u_h(1) - 3u_h'(1) = 0.005$$

So the natural B.C is satisfied by our solution to Galerkin.

a(u, v)

21 January 2022 16:02

```
function [outcome] = alFunct(u, v, x)
    outcome = int((1 + x^2)*diff(u)*diff(v) + x*v*diff(u) - u*v*(x^2), x, 0, 1) - 6;
    return
end
```

```
Not enough input arguments.

Error in alFunct (line 2)
    outcome = int((1 + x^2)*diff(u)*diff(v) + x*v*diff(u) - u*v*(x^2), x, 0, 1) - 6;
```

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a(1, v)

21 January 2022 16:02

```
function [outcome] = nulFunct(v, x)
    outcome = int(- v*(x^2), x, 0, 1) - 6;
    return
end
```

```
Not enough input arguments.

Error in nulFunct (line 2)
    outcome = int(- v*(x^2), x, 0, 1) - 6;
```

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K matrix

21 January 2022 16:03

```
function [K] = stiffKmat(x)
    K = zeros(4, 4);
    %K(1,1) = alFunct(x,x,x);
    %K(1,2) = alFunct(x^2, x, x);
    %K(1,3) = alFunct(x^3,x,x);
    %K(1,4) = alFunct(1,x,x);
    K
    K = [alFunct(x,x,x) alFunct(x^2,x,x) alFunct(x^3,x,x) nulFunct(x,x);
        alFunct(x, x^2, x) alFunct(x^2, x^2, x) alFunct(x^3, x^2, x) nulFunct(x^2, x);
        alFunct(x, x^3, x) alFunct(x^2, x^3, x) alFunct(x^3, x^3, x) nulFunct(x^3, x);
        0 0 0 1];
    return
end
```

```
K =

     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

```
Not enough input arguments.

Error in stiffKmat (line 8)
    K = [alFunct(x,x,x) alFunct(x^2,x,x) alFunct(x^3,x,x) nulFunct(x,x);
```

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1

Wednesday, January 19, 2022 5:43 PM

Given weak form

$f : [0, 1] \to R$

$u \in \mathcal{V} = \{ w : [0,1] \to R \text{ smooth} \mid w(0) = 0 \}$

$$\int_0^1 u'(x)\, v'(x)\, dx = \int_0^1 f(x)\, v(x)\, dx$$

for any $v \in \mathcal{V}$

$$\Rightarrow \int_0^1 \underline{u'(x)\, v'(x)} - f(x)\, v(x)\, dx = 0$$

by parts for this term

$$\Rightarrow \int_0^1 - u''(x)\, v(x) - f(x)\, v(x)\, dx + u'(1)v(1) - u'(0)v(0) = 0$$

here we know $v(0) = 0$

from the simple argument every term multiplying to test function $v(x)$ should be zero as value of $v(x)$ can be chosen arbitrarily

$$-u''(x) - f(x) = 0 \qquad x \in [0, 1]$$

$$u'(1) = 0$$

we have $u(x) = -\dfrac{\sin^2(2\pi x)}{8\pi^2} \propto$ → Find if true

$f(x) = \cos(4\pi x)$

$$u'(x) = -\frac{2 \sin(2\pi x) \cdot 2\pi \cos(2\pi x)}{8\pi^2} \Rightarrow 2\sin(2\pi x)\cos(2\pi x) = \sin 4\pi x$$

$$= -\frac{1}{4\pi} \sin(4\pi x)$$

$$= -\frac{1}{4\pi} \sin(4\pi x)$$

$\Rightarrow u'(1) = \frac{-1}{4\pi} \sin(4\pi \cdot 1) \Rightarrow$ as $\sin(4\pi) = 0$

$= 0$ $\quad \therefore$ boundary is satisfied

$u''(x) = \frac{-1}{4\pi} \times 4\pi \cos 4\pi x = -\cos 4\pi x$

$\Rightarrow$ ~~F~~From the strong form $-u''(x) - f(x) = 0$

$\Rightarrow$ as $f(x) = \cos 4\pi x$

$$-(-\cos 4\pi x) - \cos 4\pi x = 0$$

$\therefore$ strong form is satisfied when

$$u(x) = -\frac{(\sin(2\pi x))^2}{8\pi^2}$$

hence this is an exact solution.

$$\begin{bmatrix} a(N_0,N_0) & & \\ & \ddots & \\ & & \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} \int_0^1 \cos(4\pi x) \times iP_0(x)\, dx \\ iP_1(x) \\ iP_2(x) \\ iP_3(x) \end{bmatrix}$$

We know $\int_0^1 P_n(x)\, P_m(x)\, dx = \delta_{mn}$

from the weak form

$$\int_0^1 u'(x)\, v'(x)\, dx = \int_0^1 f(x)\, v(x)\, dx$$

here $\mathcal{S} = \mathcal{V}$, hence our system of equations will be, i.e our basis functions

$$\mathcal{W}_h = \left\{ w = \sum_{k=0}^{n} c_k\, iP_k(x) \;\middle|\; (c_0, \dots, c_n) \in \mathbb{R}^{n+1} \right\}$$

which implies functions

$iP_0(x)$, $iP_1(x)$ ..., $iP_n(x)$

are in $\mathcal{S}_h$ & $\mathcal{V}_h$

$$\mathcal{W}_h = \text{span}\left(\bigcup_{k=0}^{n} iP_k(x) \right)$$

$$\mathcal{V}_h = \mathcal{W}_h \cap \mathcal{V} = \{ w_h \subset \mathcal{W}_h \mid w_h(0) = 0 \}$$

$$\mathcal{S}_h = \mathcal{W}_h \cap \mathcal{S} = \{ w_h \in \mathcal{W}_h \mid w_h(0) = 0 \}$$

$$iP_k(x) = \sqrt{2k+1}\, (-1)^{k+1} \sum_{i=0}^{k} \binom{k}{i} \binom{k+i}{i} \frac{(-x)^{i+1}}{i+1}$$

$$iP_k(0) = 0 \quad \forall \quad k = \{0, 1, \dots n\}$$

so both $\mathcal{V}_h$ & $\mathcal{S}_h$ have same span

$$\mathcal{V}_h = \mathcal{S}_h = \text{span}\left(\bigcup_{k=0}^{n} iP_k(x) \right)$$

we also have $a(u, v) = \int_0^1 u'(x) v'(x)\, dx$

which is symmetric bilinear

$$u_h(x) = \sum^{n} u_b\, N_b(x) \quad \&$$

$u_h(x) = \sum_{b=1}$

$v_h(x) = \sum_{a=1}^{n} u_a N_a(x)$ where

N_a & N_b are $i\,P(x)$ & functions

$$a(N_b, N_a) = \int_0^1 N_b'(x) \; N_a'(x) \; dx$$

$$= \int_0^1 i P_b'(x) \; i P_a'(x) \; dx$$

we know $i P_b'(x) = P_b(x)$ & similar for a

$$= \int_0^1 P_b(x) \, P_a(x) \, dx$$

from orthogonality theorm

$$\int_0^1 P_b(x) \; P_a(x) \; dx = \delta_{ab} \Rightarrow \begin{cases} 1 & a=b \\ 0 & a \neq b \end{cases}$$

$$\therefore \quad a(N_b, N_a) = \begin{cases} 1 & a=b \\ 0 & a \neq b \end{cases}$$

(a) given to compute ${}^1u_h$, ${}^3u_h$, ${}^5u_h$, ${}^7u_h$

i.e for ${}^1u_h$

$$K = \begin{bmatrix} K_{00} & k_{01} \\ k_{10} & k_{11} \end{bmatrix}$$

from orthogonality

$k_{ab} = a(N_b, N_a) = \begin{cases} 1 & a=b \\ 0 & a \neq b \end{cases}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ it is always identity No matter what 'n' we have i.e 1, 3, 5, 7

load vector is

$$l(N_a) = \int_0^1 \cos(4\pi x) \; i P_a(x) \; dx \quad \forall a = \{0, 1, \ldots, n\}$$

this is computed using Matlab

$$KU = F$$

$$KU = F$$

$$U = K^{-1} F \qquad K \to \text{identity} \quad \text{so}$$

$$U = F = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix} \to \text{coefficients in}$$

$$u_n = \sum_{k=0}^{n} C_k \, i P_k(x)$$

$$^1u_n = (0) \times i P_0(x) + (0.0219) \times i P_1(x)$$

$$^3u_n = (0)\, i P_0(x) + (0.0219)\, i P_1(x) + (0)\, i P_2(x) + (0.0208)\, i P_3(x)$$

$$^5u_n = (0)\, i P_0(x) + (0.0219)\, i P_1(x) + (0)\, i P_2(x) + (0.0208)\, i P_3(x) + (0)\, i P_4(x) + (-0.0442)\, i P_5(x)$$

$$^7u_n = (0)\, i P_0(x) + (0.0219)\, i P_1(x) + (0)\, i P_2(x) + (0.0208)\, i P_3(x) + (0)\, i P_4(x) + (-0.0442)\, i P_5(x) + (0)\, i P_6(x) + (0.0169)\, i P_7(x)$$

(b) Yes $\to$ $^n u_n$ is visually converging

Mainly because the analytical solution is ... can be represented

Mainly because the analytical ...

$\sin^2(x)$ and $\sin$ can be represented

as function of (Taylor series)

$$\frac{1}{2}\left[-\sum_{n=1}^{\infty}\frac{(-1)^n(2x)^{2n}}{(2n)!}\right] = c_1 x^2 - c_2 x^4 + \dots$$

and this is convergent.

if we have $(c_1, c_2 \dots c_n) \neq 0$

then our series is valid approximation of $\sin^2(x)$ function.

And in our case as we have all functions of power of x in the series i.e

$$\bigcup_{k=0}^{n} i\, P_k(x) \approx \bigcup_{k=0}^{n} x^{k+1}$$

we will get a valid approximation for $\sin^2(x)$ function.

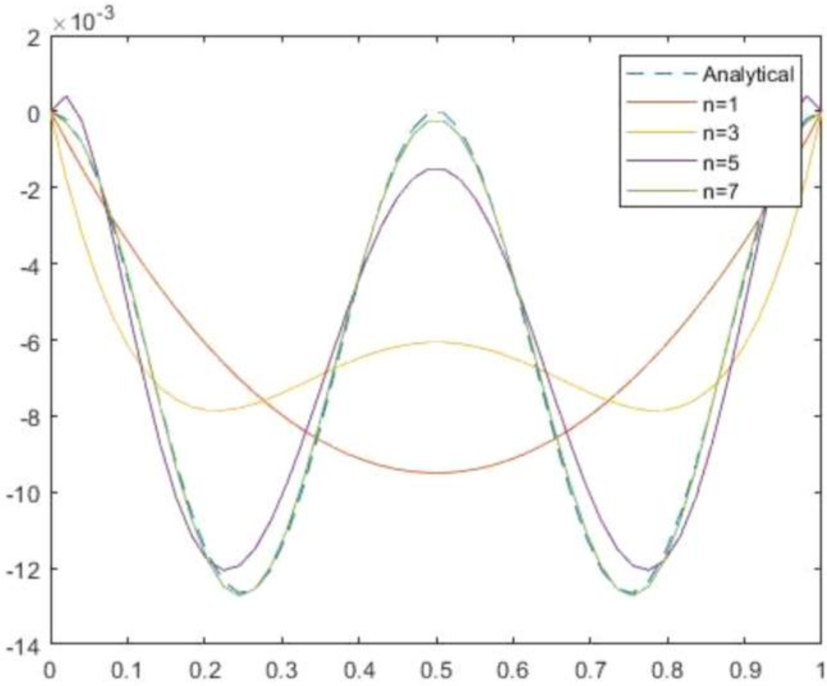
Code Problem 2

21 January 2022 15:51

```
syms x
n_pts = 50;
x_vec = linspace(0, 1, n_pts);
an_sol = - (sin(2*pi*x_vec)/(2*pi)).^2 / 2;
figure
plot(x_vec, an_sol, '--', 'DisplayName','Analytical')
hold on
for ii = 0:3
    k = 2*ii + 1;
    K_mat = zeros([k+1, k+1]);
    F_mat = zeros([k+1, 0]);
    for jj = 0:k
        K_mat(jj+1, jj+1) = int((Legendre(jj, x) * Legendre(jj, x)), 0, 1);
        F_mat(jj+1, 1) = integral_F(jj, x);
    end
    disp(F_mat)
    disp(K_mat)
    func_mat = zeros([n_pts, 1]);
    for jj = 1:n_pts
        for kk = 0:k
            func_mat(jj, 1) = func_mat(jj, 1) + F_mat(kk+1, 1)*iLegendre(kk, x_vec(jj));
        end
    end
    plot(x_vec,func_mat, 'DisplayName',strcat('n=',num2str(k)))
    hold on
end
hold off
legend
```

```
         0
    0.0219

     1     0
     0     1

         0
    0.0219
         0
    0.0208

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1

         0
    0.0219
         0
    0.0208
         0
   -0.0442

     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
     0     0     0     1     0     0
     0     0     0     0     1     0
     0     0     0     0     0     1

         0
    0.0219
         0
```

```
  0.0208
       0
 -0.0442
       0
  0.0169

   1     0     0     0     0     0     0     0
   0     1     0     0     0     0     0     0
   0     0     1     0     0     0     0     0
   0     0     0     1     0     0     0     0
   0     0     0     0     1     0     0     0
   0     0     0     0     0     1     0     0
   0     0     0     0     0     0     1     0
   0     0     0     0     0     0     0     1
```



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Now given $W_h = \left\{ w = \sum_{k=2}^{n} c_k \, {}^iP_k(x) \,\middle|\, (c_0, \dots c_n) \in \mathbb{R}^{n+1} \right\}$

(a) Similar to previous problem

$$\mathcal{V}_h = \mathcal{S}_h$$

K = Identity matrix

and as we start from $k = 2$

${}^3u_h \rightarrow$ will have only 2 basis functions i.e

$${}^3u_h = c_2 \, {}^iP_2(x) + c_3 \, {}^iP_3(x)$$

&

$$l(N_a) = \int_0^1 \cos(4\pi x) \, {}^iP_a(x) \, dx \quad \forall \, a = \{2, 3 \dots n\}$$

Similar to procedure in (2) we get the following solutions for ${}^nu_h$

$${}^3u_h = (0) \, {}^iP_2(x) + (0.0208) \, {}^iP_3(x) = 0.0208 \times \sqrt{7} \sum_{k=0}^{3} \binom{3}{k} \binom{3+k}{k} \frac{(-x)^{k+1}}{k+1}$$

$${}^5u_h = (0) \, {}^iP_2(x) + (0.0208) \, {}^iP_3(x) + (0) \, {}^iP_4(x) + (-0.0442) \, {}^iP_5(x)$$

$$= (0.0208) \sqrt{7} \sum_{k=0}^{3} \binom{3}{k} \binom{3+k}{k} \frac{(-x)^{k+1}}{k+1} - 0.0442 \sqrt{11} \sum_{k=0}^{5} \binom{5}{k} \binom{5+k}{k} \frac{(-x)^{k+1}}{k+1}$$

$${}^7u_h = (0.0208) \, {}^iP_3(x) - (0.0442) \, {}^iP_5(x) + (0.0169) \, {}^iP_7(x)$$

$$= (0.0208) \sqrt{7} \sum_{k=0}^{3} \binom{3}{k} \binom{3+k}{k} \frac{(-x)^{k+1}}{k+1} - 0.0442 \sqrt{11} \sum_{k=0}^{5} \binom{5}{k} \binom{5+k}{k} \frac{(-x)^{k+1}}{k+1}$$

$k+1$

$$= (0.0208)\sqrt{7} \sum_{k=0}^{3} \binom{3}{k}\binom{3+k}{k} \frac{(-x)^{k+1}}{k+1}$$

$$+ \; 0.0169\sqrt{15} \sum_{k=0}^{7} \binom{7}{k}\binom{7+k}{k} \frac{(-x)^{k+1}}{k+1}$$

(b) In this case it will not be convergent as we are always missing the term

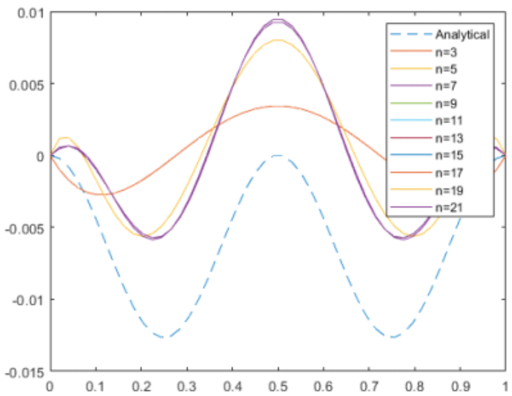
$$(0.0219)\, \varepsilon P_1(x) = (0.0219)\sqrt{3} \sum_{k=0}^{1} \binom{1}{k}\binom{1+k}{k} \frac{(-x)^{k+1}}{k+1}$$

$$= 0.0219\sqrt{3}\left[-x + x^2\right]$$

& we know that legendre polynomials are linearly independent, which makes their integrals linearly independent (As they are polynomial in nature not exponential or logarithmic)

So $iP_1(x)$ can never be recovered from linear combination of $\sum_{k=2}^{n} c_k \, iP_k(x)$

$\Rightarrow$ So This series will not form complete basis for $\sin^2 x$ function & never converge no matter how big of 'n' we take.



Analytical
n=3
n=5
n=7
n=9
n=11
n=13
n=15
n=17
n=19
n=21
0.01
0.005
0
-0.005
-0.01
-0.015
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

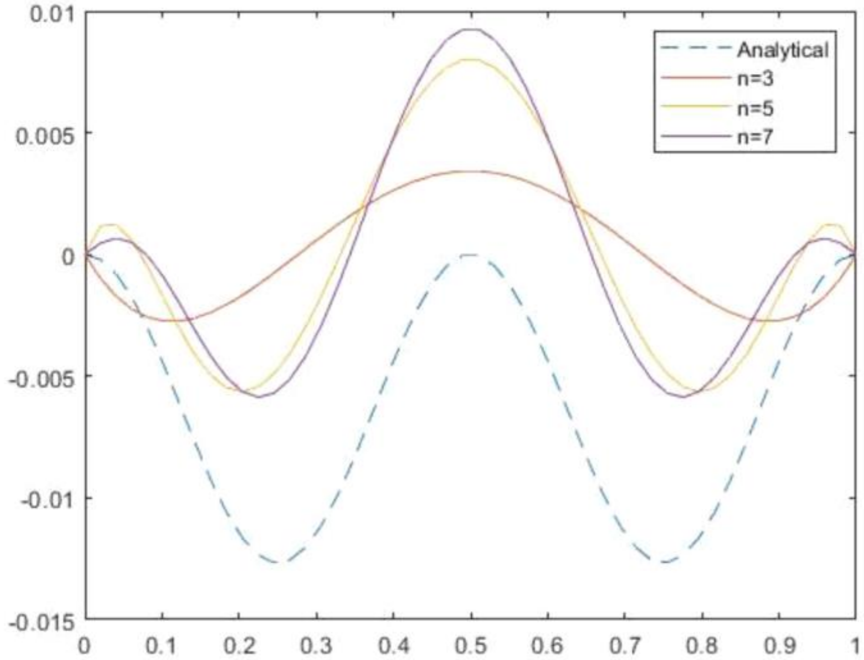
Code Problem 3

21 January 2022 15:50

```
syms x
n_pts = 50;
x_vec = linspace(0, 1, n_pts);
an_sol = - (sin(2*pi*x_vec)/(2*pi)).^2 / 2;
figure
plot(x_vec, an_sol, '--', 'DisplayName','Analytical')
hold on
for ii = 1:3
    k = 2*ii + 1;
    k
    K_mat = zeros([k-1, k-1]);
    F_mat = zeros([k-1, 0]);
    for jj = 1:k-1
        K_mat(jj, jj) = int((Legendre(jj+1, x) * Legendre(jj+1, x)), 0, 1);
        F_mat(jj, 1) = integral_F(jj+1, x);
    end
    disp(F_mat)
    disp(K_mat)
    func_mat = zeros([n_pts, 1]);
    for jj = 1:n_pts
        for kk = 1:k-1

            func_mat(jj, 1) = func_mat(jj, 1) + F_mat(kk, 1)*iLegendre(kk+1, x_vec(jj));
        end
    end
    plot(x_vec,func_mat, 'DisplayName',strcat('n=',num2str(k)))
    hold on
end
hold off
legend
```

```
k =

     3

         0
    0.0208

     1     0
     0     1


k =

     5

         0
    0.0208
         0
   -0.0442

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1


k =

     7

         0
```

```
    0.0208
         0
   -0.0442
         0
    0.0169

     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
     0     0     0     1     0     0
     0     0     0     0     1     0
     0     0     0     0     0     1
```



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Legendre Function

21 January 2022 15:59

```
function [outcome] = Legendre(n, x)
    res = 0;
    coeff = ((-1)^n)*sqrt(2*n + 1);
    for k = 0:n
        res = res + (nchoosek(n,k) * nchoosek(n+k,k) * (-x)^k);
    end
    outcome = res*coeff;
    return
end
```

```
Not enough input arguments.

Error in Legendre (line 3)
    coeff = ((-1)^n)*sqrt(2*n + 1);
```

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Integral Legendre Function

21 January 2022 16:00

```
function [outcome] = iLegendre(n, x)
    res = 0;
    coeff = ((-1)^(n+1))*sqrt(2*n + 1);
    for k = 0:n
        res = res + (nchoosek(n,k) * nchoosek(n+k,k) * (-x)^(k+1))/(k+1);
    end
    outcome = res*coeff;
    return
end
```

```
Not enough input arguments.

Error in iLegendre (line 3)
    coeff = ((-1)^(n+1))*sqrt(2*n + 1);
```

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Load vector

21 January 2022 16:00

```
function [outcome] = integral_F(n, x)
    outcome = int((cos(4*pi*x) * iLegendre(n, x)), 0, 1);
    return
end
```

```
Not enough input arguments.

Error in integral_F (line 2)
    outcome = int((cos(4*pi*x) * iLegendre(n, x)), 0, 1);
```

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