

CP-2.

Strong form:

Find $u \in C^2(\Omega)$ such that:

$$-k \Delta u = f \quad \text{in } \Omega$$

$$u = g_1 \quad \text{on } \Gamma_{g_1}$$

$$u = g_2 \quad \text{on } \Gamma_{g_2}$$

$$k \nabla u \cdot \vec{n} = h_1 \quad \text{on } \Gamma_{h_1}$$

$$k \nabla u \cdot \vec{n} = h_2 \quad \text{on } \Gamma_{h_2}$$

$$k \nabla u \cdot \vec{n} = h_3 \quad \text{on } \Gamma_{h_3}$$

$$k \nabla u \cdot \vec{n} = h_4 \quad \text{on } \Gamma_{h_4}$$

Define

$$S = \{ u : \Omega \rightarrow \mathbb{R} \text{ smooth such that } u(x) = g_1 \text{ for } x \in \Gamma_{g_1}; u(x) = g_2 \text{ for } x \in \Gamma_{g_2} \}.$$

$$V = \{ v : \Omega \rightarrow \mathbb{R} \text{ smooth } | v(x) = 0 \text{ for } x \in \Gamma_{g_1} \cup \Gamma_{g_2} \}.$$

Find $u \in S$ such that:

$$a(u, v) = \ell(v) \quad \text{for all } v \in V$$

where $a(u, v) = k \int_{\Omega} \nabla u \cdot \nabla v \, d\Omega$

$$\ell(v) = \int_{\Omega} f v \, d\Omega + \int_{\partial \Omega_N} H v \, d\Gamma$$

where $\partial \Omega_N = \{ \Gamma_{h_1} \cup \Gamma_{h_2} \cup \Gamma_{h_3} \cup \Gamma_{h_4} \}$

$$H = \begin{cases} h_1 & x \in \Gamma_{h_1} \\ h_2 & x \in \Gamma_{h_2} \\ h_3 & \vdots \\ h_4 & \vdots \end{cases}$$

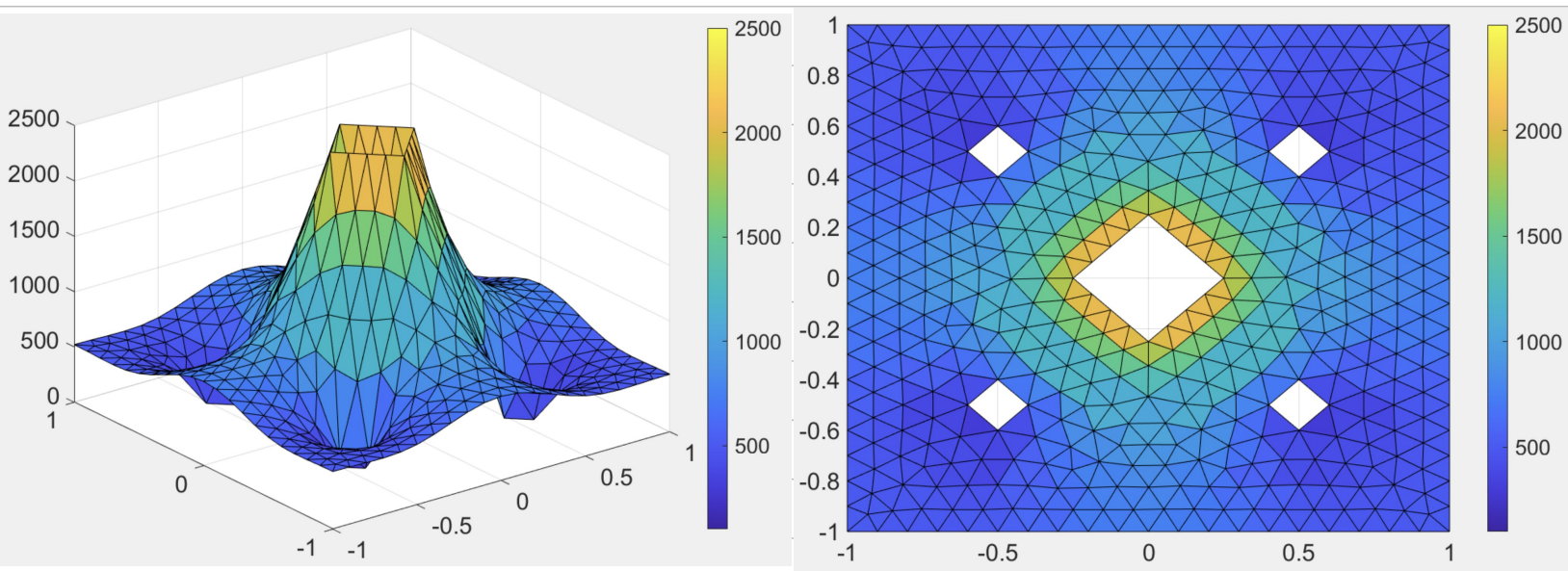
Discretized Form:

Let $\mathcal{W}_n = \text{span}(\{N_1, \dots, N_{e+1}\})$.

$$S_n = \{u_n \in \mathcal{W}_n \mid u_n(x_a^e) = q(x_a^e) \forall x_a^e \in \mathcal{J}_q\}$$

$$\mathcal{V}_n = \{v_n \in \mathcal{W}_n \mid v_n(x_a^e) = 0 \forall x_a^e \in \mathcal{J}_q\}$$

1.



2. & 3

See implementations of function. uValue.

function PI Function.

4. At $(0, 0.9)$,

$$u_h = 769.0345^\circ\text{C}$$

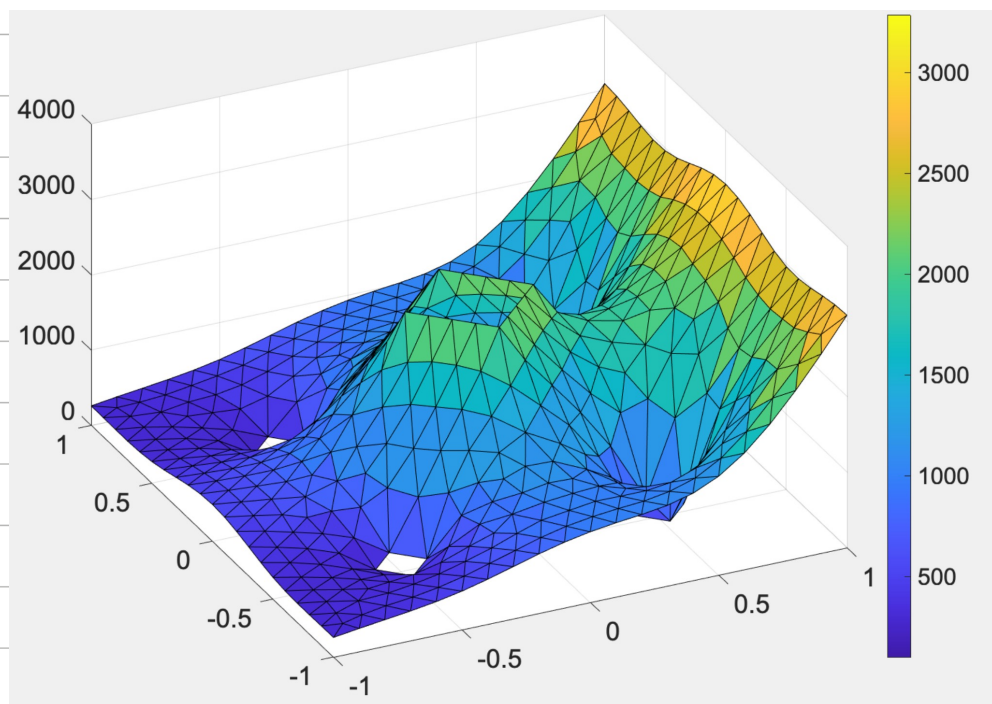
$$\frac{\partial u_h}{\partial x_1} = -2.7517^\circ\text{C}/\text{ft}; \quad \frac{\partial u_h}{\partial x_2} = -340.6147^\circ\text{C}/\text{ft}$$

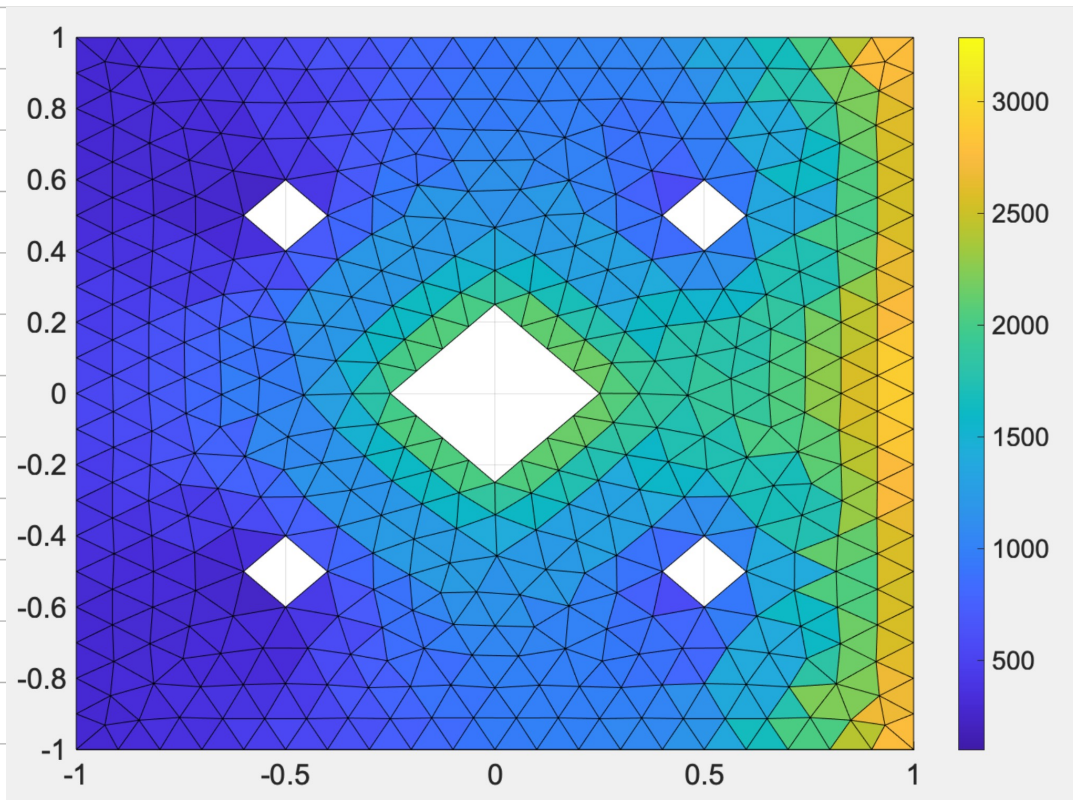
At $(0.25, 0.25)$,

$$u_h = 1.5810e3^\circ\text{C}$$

$$\frac{\partial u_h}{\partial x_1} = -3.4037e3^\circ\text{C}/\text{ft}; \quad \frac{\partial u_h}{\partial x_2} = 3.4069e3^\circ\text{C}/\text{ft}$$

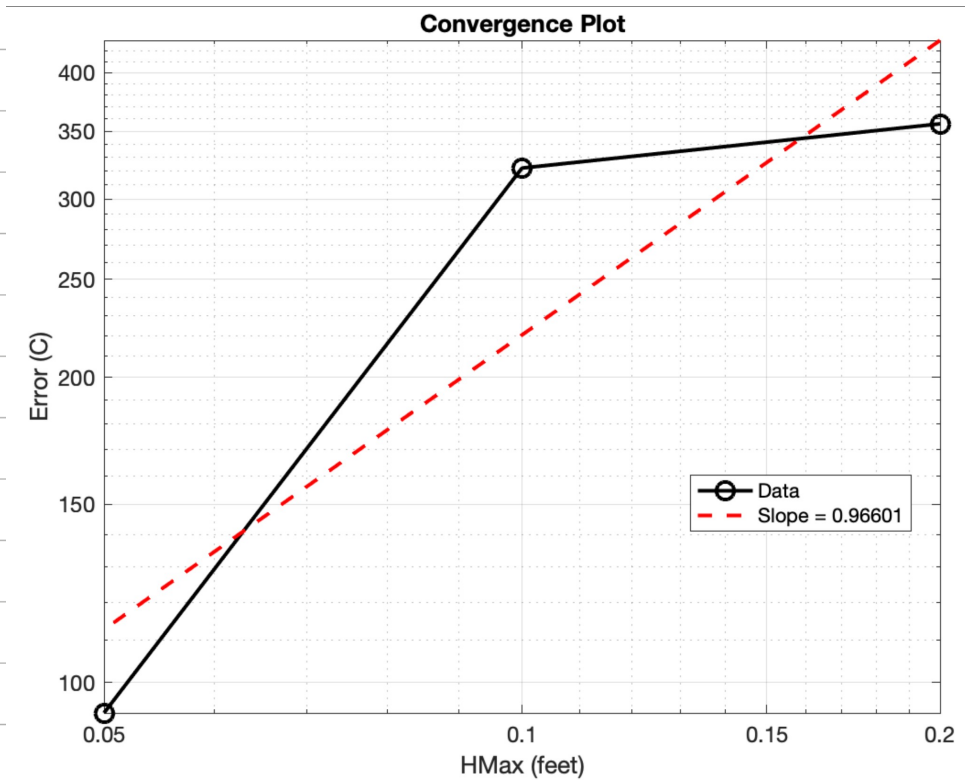
5.





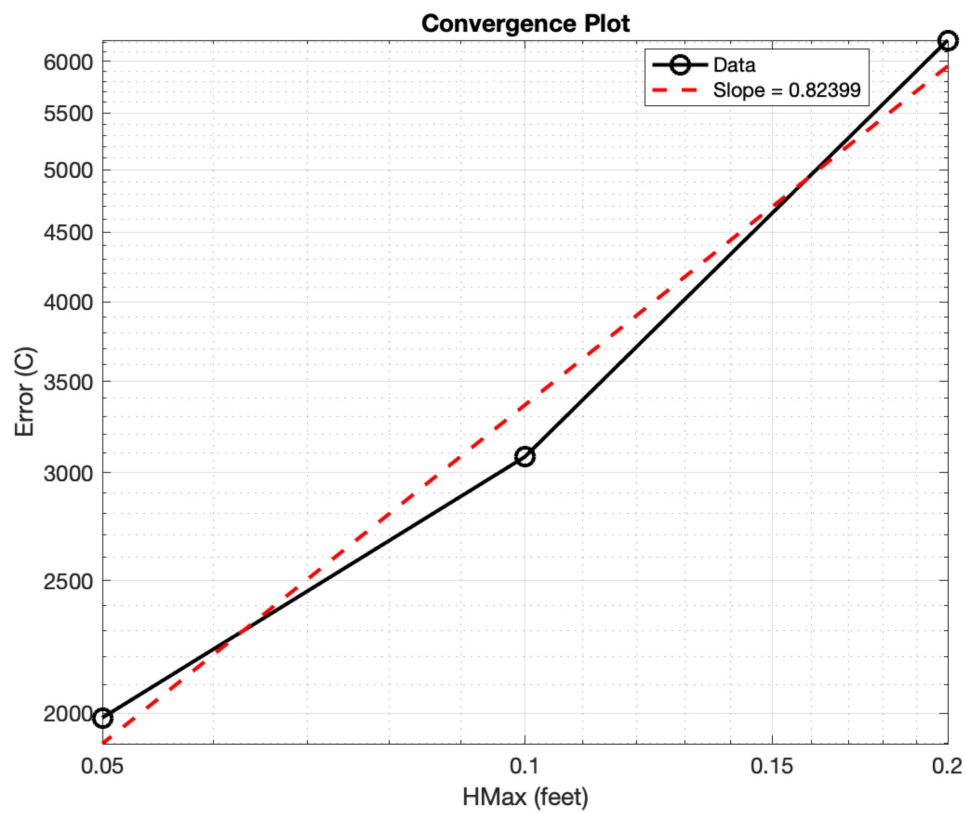
b. (a).

$$e_{0,2}(u) = \left[\sum_{i,j=0}^N u(X_{ij}) \right]^{\frac{1}{2}}$$



The f

$$e_{1,2}(u) = \left[\sum_{i,j=0}^N \|\nabla u(X_{ij})\|^2 \right]^{1/2}$$



(b).

