CP-3.

The weak form of the isotropic linear Elasticity problem can be given as:

Given the domain Ω and the following spaces:

$$\mathcal{W} = \{\vec{w} : \Omega \to \mathbb{R}^2 \mid \vec{w} \text{ is smooth vector field}\}$$

$$S = \{\vec{w} \in \mathcal{W} \mid \vec{w} = \vec{g} \text{ on } \partial\Omega_D\}$$

$$\mathcal{V} = \{\vec{v} \in \mathcal{W} \mid \vec{v} = 0 \text{ on } \partial\Omega_D\}$$

Find $\vec{w} \in S$ such that

$$a(\vec{w}, \vec{v}) = \ell(\vec{v}) \qquad \forall \vec{v} \in \mathcal{V}$$

where

$$a(\vec{w}, \vec{v}) = \int_\Omega \frac{E}{1+\nu}\left(\varepsilon(\nabla\vec{w}) : \varepsilon(\nabla\vec{v}) + \frac{\nu}{1-2\nu}\operatorname{div}\vec{w}\operatorname{div}\vec{v}\right)$$

$$\ell(\vec{v}) = \int_\Omega \vec{b}\cdot\vec{v}\, d\Omega + \int_{\partial\Omega_N} \vec{H}\cdot\vec{v}\, d\Omega.$$

$$\frac{dN_1}{dx_1} = \sum_{i=1}^{3} \frac{\partial N_1}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial x_1}$$

$$\frac{d\lambda_i}{dx_i} = \begin{bmatrix} \frac{\partial \lambda_1}{\partial x_1} & \frac{\partial \lambda_2}{\partial x_1} & \frac{\partial \lambda_3}{\partial x_1} \\ \frac{\partial \lambda_1}{\partial x_2} & \frac{\partial \lambda_2}{\partial x_2} & \frac{\partial \lambda_3}{\partial x_2} \end{bmatrix}$$

$$dN_1 = \begin{bmatrix} \frac{\partial N_1}{\partial \lambda_1} & \frac{\partial N_1}{\partial \lambda_2} & \frac{\partial N_1}{\partial \lambda_3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda_1}{\partial x_1} & \frac{\partial \lambda_1}{\partial x_2} \\ \frac{\partial \lambda_2}{\partial x_1} & \frac{\partial \lambda_2}{\partial x_2} \\ \frac{\partial \lambda_3}{\partial x_1} & \frac{\partial \lambda_3}{\partial x_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial N_1}{\partial x_1} + \frac{\partial N_1}{\partial x_1} + \frac{\partial N_1}{\partial x_1} & \frac{\partial N_1}{\partial x_2} + \frac{\partial N_1}{\partial x_2} + \frac{\partial N_1}{\partial x_2} \\ 0 & 0 \end{bmatrix}$$

$$\begin{array}{ccc} N_1 & \cdots & N_6 \end{array}$$

$$\begin{bmatrix} \frac{\partial N_1}{\partial x_1} & \frac{\partial N_1}{\partial x_2} & & \frac{\partial N_6}{\partial x_1} & \frac{\partial N_6}{\partial x_2} \\ 0 & 0 & & 0 & 0 \end{bmatrix}$$

$$\varepsilon(\nabla u) = \frac{1}{2}(\nabla u + \nabla u^T)$$

$$= \left[\begin{matrix} \frac{\partial N_1}{\partial x_1} & \frac{1}{2}\frac{\partial N_1}{\partial x_2} \\ 0 & 0 \end{matrix} \middle| \begin{matrix} \frac{\partial N_2}{\partial x_1} & \frac{1}{2}\frac{\partial N_2}{\partial x_2} \\ 0 & 0 \end{matrix} \right| \cdots$$

ε_{11}