CP-2.

Strong form:

Find $u \in C^2(\Omega)$ such that:

$$-k\Delta u = f \quad \text{in } \Omega$$

$$u = g_1 \quad \text{on } \Gamma_{g_1}$$
$$u = g_2 \quad \text{on } \Gamma_{g_2}$$

$$k\nabla u \cdot \hat{n} = h_1 \quad \text{on } \Gamma_{h_1}$$
$$k\nabla u \cdot \hat{n} = h_2 \quad \text{on } \Gamma_{h_2}$$
$$k\nabla u \cdot \hat{n} = h_3 \quad \text{on } \Gamma_{h_3}$$
$$k\nabla u \cdot \hat{n} = h_4 \quad \text{on } \Gamma_{h_4}$$

Define $S = \{ u : \Omega \to \mathbb{R}$ smooth such that $u(x) = g_1$ for $x \in \Gamma_{g_1}$; $u(x) = g_2$ for $x \in \Gamma_{g_2} \}$.

$V = \{ v : \Omega \to \mathbb{R}$ smooth $\mid v(x) = 0$ for $x \in \Gamma_{g_1} \cup \Gamma_{g_2} \}$

Find $u \in S$ such that:

$$a(u, v) = \ell(v) \quad \text{for all } v \in V$$

where $a(u, v) = k\int_\Omega \nabla u \cdot \nabla v \, d\Omega$

$$\ell(v) = \int_\Omega f v \, d\Omega + \int_{\partial\Omega_N} H v \, d\Gamma$$

where $\partial\Omega_N = \{ \Gamma_{h_1} \cup \Gamma_{h_2} \cup \Gamma_{h_3} \cup \Gamma_{h_4} \}$

$$H = \begin{cases} h_1 & x \in \Gamma_{h_1} \\ h_2 & x \in \Gamma_{h_2} \\ h_3 & \\ h_4 & \vdots \end{cases}$$

Discretized Form:

Let $\mathcal{W}_h = \text{span}(\{N_1, \cdots, N_{|\mathcal{E}|+1}\})$.

$$S_h = \{u_h \in \mathcal{W}_h \mid u_h(x_a^e) = g(x_a^e) \ \forall \ x_a^e \in \mathcal{J}_g\}$$

$$\mathcal{V}_h = \{v_h \in \mathcal{W}_h \mid v_h(x_a^e) = 0 \ \forall \ x_a^e \in \mathcal{J}_g\}$$

1.