Strong form:

Find $u \in C^2(\Omega)$ such that:

$$-k\Delta u = f \quad \text{in } \Omega$$
$$u = g_1 \quad \text{on } \Gamma_{g_1}$$
$$u = g_2 \quad \text{on } \Gamma_{g_2}$$
$$k\nabla u \cdot \check{n} = h_1 \quad \text{on } \Gamma_{h_1}$$
$$k\nabla u \cdot \check{n} = h_2 \quad \text{on } \Gamma_{h_2}$$
$$k\nabla u \cdot \check{n} = h_3 \quad \text{on } \Gamma_{h_3}$$
$$k\nabla u \cdot \check{n} = h_4 \quad \text{on } \Gamma_{h_4}$$

Define

$S = \{u : \Omega \to \mathbb{R}$ smooth such that $u(x) = g_1$ for $x \in \Gamma_{g_1}$; $u(x) = g_2$ for $x \in \Gamma_{g_2}\}$.

$V = \{v : \Omega \to \mathbb{R}$ smooth $\mid v(x) = 0$ for $x \in \Gamma_{g_1} \cup \Gamma_{g_2}\}$

Find $u \in S$ such that:

$$a(u, v) = \ell(v) \quad \text{for all } v \in V$$

where $a(u, v) = k\int_\Omega \nabla u \cdot \nabla v \, d\Omega$

$$\ell(v) = \int_\Omega f v \, d\Omega + \int_{\partial\Omega_N} H v \, d\Gamma$$

where $\partial\Omega_N = \{\Gamma_{h_1} \cup \Gamma_{h_2} \cup \Gamma_{h_3} \cup \Gamma_{h_4}\}$.

$$H = \begin{cases} h_1 & x \in \Gamma_{h_1} \\ h_2 & x \in \Gamma_{h_2} \\ h_3 & \vdots \\ h_4 & \end{cases}$$

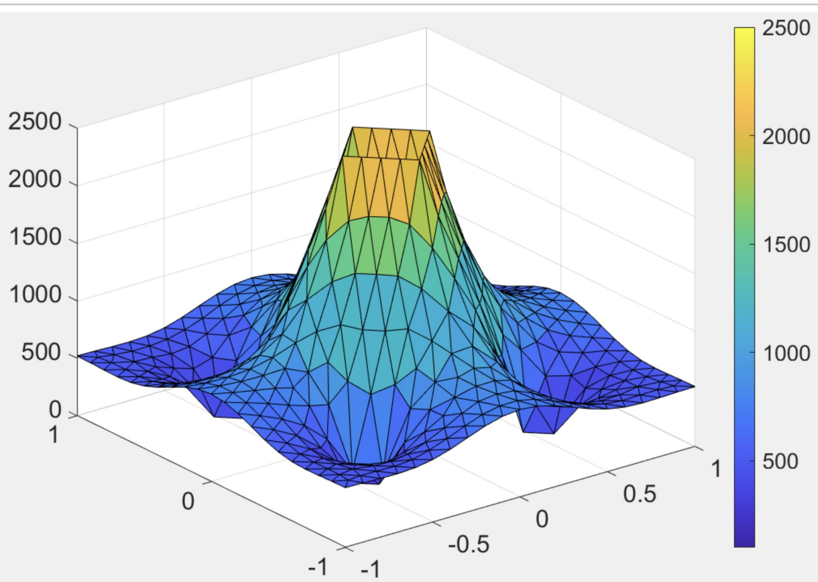
Discretized Form:

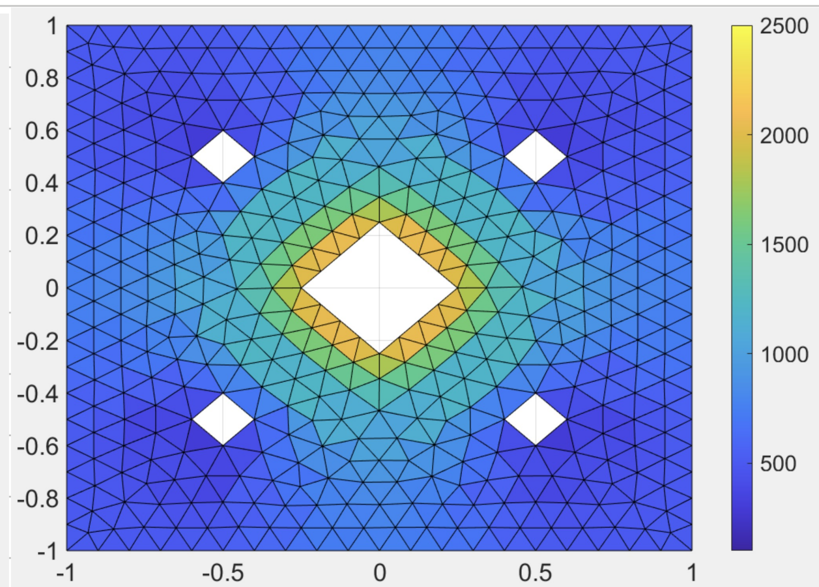
Let $W_h = \text{span}(\{N_1, \cdots, N_{el+1}\})$.

$$S_h = \{u_h \in W_h \mid u_h(x_a^e) = g(x_a^e)\ \forall x_a^e \in \Gamma_g\}$$

$$V_h = \{v_h \in W_h \mid v_h(x_a^e) = 0\ \forall x_a^e \in \Gamma_g\}$$

1.





2.