CP-3.

The weak form of the isotropic linear Elasticity problem can be given as:

Given the domain Ω and the following spaces:

$$\mathcal{W} = \{ \vec{w} : \Omega \to \mathbb{R}^2 \mid \vec{w} \text{ is smooth vector field} \}$$

$$S = \{ \vec{w} \in \mathcal{W} \mid \vec{w} = \vec{g} \text{ on } \partial\Omega_D \}$$

$$\mathcal{V} = \{ \vec{v} \in \mathcal{W} \mid \vec{v} = 0 \text{ on } \partial\Omega_D \}$$

Find $\vec{w} \in S$ such that

$$a(\vec{w}, \vec{v}) = \ell(\vec{v}) \qquad \forall \vec{v} \in \mathcal{V}$$

where

$$a(\vec{w}, \vec{v}) = \int_\Omega \frac{E}{1+\nu} \left(\varepsilon(\nabla\vec{w}) : \varepsilon(\nabla\vec{v}) + \frac{\nu}{1-2\nu} \operatorname{div}\vec{w} \operatorname{div}\vec{v} \right)$$

$$\ell(\vec{v}) = \int_\Omega \vec{b} \cdot \vec{v} \, d\Omega + \int_{\partial\Omega_N} \vec{H} \cdot \vec{v} \, d\Omega.$$