Quantum Control Lab Report

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Section 1 - Theory

In this experiment, we seek to set up, manipulate, and measure the properties of a quantum system, and measure how changes in our setup parameters affect the quantum system. The particular quantum system we will study in the experiment to follow is based on the physical angular spin of nuclear protons in some substance. This phenomenon is often called Nuclear Magnetic Resonance.

At a ground state, prior to being manipulated by any quantum control technique, the spin orientation of a large number of protons in a system at rest will be approximately random. This is due to the steady state equilibrium to which the system will always revert at long time scales because of the coupling of the system to the outside thermal environment. This could be visualized as a three-dimensional area loosely filled with protons all spinning in random directions. The axis of rotation of these protons will be defined by an external magnetic field, but the handedness of their spin will still be random. If forced into a state of off-axis-alignment with this external field . Each of these precessing protons will generate their own individual magnetic field component. However, a measurement of the net magnetic field of the system in this state will yield no signal since, on average, each individual magnetic field vector will cancel with another.

However, exciting the system with a short-duration uniform magnetic field–which is separate and orthogonal to the first magnetic field that generates the precession–will cause each of these “dipoles” to drift their rotation axis towards that of the applied external magnetic field. Such a signal is referred to as a “Preparation Pulse.” Since the direction of rotation of each proton is random, the actual alignment may be with-the-field or against-the-field, because the dipole polarization is effectively reversed in either case. If the Preparation Pulse flips the protons by exactly $\pi/2$, then a maximal degree of alignment is achieved. For an instant, the precession of all protons will generate individual magnetic field vectors which will constructively add, resulting in a non-zero and measurable net B field. This measured field, however, will quickly dissipate due to the fact that some–approximately half–of the protons will begin to precess in exactly the opposite direction of the other protons in one instant. Very soon, the protons will achieve perfect destructive interference.

One additional pulse–a π-pulse this time–timed at just this moment would effectively reverse the direction of each proton precession. Each proton would rotate in exactly the opposite direction, back towards where each started. Each proton will individually sweep back past their original, once aligned, state. At this particular point in time, a magnetic field “Echo” would be measurable. Such a pulse is called an “Intervention Pulse.”

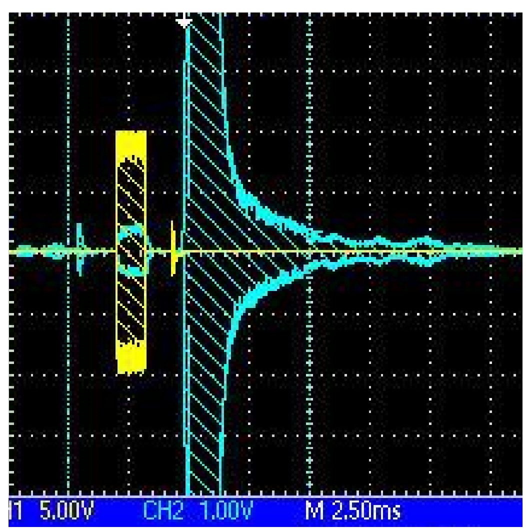
The characteristic time between the initial Preparation pulse, the Intervention pulse, and time when the Echo is measured–as well as the optimal strength, duration, and frequency of the Preparation and Intervention pulses themselves–are highly dependent on the individual material being studied. This makes Nuclear Magnetic Resonance a very useful material characterization tool that is often used in material science, chemistry, and physics to uncover the composition of unknown compounds. In addition, the particular quantum state set up by the precession and echo of nuclear spins is a strong candidate as a physical implementation of qubits. It is being actively studied for its applications in quantum computing and quantum information.

Section 2 - Experimental Setup

Our implementation of this NMR study uses TeachSpin's Quantum Control experiment. The source of our protons comes from ordinary tap water. Our control system is in three parts. The "Sample" solenoid holds the water which contains the population of protons we will be manipulating. This solenoid is responsible for generating the Preparation and Intervention pulses that will adjust the alignment of the protons. It is also responsible for capturing the generated magnetic flux from the proton precession so we can read out the magnetic field strength as current and voltage. A different, orthogonally oriented, solenoid is responsible for generating the constant and uniform B field for the protons to precess around. This coil is called the "Main" solenoid. A third and final coil, the "Pilot" coil, is used only to calibrate the device by generating predictable magnetic flux pulses so we can reliably optimize our pulse frequencies. We began by finding the optimal frequencies of pulses to send into the sample coil and the steady B field to ensure we had the largest signal.

a) <u>Preparation Pulse Optimization</u>

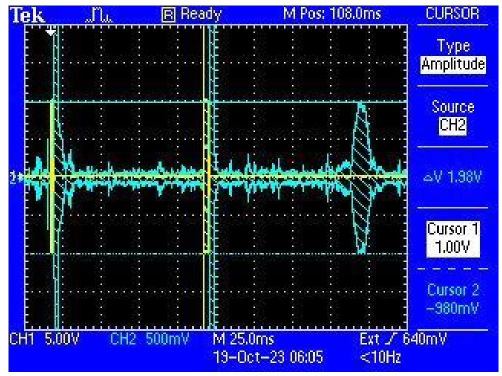
The preparation pulse is an AC signal sent into the sample solenoid designed to orient the spins of protons. We optimized the preparation pulse so the proton precession signal would be the largest, giving the best data. We optimized the pulse under two conditions, number of cycles and strength.



In order to clearly see the signal of the proton precession, the preparation pulse needs to end while the spin orientation of the proton is not aligned with the steady magnetic field generated by the main coil. If the spin orientation of the protons are left perfectly aligned with the main coil's field, there will be very little precession, and no measurable signal. Therefore we tuned the signal to theoretically leave the spin orientation opposite the B field generated by the main coil.

The preparation pulse is sent in (yellow signal) and the inductance on the sample coil would produce a measurable potential (blue). The transient immediately following the preparation pulse is the change in magnetic field due to the signal itself. The tail to the right is the Larmor precession of the protons spin. The signal starts off strong and decays in milliseconds due to the ambient B fields interfering with the generated magnetic field.

b) <u>Intervention Pulse Optimization</u>



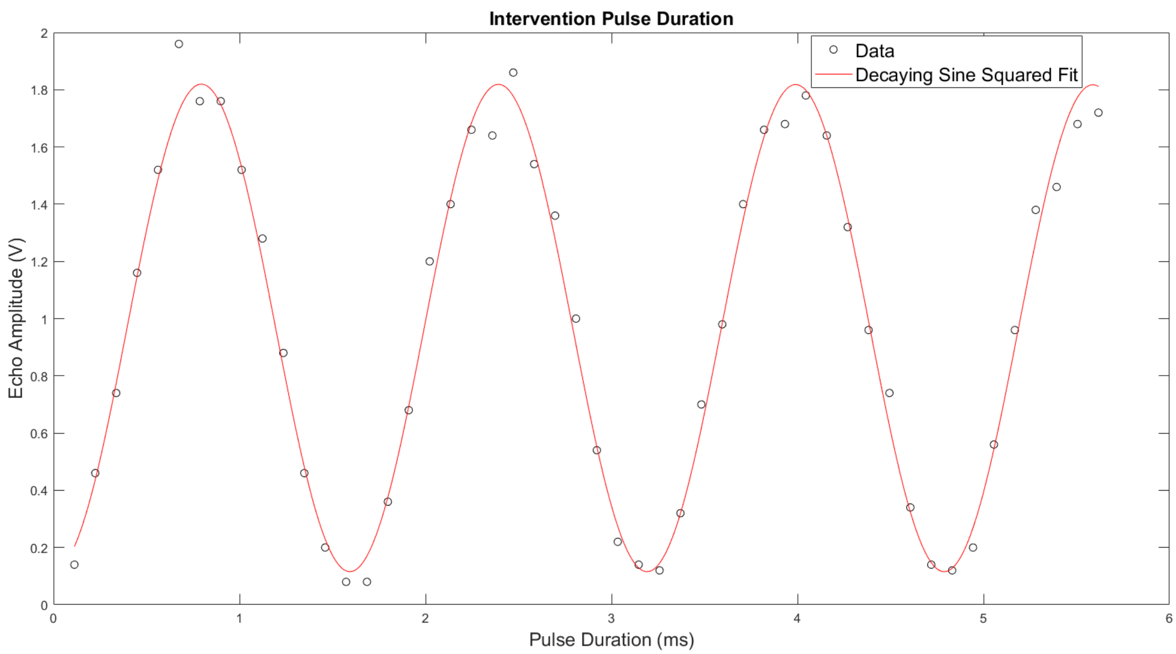
The intervention pulse is a signal sent after a time tau. The goal of this pulse is to act as a mirror image to the preparation pulse. The intervention pulse realigns the spins of the protons. After the preparation pulse, the protons experience Larmor precession. This is seen because a signal appears at 2 tau showing that by sending the intervention pulse, we were regaining some of the signal generated by the precession of the protons back. Due to ambient B fields and proton-proton interactions, the precession rates occur at slightly different rates. As the signal dies down, the precession rates appear to randomize beyond the noise floor. The intervention pulse is the "inverse" of the preparation pulse. By sending a signal with a phase shift, it is possible to obtain an "echo" of the spin. The signal on the right of the oscilloscope shows the protons precession as they begin to realign. The protons began to precess at the same frequency, inducing a measurable change in potential. That signal then decays just as it did after the preparation pulse giving the echo signal on the right a symmetric lobe.

Section 3 – Experiment 1 – Intervention Pulse Study

a) **Effect of Pulse Duration**

In our first experiment, we varied the number of cycles–and therefore the duration–of our AC Intervention pulse. This factor contributes to how strongly the proton dipole orientations are changed by, and thus will affect how close we are to an ideal π-rotation. This mechanism should also work if a π-pulse is overshot, and instead a 3π-pulse is achieved (perhaps with some loss), so the data is expected to follow some sort of sinusoidal or sine-squared relationship.

We varied the number of cycles and measured the amplitude of the NMR echo in volts. The data is below:

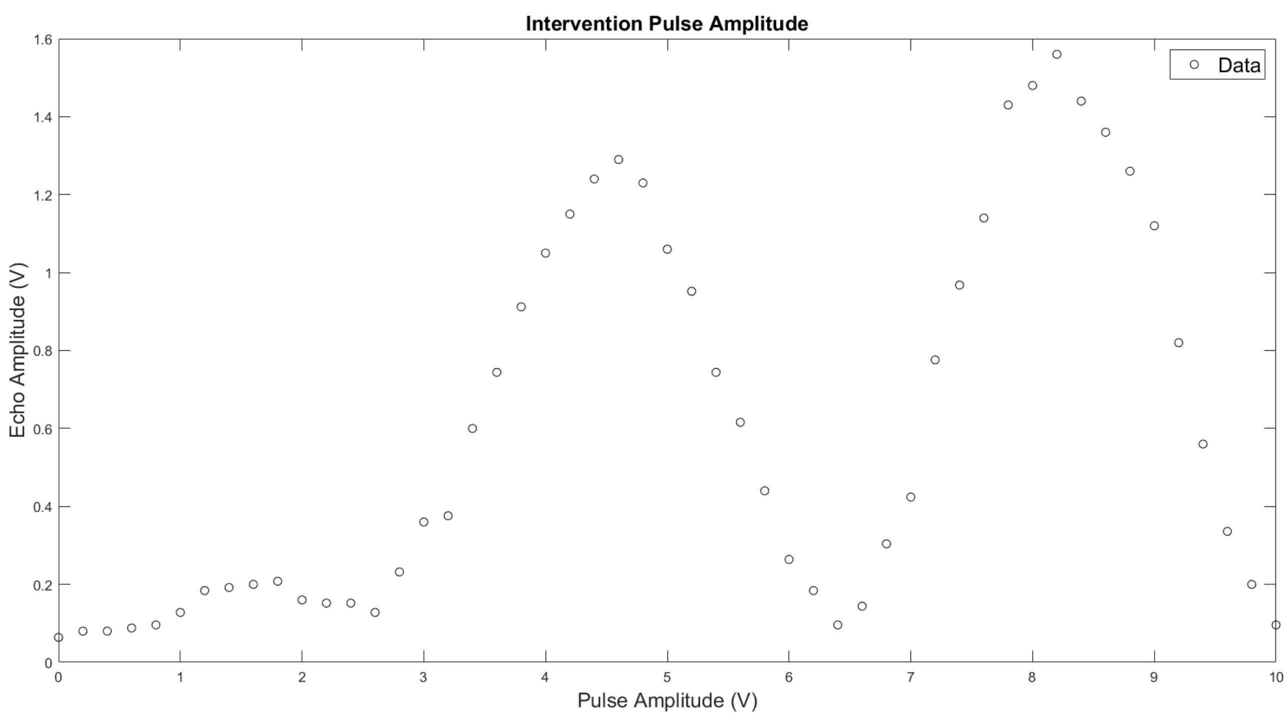


This data very closely follows a sine-squared fit. This is to be expected, and represents the periodic nature of achieving 2π rotations past the optimal π-pulse, which we achieved at an intervention pulse duration of approximately 0.8 ms (using an amplitude of 10V and frequency of 89kHz).

Looking at the overall decay envelope, it can be seen that the echo signal amplitude very gradually decreases with increasing duration. Although the effect is extremely small in our study, it can still be seen, and we attribute this to the natural imperfections of our sample and environment. The longer the signal was on, the more opportunity each proton had to interact with surrounding protons as well as the ambient B fields. If a proton were to interact with a force outside of the signal we generated, the procession of said proton would vary inhomogeneously with the rest of the sample. If the pulse duration were to increase to the order of seconds, we would expect to see a noticeable change in the peaks of the echo signal. In a perfectly theoretical environment, the duration should not affect the peak signal. The ambient B fields along with the fields generated by each proton changing orientation causes the signal to decay. The longer the protons are experiencing the signal, the more time each proton has to become inhomogenous with the rest of the spins, reducing our precession signal.

b) <u>Effect of Pulse Amplitude</u>

Next, we varied the intervention pulse amplitude and measured the Echo response in Volts. We do this at a constant pulse duration and frequency. Similar to duration, the pulse amplitude should affect how strongly the proton dipole orientations are rotated, and will therefore affect the overall rotation angle. Similarly the data is expected to follow some sort of sinusoidal or sine-squared relationship. Here is our experimental data:



This data does indeed follow a sin-squared periodic function. However, we also see a strongly increasing exponential-type envelope. This is somewhat unexpected to us since we would have expected a small exponential *decay* model, similar to the results we see from experiment 1a. This envelope does not appear to follow any standard fit model, so we do not yet have a good fit for this dataset.

One possible explanation for this is the storage of electrical potential energy as some other form of energy in the system, which is then converted back into magnetic flux upon the recombination of the spin-echo and read out in our signal. Higher input voltage would then correspond directly to greater energy that the system stores and then greater output signal. This, however, implies that there is some unaccounted for form of potential or kinetic energy at play to store energy as, which seems unlikely. Also, this absorption and release of energy would need to operate on the exact time scale to that of the spin-echo, itself, which would be miraculous at best.

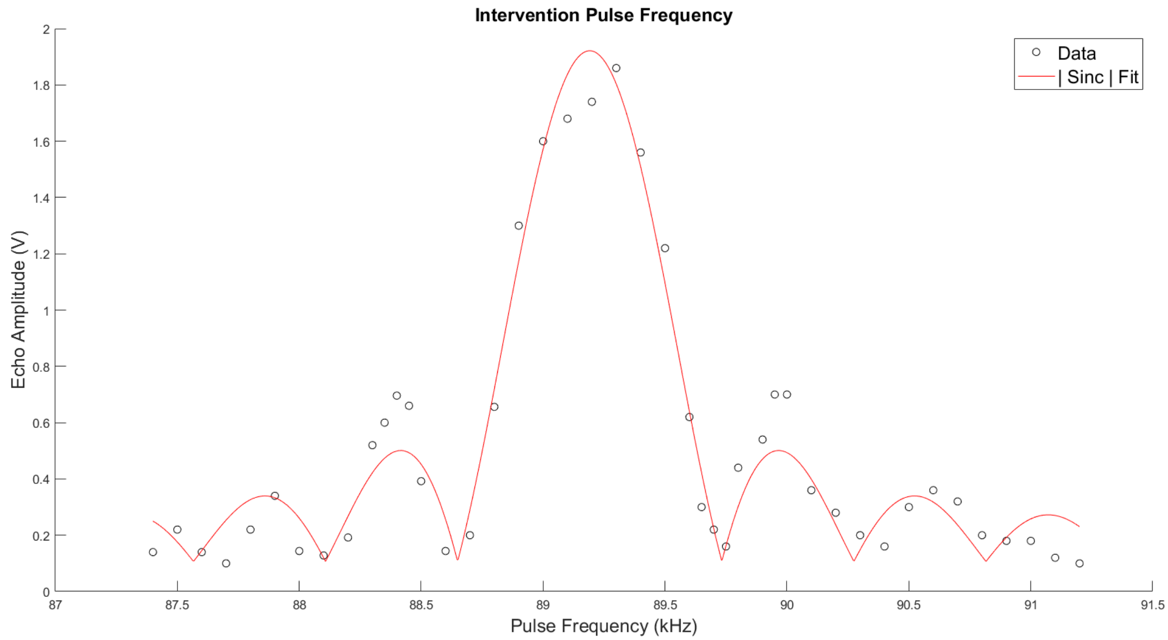
It is also possible that our experimental setup was flawed. The particular oscilloscope we used for measurement was sometimes found to fail in interesting modes throughout the course of this experiment. For instance, at times the scope would be triggered by channels which it did not appear to be configured to trigger on, perhaps due to faulty grounding or shorts internal to the oscilloscope itself. It is not inconceivable that our output signal may have coupled to some unpredictable impedance drain for the duration of this particular measurement, causing the envelope we see.

In either case, the periodic-nature of the underlying physics is still clearly on display. The first peak represents the first π-pulse, and the subsequent peaks represent 2π rotations of this peak. This is fundamentally the same physics as explained in pulse duration investigation since the overall magnitude of rotation angle is proportional to the product of the intervention pulse Duration * Amplitude. In theory, both hold proportionally equal weight.

c) <u>Effect of Pulse Frequency</u>

Finally, we will measure the effect that the Intervention pulse *frequency* has on the amplitude of the spin-echo. We will do this by holding constant the pulse duration and pulse amplitude, and changing the frequency of the intervention pulse by a small perturbation around the peak frequency expected at ~89 kHz.

It is expected to see some form of beats pattern about the peak frequency with several off-resonance peaks. This is because frequencies away from the ideal characteristic frequency should not couple to the spin-system of the protons as efficiently. As you may recall, we interface to this system using a solenoid inductor to pick up the generated magnetic flux of the spin-echo. The LC resonance of this circuit has only one single ideal peak where this maximal LC resonator coupling is achieved. Using a different frequency attempts to excite different resonance modes, and will be attenuated with varying degrees of interference. Our experimental data is shown below:



Our data does indeed follow this prediction. We see a maximal peak near the characteristic resonant frequency of 89 kHz, and several much smaller side-lobes to the left and right. A decent, but still not perfect, fit for our data is a function of the form abs(sinc(f)). One interesting insight from this data is that the peak frequency appears to actually occur at about f0 = 89.18 kHz, which is slightly higher from our iteratively tuned frequency of 89.00 kHz. This implies that we could have actually achieved slightly greater signal magnitudes for studies in other parts of this report with further fine-tuning of our characteristic frequency. We chose to continue to use f0 = 89.00 kHz to maintain maximum continuity of our results.

We have already characterized the primary variables that contribute to the amount that the proton spins get rotated by: namely the intervention pulse Duration * Amplitude product. Intervention frequency plays a crucial role in the coupling efficiency of *applying* this product to enact changes in the spin system. It can be thought of as how efficient our instrumentation is at actually achieving these pulse rotations.

As theory predicts, the central peak of our data is a result of the Rabi Lineshape. This is defined as the frequency at which a two-level system will fluctuate between the two energy levels. Applying a pulse at exactly this frequency will result in maximal coupling between these two energy levels, and thus will “excite” a maximum number of proton spins with the desired spin rotation. The outer side-lobes are likely due to the contribution of multiple phenomena, just one of which being the beats frequencies of the LC resonator in our instrumentation as mentioned above. In reality, we do not yet have enough data to draw further conclusions about the side lobes from this experiment alone. Additional testing, especially by taking measurements at different Duration * Amplitude products, would yield highly insightful data about how the side-lobes behave with respect to different variables.

Section 4 – Experiment II – Qubit Experiments

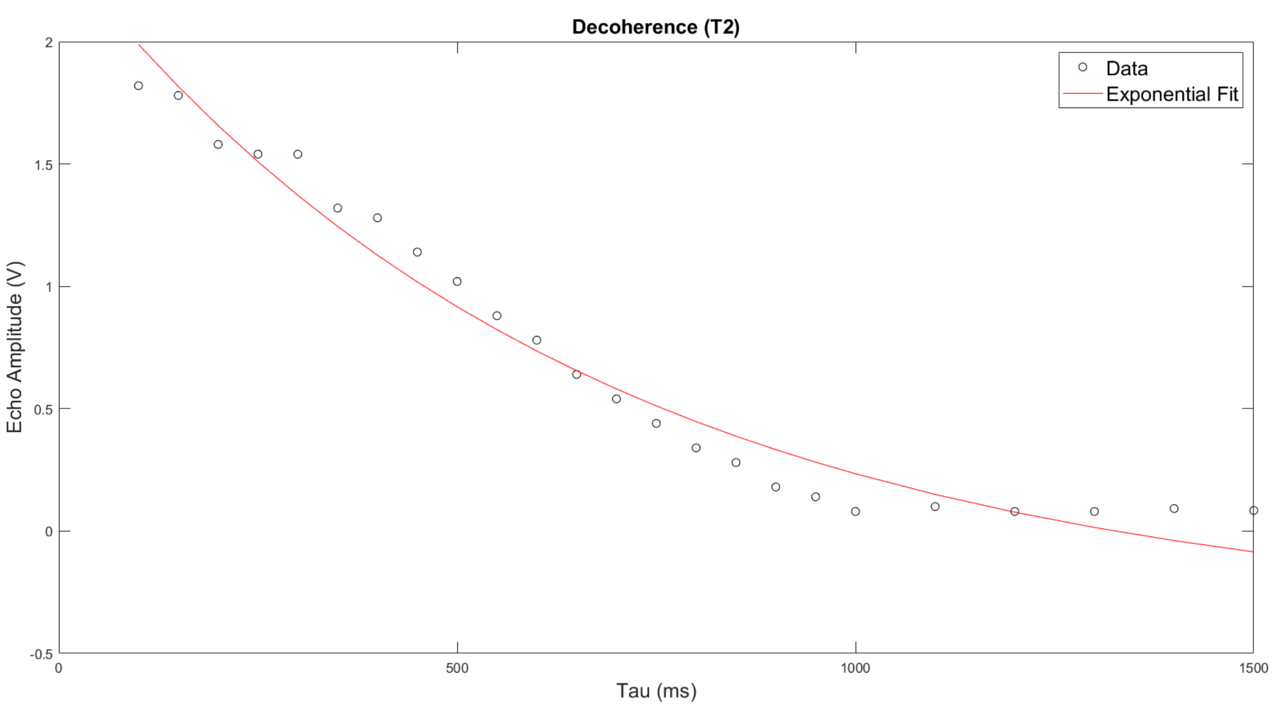
One of the primary uses of NMR systems like we have thus far characterized is for quantum computing. The spin-state system can be exploited to store quantum information. The primary parameters which are important for the usage of a quantum system as a qubit are its Thermalization time (T1), which is the time that it takes for the system to return from an excited state to the ground state; and the Decoherence time (T2), which is the amount of time that the system is able to coherently hold quantum information. We will measure the relevant quantities in the following section of the experiment.

a) <u>Measure Decoherence Time (T2)</u>

The decoherence time (or T2) is the amount of time that it takes for the system to lose the quantum state that it has stored. In the context of our experiment, T2 is a quantifier of how long we can delay the measurement of our spin-echo before we can't measure it confidently anymore. We will experimentally measure this duration by varying the time, Tau, between the preparation pulse and intervention pulse, which is the same time between the intervention pulse and final spin-echo. As a function of tau, theory predicts that the resulting spin-echo signal should follow an exponential decay of the form:

V = exp(2*tau/T2)

Our experimental data is below.

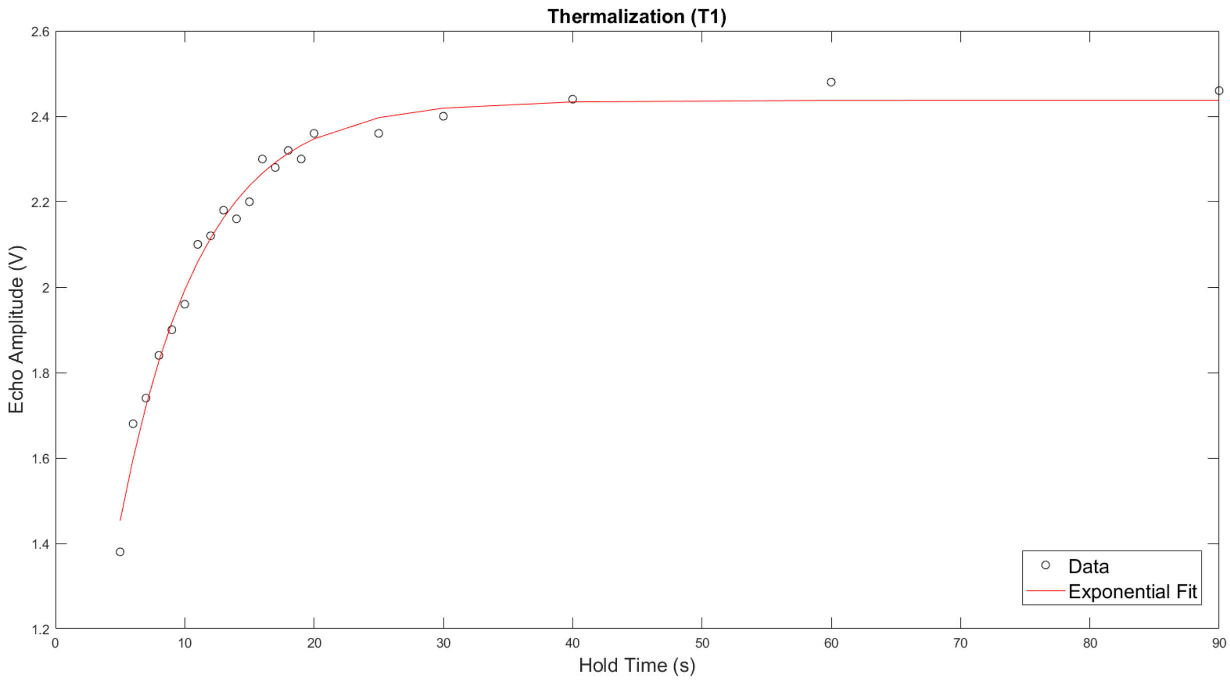


We clearly see from the data that small values of tau (small delays from P-to-I-to-Echo) result in much larger measured echo-signal compared to longer values of tau. This is because when the protons are allowed to interact with the environment for longer periods of time, some of the protons begin to re-thermalize and return to their natural ground-state, thus "losing" some of the signal in the process. Minimizing the amount of time between the pulses, therefore, minimizes the amount of "lost" signal. This signal "loss" also arises from a form of spin-spin interference between the particles that has an increasing contribution with longer time periods.

The data we acquired can be (very roughly) fit to an exponential decay, as predicted by theory. The fit yields a decoherence time of T2 = 1.32s extracted from the exponential time constant embedded in the fit. This is a relatively long T2 time when compared to most other traditional qubit implementations which often have T2's on the order of μs, depending on the implementation. This makes sense in the context of our experiment, since we are also using relatively long control pulse durations and relatively weak B-fields: for example we use mili-second pulses, while standard NMR instrumentation often utilizes μs pulses. The overall "time-scale" of our experiment is somewhat more macroscopic.

b) Measure Thermalization Time (T1)

The second major quantity of interest in the discussion of qubit technology is T1, the Thermalization time. This is the time that it takes a qubit to fall from an excited state back to the ground state. In our experiment, T1 is a quantifier of how long it takes for the spin states of the protons to return to their baseline level. To measure this, we vary a HOLD time parameter, which denotes the period related to how frequently our system will generate new pulses. This will affect how long the system has to thermalize to the external thermal bath before generating a new pulse. We then measure the amplitude of the resulting spin-echo pulses in Volts. Our experimental data is below:



The data we have collected clearly shows an exponential increase in signal strength at time-scales on the order of 1s-20s, with a relatively flat equilibrium at large time scales. This is due to the resetting of proton spin-states, and provides evidence of a re-thermalization process occurring. At very low HOLD times, there is not enough time for thermalization to occur through a Boltzmann process. Since many of the protons may not have yet "forgotten" their prior spin information by the time the next quantum system is set up, the new spin-states become biased towards one state over another. This unravels the dynamics of the NMR spin-echo phenomenon, causing the signal to trend towards zero.

Fitting the data to an increasing exponential function, we acquire a Thermalization time constant of T1 = 6.27s. Again, this time scale is quite large but is explained by the overall scale of our experimental instrumentation. Compared to our T2 time (T2 = 1.32s), we see that T1 > T2. This is a well expected result that holds for all qubit implementations. It arises from the fact that the qubit must be able to resist rethermalization at least as long as it can retain its quantum information. We also see that our T1 and T2 measurements are roughly on the same order. This tells us that the physics governing both interactions is largely the same: the Boltzmann rethermalization of spin states is likely the leading governing factor in both measurements.

Section 5 – Conclusion

Overall, in this report we have presented findings about the close study of one particular implementation of a quantum system. This system exploits the precession of protons in an external magnetic field to achieve and measure a “Spin-Echo” in a phenomenon called Nuclear Magnetic Resonance. Our experimental findings were highly enlightening into the nuances of this realm of study, and it has provided us with insights into the larger world of quantum information science. We have discussed many interesting dynamics of the control of this quantum system, including the optimization of the Preparation and Intervention pulses which ultimately construct our quantum system. We also studied this system as a qubit in the context of quantum computing, and measured Decoherence and Thermalization times of $T2 = 1.32s$ and $T1 = 6.27s$. This provides some idea of the behavior of quantum systems as in the context of quantum information. Overall, this experiment provided great insight into the world of quantum mechanics.