Quantum Optics Lab Report

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Section 1 - Theory

In this experiment, we seek to probe a quantum phenomena called "parametric downconversion." Parametric downconversion is a phenomenon in the field of quantum optics offering unique insights into the quantum nature of light and its applications in various quantum technologies. This phenomenon involves the generation of entangled photon pairs through the incidence of an initial "pump" photon on a nonlinear crystal optic. When the pump photon strikes the crystal, it splits into two lower-energy photons known as the "signal" and "idler" photons in a process dictated by conservation of energy and momentum. These signal and idler photons are correlated in properties, such as energy, momentum, and polarization, and are thus considered quantum mechanically entangled. In this experiment, we aim to explore the underlying principles of parametric downconversion.

When incident on a nonlinear optic, the wavelength of a single incident photon does necessarily need to be conserved as a single outgoing photon, like it is with linear optics. Instead, a single incident photon may split into multiple different downstream photons, each with their own energy and momentum. They are constrained such that the overall total energy and momentum of the downstream photons equals the energy and momentum of the incident photon. This gives rise to the possibility of incident photons splitting into multiple downstream photons exhibiting different energies to that of the incident photon. This process is called parametric downconversion, since down-converted photons will always have energy less-than-or-equal-to that of the incident photon. One critical consequence of this phenomena is the fact that these down-converted photons may interact differently with optical devices since they have different wavelengths. In certain media, such as the crystals we will be using in this experiment, the index of refraction–and thus the refraction angle–is dependent on the photon wavelength.

Working from conservation of energy, we can start to formulate this. Let E_p, E_s, and E_i represent the energy of the incident Pump photon, and downconverted Signal and Idler photons respectively. Conservation of energy of this system can be written as:

$$E_p = E_s + E_i$$

$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$

$$\omega_p = \omega_s + \omega_i$$

Furthermore, by using the relationship $c = f\lambda$, we get the simple result:

$$1/\lambda_p = 1/\lambda_s + 1/\lambda_i$$

If the signal and idler photons are equal and opposite copies of each other, then we get the even stronger statement:

$$2\lambda_p = \lambda_s = \lambda_i$$

or equivalently,

$$\omega_p = 2\omega_s = 2\omega_i$$

This is the wavelength of downconverted light expected from splitting a single incident pump photon into 2 photons of equal energy.
Additionally, from conservation of momentum, we get the phase matching condition:

$$\mathbf{k}_\mathrm{p} = \mathbf{k}_\mathrm{s} + \mathbf{k}_\mathrm{i}$$

This can be casted into a form containing the propagation angles of of the down-converted photons measured relative to propagation direction of incident light:

Parallel components: $$n_\mathrm{p}\omega_p = n_s\, \omega_s\, cos\theta_s + n_i\, \omega_i\, cos\theta_i$$

Perpendicular components: $$0 = n_s\, \omega_s\, sin\theta_s + n_i\, \omega_i\, sin\theta_i$$

Where n_p, n_i, and n_s are the wavelength-dependent indices of refraction in the crystal.
Imposing the condition $\omega_p = 2\omega_s = 2\omega_i$ for equal and opposite photons, we get:

$$n_p = n_s\, cos\theta_s$$

In reality, we expect the signal and idler photons to be ejected from the downconverting crystal in radially symmetric cones aligned with the propagation direction of the pump photon, and with half-angles defined by the relationship above. In this experiment, we only collect the particular photons that intersect with the horizontal plane, which yields just two photon trajectories at angles +- θ_s . Crucially, each photon sent down the $+\theta_s$ trajectory will necessarily correlate with an entangled photon traveling down the $-\theta_s$ trajectory in order to conserve energy and momentum.

It is important to realize that the indices of refraction of the down-converted photons are different from the initial photon's due to a wavelength dependence of the index of refraction in these materials. For Beta-Barium-Borate, a popular downconverting crystal, this dependence can be modeled by:

$$n_o(\lambda)= [\ 2.7359 + 0.01878/(\lambda_s - 0.01822) - 0.01354\ \lambda_s\ ]^{1/2}$$

$$n_e(\lambda)= [\ 2.3753 + 0.01224/(\lambda_s - 0.01667) - 0.01516\ \lambda_s\ ]^{1/2}$$

Where n_o and n_e are the ordinary and extraordinary indices of refraction corresponding to different modes of refraction through the crystal, and λ is measured in μm.

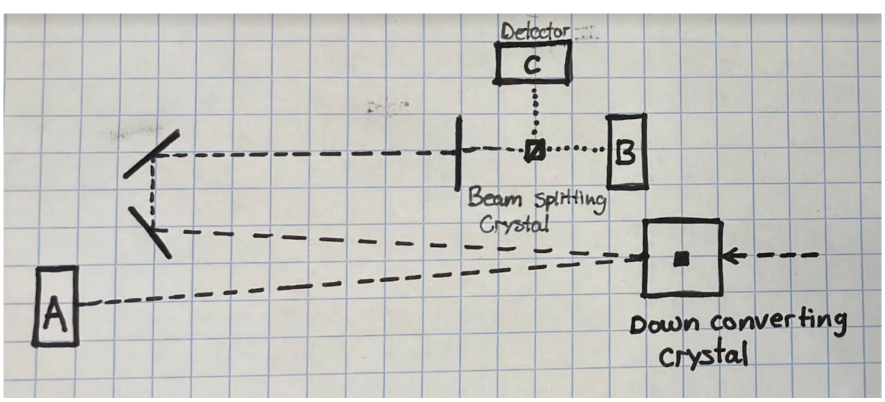
From this we can deduce the existence of the orientation dependence of the downconversion crystal. As the crystal is rotated, the orientation of the optical axis of the crystal is rotated. The ordinary and extraordinary indices of refraction describe refraction behavior relative to this optical axis, so they too are dependent on the orientation of the crystal. We see from above that the propagation angle of the down-converted photons is dependent on the indices of refraction. Thus, we transitively deduce that the orientation of the down-converting crystal affects the propagation direction of the outgoing photons. As we rotate the crystal, we expect the optimal position and orientation of a detector to count these down-converted photons to shift slightly.

Section 2 - Experimental Setup

To investigate the phenomena of parametric downconversion, we use a tabletop optics station. A primary laser emitting 405 nm blue light is positioned furthest upstream. In its path is the downconverting crystal (DC) which is responsible for producing our entangled signal and idler photon pairs. As detectors, we use an array of Avalanche Photodiodes (APDs) configured to be sensitive to the 810 nm range using bandpass filters.

When a photon strikes an electron in the ADP, that electron is exited and “kicked” free. The APD has an internal electric potential such that when an electron is freed by a striking photon, that electron gains a certain amount of kinetic energy. The electron with that gained energy strikes another electron, freeing that electron to do the same. Each electron strikes a new electron generating a very sudden flow of electrons. The end result is an “avalanche” of electrons flowing towards the potential in the APD measured as a short-lived current pulse before the APD resets again.

Our “detectors” are really converging lenses which are connected to individual APD channels using fiber optic cables. Detector A is positioned to measure the signal photon approximately 3 degrees counter-clockwise from the axis normal to the DC. Detector B is configured to measure the idler photon, which is emitted approximately 3 degrees clockwise from this axis. Later on, we will also introduce another detector, Detector C, and a beam splitter between Detectors B and C that will split idler photons down one of these paths. The data from the APDs are sent to an FPGA, which relays the data to the computer, but is also capable of measuring coincidence counts between any two APD channels.



To detect coincidences, a signal would need to appear on two detectors within a set time period (configured as 6 ns). For example, once the photons leave the down converting crystal, some of them will strike Detector A. At that point, the FPGA will measure a count, indicating that a photon has struck the detector. If it receives another count, this time from Detector B, within the 6 ns time frame then it interprets that as a coincidence.

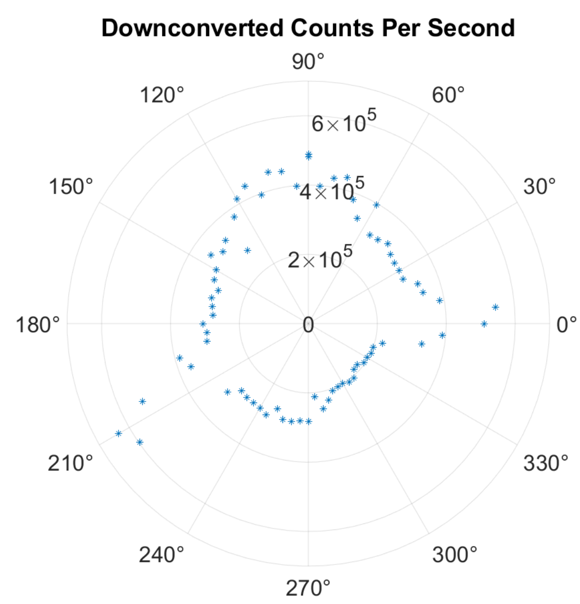
The extra distance that the photons travel to get to detector B was measured as 19 inches. The time it would take photons traveling the speed of light to travel 19 inches is roughly 1.5 ns. This means that proper coincidence photons should still have plenty of time to travel the extra path length and appear as coincidences within the 6ns window, so the path length difference in our experimental setup should make no difference to our data. There will of course be noise in the form of random accidental coincidences from two non-correlated photons hitting both detectors within the 6ns time window.

Section 3 - Experiment & Results

In the following section, we will discuss our experimental findings using statistical analysis techniques. We seek to show that a) we can in fact measure down-converted photons, and the count rate incident on a stationary detector will depend on the orientation of the down-converting crystal, and b) that the photons we measure are in fact correlated to equal and opposite photons, and can not merely be explained away as accidental coincidences.

a) **Measuring Down-Converted Photons**

To measure the number of down-converted photons from the crystal per unit time, we adjust the position and orientation of detector A until we find where non-zero detector counts are found. We then iteratively adjust all of the optics, including the orientation of the downconverting crystal, until counts are optimized. The final position of detector A is roughly 1.5 inches offset from the main axis of the crystal, which equates to ~3 degrees of angle offset from the crystal. We then measure the effect of crystal orientation by spinning the crystal in small increments and measuring count rate measured by the APD. The data we acquired is to the right:



We see that there is a large peak at ~210 degrees, and another smaller distinct peak at ~0 degrees. These peaks correspond to orientations where the diffraction angle of the down-converted photons most closely matches the alignment angle of the detector, and as a result a maximized amount of photons is sent into the detector. At other crystal orientations, the photons are diffracted by some other angles which the detector was not optimized to detect without realignment.

We also see that the off-peak signal roughly follows an ellipsoid with non-zero eccentricity rather than a perfect circle. This is due to the overall conical photon trajectory space which precesses with the rotation of the crystal. This cone swept by the half-angle θ_s is not perfectly aligned with the rotation axis of the crystal. This off-axis conical rotation causes a parallaxing effect when mapped to the horizontal plane we measure on. The optimal position of the detector, and therefore also its optimal orientation angle, shift slightly when the crystal is rotated. Since we keep the position of the detector constant for this measurement, the count rate measured by the detector rises and falls due to this geometric fluctuation. To experimentally undo this parallaxing effect, the detector would have to be re-aligned with each crystal orientation angle.

This parallaxing is also a very likely explanation for the off-symmetry in the two peaks that we measure. The peaks are offset by 210 degrees, rather than 180 degrees as would perhaps be expected for a symmetric crystal lattice; and the magnitude of the two peaks differ by as much as $2\text{x}10^5$ counts per second. Since we expect the conical trajectory of the down-converted photons to precess some slight amount within with a full rotation, we can expect a very slight shift in optimal position and orientation of the detector in accordance with this precession at these anti-symmetric positions. The contribution of the small precession causes this optimal peak to completely miss the detector and appear as a low. However, we can capture a slightly different conic section by rotating the crystal slightly. It just so happens that an additional 30 degrees results in a conic section solution that intersects with our detector as configured; but the detector may still not be perfectly aligned, hence the decrease in overall magnitude for this secondary peak.

Regardless, this data certainly at least provides evidence for the existence and feasibility of measurement of down-converted photons which arise from the splitting of higher energy photons through a nonlinear optic.

b) Measuring Correlated Photon Pairs

Perhaps the most quintessential theme in the phenomenon of parametric downconversion is the fact that down-converted photons must come in entangled pairs to conserve energy and momentum. To test this theory, we will simultaneously measure the count rate from the supposedly-entangled signal and idler photons. The count rate of simultaneously occurring counts will be called "coincidence" counts. We hope to find that the measured coincidence count rate is statistically impossible to explain as mere accidental counts, and thus can only be explained through the downconversion phenomenon. First, we set up the experiment so that we measure downconverted signal photons from detector A, and idler photons from detector B. The count rate of all detectors in this configuration is below the form $x \pm \sigma_{stdev}$.

$$\textbf{Detector A:} \quad 6.27 \times 10^5 \pm 0.04 \times 10^5 \ s^{-1}$$
$$\textbf{Detector B:} \quad 2.12 \times 10^5 \pm 0.02 \times 10^5 \ s^{-1}$$
$$\textbf{Coincidence Counts:} \quad 1700 \pm 10 \ s^{-1}$$

The large discrepancy in the count rate between Detector A and B is most likely due to the several extra optical components that idler photons interact with to reach Detector B that the signal photons do not interact with to reach Detector A. Certainly, some experimental error comes from slight misalignment of these extra optics. This error does *not* come from significant differences in the sensitivity of the individual APD detectors for channel A and B used here. Our APD channels A and B were found to have roughly the same sensitivity when the same signal was analyzed on both detectors, so no multiplicative adjustment was applied for the following calculations.

To see if this coincidence count rate is meaningful, we analyze the probability of achieving this count rate from accidental coincidences alone. The FPGA we use for data acquisition in this experiment is configured to measure a coincidence count only when photons strike both detectors within a short total time window of duration 6ns. The expected number of accidental counts can then be found by multiplying the count rate of both detectors by the time window that the FPGA uses to correlate these coincidence counts. This yields:

$$\textbf{Expected Accidental Count Rate} = (6.27 \times 10^5 \ s^{-1}) \bullet (6 \ ns) \bullet (2.12 \times 10^5 \ s^{-1}) = 797.5 \ s^{-1}$$

Let A and B be the count rate of Detector A and B respectively, and let α be the accidental count rate. Also let σ_A, σ_B, and σ_α represent the standard deviation of these quantities. To propagate the error of our measurements, we can perform the following calculation:

$$\sigma_\alpha / \alpha = \sqrt{ (\sigma_A / A)^2 + (\sigma_B / B)^2 }$$

From our data, we calculate:

$$\sigma_\alpha = 9.08 \ s^{-1}$$

This standard deviation represents the standard amount we would expect our coincidence count rate to deviate from our calculated accidental count rate of 797.5 s^{-1}, given all counts are *purely accidental in nature*. However, our measured coincidence rate is 1700 s^{-1}. This value deviates from the calculated value by $99.4 \times \sigma_\alpha$. The probability of this occurring is < 0.0001%, meaning it is statistically impossible to ever occur. This provides concrete evidence that accidental coincidences alone could not possibly explain the measured count rate, and that the downconversion phenomenon in fact produces photons in entangled pairs which must coexist.

Next, we introduce a new detector, Detector C, and a beam splitter that can distribute photons between Detectors B and C. In theory, we expect to measure the coincidences between Detectors A and B *or* A and C. However, we expect a very low coincidence count rate between detectors B and C. This is because the single photon must ultimately choose a single path of propagation upon incidence of the beam splitter, and could not possibly traverse both simultaneously. Below is count rate data measured from this configuration. It is important to note that multiplicative adjustment factors of 1.9 and 1.7 were applied to the count rates for Detectors A and B, since the APD Detector C channel was found to be significantly more sensitive to photon counts.

Detector A: $17.06 \times 10^5 \pm 0.08 \times 10^5 \ s^{-1}$
Detector B: $4.27 \times 10^5 \pm 0.03 \times 10^5 \ s^{-1}$
Detector C: $1.09 \times 10^5 \pm 0.02 \times 10^5 \ s^{-1}$

A and B Coincidences: $2850 \pm 20 \ s^{-1}$
A and C Coincidences: $2200 \pm 20 \ s^{-1}$
B and C Coincidences: $340 \pm 5 \ s^{-1}$

Much like calculations before, we calculate a coincidence count rate based purely on accidental counts between Detectors B and C of:

$$\textbf{Expected Accidental Count Rate} = (4.27 \times 10^5\ s^{-1}) \bullet (6\ ns) \bullet (1.09 \times 10^5\ s^{-1}) = 279.2\ s^{-1}$$

Clearly there is some experimental error, but we see that, as expected, the number of counts corresponding to AB and AC coincidences is roughly equal. This provides evidence for photon current traveling down both paths with roughly equal probability. Furthermore, the count rate measured for BC coincidences is nearly the same as the calculated accidental coincidences expected between Detectors B and C. This means that the non-zero BC coincidence count rate we measured could quite nearly be explained as an accidental phenomenon alone. This provides evidence for individual photons in fact exclusively selecting one path or the other through the beam splitter, but not both.

Section 4 - Conclusion

This experimental investigation into parametric downconversion has yielded insightful results affirming the quantum entanglement of correlated photon pairs. The dependence of crystal orientation on indices of refraction demonstrated the importance of precise alignment for optimal downconversion efficiency for our nonlinear crystal. The experimental setup, featuring a detector based on Avalanche Photodiodes, accurately detected the 810 nm downconverted photons emitted from a 405 nm laser incident on a downconverting crystal. Through repeated optimization of crystal orientation, two distinct peaks in the count rate demonstrated the crystal's sensitivity to its fine-orientation. Coincidence measurements further substantiated the nature of the down converted photons. These observations provided crucial evidence supporting the entangled nature of the down converted photons.