

embedded scripting language

Fixed Point Support Library

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Abstract

The "PAWN" programming language has only native support for 32-bit integers. This document describes an extension to the PAWN run-time environment that adds support for "fixed point" rational values.

The software that is associated with this application note can be obtained from the company homepage, see section "Resources"

Introduction	1
Implementing the library	. 4
Usage	. 5
Native functions	. 6
CUSTOM OPERATORS	.8
RESOURCES	. 9
Index	11

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Introduction

The "PAWN" programming language is a simple C-like extension/scripting language. The only data type that it supports is a 32-bit integer, called a *cell*. The PAWN programming language is described in its manual and it is freely available; see the section "Resources" for more information. This Fixed Point support library adds "fixed point decimal arithmetic" and fixed point rational values to the "PAWN" programming language.

In computer applications, rational values have limited precision, regardless of how they are implemented. The advantage of fixed point arithmetic over the more common floating point arithmetic is that fixed point defines the precision in an exact and intuitive manner.

It is well known, for example, that the value 0.1 cannot be represented exactly in the floating point format standardized in IEEE 754* —which is the ubiquitous floating point format. This imperfection leads to the following PAWN program to give a surprising result:

Listing floating point arithmetic example, giving a surprising result

```
#include <float>
main()
   {
   new Float: a = 0.0
   new Float: b = 1.0

   for (new i = 0; i < 10; i++)
        a += 0.1

   if (a == b)
        printf("%f and %f are equal\n", a, b)
   else
        printf("%f is not the same as %f\n", a, b)
}</pre>
```

The above program is in PAWN, but the phenomenon is shared by all languages that use the IEEE 754 floating point format.[†] However, if the program were

^{*} IEEE 754 implements binary floating point and the value 0.1 is an infinitely recurring fraction in binary base.

I have verified this with a similar program in C, using both the single-precision and double-precision floating point formats (see section "Resources" for the source code). A tutorial for the Python language also mentions this pitfall and amplifies that it is "not a bug" in Python.

translated to use the fixed point library instead of the floating point library, it would give the intuitive (and correct!) answer that ten times 0.1 is indeed 1.0.

The above program shows that tiny errors in floating point values can accumulate to larger, noticeable errors after repeated operations. Sometimes, the intermediate incorrectness is hidden by extra "guard digits" in internal CPU registers and rounding in library functions. However, even single operations can cause errors. From the "Decimal Arithmetic FAQ, Part 1" (see section "Resources") comes the following example:

Consider the calculation of a 5% sales tax on an item (such as a \$0.70 telephone call), which is then rounded to the nearest cent. Using double binary floating-point, the result of $$0.70 \times 1.05$ is 0.7349999999999998667732370449812151491641998291015625; the result should have been <math>0.735 (which would be rounded up to \$0.74) but instead the rounded result would be \$0.73

Fixed point arithmetic, and especially *decimal* arithmetic is appropriate for applications where the precision is important and all numbers lie in a limited range; commercial and financial applications are a prime example, but graphic/programs also benefit from fixed point decimal arithmetic.

The fixed point format used in this library uses three decimal digits and stores the values in two's complement, giving a range of -2147483 to +2147482 with 3 digits behind the decimal point. Fixed point arithmetic also goes by the name "scaled integer" arithmetic. Basically, a fixed point number is the numerator of a fraction where the denominator is implied. For this library, the denominator is 1000—therefore, the integer value 12345 stands for $\frac{12345}{1000}$ or 12.345.

In rounding behaviour, however, there is a subtle difference between fixed point arithmetic and straight-forward scaled integer arithmetic: in fixed point arithmetic, it is usually intended that the least significant digit should be rounded before any subsequent digits are discarded; but many scaled integer arithmetic implementations just "drop" any excess digits. In other words, $\frac{2}{3}$ in fixed point arithmetic results in 0.667, which is more accurate than the scaled integer result of 0.666.

Apart from the decimal arithmetic model (used by this library), there are also fixed point packages that use a binary model, where the fractional part is specified in the number of "bits" rather than "digits". The binary model has significantly different properties than the decimal model, and it is used for entirely different purposes. In discussions on fixed point arithmetic (e.g. versus floating-point arithmetic), one should be careful about the radix of the fixed point format that is discussed. As a side note, fixed point decimal arithmetic is sometimes (incorrectly) referred to as

"BCD" arithmetic —the Binary Coded Decimals format stores one digit in each nibble (half byte, or four bits). Fixed-point arithmetic and BCD format may be combined, but the two are not the same. As already noted, this library uses two's complement rather than BCD.

This appendix assumes that the reader understands the PAWN language. For more information on PAWN, please read the manual "The PAWN booklet — The Language" which is available from the company homepage.

Implementing the library

The fixed point support library consists of the files FIXED.C and FIXED.INC. The C file may be "linked in" to a project that also includes the PAWN Abstract Machine (AMX.C), or it may be compiled into a DLL (Microsoft Windows) or a shared library (Linux). The .INC file contains the definitions for the PAWN compiler of the native functions in FIXED.C, as well as several user-defined operators. In your PAWN programs, you may either include this file explicitly, using the #include preprocessor directive, or add it to the "prefix file" for automatic inclusion into any PAWN program that is compiled.

The FIXED.INC also sets the rational number format for the PAWN compiler to a fixed point number with three decimal digits (using #pragma rational). This may lead to a conflict if a different rational number format was already set. Specifically, you may not be able to use this fixed point extension module together with a floating point module. Such conflicts can be resolved by removing the #pragma rational directive from either module.

The "Implementor's Guide" for the PAWN toolkit gives details for implementing the extension module described in this application note into a host application. The initialization function, for registering the native functions to an abstract machine, is amx_FixedInit and the "clean-up" function is amx_FixedCleanup. In the current implementation, calling the clean-up function is not required.

If the host application supports dynamically loadable extension modules, you may alternatively compile the C source file as a DLL or shared library. No explicit initialization or clean-up is then required. Again, see the Implementor's Guide for details.

The extension module AMXCONS.C (console input/output) has some support for fixed point values. You have to enable this support by compiling the file with the FIXEDPOINT macro defined.

Usage

Depending on the configuration of the PAWN compiler, you may need to explicitly include the FIXED.INC definition file. To do so, insert the following line at the top of each script:

```
#include <fixed>
```

The #pragma rational setting in FIXED.INC allows you to specify rational literal numbers directly. For example:

```
new Fixed: amount = 123.45
amount += 78.90
```

To convert from integers to fixed point values, use one of the functions fixed or fixedstr. The function fixed creates a fixed point number with the same integral value as the input value and a fractional part of zero. Function fixedstr makes a fixed point number from a string, which can include a fractional part.

A user-defined assignment operator is implemented to automatically coerce integer values on the right hand to a fixed point format on the left hand. That is, the lines:

```
new a = 10
new Fixed: b = a
are equivalent to:
new a = 10
new Fixed: b = fixed(a)
```

To convert back from fixed point numbers to integers, use the functions fround and ffract. Function fround is able to round upwards, to round downwards, to "truncate" and to round to the nearest integer. Function ffract gives the fractional part of a fixed point number, but still stores this as a fixed point number.

The common arithmetic operators: +, -, * and / are all valid on fixed point numbers, as are the comparison operators and the ++ and -- operators. The modulus operator % is forbidden on fixed point values.

The arithmetic operators also allow integer operands on either left/right hand. Therefore, you can add an integer to a fixed point number (the result will be a fixed point number). This also holds for the comparison operators: you can compare a fixed point number directly to an integer number (the return value will be true or false).

Native functions

Fixed:fixed(value)

Create a fixed point number with the same (integral) value as the parameter value.

Fixed:fixedstr(const string[])

Create a fixed point number from a string. The string may specify a fractional part, e.g., "123.45".

Fixed:fmul(Fixed:oper1, Fixed:oper2)

Multiply two fixed point numbers. The user-defined * operator forwards to this function.

Fixed:fdiv(Fixed:dividend, Fixed:divisor)

Fixed point division. The user-defined / operator forwards to this function.

Fixed:fmuldiv(Fixed:oper1, Fixed:oper2, Fixed:divisor)

Multiply two fixed point numbers, then divide it by a third number ("divisor"). This routine avoids rounding to a fixed number of decimals halfway the multiply and divide. Therefore, the result of fmuldiv(a, b, c) may have higher precision than "(a * b) / c".

Fixed:ffract(Fixed:value)

Returns the fractional part of value, in fixed point format. For example, if the input value is "3.14", ffract returns "0.14".

fround(Fixed:value, fround_method:method=fround_round)

Round a fixed point number and return the value as an integer. The rounding method may be one of:

fround_round round to the nearest integer, where a fractional part of exactly 0.5 rounds upwards (this is the default);

fround_floor round downwards;
fround_ceil round upwards;

fround_tozero round downwards for positive values and upwards for negative values ("truncate");

fround_unbiased round to the nearest *even* integer number when fractional part is exactly 0.5 (the values "1.5" and "2.5" both round to "2"). This is also known as "Banker's rounding".

When rounding negative values upwards or downwards, note that -2 is considered smaller than -1.

Fixed:fpower(Fixed:value, exponent)

Returns value raised to the power exponent. The base value and the function result are in fixed point format, but the exponent must be an integer value. The exponent may be zero or negative.

For exponents higher than 2 and fractional values, the fpower function may have higher precision than repeated multiplication, because the function tries to calculate with an extra digit. That is, the result of fpower(3.142, 4) is probably more accurate than 3.142 * 3.142 * 3.142 * 3.142.

Fixed:fsqroot(Fixed:value)

Returns the square root of the input number.

Fixed:fabs(Fixed:value)

Returns the absolute value of the input value.

Custom operators

All custom operators are declared "native" or "stock". Operators that you do not use in your script take no space in the P-code file.

```
Fixed:operator*(Fixed:oper1, Fixed:oper2)
Fixed:operator/(Fixed:oper1, Fixed:oper2)
Fixed:operator=(oper)
Fixed:operator++(Fixed:oper)
Fixed:operator—(Fixed:oper)
Fixed:operator*(Fixed:oper1, oper2)
                                          ("*" is commutative)
Fixed:operator/(Fixed:oper1, oper2)
Fixed:operator/(oper1, Fixed:oper2)
                                          ("+" is commutative)
Fixed:operator+(Fixed:oper1, oper2)
Fixed:operator-(Fixed:oper1, oper2)
Fixed:operator-(oper1, Fixed:oper2)
bool:operator>(Fixed:oper1, oper2)
bool:operator>(oper1, Fixed:oper2)
bool:operator>=(Fixed:oper1, oper2)
bool:operator>=(oper1, Fixed:oper2)
bool:operator<(Fixed:oper1, oper2)
bool:operator<(oper1, Fixed:oper2)
bool:operator<=(Fixed:oper1, oper2)
bool:operator<=(oper1, Fixed:oper2)
                                          ("==" is commutative)
bool:operator==(Fixed:oper1, oper2)
                                          ("!=" is commutative)
bool:operator!=(Fixed:oper1, oper2)
```

Resources

The PAWN toolkit can be obtained from **www.compuphase.com** in various formats (binaries and source code archives). The manuals for usage of the language and implementation guides are also available on the site in Adobe Acrobat format (PDF files).

The limitations of IEEE 754 floating point arithmetic are well documented, but not very widely known. An introductory article on the pitfalls of floating point arithmetic is "The Perils of Floating Point" by Bruce M. Bush, available on www.lahey.com/float.htm. A more in-depth article, which is the source of one of the examples in this application note, is IBM's "Decimal Arithmetic FAQ, Part 1" at www2.hursley.ibm.com/decimal/decifaq1.html.

The source code in C that demonstrates the limited precision of IEEE 754 floating point arithmetic (using double precision floating point) is below. The "volatile" variable attribute is to avoid the compiler from "hiding" the error from view: an optimizing C/C^{++} compiler may keep the accumulated result in an internal processor register and avoid the incremental truncation error —a highly optimizing C/C^{++} compiler might even detect the pattern and set "a" to 1.0 without looping at all.

Listing floating point surprise, in C

```
#include <stdio.h>
{
    volatile double a = 0.0;
    volatile double b = 1.0;
    int i;
    for (i = 0; i < 10; i++)
        a += 0.1;
    if (a == b)
        printf("%f and %f are equal\n", a, b);
    else
        printf("%f is not the same as %f\n", a, b);
    return 0;
}</pre>
```

Index

- \diamond Names of persons (not products) are in italics.
- ⋄ Function names, constants and compiler reserved words are in typewriter font.

! A	#pragma rational, 4, 5 Absolute value, 7	fabs, 7 fdiv, 6 ffract, 5, 6 fixed, 5, 6 fixedstr, 5, 6
D	Abstract Machine, 4 Adobe Acrobat, 9	Floating point, 1, 2, 9 fmul, 6 fmuldiv, 6 Forbidden operators, 5
В	Banker's rounding, 6 Base 10, See Decimal arithmetic Base 2, See Binary arithmetic Binary arithmetic, 2 Binary Coded Decimals, 2 Bush, B.M., 9	fpower, 7 fround, 5, 6 fsqroot, 7 Host application, 4
C		L IEEE 754, 1, 9 Linux, 4 Literal numbers, 5
D	Decimal arithmetic, 2 DLL, 4	Modulus, 5
_	Dropped digits, 2 Dynamically loaded module, 4	Native functions, 4 registering, 4 Nibble, 2
Е	Exponentiation, 7 Extension module, 1, 4 dynamically loaded, 4	Operators forbidden, 5 user-defined, 4, 5, 8

P	Precision, 1 Prefix file, 4	Scaled integer, 2 Shared library, 4	
	Preprocessor directive, 4 Python, 1	Square root, 7	
R	Registering, 4 Rounding, 2	T Two's complement, 2 U User-defined operators, 4, 5, 8	