Flight Efficiency in European Airspace

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Outline

1. The Problem
2. Data analysis algorithms
3. Optimisation strategies
   - Discrete approach
   - Continuous problem approach

- Busy European airspace
- Civil aircraft routing inefficiencies:
    - Military airspace
    - Waypoints
- Investigate optimisation strategies
    - Flexible time access to military airspace
    - Modification of military airspace
    - Quantification of benefits associated with each strategy

Sample data:

- Over $2,000$ European airports
- Circa $33,000$ flights from a single day (departure airport/time, arrival airport/time, cruising altitude)
- Military airspace (location, altitudes)

Cost

- Costs based on path flown
- Fuel costs
- Delay costs

Data: Number of intersections

- Plot great circle of flight routes, assume uniform speed and height
- Plot paths of intervals of half an hour
- Assume all military airbases are available
- Work out number of intersections with each military airbase in that half an hour
- Plot flights paths for a given day: movie
- To represent data we colour each airbase according to number of intersections; see movie!

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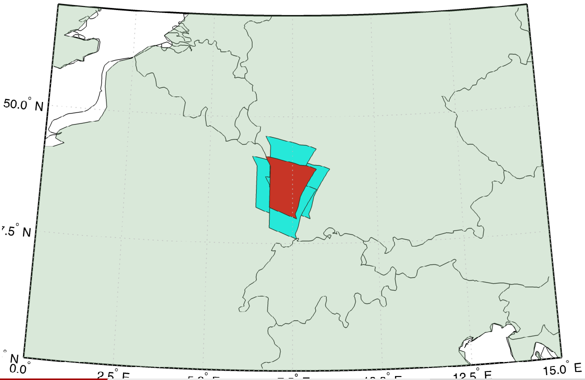
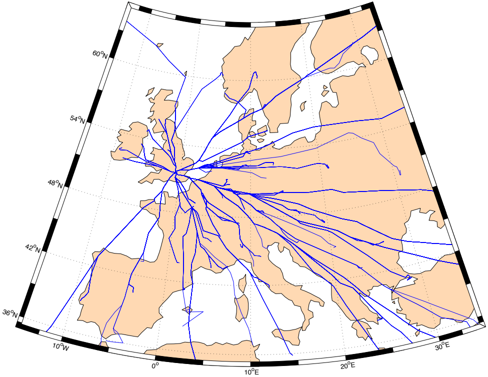
Moving airbases

- Investigate effect of moving one airbase by small amount of latitude/longitude
- Corresponds to moving airbases by about 30km along compass points as seen below



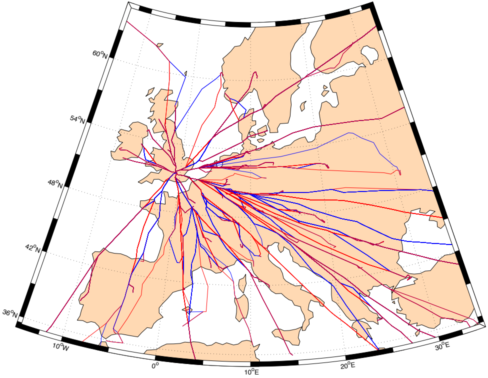
Optimal routing using actual waypoints/segments

Flights from LHR avoiding military space (blue):

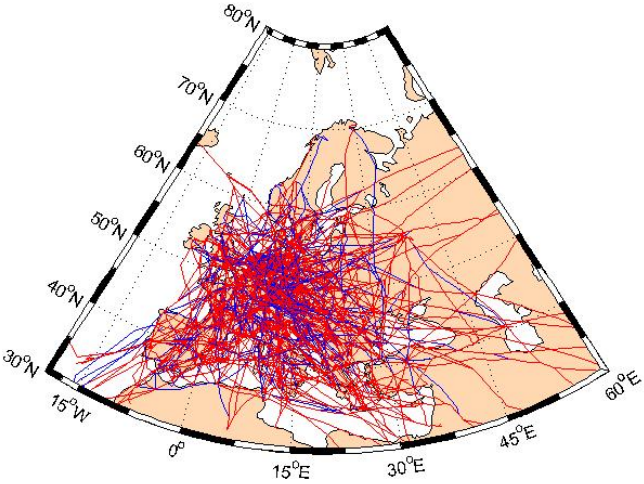


Optimal routing using actual waypoints/segments

Flights from LHR allowing access to all military space (red):



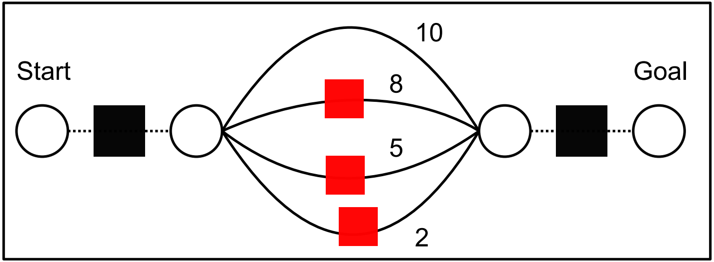
And now on the whole of Europe . . .



Method explicitly outputs a cost saving of $30 per flight.

Discrete path planning

- What is the cheapest path from A to B?



- Dijkstra's algorithm (or A*)
- How to calculate gains from removing any one obstacle without having to re-run the search for each case?

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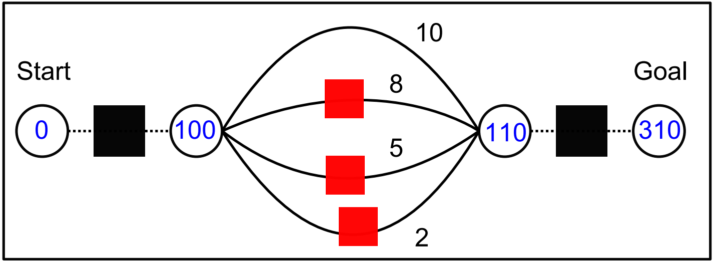

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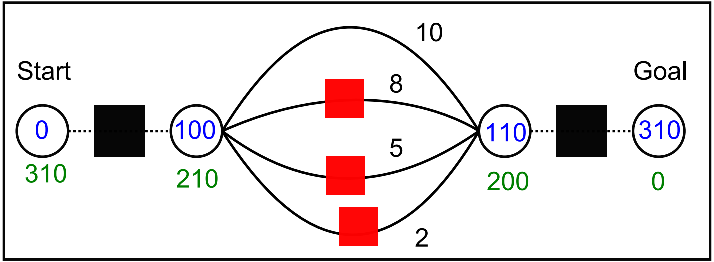

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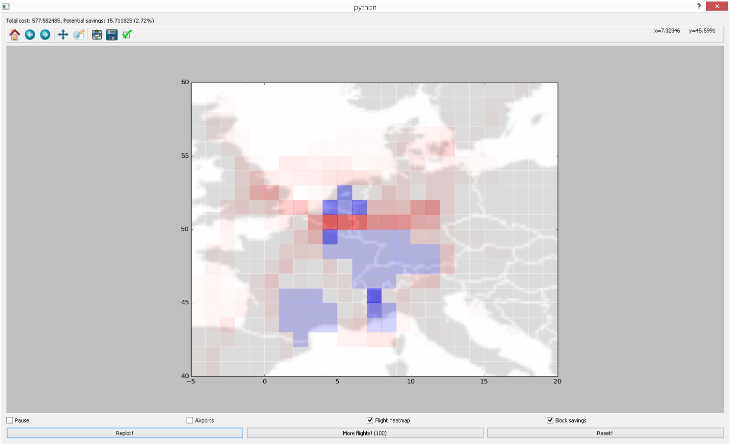

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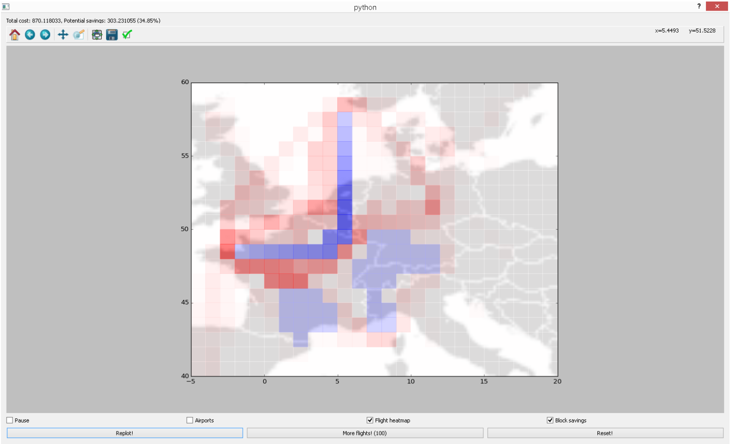


Linear programming

The shortest path problem is

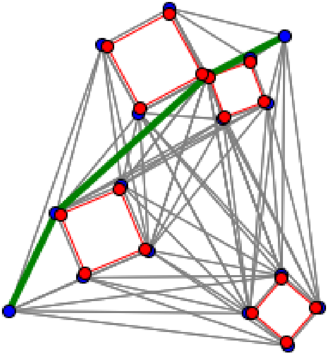
$$\min_{x_{i,j,f}} \sum_{i,j} C_{i,j,f} x_{i,j,f} \tag{1}$$

subject to:

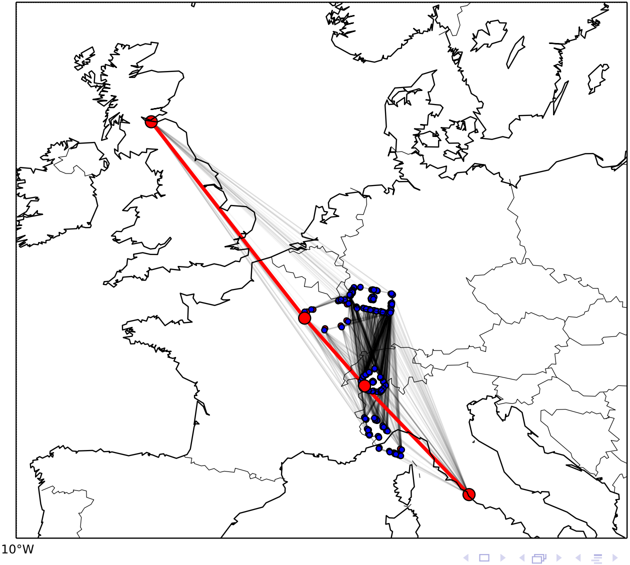
$$\sum_j x_{i,j,f} - x_{j,i,f} = \begin{cases} 1, & \text{if } i = \text{departure airport for flight } f, \\ -1, & \text{if } i = \text{arrival airport for flight } f, \\ 0, & \textit{otherwise}. \end{cases} \quad \forall\ i, f. \tag{2}$$

$$x_{i,j,f} \geq 0, \ \forall\ i, j, f. \tag{3}$$

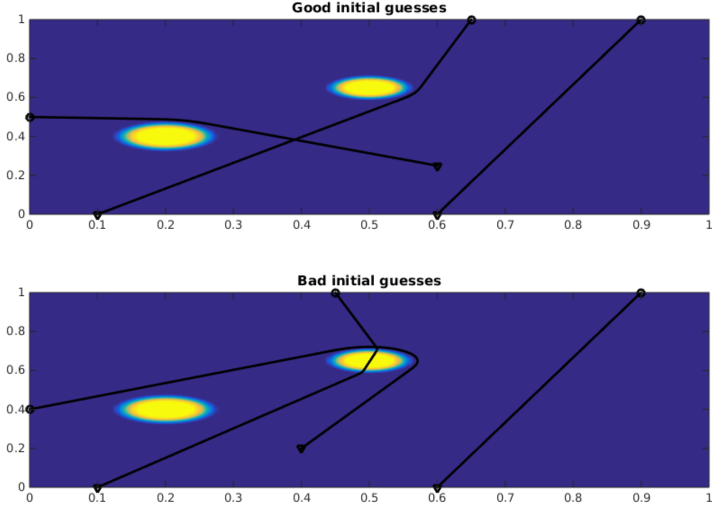
Shortest Path on Visibility Graph – Toy Examples



Visibility Graph for Europe Data



Continuous pathfinding approach



Continuous pathfinding approach

Curve $\gamma : [0, 1] \to \mathbb{R}^2$, such that $\gamma(0) = \mathbf{0}$, $\gamma(1) = \mathbf{x}$,
Cruise speed v_c, Wind velocity field $\mathbf{w} : \mathbb{R}^2 \to \mathbb{R}^2$.
Parametrisation: $\gamma(t) = t\mathbf{x} + a(t)\mathbf{x}^\perp, \quad a(0) = a(1) = 0.$

$$\mathrm{Time}[a] = T[a] = \int_0^1 \frac{\sqrt{1 + \dot{a}(t)^2}}{v(t, a, \dot{a}, \mathbf{w})} \, \mathrm{d}t,$$

where

$$v(t) = \left(|v_c|^2 + 2(\mathbf{x} + \dot{a}(t)\mathbf{x}^\perp) \cdot \mathbf{w}(t\mathbf{x} + a(t)\mathbf{x}^\perp) - |\mathbf{w}(t\mathbf{x} + a(t)\mathbf{x}^\perp)|^2 \right)^{1/2}.$$

Euler-Lagrange equation: subject to $a(0) = a(1) = 0$.

$$\frac{d}{dt} \left(\frac{\dot{a}(t)}{v(t)\sqrt{1 + \dot{a}(t)^2}} - \frac{2\mathbf{x}^\perp \cdot \mathbf{w}(t\mathbf{x} + a(t)\mathbf{x}^\perp)}{v(t)^3} \right) + \frac{2(\mathbf{x} + \dot{a}(t)\mathbf{x}^\perp - \mathbf{w}(t\mathbf{x} + a(t)\mathbf{x}^\perp)) \cdot \nabla \mathbf{w}(t\mathbf{x} + a(t)\mathbf{x}^\perp) \cdot \mathbf{x}^\perp}{v(t)^3} = 0.$$

Future ideas

- Find the path of several flights at once
- Include sector capacity constraints
- Include scheduling constraints
- Last two options require to include time in the optimisation.
- Optimize overall network efficiency.

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