Noise Generation by Aircraft

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Introduction

Although modern aircraft are now significantly quieter than early aircraft, there is still room for improvement. With demand for aircraft travel ever increasing, regulations controlling the effective perceived noise level are becoming tighter. The regulations on aircraft engines are becoming stricter in two aspects, the noise generated and fuel efficiency.

The main contributions to the total noise of the aircraft come from the engine (fan, compressor, combustor, turbine and jet) and the airframe itself. Our aim is to study the noise from the fan of the engine and the noise from the leading (front) edge of the wing.

We can use the formula $L = 20\log_{10}(p/p_{ref})$ to calculate the sound level in decibels, where p_{ref} is usually $20dB$.

Fan Noise

Fan Noise comes from the aerodynamic interaction between the fan blades and stators as air flows between them, and is influenced by many factors such as position and number of blades.

We firstly model the engine as an infinite duct and the blades as some rotating surface. We also assume in our model that the air has nice properties such as being inviscid, compressible and a perfect gas. The air approximately moves (or has a base flow) in a helix since it has velocity from the rotating fan blades and aircraft moving through the air. We assume the duct walls can have lining on them to absorb and reduce some of the noise.

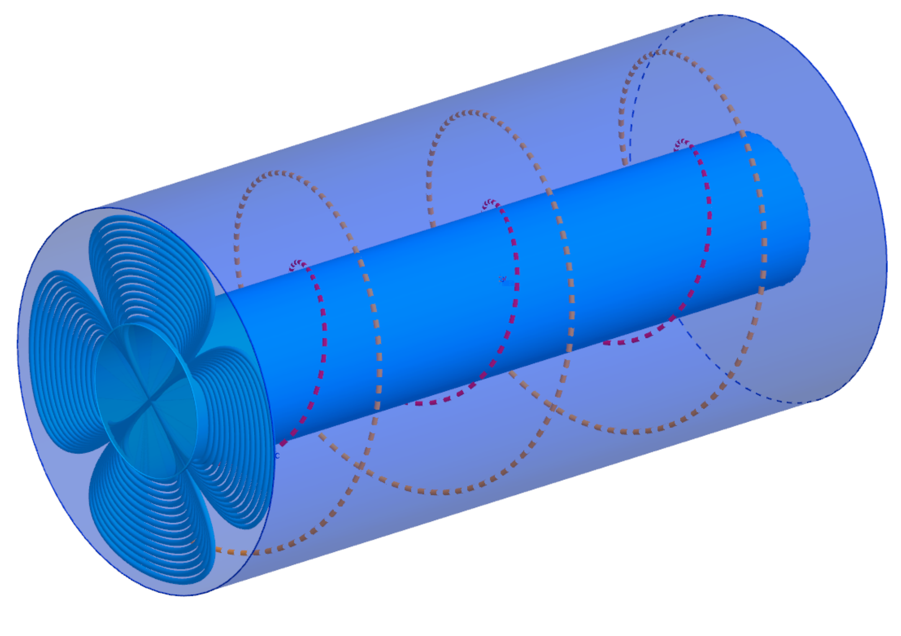


Fig. 2: Our model of the engine

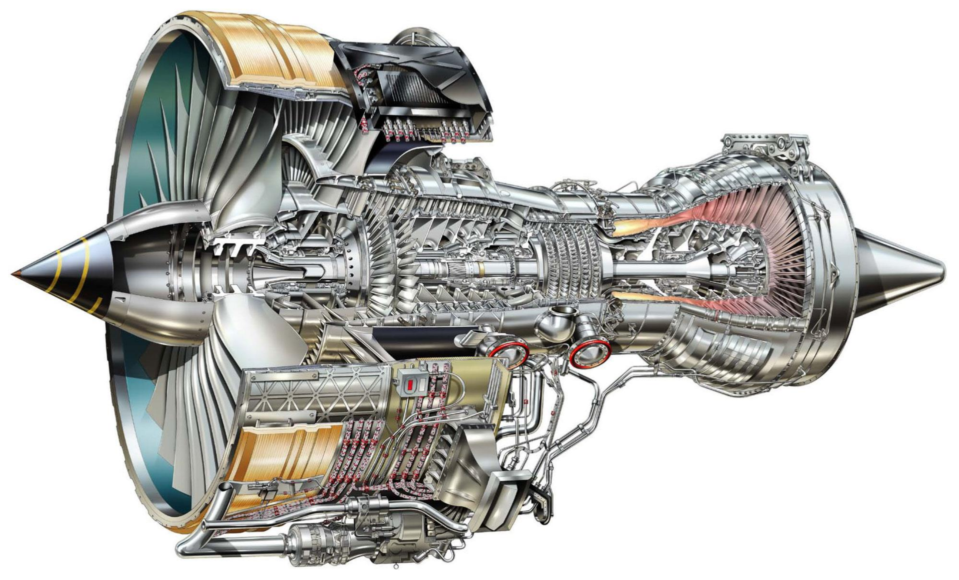


Fig. 1: Rolls Royce Trent 700 Engine

We want to solve the Navier–Stokes differential equations by assuming that the air looks like:

$$\begin{aligned} \bar{u} &= U + u \quad \text{velocity} \\ \bar{p} &= P + p \quad \text{pressure} \\ \bar{\rho} &= R + \rho \quad \text{pressure} \end{aligned}$$

U, P and R come from the helix shape that the air approximately takes, and we assume are much larger than u, p and ρ.

This gives us a differential equation to solve for the pressure $\bar{p}$, given by

$$\mathcal{F}(\bar{p}) = \mathcal{S}, \tag{1}$$

where $\mathcal{F}$ is a sixth order partial differential equation and $\mathcal{S}$ is a source term which depends on factors such as the blade shape and position.

Many people are using CFD (computational fluid dynamics) to solve (1), which involves solving it numerically. This often takes a long time and can be hard to understand the effects of different parameters. We wish to find an analytic answer, so we can write down the solution of the equation.

To solve the equation analytically, we assume that the frequency of the blades is large (since they are rotating very fast!). We then introduce a Fourier transform which reduces the equation to an approximate second order ordinary differential equation using the WKB method, which is approximately of the form

$$p''(r) + q(r,k)p(r) = 0,$$

where q depends on both the frequency of the blades k and the duct radius r. The behaviour of q can be complicated depending on the parameters, and some example of q are below. Once we know the behaviour of q we can compute p.

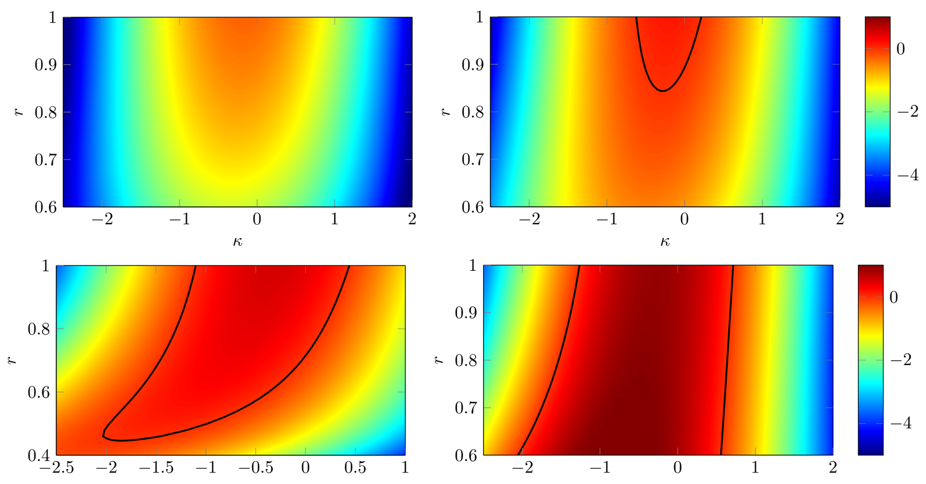
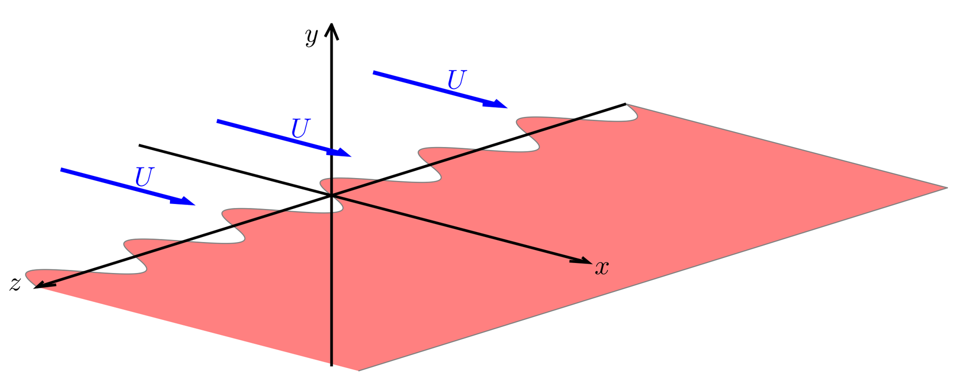


Fig. 5: Contour plots of q, black line is where $q = 0$.

Aerofoil Noise

Aerofoil Noise comes from the interaction between the leading edge and trailing (rear) edge with the air as it flows either side of the wing. We are interested in what happens to the noise if we introduce a serration on the usually straight leading edge, for example a sine wave.



We begin by approximating the aerofoil as a thin plate defined by $x > f(z)$ for some function f, as seen in Figure.

The inspiration for using the serrations comes from the shape of a whales fin as seen below.

Fig. 4: http://tinyurl.com/whalefin

To calculate the pressure from the wing we use a formula by Howe involving Green's functions.

We model the air having velocity

$$\boldsymbol{u} = (U, 0, 0) + \nabla \times \Phi,$$

where Φ is a sum over N eddies, whose given by

$$\Phi(x,t) = \sum_{i=1}^{N} A_i e^{-\alpha_i r_i}\boldsymbol{e}_x + B_i e^{-\beta_i r_i}\boldsymbol{e}_y + C_i e^{-\gamma_i r_i}\boldsymbol{e}_z$$

with

$$r_i(\boldsymbol{x}, t) = (x - x_i - Ut)^2 + (y - y_i)^2 + (z - z_i)^2.$$

We calculate the pressure for a single eddy below and right, and also when the wing is at an angle of attack.

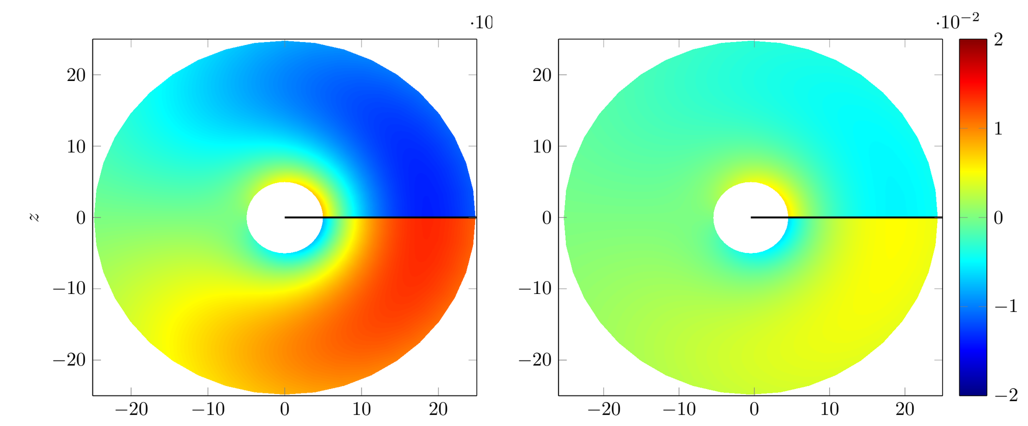
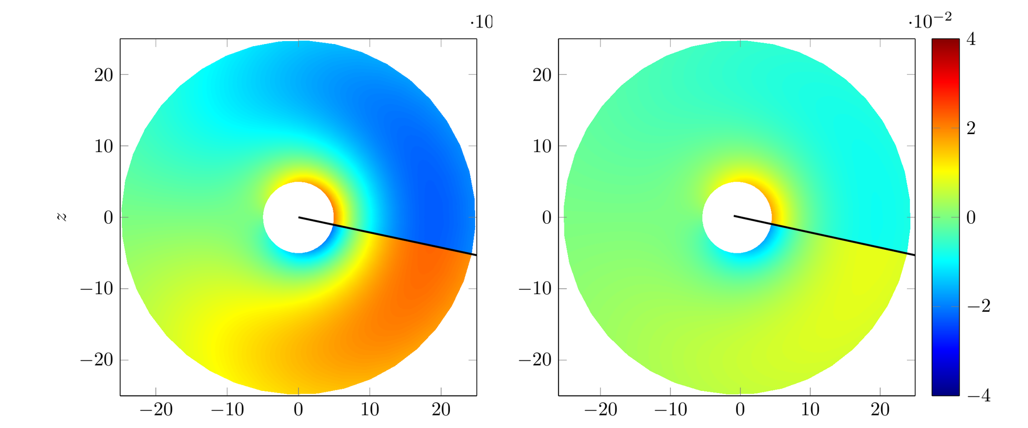


Fig. 6: Effect of serrated edges and angle of attack on the pressure. Left pictures are with a straight edge and right pictures have serrations $f(z) = 0.5\sin(z)$. Pictures above are at $\alpha = 0$ angle of attack and the ones below are at $\alpha = \pi/15$ (12°)



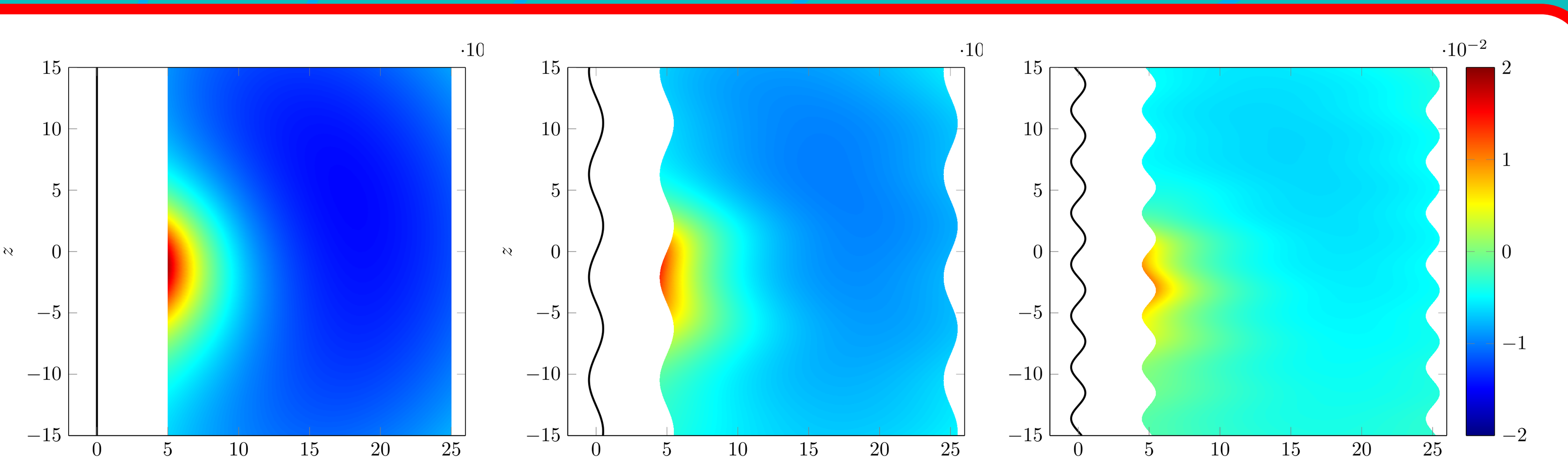


Fig. 8: Pressure at zero angle of attack with serrations of the form $f(z) = A\sin(0.5x)$, where $A = 0$ (left), $A = 0.75$ (centre) and $A = 1.5$ (right).

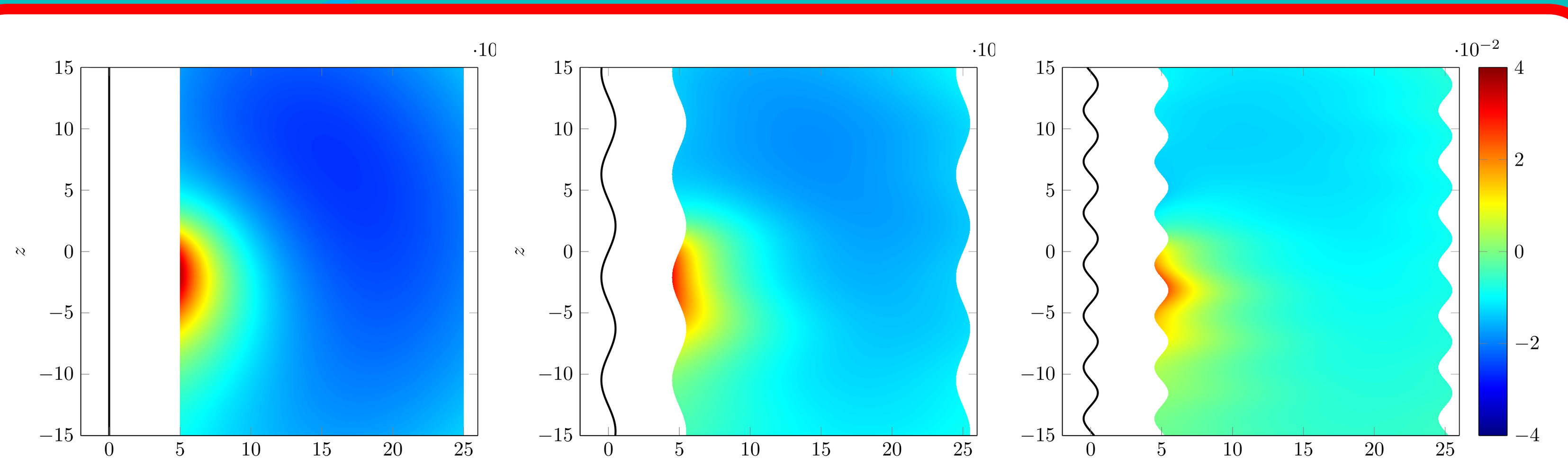


Fig. 9: Pressure at angle of attack $\alpha = \pi/15$ (12°) with serrations of the form $f(z) = A\sin(0.5x)$, where $A = 0$ (left), $A = 0.75$ (centre) and $A = 1.5$ (right).

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