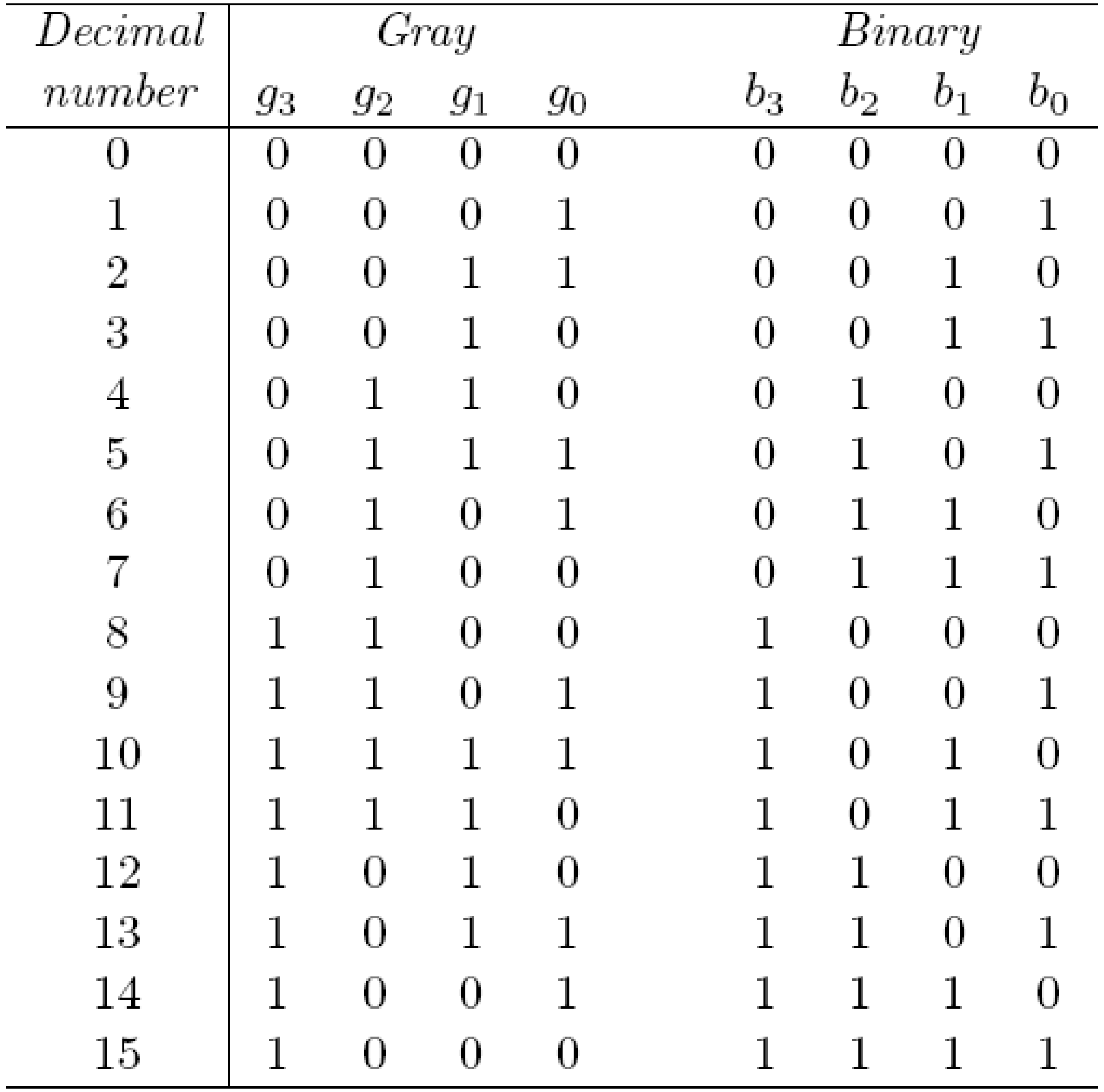
Karnaugh Maps (K-Maps)

Gray Code



Decimal number	*Gray* g_3	g_2	g_1	g_0	*Binary* b_3	b_2	b_1	b_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	1	0	0	1	0
3	0	0	1	0	0	0	1	1
4	0	1	1	0	0	1	0	0
5	0	1	1	1	0	1	0	1
6	0	1	0	1	0	1	1	0
7	0	1	0	0	0	1	1	1
8	1	1	0	0	1	0	0	0
9	1	1	0	1	1	0	0	1
10	1	1	1	1	1	0	1	0
11	1	1	1	0	1	0	1	1
12	1	0	1	0	1	1	0	0
13	1	0	1	1	1	1	0	1
14	1	0	0	1	1	1	1	0
15	1	0	0	0	1	1	1	1

K-Maps

- A visual way to simplify logic expressions
- It gives the most simplified form of the expression
- A Karnaugh map is a graphical method used to obtained the most simplified form of an expression in a standard form (Sum-of-Products or Product-of-Sums).
- The simplest form of an expression is the one that has the minimum number of terms with the least number of literals (variables) in each term, thus the function will be implemented with the minimum number of gates with minimum number of input.

Three-Variable K-Maps

$$f = \sum(0,4) = \overline{B}\,\overline{C}$$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**0**
1	**1**	**0**	**0**	**0**

$$f = \sum(4,5) = A\,\overline{B}$$

A \ BC	00	01	11	10
0	**0**	**0**	**0**	**0**
1	**1**	**1**	**0**	**0**

$$f = \sum(0,1,4,5) = \overline{B}$$

A \ BC	00	01	11	10
0	**1**	**1**	**0**	**0**
1	**1**	**1**	**0**	**0**

$$f = \sum(0,1,2,3) = \overline{A}$$

A \ BC	00	01	11	10
0	**1**	**1**	**1**	**1**
1	**0**	**0**	**0**	**0**

$$f = \sum(0,4) = \overline{A}\,C$$

A \ BC	00	01	11	10
0	**0**	**1**	**1**	**0**
1	**0**	**0**	**0**	**0**

$$f = \sum(4,6) = A\,\overline{C}$$

A \ BC	00	01	11	10
0	**0**	**0**	**0**	**0**
1	**1**	**0**	**0**	**1**

$$f = \sum(0,2) = \overline{A}\,\overline{C}$$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**1**
1	**0**	**0**	**0**	**0**

$$f = \sum(0,2,4,6) = \overline{C}$$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**1**
1	**1**	**0**	**0**	**1**

Three-Variable K-Maps

A \ BC	00	01	11	10
0		1		
1	1		1	1

A \ BC	00	01	11	10
0	1		1	1
1	1			1

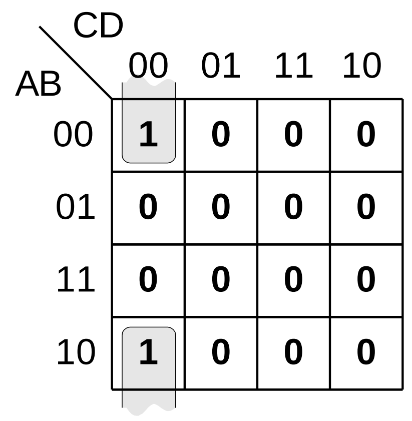
A \ BC	00	01	11	10
0			1	1
1	1	1		

A \ BC	00	01	11	10
0			1	
1	1		1	1

A \ BC	00	01	11	10
0		1	1	1
1		1	1	

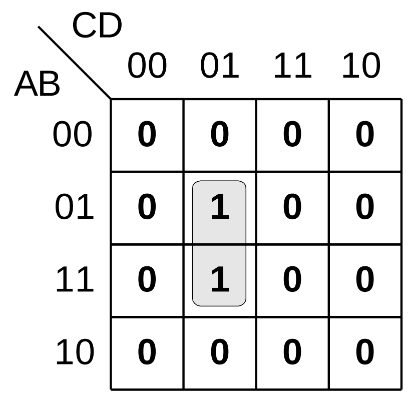
A \ BC	00	01	11	10
0				
1				

Four-Variable K-Maps



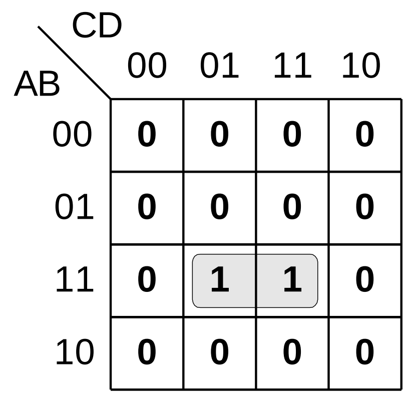
AB \ CD	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	0	0	0

$$f = \sum(0,8) = \overline{B} \bullet \overline{C} \bullet \overline{D}$$



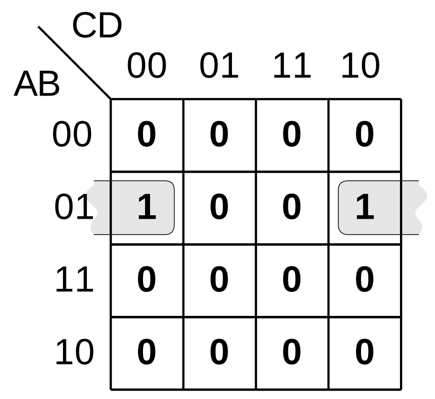
AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	0	0
10	0	0	0	0

$$f = \sum(5,13) = B \bullet \overline{C} \bullet D$$



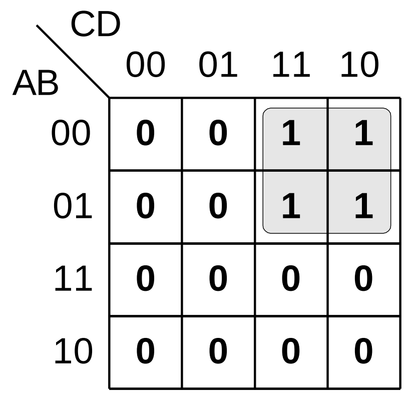
AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	1	0
10	0	0	0	0

$$f = \sum(13,15) = A \bullet B \bullet D$$



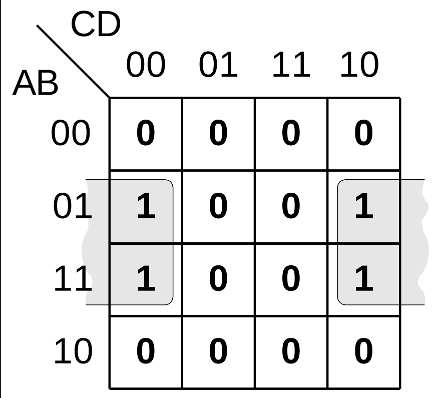
AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(4,6) = \overline{A} \bullet B \bullet \overline{D}$$



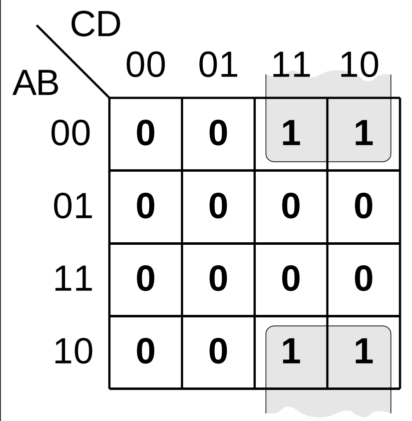
AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(2,3,6,7) = \overline{A} \bullet C$$



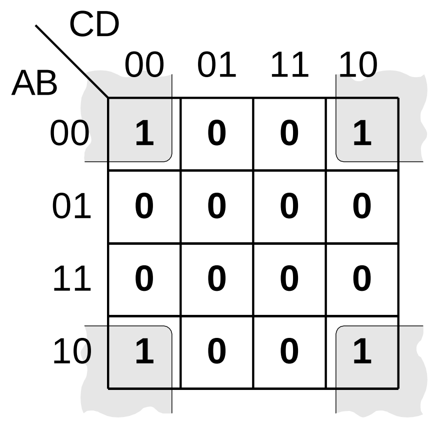
AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$$f = \sum(4,6,12,14) = B \bullet \overline{D}$$



AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

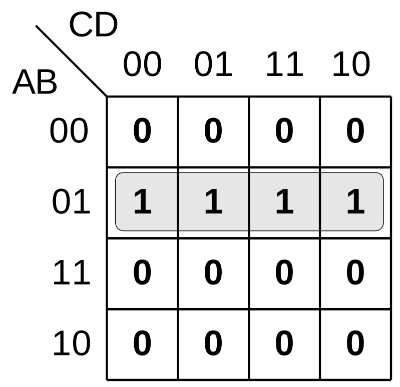
$$f = \sum(2,3,10,11) = \overline{B} \bullet C$$



AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

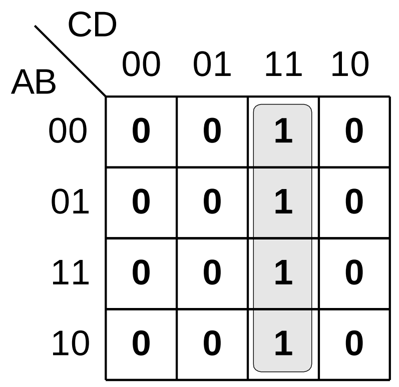
$$f = \sum(0,2,8,10) = \overline{B} \bullet \overline{D}$$

Four-Variable K-Maps



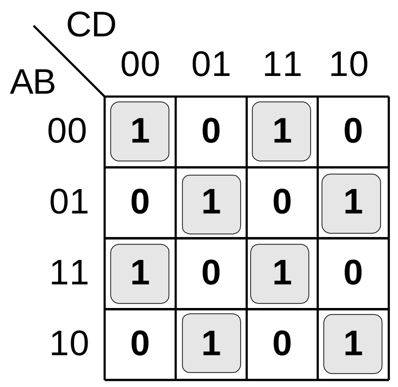
AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$f = \sum(4,5,6,7) = \overline{A} \bullet B$



AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

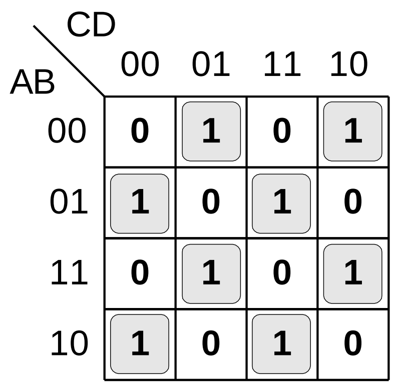
$f = \sum(3,7,11,15) = C \bullet D$



AB \ CD	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

$f = \sum(0,3,5,6,9,10,12,15)$

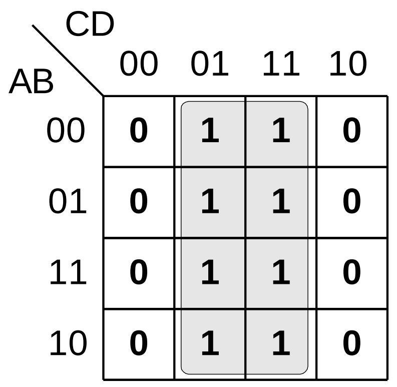
$f = A \otimes B \otimes C \otimes D$



AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$f = \sum(1,2,4,7,8,11,13,14)$

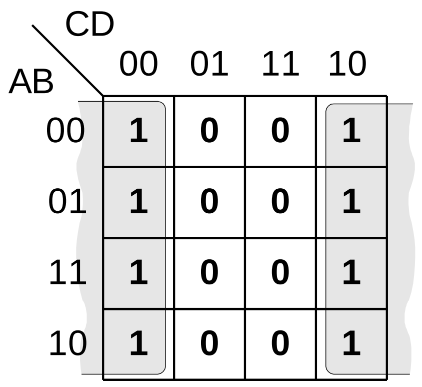
$f = A \oplus B \oplus C \oplus D$



AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$f = \sum(1,3,5,7,9,11,13,15)$

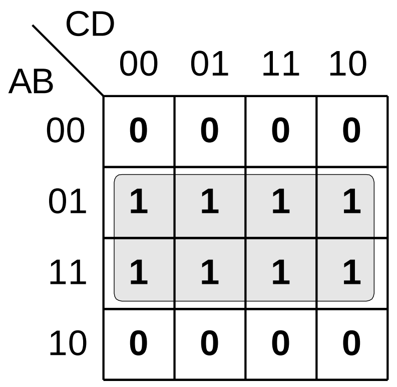
$f = D$



AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$f = \sum(0,2,4,6,8,10,12,14)$

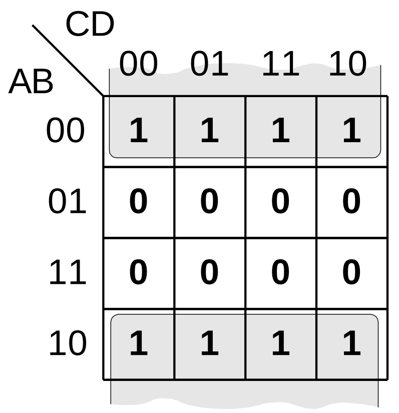
$f = \overline{D}$



AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$f = \sum(4,5,6,7,12,13,14,15)$

$f = B$



AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$f = \sum(0,1,2,3,8,9,10,11)$

$f = \overline{B}$

Design of combinational digital circuits

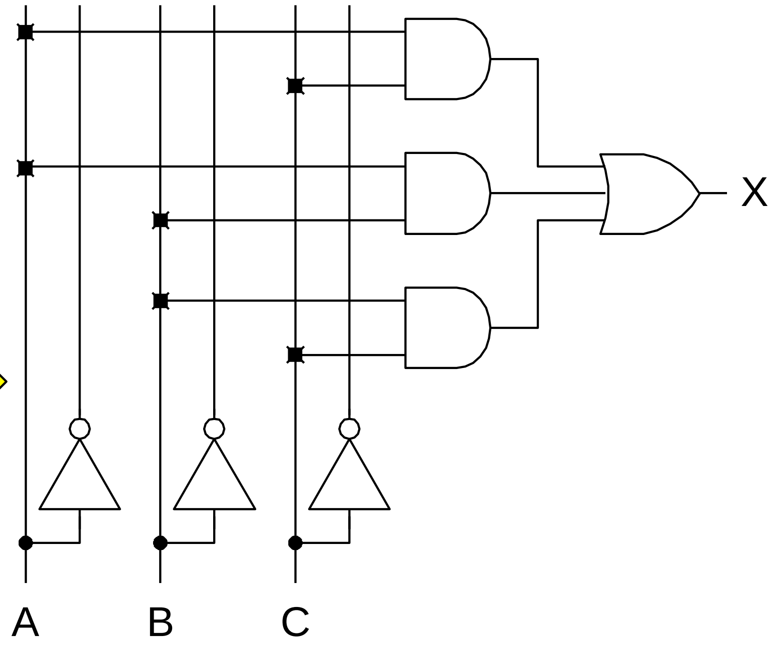
- Steps to design a combinational digital circuit:
  - From the problem statement derive the truth table
  - From the truth table derive the unsimplified logic expression
  - Simplify the logic expression
  - From the simplified expression draw the logic circuit
- Example: Design a 3-input (A,B,C) digital circuit that will give at its output (X) a logic 1 only if the binary number formed at the input has more ones than zeros.

	Inputs			Output
	A	B	C	X
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$$X = \sum(3, 5, 6, 7)$$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$X = AC + AB + BC$$

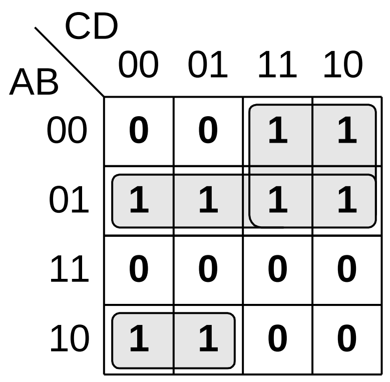


Design of combinational digital circuits

- Example: Design a 4-input (A,B,C,D) digital circuit that will give at its output (X) a logic 1 only if the binary number formed at the input is between 2 and 9 (including).

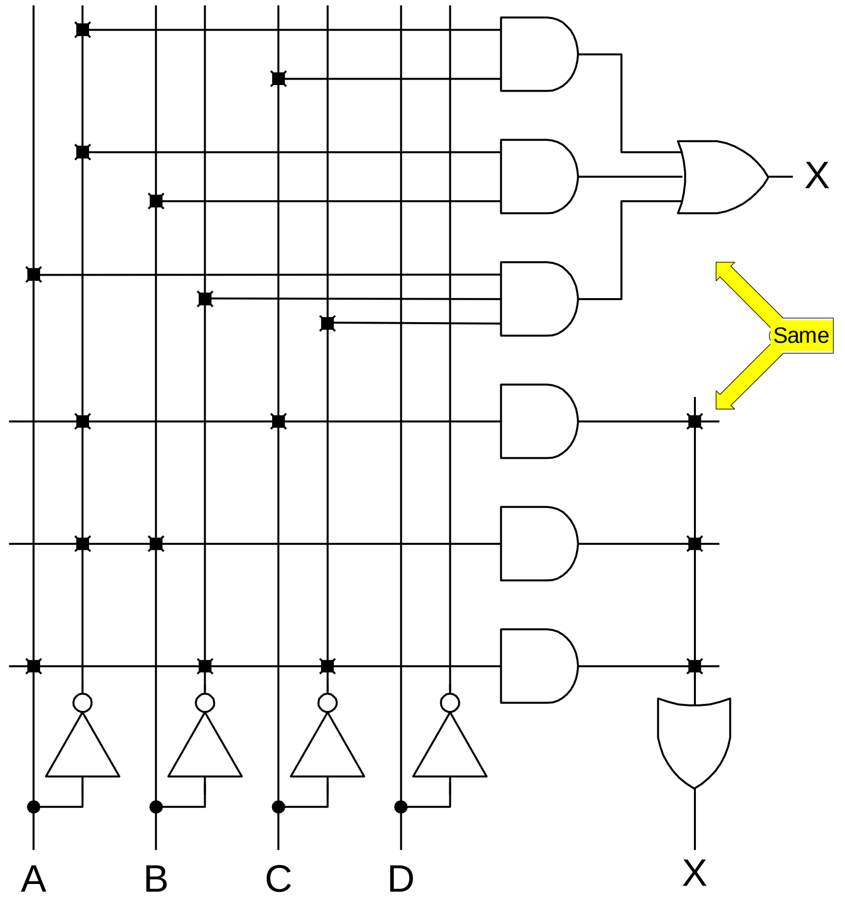
	Inputs				Output
	A	B	C	D	X
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

$$X = \sum(2,3,4,5,6,7,8,9)$$



AB \ CD	00	01	11	10
00	**0**	**0**	**1**	**1**
01	**1**	**1**	**1**	**1**
11	**0**	**0**	**0**	**0**
10	**1**	**1**	**0**	**0**

$$X = \overline{A}C + \overline{A}B + A\overline{B}\,\overline{C}$$


X
Same
A
B
C
D
X