

Combinational Logic

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Introduction

- A combinational circuit consists of logic gates whose outputs, at any time, are determined by combining the values of the inputs.
- For n input variables, there are 2^n possible binary input combinations.
- For each binary combination of the input variables, there is one possible output.

Introduction

- Hence, a combinational circuit can be described by:
1. A truth table that lists the output values for each combination of the input variables, or
 2. m Boolean functions, one for each output variable.



Introduction



Figure: Combinational Circuits

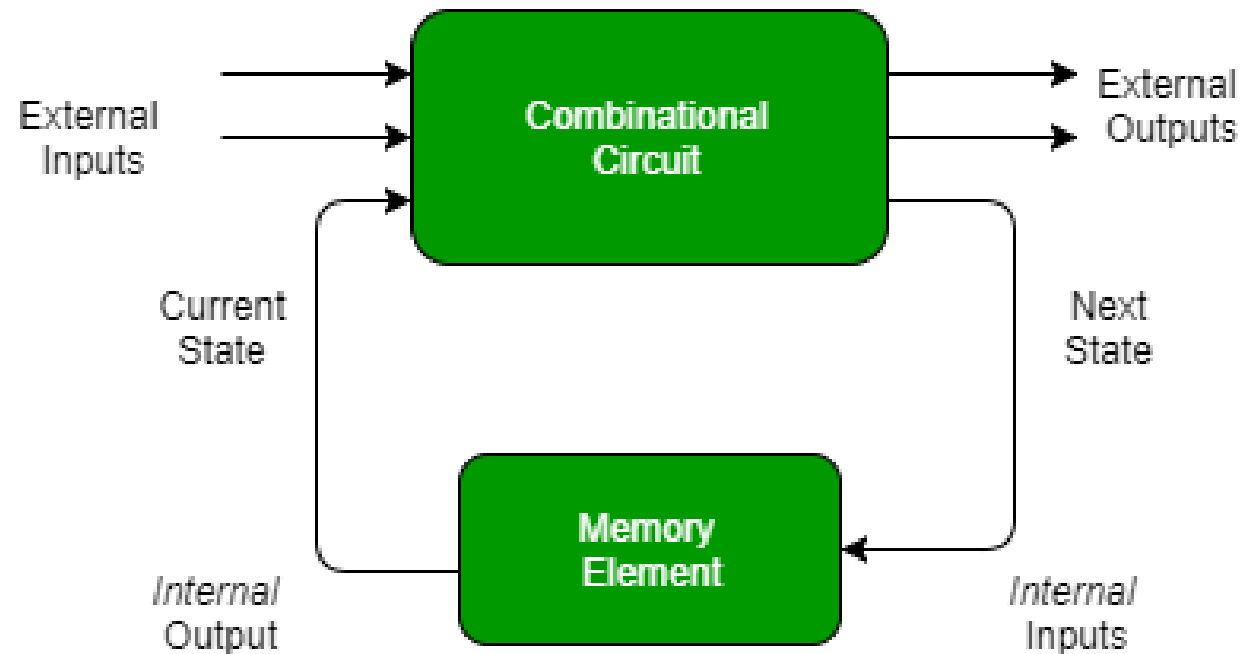


Figure: Sequential Circuit

Design procedure

The procedure involves the following steps:

- State the problem.
- Determine no. of available input variables and required output variables.
- Assign letter symbols to the input and output variables.
- Derive the truth table that defines the required relationship between inputs and outputs.
- Obtain simplified Boolean function for each output.
- Draw the logic diagram.

Adders (half adder)

- It is an arithmetic combinational logic circuit designed to perform addition of two single bits.
- It contains two inputs and produces two outputs.
- Inputs are called Augend and Added bits and Outputs are called **Sum and Carry**.
- Addition of single bits

$$0+0= 0$$

$$0+1= 1$$

$$1+0= 1$$

$$1+1=10$$

The result of $1+1$ is 10, where '1' is carry-output (C_{out}) and '0' is Sum-output

Adders (half adder)

Truth Table of Half Adder:

Inputs		Outputs	
A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Adders (half adder)

K-map for output variable Sum 'S':

A \ B	0	1
0		1
1	1	

$A'B$

$B'A$

$$A'B + B'A = A \text{ xor } B$$

K-map is of Sum of products form. The equation obtained is

$$S = AB' + A'B \implies S = A \text{ xor } B$$

K-map for output variable Carry 'C':

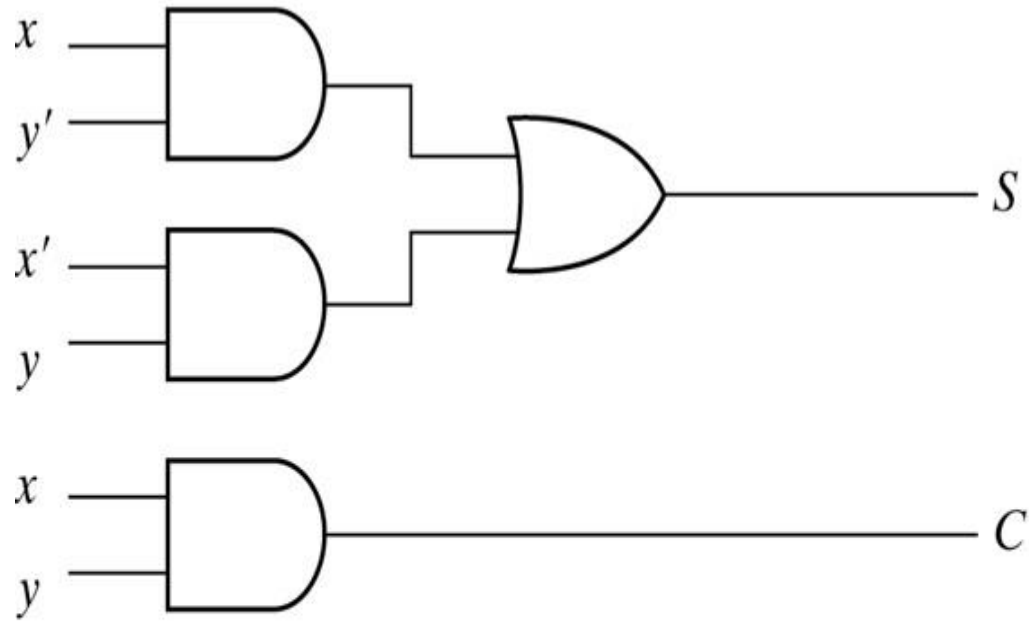
A \ B	0	1
0		
1		1

AB

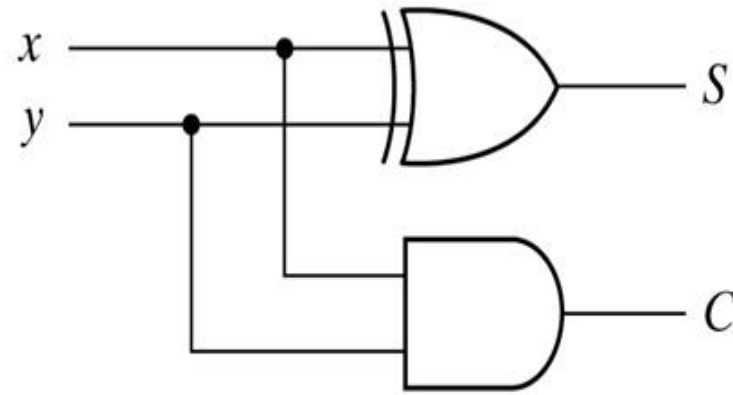
The equation obtained from K-map is

$$C = AB$$

Implementation of HALF ADDER

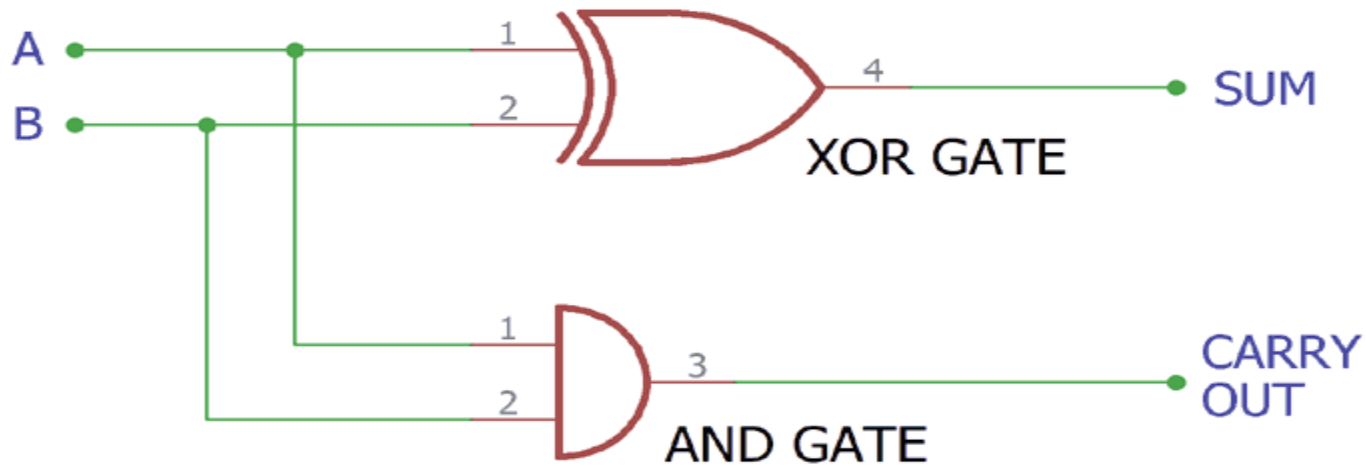
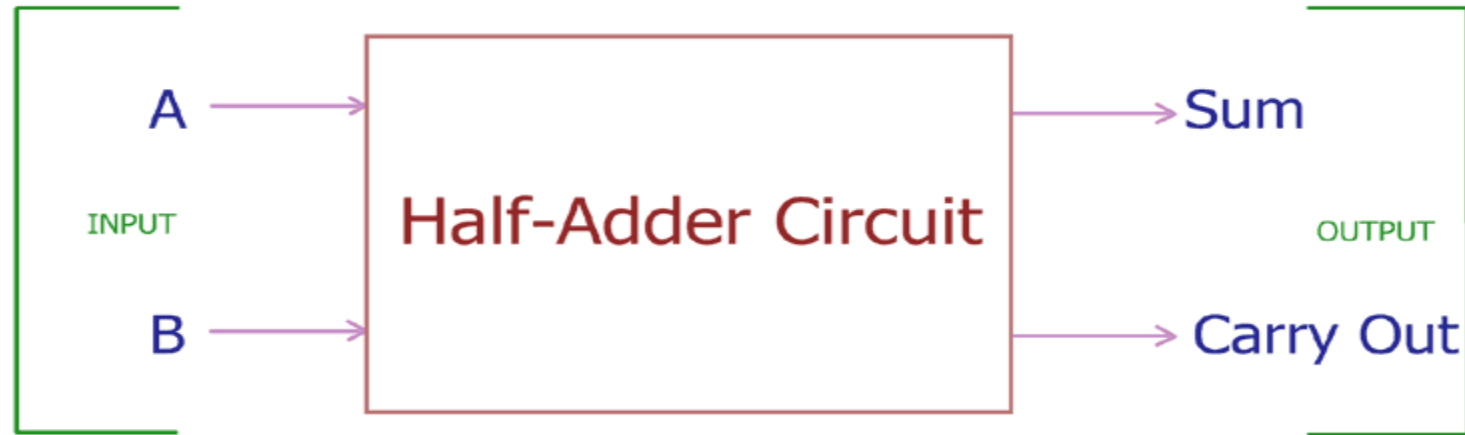


(a) $S = xy' + x'y$
 $C = xy$



(b) $S = x \oplus y$
 $C = xy$

Implementation of HALF ADDER



Limitations: Adding of Carry is not possible in Half adder.

Adders (full adder)

- To overcome the above limitation faced with Half adders, Full Adders are implemented.
- It is an arithmetic combinational logic circuit that performs addition of three single bits.
- It contains three inputs (A , B , C_{in}) and produces two outputs (Sum and C_{out}).
- Where, $C_{in} \rightarrow$ Carry In and $C_{out} \rightarrow$ Carry Out

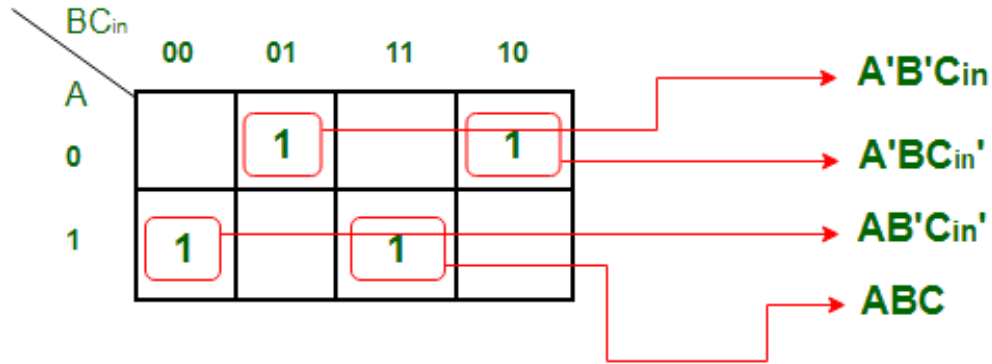
Adders (full adder)

Truth table of Full Adder:

Inputs			Outputs	
A	B	C _{in}	Sum	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Adders (full adder)

K-map Simplification for output variable Sum 'S' :



The equation obtained is

$$S = A'B'C_{in} + AB'C_{in}' + ABC + A'BC_{in}'$$

The equation can be simplified as

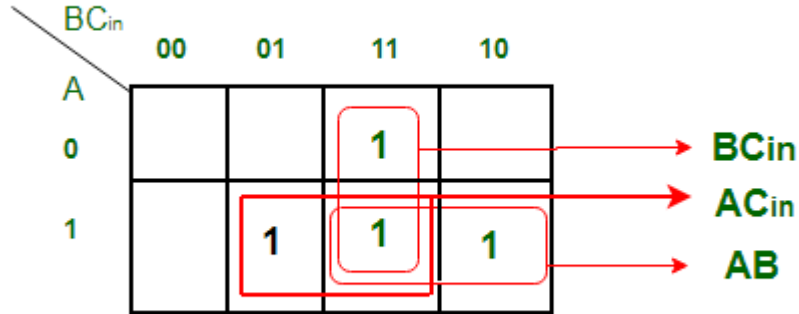
$$S = B'(A'C_{in} + AC_{in}') + B(AC + A'C_{in}')$$

$$S = B'(A \text{ xor } C_{in}) + B(A \text{ xor } C_{in})'$$

$$S = A \text{ xor } B \text{ xor } C_{in}$$

Adders (full adder)

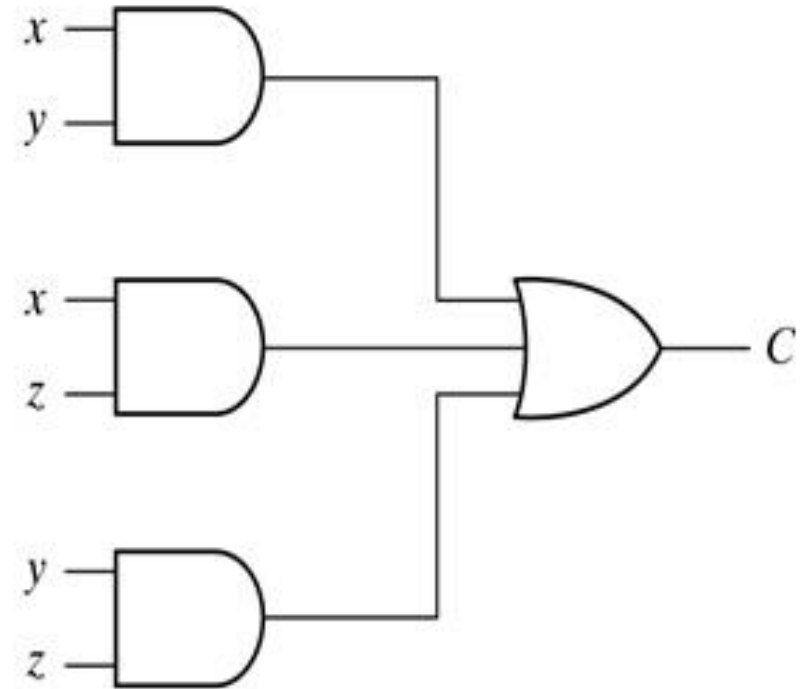
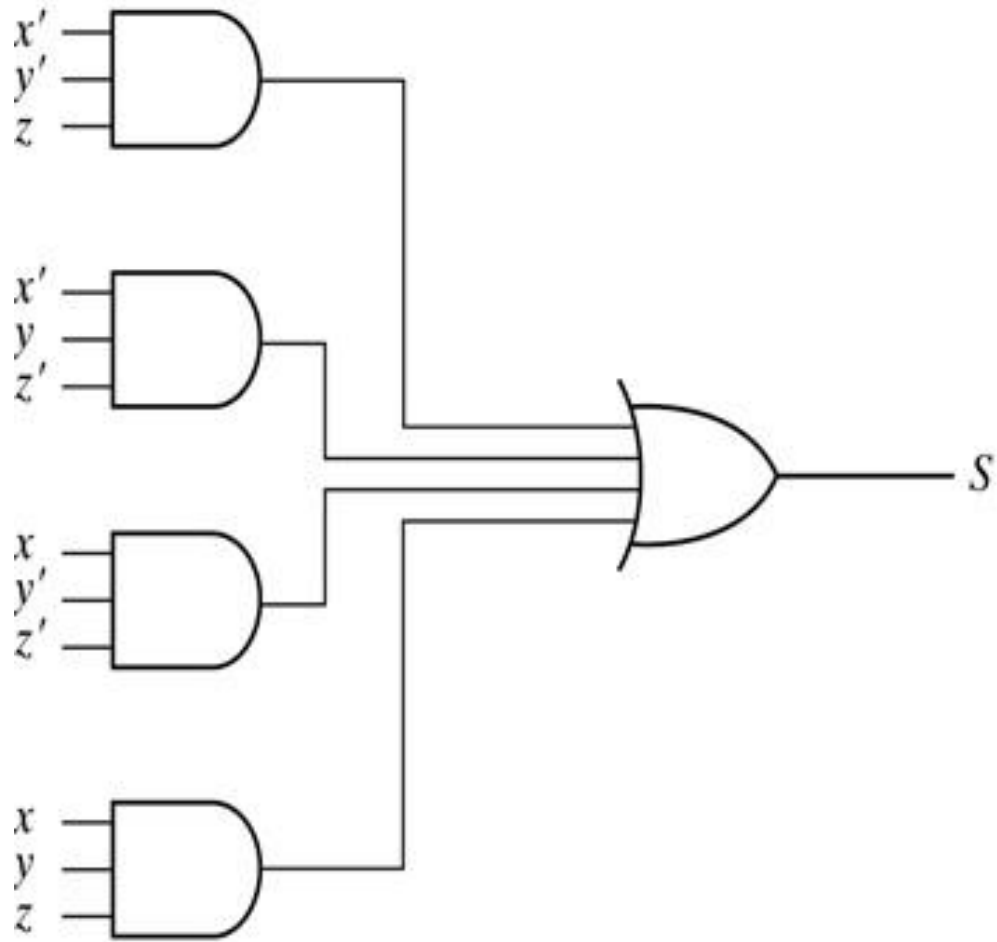
K-map Simplification for output variable 'Cout'



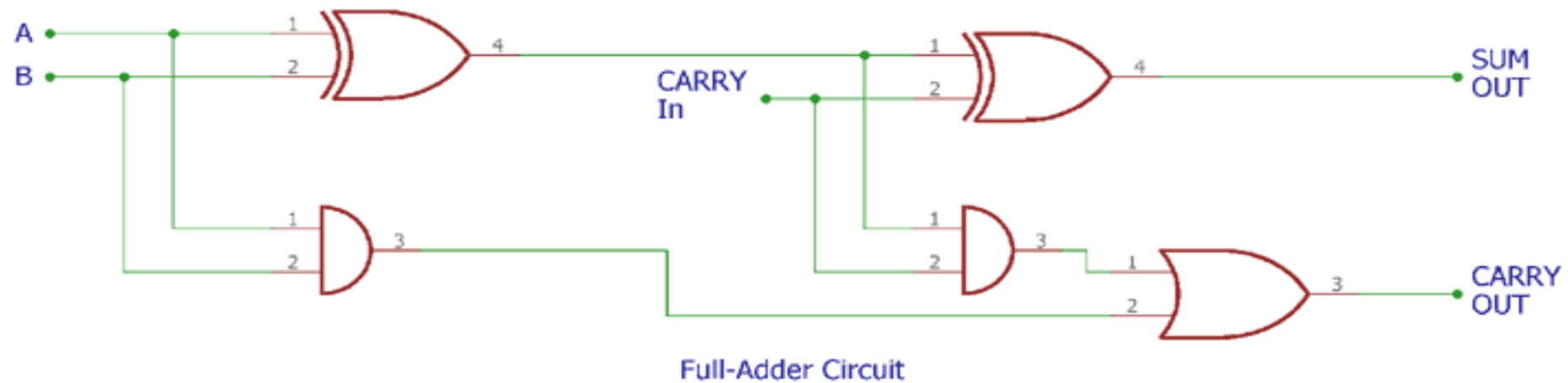
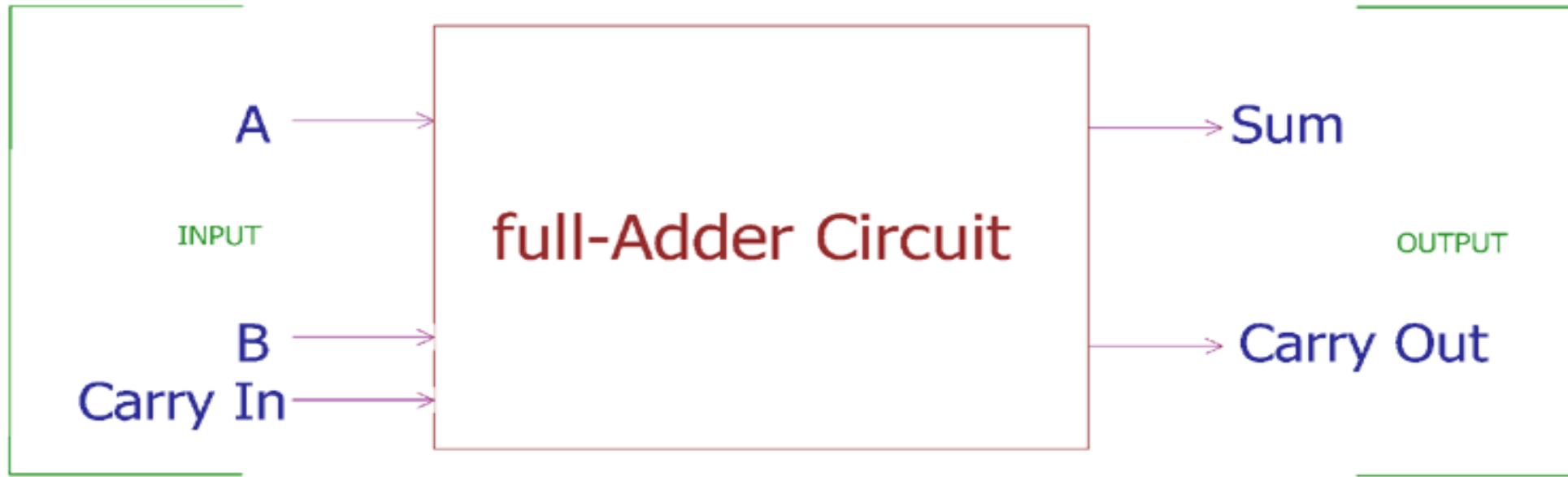
The equation obtained is

$$C_{out} = BC_{in} + AB + AC_{in}$$

Adders (full adder)

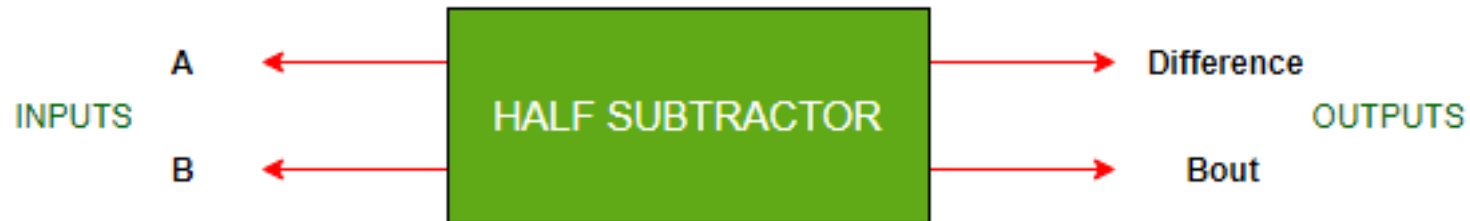


Adders (full adder)



Half Subtractor

- It is a combinational logic circuit designed to perform the subtraction of two single bits.
- It contains two inputs (A and B) and produces two outputs (Difference and Borrow-output).



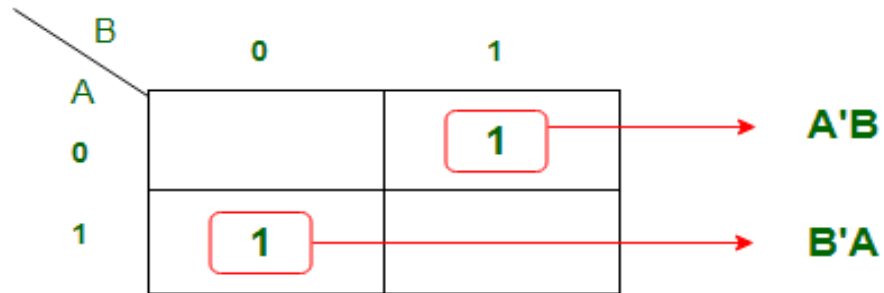
Half Subtractor

Truth Table of Half Subtractor:

Inputs		Outputs	
A	B	D	B _o
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

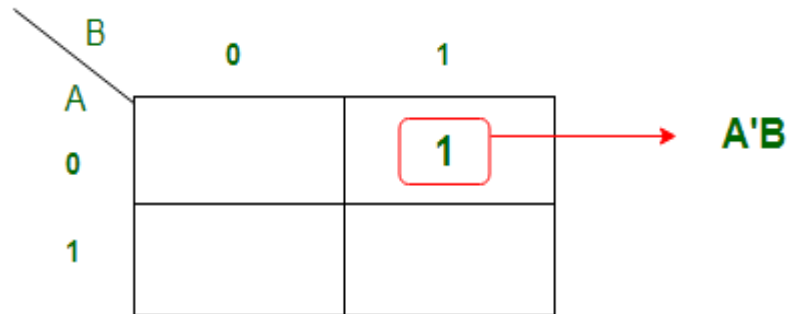
Half Subtractor

K-map Simplification for output variable 'D':



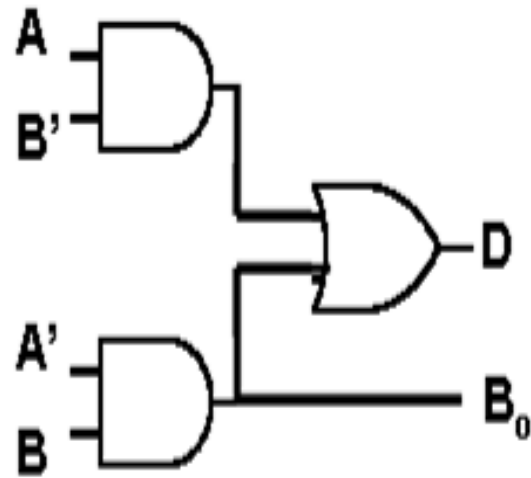
$$A'B + B'A = A \text{ xor } B$$

K-map Simplification for output variable 'Bout' :



Half Subtractor

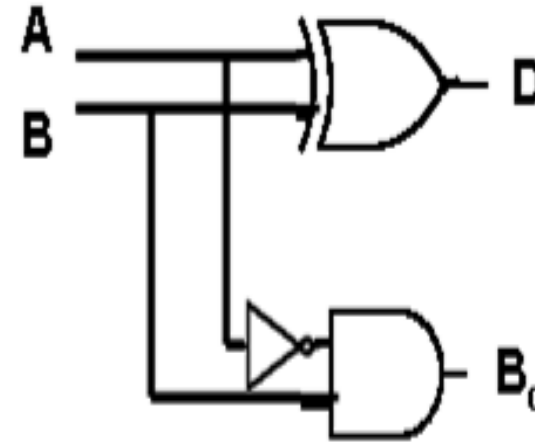
Implementation of HALF SUBTRACTOR



(a)

$$D = A'B + AB'$$

$$B_0 = A'B$$



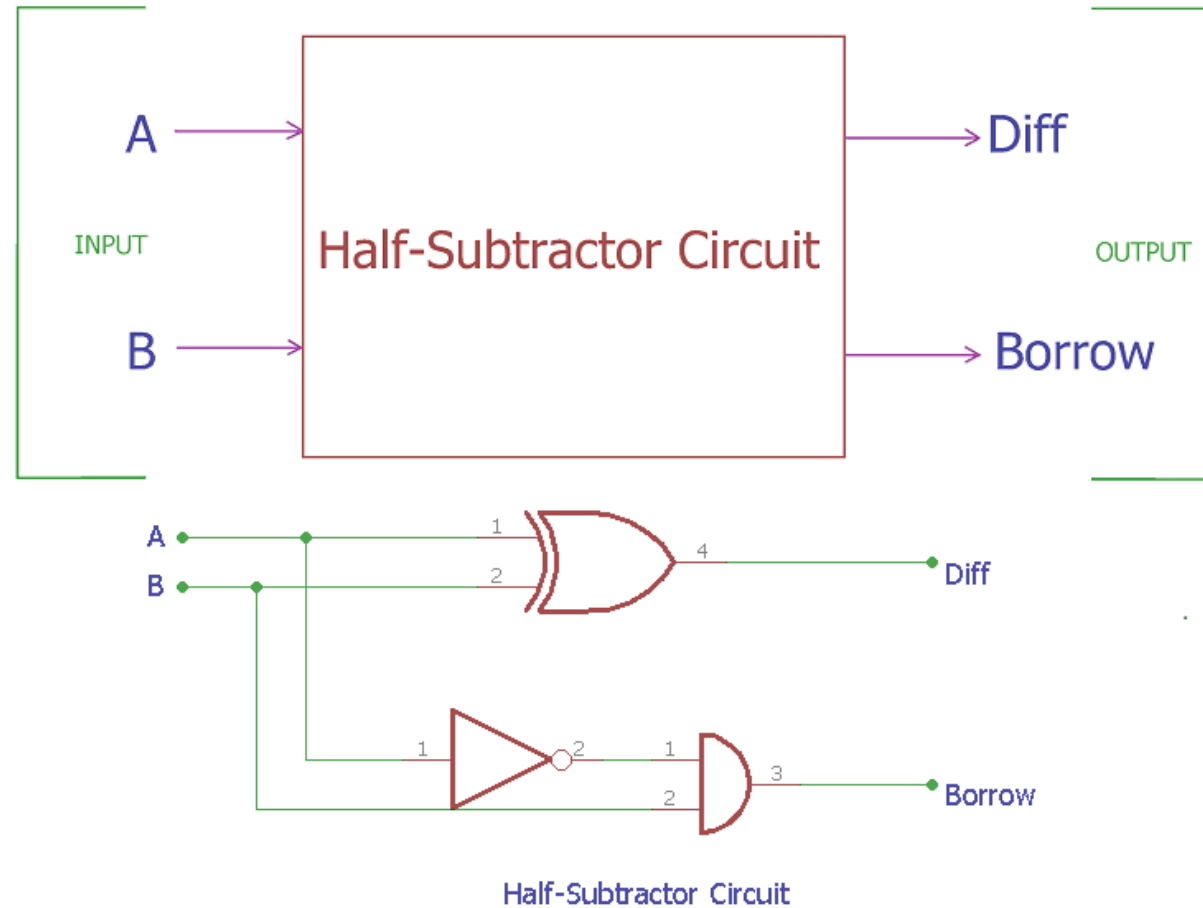
(b)

$$D = A'B + AB'$$

$$B_0 = A'B$$

Half Subtractor

Implementation of HALF SUBTRACTOR



Full Subtractor

- It is a Combinational logic circuit designed to perform subtraction of three single bits.
- It contains three inputs(A , B , B_{in}) and produces two outputs (D , B_{out}).
- Where, A and B are called **Minuend** and **Subtrahend** bits.
- And, $B_{in} \rightarrow$ Borrow-In and $B_{out} \rightarrow$ Borrow-Out

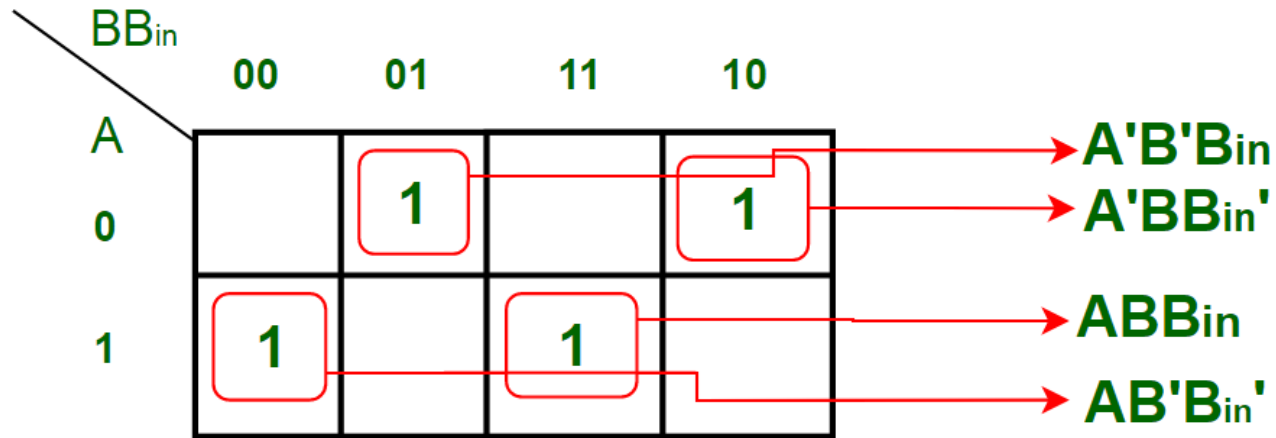
Full Subtractor

Truth Table of Full Subtractor:

Inputs			Outputs	
A	B	B _{in}	D	B _{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Full Subtractor

K-map Simplification for output variable 'D' :



The equation obtained from above K-map is : $D = A'B'B_{in} + AB'B_{in}' + AB B_{in} + A'BB_{in}'$
which can be simplified as,

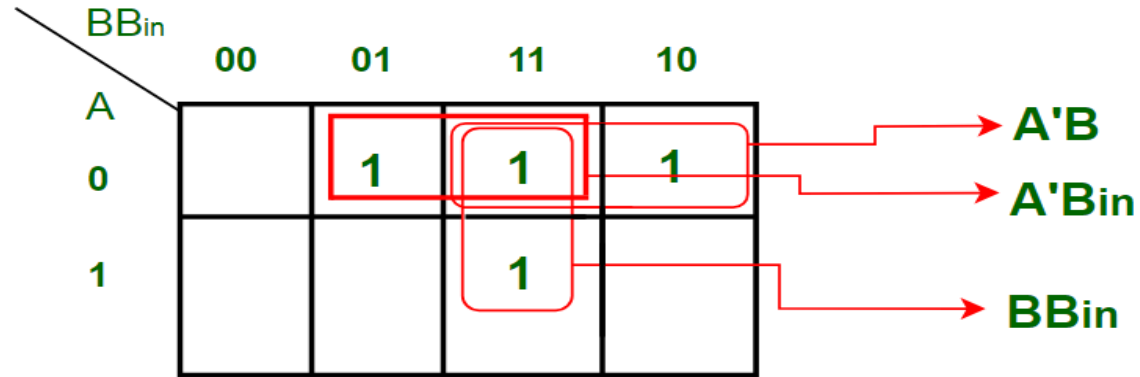
$$D = B'(A'B_{in} + AB_{in}') + B(AB_{in} + A'B_{in}')$$

$$D = B'(A \text{ xor } B_{in}) + B(A \text{ xor } B_{in})'$$

$$D = A \text{ xor } B \text{ xor } B_{in}$$

Full Subtractor

K-map Simplification for output variable 'Bout' :

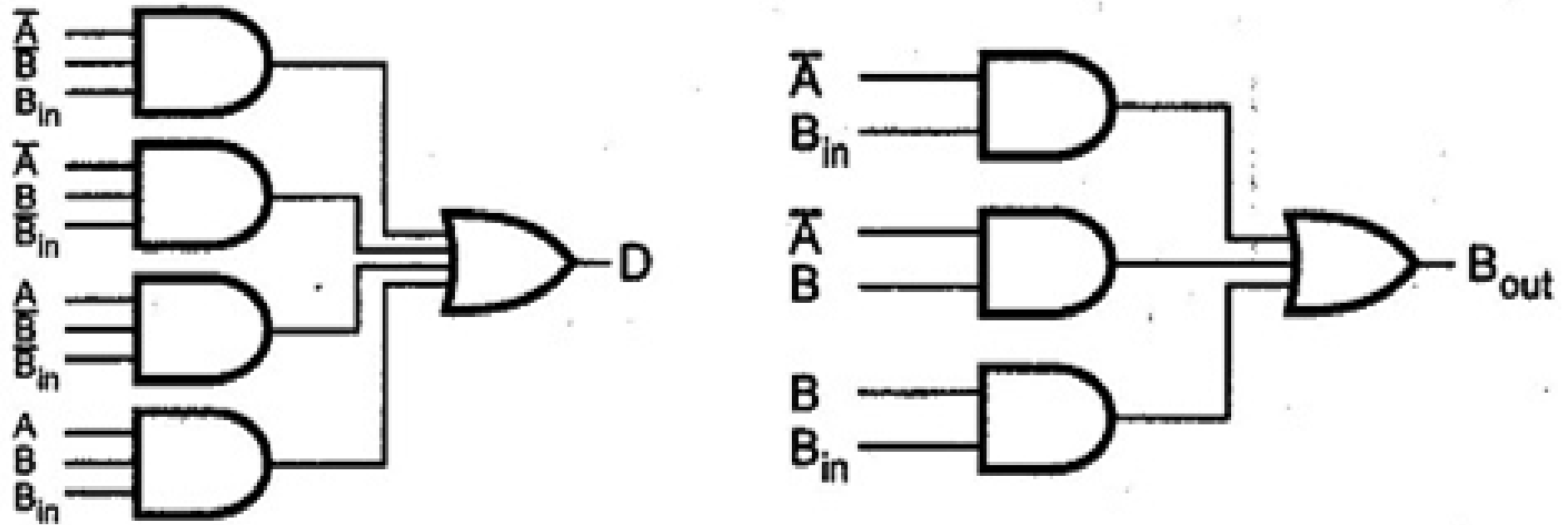


The equation obtained is: $B_{out} = BB_{in} + A'B + A'B_{in}$

Full Subtractor

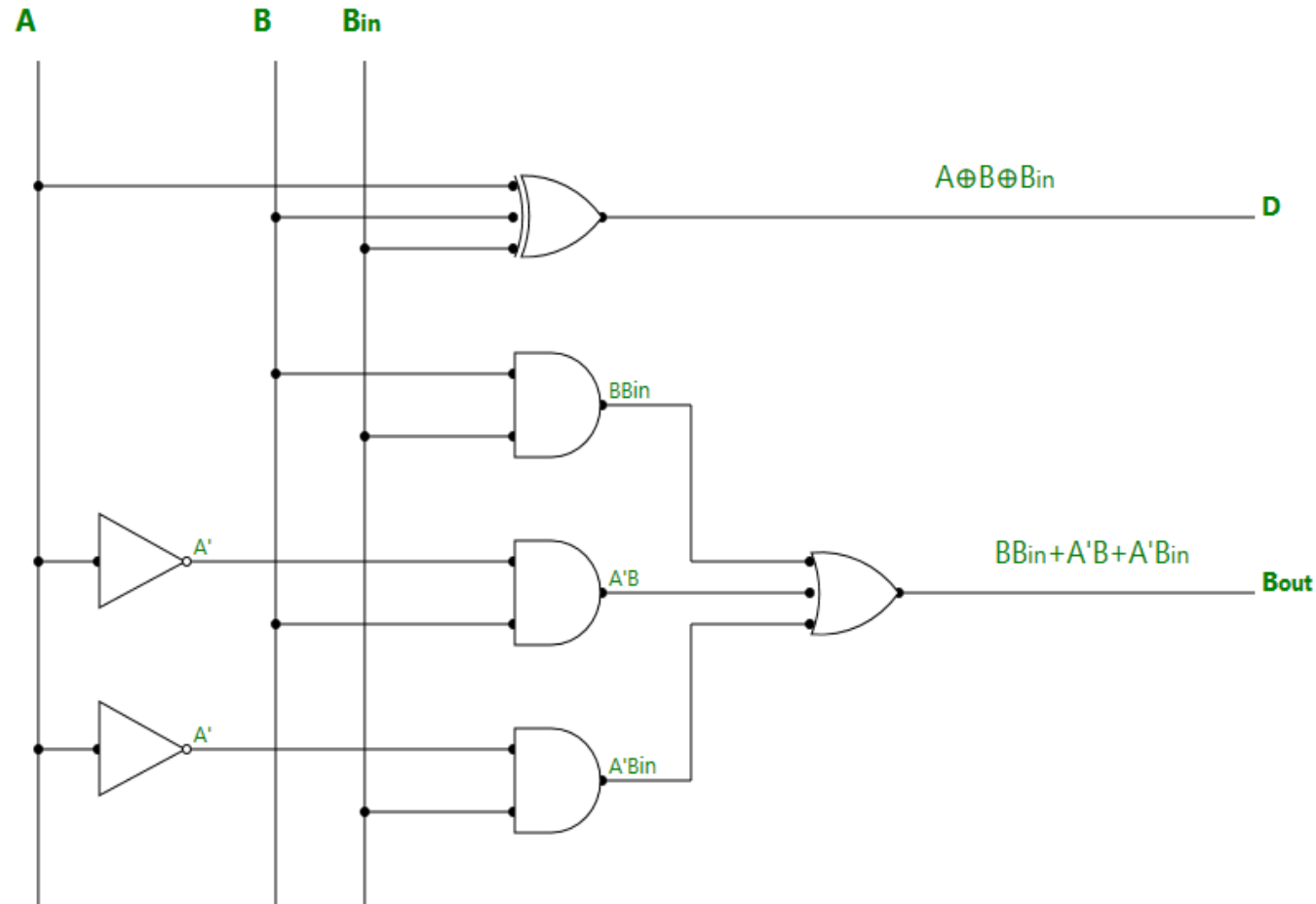
Logic Diagram of Full Subtractor:

Logic Diagram



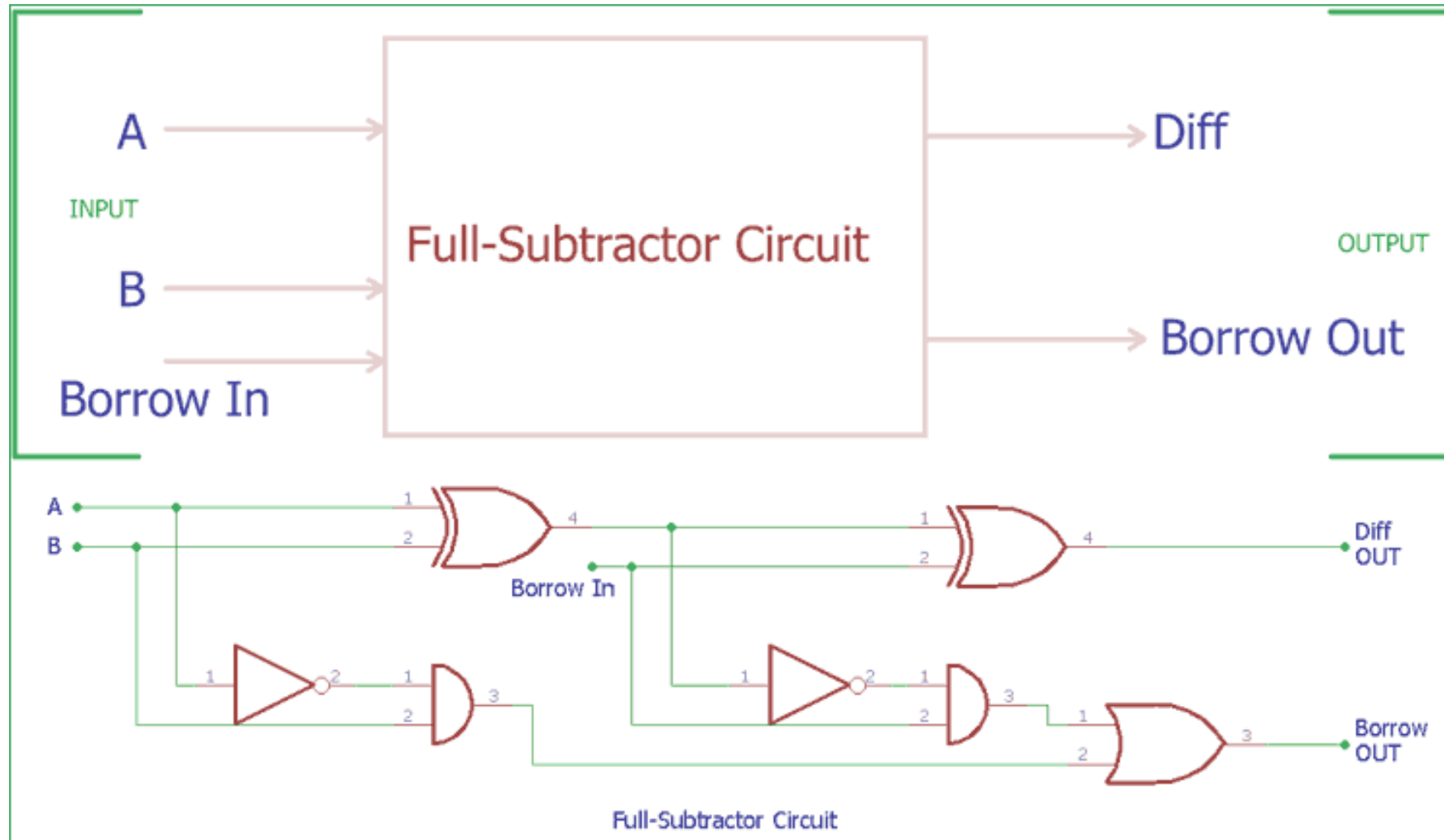
Full Subtractor

Logic Diagram of Full Subtractor:



Full Subtractor

Logic Diagram of Full Subtractor:

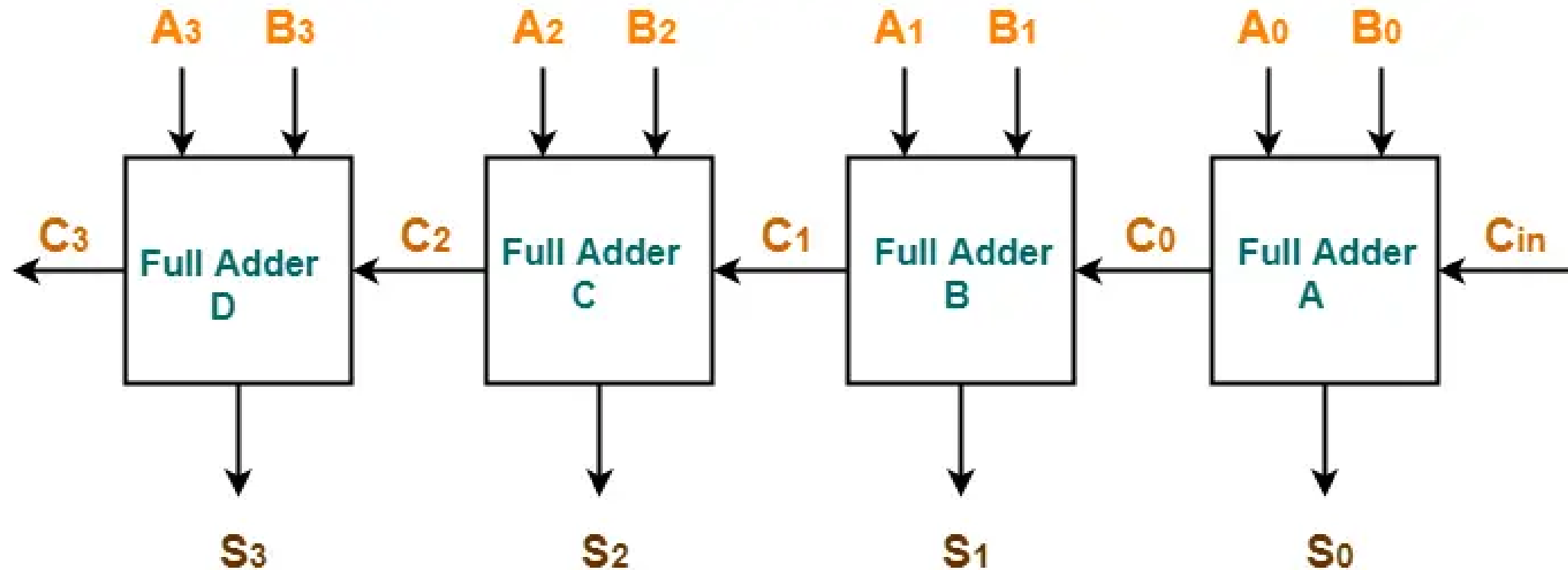


Applications

- Arithmetic Operations
- ALU (Arithmetic Logic Unit)
- Digital Signal Processing (DSP)
- Memory Addressing
- Data Compression and Encryption
- Digital Counters and Timers
- Control Systems
- Image and Video Processing
- Data Address Calculation
- Floating-Point Arithmetic
- Error Detection and Correction

Ripple carry adder

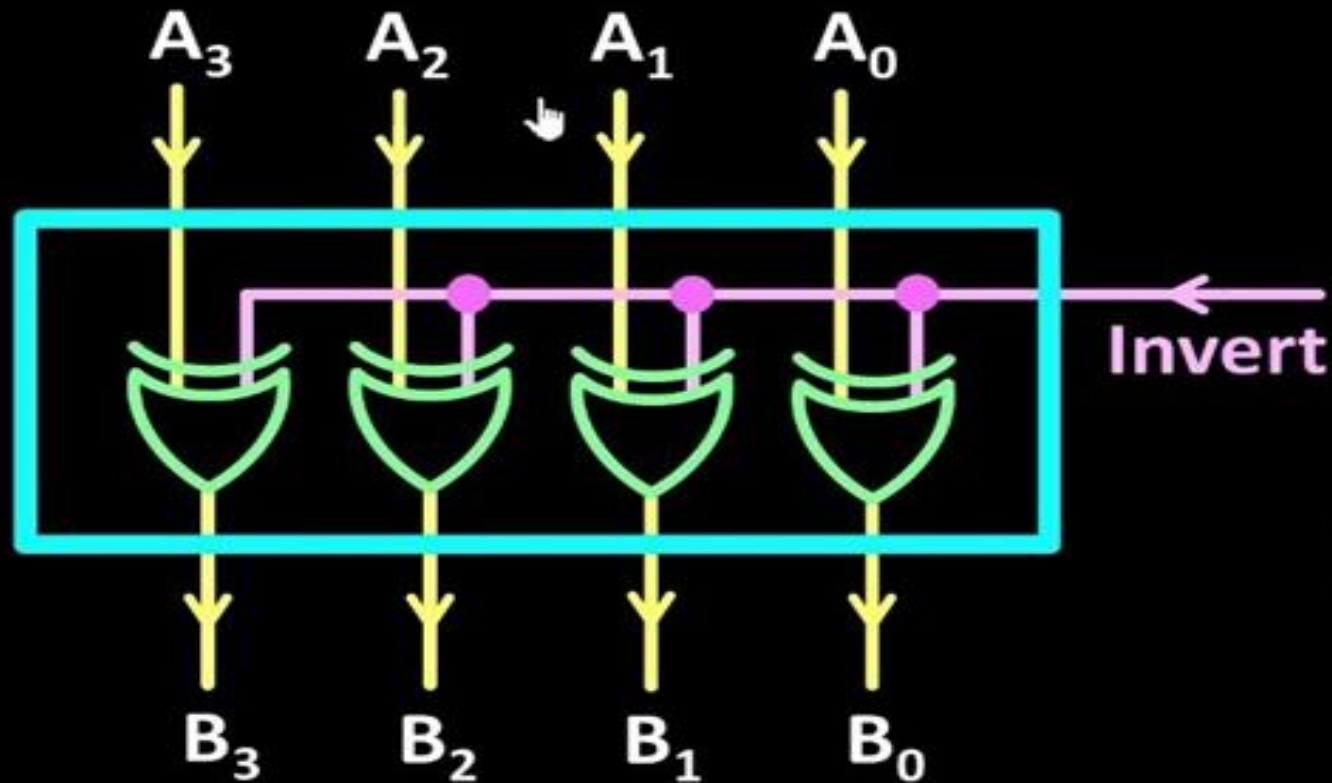
A ripple carry adder is a digital circuit that produces the arithmetic sum of two binary numbers. It can be constructed with full adders connected in, with the carry output.



4-bit Ripple Carry Adder

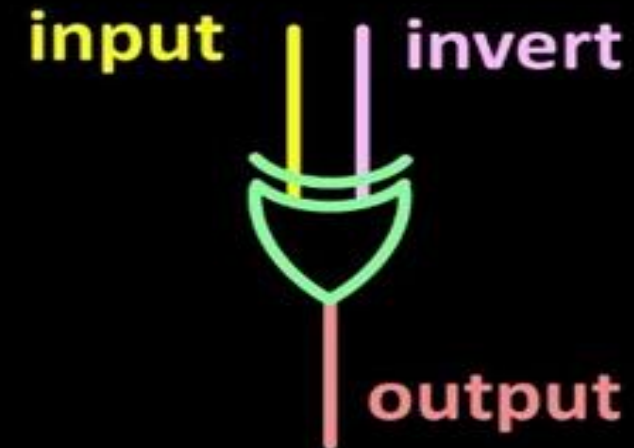
CONTROLLED INVERTER Circuit

Controlled Inverter



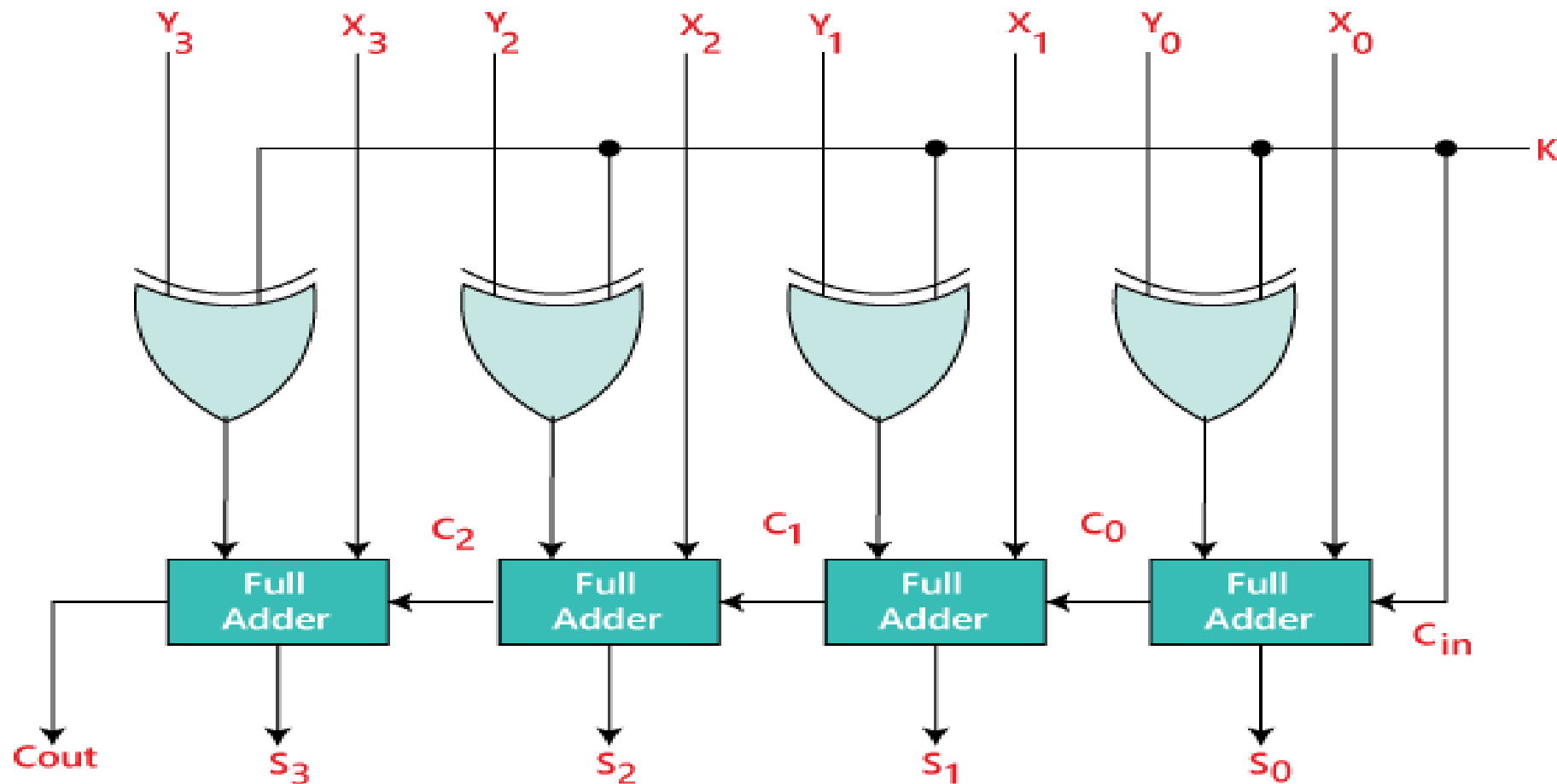
Truth table of *XOR* gate

x	y	z
0	0	0
1	0	1
0	1	1
1	1	0



Binary Adder-Subtractor

A Binary Adder-Subtractor is capable of both the addition and subtraction of binary numbers in one circuit itself. The operation is performed depending on the binary value the control signal holds. It is one of the components of the ALU (Arithmetic Logic Unit).



Carry Look-Ahead Adder

it is the circuit that performs binary addition the fastest by utilizing the Carry Generate and Carry Propagate ideas.

