* Minimization of K-Map.

- Use for minimization of Boolean expression
  1) By using Boolean laws
  2) using K-Map.

* Given function $f(A,B) = \Sigma_m(2,3)$

	A	B	F
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1

$f(A,B) = AB' + AB.$

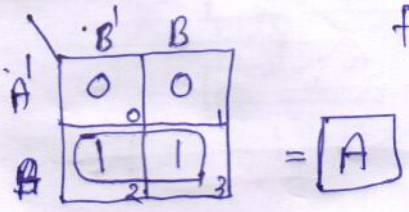
$AB' + AB$

* using laws :-

$$f(A,B) = AB' + AB$$
$$= A(B' + B)$$

$$\boxed{F(A,B) = A}$$

* K-Map :- Graphical Representation.



find the constant value

$= \boxed{A}$

* K-Map (Karnaugh Map)

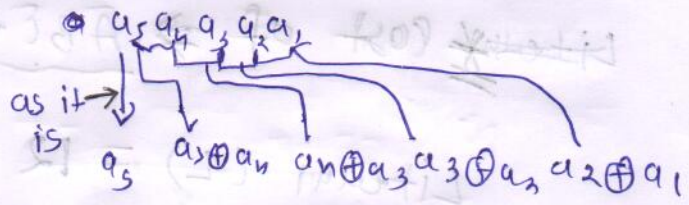
- K-map is a graphical representation used for ~~simplification~~ simplifying the boolean expressions.
- for boolean expression consisting of n-variables, number of cells required in K-map = 2^n cells.
- K-map is based on Gray code (unit distance code)
- K-map is based on three types of Input value (0, 1, don't care).
- Using Gray code common deriving is easy.

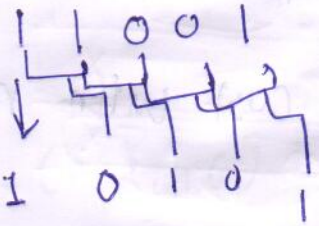
* Gray code * [unit distance code]

8421	
0 0000	0000
1 0001	0001
2 0010	0011
3 0011	0010
4 0100	0110
5 0101	0111
6 0110	0101
7 0111	0100
8 1000	1100
9 1001	1101
10 1010	1111

Conversion

$(11001)_B = (\quad)_G$





XOR

0	0	0
0	1	1
1	0	1
1	1	0

$(1\,0\,1\,0\,1)_G \longrightarrow (11001)_B$

$1 \oplus 0 \to 1 \quad 0 \oplus 0 \to 0 \quad 0 \oplus 1 \to 1$

$1\ 1\ 0\ 0\ 1$

$(11001)_B = (10101)_G$

* AB

00

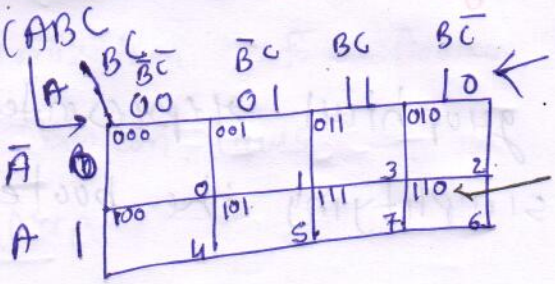
01

11

10

ABC

F(ABC)



Gray code.

minterm Representation

* simplify :- $F(ABC) = m_2 + m_3 + m_6 + m_7$

$$F(ABC) = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$

$$= \bar{A}B(\bar{C}+C) + AB(\bar{C}+C)$$

$$= \bar{A}B + AB$$

$$= B(\bar{A}+A)$$

$$= \boxed{B}$$

~~Literal~~ cost of $\Rightarrow \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$

Literal (L) = 12

Gate input cost (G) = 12 + 4 = 16

Gate input cost with not (GN) = 16 + 2 = 18

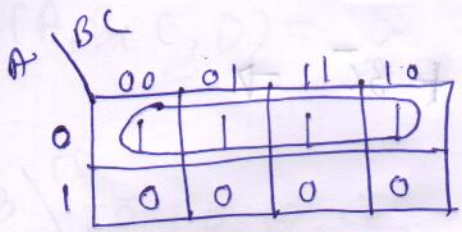
* cost of = B

L = 1

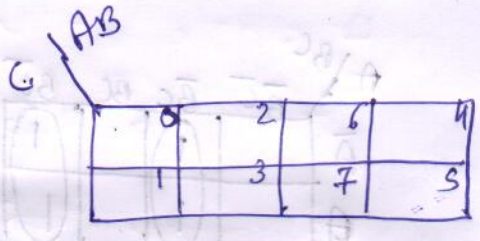
G = 0

GN = 0

* $F(ABC) = \Sigma_m(0,1,2,3)$

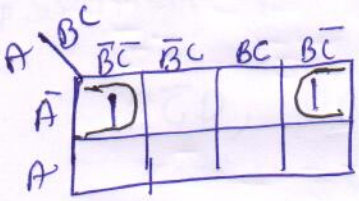


$\Rightarrow \bar{A}$

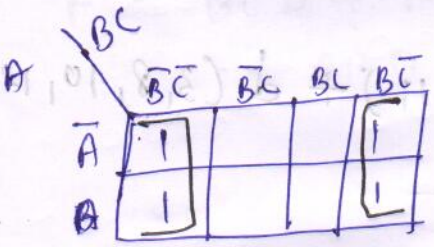


- maximum group 1, 2, 4, 8, because we get most optimized value

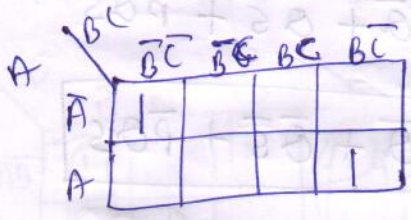
* How to make pairs :- [1, 2, 4, 8, 16 ...]



$F = \bar{A}\bar{C}$

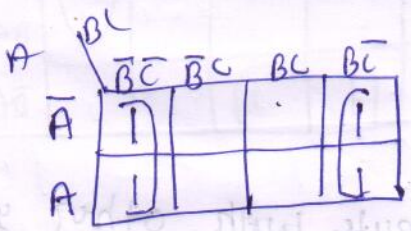


$F = \bar{C}$



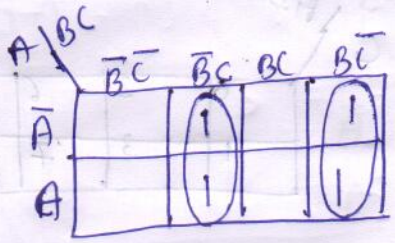
$F = \bar{A}\bar{B}\bar{C} + AB\bar{C}$

$\Sigma_m = (0,2,4,6)$



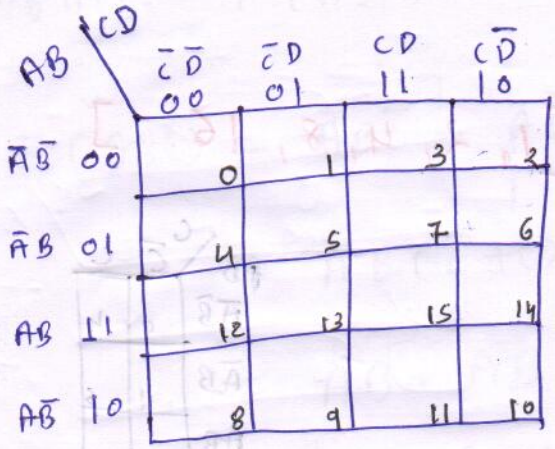
$F = \bar{C}$

* $F(ABC) = \sum_m (1, 2, 5, 6)$



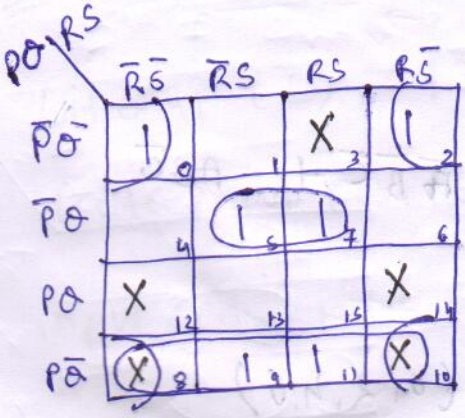
$F = \bar{B}C + B\bar{C} \Rightarrow$

* 4-Variable K-Map *



Ex $F(P,Q,R,S) = \sum_m (0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$

Focus only 1,



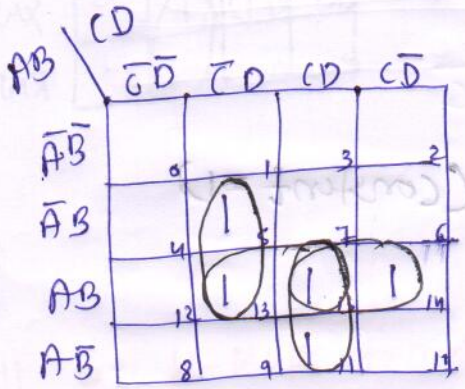
$F = P\bar{Q} + \bar{Q}\bar{S} + \bar{P}QS$

$\boxed{F = P\bar{Q} + \bar{Q}\bar{S} + \bar{P}QS}$

- Assertial Prime implicant
  - → Find a 1 that not participate with other group
  - → above Example → 3 = assertial Prime implicant.

* How many Essential Prime implicants.

* $F(A,B,C,D) = \sum_m (5, 11, 13, 14, 15)$.



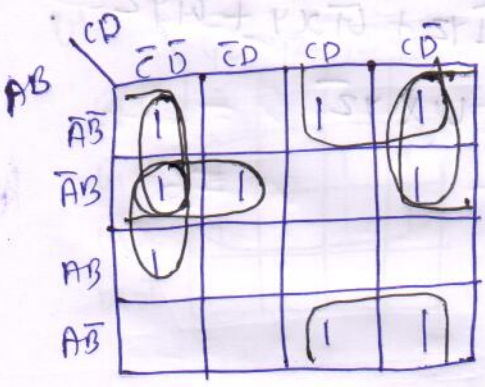
⇒ Prime implicants = 4 (PIm)

⇒ Essential Prime implicants ⇒ (EsPIm) = 3

$B\bar{C}D$, ABC, ACD, → (EsPIm).

$F = B\bar{C}D + ABC + ACD + ABD$ → (PIm)

* $F(A,B,C,D) = \sum_n (0, 2, 3, 4, 5, 6, 10, 11, 12)$.



$F = \bar{B}C + \bar{A}\bar{D} + \bar{A}B\bar{C} + B\bar{C}\bar{D}$

* Four Variable terms.

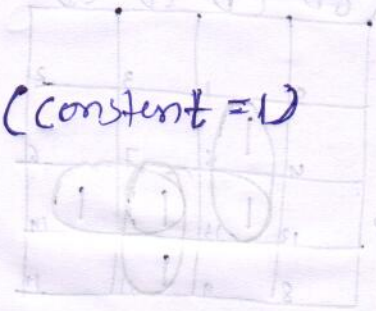
A single 1 = 4 Variables

Two 1's = 3 Variables

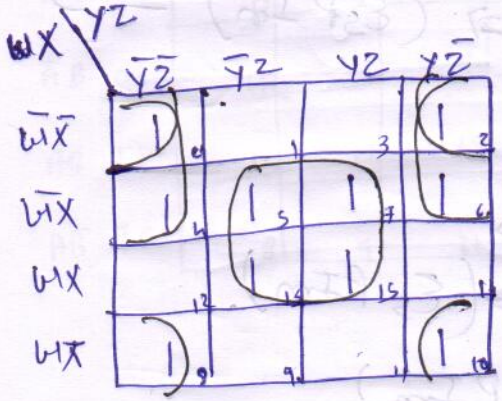
Four 1's = 2 variables

Eight 1's = 1 Variables

sixteen 1's = 0 variables (constant = 1)



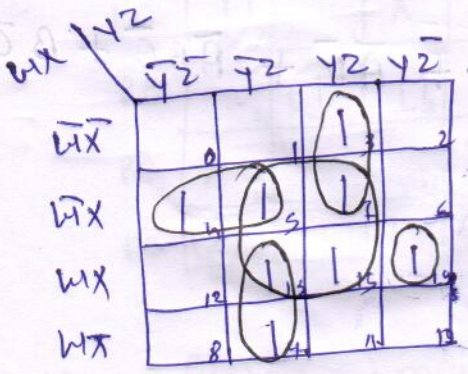
* $F(wxyz) = \Sigma m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$



$F = xz + \bar{w}\bar{z} + \bar{x}\bar{z}$

$F = xz + \bar{x}\bar{z} + \bar{w}x$

* $F(wxyz) = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$

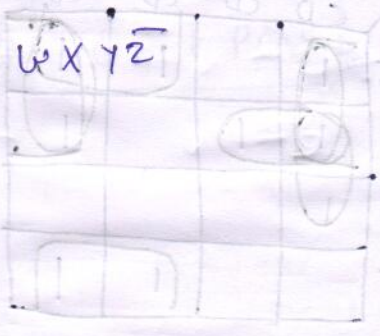


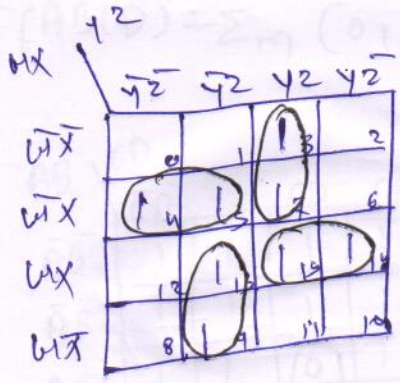
$F = xz + \bar{w}yz + \bar{w}x\bar{y} + w\bar{y}z + wxy\bar{z}$

$L = 15$

$G = 20$

$GN = 23$



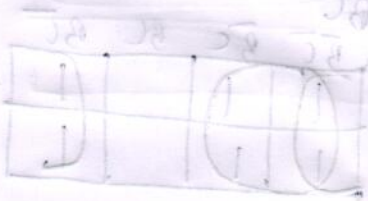


$F = \bar{w}yz + wxy + w\bar{y}z + \bar{w}x\bar{y}$

$L = 12$

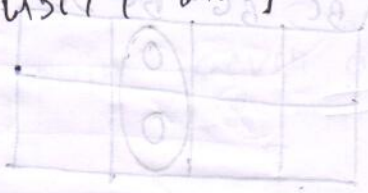
$G = 16$

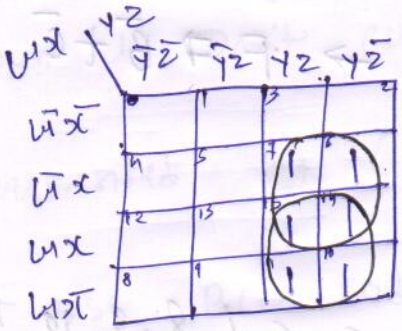
$GN = 18$



* Fill a K-Map from boolean equation

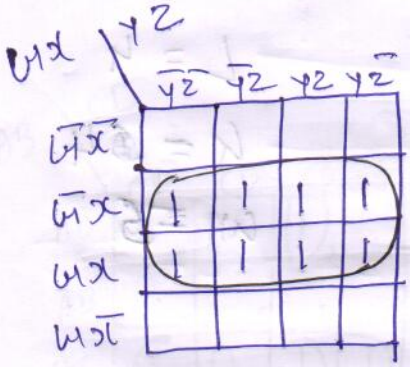
$F(w, x, y, z) = x\bar{y}\bar{z} + xyz + w\bar{x}\bar{y} + wx\bar{y}$





$F = xy + wy$

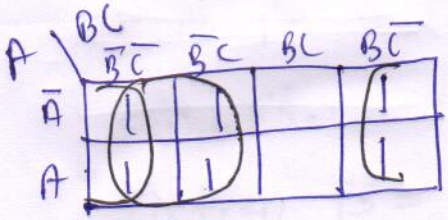
* $F(w, x, y, z) = xy + x\bar{y} + wxyz$



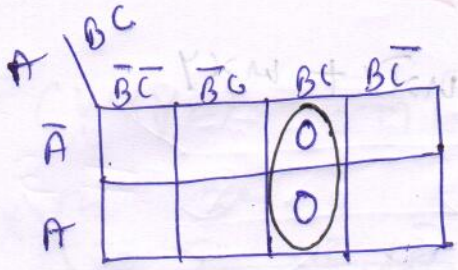
$F = x$

* K-Map for maxterm.

* $F(ABC) = \Sigma_m (0, 1, 2, 4, 5, 6)$.



$F = \bar{B} + \bar{C}$

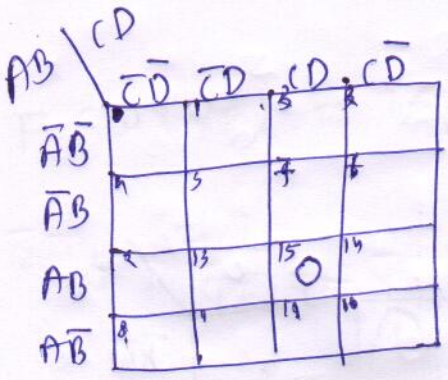


$\bar{F} = BC$

$\downarrow$

$= F = \overline{BC} \rightarrow F = \bar{B} + \bar{C}$

* $F(ABCD) = \Sigma_m (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)$



$\bar{F} = AB\bar{C}D$

$\bar{\bar{F}} = \overline{AB\bar{C}D}$

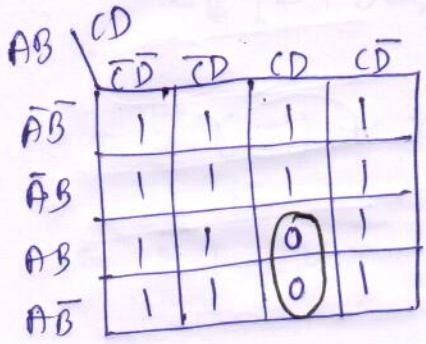
$\boxed{F = \overline{AB\bar{C}D}}$

$L = 4$

$W = 4$

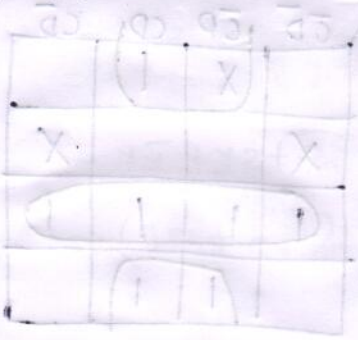
$CM = 5$

* $F(ABCD) = \Sigma_m(0,1,2,3,4,5,6,7,8,9,10,12,13,14)$.



$\bar{F} = AC\bar{D}$

$F = \overline{AC\bar{D}}$



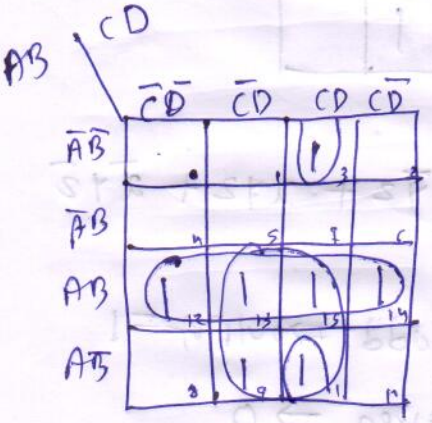
* Don't care *

- input values for the minterm, will never occur, known as Don't care.
- Represented by ('X').

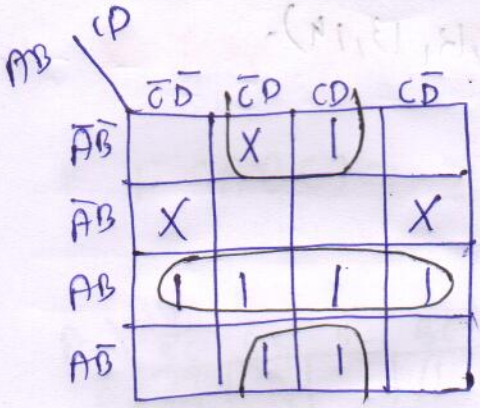
- we use BCD code, 6 code is never utilized it's called don't care.

Ex $F(A,B,C,D) = \Sigma_m(3,9,11,12,13,14,15) + \Sigma_d(1,4,6)$.

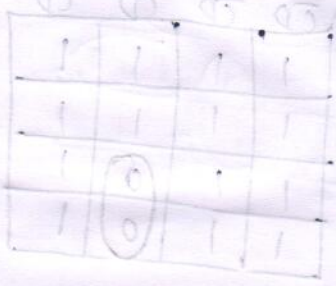
↓ never utilized in system



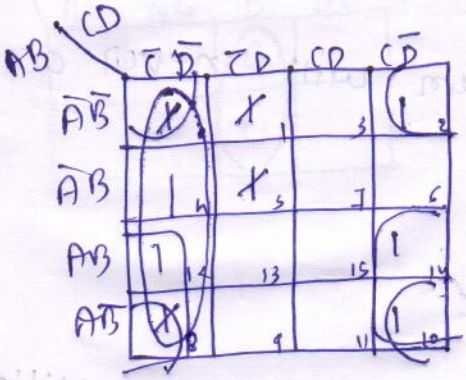
$F = AB + AD + \bar{B}CD$



$F = A\bar{B} + \bar{B}D$



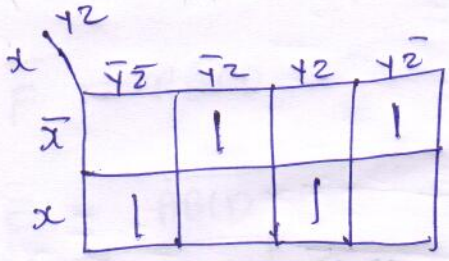
* $F(A,B,C,D) = \Sigma_m(2,4,10,12,14) + \Sigma_d(0,1,5,8)$



$F = \bar{C}\bar{D} + \bar{B}\bar{D} + A\bar{D}$

* Function Realization :-

X	Y	Z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	0	1



$F = x\bar{Y}\bar{Z} + \bar{x}\bar{Y}Z + xYZ + \bar{x}Y\bar{Z}$

$\Rightarrow$ no of 1's odd $\Rightarrow$ output = 1

1's even $\Rightarrow$ 0

→ This known as Even. parity check, Check for intigrity of data. Use in security system.

$$F = x\bar{y}\bar{z} + \bar{x}\bar{y}z + xyz + \bar{x}y\bar{z}$$

$$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$$

$$= \bar{x}(T) + x(\bar{T})$$

$$= \bar{x}T + x\bar{T}$$

$$= x \oplus T$$

$$\overline{\bar{y}z + y\bar{z}}$$

$$= (y + \bar{z}) \cdot (\bar{y} + z)$$

$$= y\bar{y} + yz + \bar{z}\bar{y} + \bar{z}z$$

$$= (yz + \bar{z}\bar{y})$$

$$T = \bar{y}z + y\bar{z}$$

$$T = y \oplus z$$

$$\boxed{F = x \oplus y \oplus z}$$

↳ used for Even parity check.