Introduction to Canonical Forms

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Overview

- What are Canonical Forms?
- Minterms and Maxterms
- Index Representation of Minterms and Maxterms
- Sum-of-Minterm (SOM) Representations
- Product-of-Maxterm (POM) Representations
- Representation of Complements of Functions
- Conversions between Representations

What are Canonical Forms

- Boolean function expressed as a **sum of minterms** or **product of maxterms** are said to be canonical form.
- It is useful to specify Boolean functions in a form that:
  - Allows comparison for equality.
  - Has a correspondence to the truth tables
- Canonical Forms in common usage:
  - Sum of Minterms (SOM)
  - Product of Maxterms (POM)

Standard form: Sum Of Product (SOP) : AB’ + AC’ + ABC
Product Of Sum (POS) : (A + B’) ▪ (A +C’) ▪ (A + B + C)

SUM -> OR gate
PRODUCT -> AND gate

Minterms

- **Minterms** are AND terms with every variable present in either true or complemented form.

- Given that each binary variable may appear normal (e.g., x) or complemented (e.g., ~x), there are 2^n minterms for n variables.

- Example: Two variables (X and Y) produce 2 x 2 = 4 combinations:

  XY (both normal)
  $X\overline{Y}$ (X normal, Y complemented)
  $\overline{X}Y$ (X complemented, Y normal)
  $\overline{X}\,\overline{Y}$ (both complemented)

- Thus there are four minterms of two variables.

Minterms

- For three variables function, eight minterms are possible as listed in the following table in Figure

A	B	C	Minterm
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

Minterms

Steps to obtain SUM OF MINTERMS

- Find out how many variables is there in equation

- List out all minterms for given variables. i.e m0 to m7 for 3 variables

- Check the given equation is in **SOP FORM** or not? If not then convert it into **SOP form**. i.e **ab + bc + ac’** form

- Find out missing variable in each term & add that variable in that term. i.e for A,B,C: B is missing in AC’. Add B in AC’ using AC’ = AC’(B+B’) = ABC’ + AB’C’

- ADD ALL MISSING VARIABLES IN EACH TERMS OF THE EQUATION

- ARRANGE ALL **OBTAINED MINTERMS** IN ASCENDING ORDERS. i.e from m0 to m7 for 3 variables

Minterms

Sum of Minterms $F = A + B'C$

$$A = A(B + B')$$
$$= AB + AB'$$
$$= AB(C + C') + AB'(C + C')$$
$$= ABC + ABC' + AB'C + AB'C'$$
$$B'C = B'C(A + A') = AB'C + A'B'C$$

$$F = ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$$
$$= ABC + ABC' + AB'C + AB'C' + A'B'C$$
$$= A'B'C + AB'C' + AB'C + ABC' + ABC$$
$$= m1 + m4 + m5 + m6 + m7$$

$$F(A,B,C) = \Sigma(1, 4, 5, 6, 7)$$

Maxterms

- Maxterms are OR terms with every variable in true or complemented form.

- Given that each binary variable may appear normal (e.g., x) or complemented (e.g., ~x), there are 2^n maxterms for n variables.

- Example: Two variables (X and Y) produce 2 x 2 = 4 combinations:

  $X + \underline{Y}$ (both normal)
  $\underline{X} + Y$ (x normal, y complemented)
  $\underline{X} + \underline{Y}$ (x complemented, y normal)
  $X + Y$ (both complemented)

Maxterms

- Like minterms, for a three-variable function, eight maxterms are also possible as listed in the following table in Figure

A	B	C	Maxterm
0	0	0	$A + B + C$
0	0	1	$A + B + C'$
0	1	0	$A + B' + C$
0	1	1	$A + B' + C'$
1	0	0	$A' + B + C$
1	0	1	$A' + B + C'$
1	1	0	$A' + B' + C$
1	1	1	$A' + B' + C'$

Maxterms

Steps to obtain PRODUCT OF MAXTERMS

- Find out how many variables is there in given equation
- List out all maxterms for given variables. i.e M0 to M7 for 3 variables
- Check the given equation is in **POS FORM** or not? If not then convert it into **POS form**. i.e **(a + b)(b + c)(a +c')** form
- Find out missing variable in each term & add that variable in that term. i.e for A,B,C: B is missing in (A + C'). Add B in (A + C') using (A + C') = (A + C')BB'
  = (A + B + C)(A + B' + C)
- ADD ALL MISSING VARIABLES IN EACH TERMS OF THE EQUATION
- ARRANGE ALL **OBTAINED MAXTERMS** IN ASCENDING ORDERS. i.e from M0 to M7 for 3 variables

Maxterms

Product of Maxterms

$$F = xy + x'z$$
$$= (xy + x')(xy + z)$$
$$= (x + x')(y + x')(x + z)(y + z)$$
$$= (x' + y)(x + z)(y + z)$$

$$x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$$
$$x + z = x + z + yy' = (x + z + y)(x + z + y')$$
$$y + z = y + z + xx' = (y + z + x)(y + z + x')$$
$$F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z')$$
$$= M0M2M4M5$$

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

Maxterms and Minterms

x	**y**	**z**	**function f1**	**function f2**
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

F1 = x'y'z + xy'z' + xyz = m1 + m4 + m7

F2 = x'yz + xy'z + xyz' + xyz = m3 + m5 + m6 + m7

Maxterms and Minterms

- Examples: Two variable minterms and maxterms.

Index	Minterm	Maxterm
0	$\bar{x}\,\bar{y}$	$x + y$
1	$\bar{x}\,y$	$x + \bar{y}$
2	$x\,\bar{y}$	$\bar{x} + y$
3	$x\,y$	$\bar{x} + \bar{y}$

- The index above is important for describing which variables in the terms are true and which are complemented.

Maxterms and Minterms

- Purpose of the Index

The index for the minterm or maxterm, expressed as a binary number, is used to determine whether the variable is shown in the true form or complemented form.

For Minterms:

“1” means the variable is “Not Complemented” and
“0” means the variable is “Complemented”.

For Maxterms:

“0” means the variable is “Not Complemented” and
“1” means the variable is “Complemented”.

Maxterms and Minterms

Index Example in Three Variables

Example: (for three variables)

Assume the variables are called X, Y, and Z.

The standard order is X, then Y, then Z.

The Index 0 (base 10) = 000 (base 2) for three variables).

All three variables are complemented for minterm 0 ($\overline{X}, \overline{Y}, \overline{Z}$) and no variables are complemented for Maxterm 0 (X,Y,Z).

- Minterm 0, called m_0 is $\overline{X}\,\overline{Y}\,\overline{Z}$.
- Maxterm 0, called M_0 is $(X + Y + Z)$.
- Minterm 6 ?
- Maxterm 6 ?

Maxterms and Minterms

- Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\,\bar{b}\,\bar{c}\,\bar{d}$	$a + b + c + d$
1	0001	$\bar{a}\,\bar{b}\,\bar{c}\,d$	?
3	0011	?	$a + b + \bar{c} + \bar{d}$
5	0101	$\bar{a}\,b\,\bar{c}\,d$	$a + \bar{b} + c + \bar{d}$
7	0111	?	$a + \bar{b} + \bar{c} + \bar{d}$
10	1010	$a\,\bar{b}\,c\,\bar{d}$	$\bar{a} + b + \bar{c} + d$
13	1101	$a\,b\,\bar{c}\,d$	?
15	1111	$a\,b\,c\,d$	$\bar{a} + \bar{b} + \bar{c} + \bar{d}$

Conversion Between Canonical Forms

$F(A, B, C) = \Sigma\,(1, 4, 5, 6, 7)$ (Sum Of Product) (Sum Of Minterms)

$F'(A, B, C) = \Sigma\,(0, 2, 3) = m0 + m2 + m3$

complement of $F' = F = (m0 + m2 + m3)'$

$= m0' \,.\, m2' \,.\, m3'$

$= M0.M2.M3$

$= \Pi\,(0, 2, 3)$ (Product Of Sum) (Product Of Maxterms)

$mj' = Mj$

$F (A, B, C) = \Sigma\,(1, 4, 5, 6, 7) = \Pi\,(0, 2, 3)$