

* Minimization of K-Map.

Use for minimization of Boolean expression

1) By using Boolean laws

2) using K-Map.

* Given function $f(A, B) = \sum m(2, 3)$

A \ B	0	1
0	0	0
1	0	1
2	1	0
3	1	1

$$f(A, B) = AB' + AB$$

$$AB' + AB$$

* using laws :-

$$f(A, B) = AB' + AB$$

$$= A(B' + B)$$

$$\boxed{f(A, B) = A}$$

* K-Map :- Graphical Representation.

find the constant value

	B'	B
A'	0	0
A	1	1

\boxed{A}

* K-Map (Karnaugh Map).

- K-map is a graphical representation used for simplifying the boolean expressions.

= for boolean expression consisting of n -variables, number of cells required in K-map = 2^n cells.

- K-map is based on Gray code (unit distance code)

- K-map is based on three types of input

Value (0, 1, don't care).

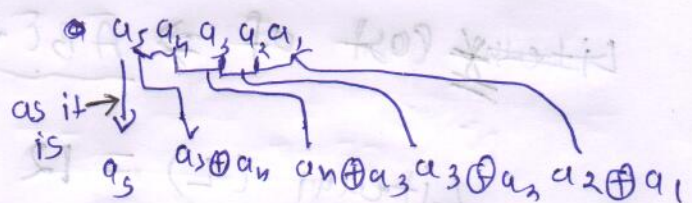
= using Gray code conversion is easy.

* Gray code * [unit distance code]

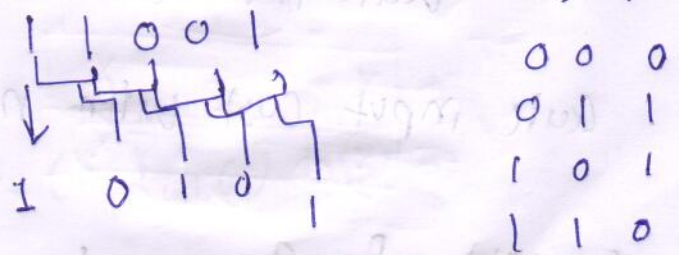
	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
	1	1	1	1

conversion

$$(11001)_B = (\quad)_G$$



XOR



$$(10101)_G \rightarrow (11001)_B$$

$$(11001)_B = (10101)_G$$

* AB

00

01

11

10

ABC

$f(ABC)$

	BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$
A	00	01	11	10
\overline{A}	000	001	011	010
A	100	101	111	110

Gray code.

min term Representation

* simplify :- $F(ABC) = m_2 + m_3 + m_6 + m_7$

$$F(ABC) = \overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$$

$$= \overline{A}B(\overline{C} + C) + AB(\overline{C} + C)$$

$$= \overline{A}B + AB$$

$$= B(\overline{A} + A)$$

$$= \boxed{B}$$

~~Literal~~ * Cost of $\Rightarrow \overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$

$$\text{Literal (L)} = 12$$

$$\text{Gate input cost (K)} = 12 + 4 = 16$$

$$\text{Gate input cost with not (GN)} = 16 + 2 = 18$$

* cost of B

$$L = 1$$

$$K = 0$$

$$GN = 0$$

* $F(ABC) = \sum_m (0, 1, 2, 3)$

A \ BC	BC			
	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$\Rightarrow \bar{A}$

C \ AB	AB			
	00	01	11	10
0	1	1	1	1
1	0	0	0	0

— maximum group 1, 2, 4, 8, because we get most optimized value

* How to make pairs: - [1, 2, 4, 8, 16...]

A \ BC	BC			
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1			1
A				

$F = \bar{A}\bar{B}$

AB \ C	\bar{C}	C
	0	1
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	1	1
$A\bar{B}$	1	1
AB	1	1

A \ BC	BC			
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1			1
A	1			1

$F = \bar{C}$

A \ BC	BC			
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1			
A				1

$F =$

AB \ C	\bar{C}	C
	0	1
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	1	1
$A\bar{B}$	1	1
AB	1	1

$\sum_m = (0, 2, 4, 6)$

A \ BC	BC			
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1			1
A	1			1

$F = \bar{C}$

* $F(ABC) = \sum_m(1, 2, 5, 6)$ $\sum = (0, 1, 2, 3, 4, 5, 6, 7)$ *

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
\overline{A}		1		1
A		1		1

$F = \overline{B}C + B\overline{C} \Rightarrow$

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
\overline{A}	0	1	0	0
A	0	0	1	0

* 4-Variable K-Map *

AB \ CD	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$ 00	0	1	3	2
$\overline{A}B$ 01	4	5	7	6
AB 11	12	13	15	14
$A\overline{B}$ 10	8	9	11	10

EX $F(P, Q, R, S) = \sum_m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$

Focus only 1,

PQ \ RS	$\overline{R}\overline{S}$	$\overline{R}S$	RS	$R\overline{S}$
$\overline{P}\overline{Q}$	1		X	1
$\overline{P}Q$		1	1	
PQ	X			X
$P\overline{Q}$	X	1	1	X

$F = \overline{P}\overline{Q} + \overline{Q}S + \overline{P}QS$

$F = \overline{P}\overline{Q} + \overline{Q}S + \overline{P}QS$

- Essential Prime implicant

→ Find a 1 that not participate with other group

→ above Example → 3 = essential Prime implicant.

* How many Essential Prime implicants.

* $F(A, B, C, D) = \sum_m (5, 11, 13, 14, 15)$.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	14	15
$A\bar{B}$	8	9	11	10

⇒ Prime implicants = 4 (PIm)

⇒ Essential Prime implicants ⇒ (E_sPIm) = 3

$\bar{B}\bar{C}D, ABC, ACD,$

$F = \bar{B}\bar{C}D + ABC + ACD + ABD$

* $F(A, B, C, D) = \sum_m (0, 2, 3, 4, 5, 6, 10, 11, 12)$.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1		5	4
$\bar{A}B$	1	3		2
AB	1			
$A\bar{B}$			11	10

$F = \bar{B}\bar{C} + \bar{A}\bar{D} + \bar{A}B\bar{C} + B\bar{C}\bar{D}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1		5	4
$\bar{A}B$	1	3		2
AB	1			
$A\bar{B}$			11	10

* Four Variable terms.

A single 1 = 4 Variables

Two 1's = 3 Variables

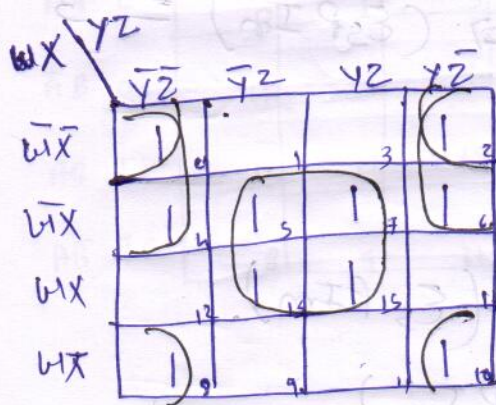
Four 1's = 2 Variables

Eight 1's = 1 Variables

sixteen 1's = 0 Variables

(Constant = 1)

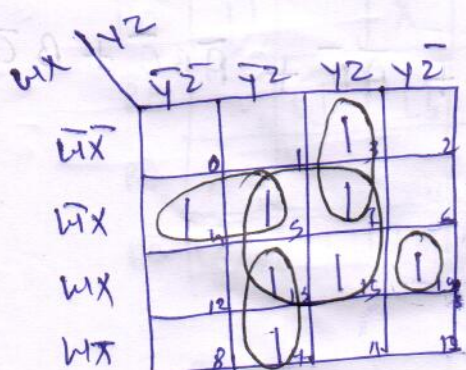
$$* F(WXYZ) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$$



$$F = XZ + \bar{W}\bar{Z} + \bar{X}\bar{Z}$$

$$F = XZ + \bar{X}\bar{Z} + \bar{W}X$$

$$* F(WXYZ) = \sum m(3, 4, 5, 7, 9, 13, 14, 15)$$



$$F = XZ + \bar{W}YZ + \bar{W}X\bar{Y} + W\bar{Y}Z + WXYZ$$

$$L = 15$$

$$Q = 20$$

$$QN = 23$$

	$\bar{y}z$	$y\bar{z}$	yz	$\bar{y}\bar{z}$
$\bar{w}x$	0	1	3	2
$\bar{w}x$	4	5	7	6
wx	12	13	15	14
wx	8	9	11	10

$$F = \bar{w}yz + wx\bar{y} + w\bar{y}z + \bar{w}x\bar{y}$$

$$L = 12$$

$$G = 16$$

$$GN = 18$$

* Fill a K-map from boolean equation

$$F(w, x, y, z) = x\bar{y}z + xy\bar{z} + w\bar{x}y + wx\bar{y}$$

	$\bar{y}z$	$y\bar{z}$	yz	$\bar{y}\bar{z}$
$\bar{w}x$	0	1	3	2
$\bar{w}x$	4	5	7	6
wx	12	13	15	14
wx	8	9	11	10

$$F = xy + wy$$

$$* F(w, x, y, z) = xy + x\bar{y} + wxyz$$

	$\bar{y}z$	$y\bar{z}$	yz	$\bar{y}\bar{z}$
$\bar{w}x$	0	1	3	2
$\bar{w}x$	4	5	7	6
wx	12	13	15	14
wx	8	9	11	10

$$F = x$$

* K-Map for maximum.

* $F(ABC) = \sum_m (0, 1, 2, 4, 5, 6)$

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
	00	01	10	11
\overline{A}	1	1	0	1
A	1	1	0	1

$F = \overline{B} + \overline{C}$

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
	00	01	10	11
\overline{A}	1	1	0	1
A	1	1	0	1

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
	00	01	10	11
\overline{A}	0	0	1	1
A	0	0	1	1

$\overline{F} = BC$

↓

$\overline{\overline{F}} = \overline{BC} \rightarrow F = \overline{B} + \overline{C}$

A \ BC	$\overline{B}\overline{C}$	$\overline{B}C$	$B\overline{C}$	BC
	00	01	10	11
\overline{A}	0	0	1	1
A	0	0	1	1

* $F(ABCD) = \sum_m (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)$

AB \ CD	$\overline{C}\overline{D}$	$\overline{C}D$	$C\overline{D}$	CD
	00	01	10	11
$\overline{A}\overline{B}$	0	1	2	3
$\overline{A}B$	4	5	6	7
$A\overline{B}$	8	9	10	11
AB	12	13	14	15

$\overline{F} = ABCD$

$\overline{\overline{F}} = \overline{ABCD}$

$F = \overline{ABCD}$

$L = 4$

$K = 4$

$CM = 5$

AB \ CD	$\overline{C}\overline{D}$	$\overline{C}D$	$C\overline{D}$	CD
	00	01	10	11
$\overline{A}\overline{B}$	0	1	2	3
$\overline{A}B$	4	5	6	7
$A\overline{B}$	8	9	10	11
AB	12	13	14	15

* $F(ABCD) = \sum m(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14)$

AB \ CD				
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	1

$\bar{F} = ACD$

$F = \overline{ACD}$

AB \ CD				
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	1

* Don't care *

- input values for the min term, will never occur, known as Don't care.
- Represented by ('X').
- we use BCD code, 6 code is never utilized it's called don't care.

AB \ CD				
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	1

Ex $F(A, B, C, D) = \sum m(3, 9, 11, 12, 13, 14, 15) + \sum d(1, 4, 6)$

AB \ CD				
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

$F = AB + AD + \bar{B}CD$

never utilized in system

Check for utilization of each bit in system. Use for utilization of each bit in system.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		X	1	
$\bar{A}B$	X			X
AB	1	1	1	1
$A\bar{B}$		1	1	

$$F = AB + \bar{B}D$$

$$D(A) = \bar{A}$$

$$D(\bar{A}) = A$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	1	0	1	1
$A\bar{B}$	1	0	1	1

$$* F(A, B, C, D) = \sum_m (2, 4, 10, 12, 14) + \sum_d (0, 1, 5, 8)$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	X	X		1
$\bar{A}B$	1	X	1	
AB	1			1
$A\bar{B}$	X			1

$$F = \bar{C}\bar{D} + \bar{B}\bar{D} + A\bar{D}$$

* Function Realization:-

X	Y	Z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	0	1

X \ YZ	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
\bar{X}		1		1
X	1		1	

$$F = X\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XYZ + \bar{X}Y\bar{Z}$$

\Rightarrow no of 1's odd \Rightarrow output = 1
 1's even \Rightarrow 0

\rightarrow This known as Even parity check,
 check for integrity of data. Use in security system.

$$F = x\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}z + \bar{x}y\bar{z}$$

$$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$$

$$= \bar{x}(T) + x(\bar{T})$$

$$= \bar{x}T + x\bar{T}$$

$$= x \oplus T$$

$$T = \bar{y}z + y\bar{z}$$

$$T = y \oplus z$$

$$\boxed{F = x \oplus y \oplus z}$$

→ used for even parity check.

$$= \overline{\bar{y}z + y\bar{z}}$$

$$= (y + \bar{z}) \cdot (\bar{y} + z)$$

$$= y\bar{y} + yz + \bar{z}\bar{y} + \bar{z}z$$

$$= (yz + \bar{z}\bar{y})$$