$\mathcal{N} =$
4
$\mathcal{O}_Z = Z^L, \mathcal{O}_X = X^L, Z = \phi_3 + i\,\phi_6, X = \phi_1 + i\,\phi_4$
(1)
??
gauge fix- ing, mass ma- trix di- ag- o- nal- iza- tion etc.
$\mathcal{O}_Z$
$\mathcal{O}_X$
chi- ral pri- mary op- er- a- tors su- per con- for- mal pri- mary

$$[D, \mathcal{O}(0)] = -i\Delta\,\mathcal{O}(0), [S^a_\alpha, \mathcal{O}(0)] = 0, [\bar{S}_{a\dot{\alpha}}, \mathcal{O}(0)] = 0 \qquad (2)$$

S^a_α
K_μ
D
scale trans- for- ma- tions
Q^a_α
P_μ

$$\exists a \in \{1, \dots, \mathcal{N}\}\,\exists \alpha \in \{1, 2\}:\ [Q^a_\alpha, \mathcal{O}(0)] = 0 \qquad (3)$$

??
??
Δ
g
$psu(2, 2|4)$
?

$$\{Q^a_\alpha, S_{b\beta}\} = -\frac{i}{2}\,\varepsilon_{\alpha\beta}(\sigma^{IJ})^a{}_b\,R_{IJ} - \frac{1}{2}\varepsilon_{\alpha\beta}\delta^a{}_b D + (\sigma^{\mu\nu})_{\alpha\beta}\delta^a{}_b\,M_{\mu\nu} \qquad (4)$$

$(\sigma^{IJ})^a{}_b$
$SU(4)$
R_{IJ}
$SO(6)$
$M_{\mu\nu}$
$SO(3, 1)$
$\mathcal{O}$
$[M_{\mu\nu}, \mathcal{O}(0)] =$
0
??
??
$\{$
${}^a_\alpha, S_{b\beta}\}, \mathcal{O}(0)] =$
$[-i\,\varepsilon_{\alpha\beta}(\sigma^{IJ})^a{}_b R_{IJ} -$
$\varepsilon_{\alpha\beta}\delta^a{}_b D, \mathcal{O}(0)] =$
0

$$\Leftrightarrow (\sigma^{IJ})^a{}_b\,[R_{IJ}, \mathcal{O}(0)] = \Delta\,\delta^a{}_b\,\mathcal{O}(0) \qquad (5)$$

$\mathcal{O}$
??
$\mathcal{O}$
??

$$[[\mathcal{O}(0), Q^a_\alpha], S_{b\beta}] = 0 \Rightarrow [\mathcal{O}(0), Q^a_\alpha] = 0 \qquad (6)$$

Q^a_α
$\mathcal{O}$
1/2
$S_{b\beta}$

$(J_1, 0, 0)$
$\Delta =$
J_1
$\bar{Q}_{3,\dot{\alpha}}$
$\bar{Q}_{4,\dot{\alpha}}$
$psu(2,2|4)$
??

$$\{\bar{Q}_{a\dot{\alpha}}, \bar{S}^{b}_{\dot{\beta}}\} = \frac{i}{2}\,\varepsilon_{\dot{\alpha}\dot{\beta}}(\sigma^{IJ})_a{}^b\,R_{IJ} - \frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}}\delta_a{}^b D + (\sigma^{\mu\nu})_{\dot{\alpha}\dot{\beta}}\delta_a{}^b\,M_{\mu\nu} \quad (9)$$

$$\Leftrightarrow (\sigma^{IJ})_a{}^b\,[R_{IJ}, \mathcal{O}(0)] = -\Delta\,\delta_a{}^b\,\mathcal{O}(0) \quad (10)$$

$(J_1,0,0)$
$\Delta =$
J_1
1/2-
BPS
op-
er-
a-
tors
$(J_1,0,0)$
$(0,J_2,0)$
$(0,0,J_3)$
??
a
α
Δ
$\mathcal{O}_Z$
$\mathcal{O}_X$
X
Z
??

$$[R^{(0)}_{14}, \mathcal{O}_X(0)] = L\,\mathcal{O}_X(0), [R^{(0)}_{36}, \mathcal{O}_Z(0)] = L\,\mathcal{O}_Z(0) \quad (11)$$

$$[D^{(0)}, \mathcal{O}_X(0)] = -i\,L\,\mathcal{O}_X(0), [D^{(0)}, \mathcal{O}_Z(0)] = -i\,L\,\mathcal{O}_Z(0) \quad (12)$$

$D^{(0)}$
$R^{(0)}_{IJ}$
$SO(6)$
$(L,0,0)$
$(0,0,L)$
$\Delta =$
$J_1 =$
L
$\Delta =$
$J_3 =$
L
$\mathcal{O}_Z$
$\mathcal{O}_X$
$\mathcal{N} =$
4
$\mathcal{O}_X$
$\mathcal{O}_Z$
??
$psu(2,2|4)$
$\mathcal{O}_X$
$\mathcal{O}_Z$
$(L,0,0)$
$(0,0,L)$
$\Delta =$
L
g
$SU(4) \simeq$
$SO(6)$
$\mathcal{O}_X$
$\mathcal{O}_Z$
L
$SO(6)$

$$\mathcal{O}_X = \Phi_X^{i_1\ldots i_L}[\phi_{i_1}\cdots\phi_{i_L}], \mathcal{O}_Z = \Phi_Z^{i_1\ldots i_L}[\phi_{i_1}\cdots\phi_{i_L}] \quad (13)$$

$\Phi_X^{i_1\ldots i_L}$
$\Phi_Z^{i_1\ldots i_L}$
$\mathcal{O}_Z$
Z
ℓ
$\ell+$
1

$\cdots Z(\phi_3+$
$i\phi_6)(\phi_3+$
$i\phi_6)Z\cdots$

$= \cdots Z(\phi_3\phi_3 - \phi_6\phi_6 + i\phi_3\phi_6 + i\phi_6\phi_3)Z\cdots$
(14)
$\Phi_Z^{\cdots 36\cdots} =$
$\Phi_Z^{\cdots 63\cdots}, \Phi_Z^{\cdots 33\cdots} +$
$\Phi_Z^{\cdots 66\cdots} =$
$0,$

$\Phi_Z^{\cdots i_\ell i_{\ell+1}\cdots} = 0 for i_\ell, i_{\ell+1} \neq 3, 6$
(15)
??
$\Phi_Z^{i_1\ldots i_L}$

$$\Phi_Z^{i_1\ldots i_\ell\, i_{\ell+1}\ldots i_L} = \Phi_Z^{i_1\ldots i_{\ell+1} i_\ell \ldots i_L}, \sum_{i_\ell=1}^{6} \Phi_Z^{i_1\ldots i_\ell\, i_\ell \ldots i_L} = 0$$
(16)
$\Phi_X^{i_1\ldots i_L}$
$SO(2,1)$
´e
$\mathcal{O}_a(x)\mathcal{O}_b(y)$
$|\vec{x}-$
$\vec{y}|$
x
y
$x_3, y_3 >$
0
$|\vec{x}-$
$\vec{y}|$
$x = (\vec{x}, x_3), \vec{x} = (x_0, x_1, x_2), |x-y|^2 = |\vec{x}-\vec{y}|^2 + |x_3-y_3|^2$
(17)
$\mathcal{O}_a(x)\mathcal{O}_b(y)$
in-
ver-
sion

$$x \to \tilde{x} = \frac{x}{|x|^2} \Rightarrow |x| \to |\tilde{x}| = \frac{|x|}{|x|^2} = \frac{1}{|x|} \Rightarrow \tilde{x} \to \frac{\tilde{x}}{|\tilde{x}|^2} = x$$
(18)
$SO(2,1)$
é
$\mathcal{O}_a(x)\mathcal{O}_b(y)$
con-
for-
mal
ra-
tio
ξ
??

$$|\tilde{x}-\tilde{y}|^2 = \frac{|x-y|^2}{x^2y^2}, \tilde{x}_3\tilde{y}_3 = \frac{x_3y_3}{x^2y^2}, \xi = \frac{|x-y|^2}{4x_3y_3} \Rightarrow \tilde{\xi} = \xi$$
(19)
$SO(1,1)$
scale
trans-
for-
ma-
tions
$x \to \tilde{x} = \lambda x, \mathcal{O}_a(x) \to \tilde{\mathcal{O}}_a(x) = \lambda^{-\Delta_a}\mathcal{O}_a(\lambda^{-1}x)$
(20)

$$\Rightarrow \mathcal{O}_a(x)\mathcal{O}_b(y) = \frac{f_{ab}(\xi)}{(2x_3)^{\Delta_a}(2y_3)^{\Delta_b}}$$
(21)
$f_{ab}(\xi)$
ξ
Δ
$f_{ab}(\xi)$
∞
$SO(4,2)$

$$\lim_{z_3\to\infty} \mathcal{O}_a(x+z)\bar{\mathcal{O}}_b(y+z) = \frac{M_{ab}}{|x-y|^{\Delta_a+\Delta_b}}$$
(22)
$\mathcal{N} =$
4
??
$f_{ab}(\xi)$

$$f_{ab}(\xi) = \xi^{-\frac{\Delta_a+\Delta_b}{2}}\left[M_{ab} + \sum^{\infty} c_{ab\ldots}\xi^n\right]$$