Reality Check 4, GPS Conditioning and Nonlinear Least Squares

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Introduction

Global Positioning System (GPS) relies on signals transmitted by satellites to accurately determine a receiver's position and timing corrections. This process involves solving a set of nonlinear equations based on the distances between the receiver and satellites, which are calculated using the speed of light and signal travel times. However, the accuracy of these calculations can be influenced by various factors, including the configuration of satellites and small errors in signal timings. Understanding how tightly or loosely grouped satellite configurations impact the numerical stability and sensitivity of GPS calculations is critical for assessing the system's reliability. These problems explore the mathematical and computational aspects of GPS, focusing on solving nonlinear equations, analyzing error propagation, and comparing the conditioning of GPS calculations under different satellite arrangements.

Question 1

Solve the system (4.37) by using a multivariate root finder in Python.

System:

$r_1(x, y, z, d) = \sqrt{(x - A_1)^2 + (y - B_1)^2 + (z - C_1)^2} - c(t_1 - d) = 0$

$r_2(x, y, z, d) = \sqrt{(x - A_2)^2 + (y - B_2)^2 + (z - C_2)^2} - c(t_2 - d) = 0$

$r_3(x, y, z, d) = \sqrt{(x - A_3)^2 + (y - B_3)^2 + (z - C_3)^2} - c(t_3 - d) = 0$

$r_4(x, y, z, d) = \sqrt{(x - A_4)^2 + (y - B_4)^2 + (z - C_4)^2} - c(t_4 - d) = 0$

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Receiver position: x = -41.773 km, y = -16.789 km, z = 6370.060 km
Time correction: d = -0.003202 seconds
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Question 2

Write a Python program to carry out the solution via the quadratic formula.

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Receiver position: x = -11484.160 km, y = -4853.894 km, z = 604.749 km
Time correction: d = 11835.185609 seconds
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(-11484.159579924179, -4853.893762908117, 604.7494113125853, 11835.18560948177)
```

Question 4

Now set up a test of the conditioning of the GPS problem. Define satellite positions (Ai,Bi,Ci) from spherical coordinates (ρ,φi,θi) as

Ai=ρCos(φi)Cos(θi)

Bi=ρCos(φi)Sin(θi)

Ci=ρSin(φi)

where ρ=26570 km is fixed, while $0 \leq \varphi i \leq \pi/2$ and $0 \leq \theta i \leq 2\pi$ for i=1,…, 4 are chosen arbitrarily.

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Satellite Positions (A, B, C):
Satellite 1: A = 0.00 km, B = 23010.29 km, C = 13285.00 km
Satellite 2: A = 9393.91 km, B = 16270.74 km, C = 18787.83 km
Satellite 3: A = 9393.91 km, B = 9393.91 km, C = 23010.29 km
Satellite 4: A = 0.00 km, B = 0.00 km, C = 26570.00 km

Satellite Ranges (R_i) and Travel Times (t_i):
Satellite 1: R = 24026.88 km, t = 0.080245 seconds
Satellite 2: R = 22520.77 km, t = 0.075221 seconds
Satellite 3: R = 21292.97 km, t = 0.071126 seconds
Satellite 4: R = 20200.00 km, t = 0.067480 seconds
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What is the maximum position error found, in meters? Estimate the condition number of the problem, on the basis of the error magnification factors you have computed.

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Change in position (Δx, Δy, Δz): 0.01 meters
Error Magnification Factor (EMF): 2.00
Condition number of the problem: 1.00
```

Question 5

Now repeat Step 4 with a more tightly grouped set of satellites. Choose all φi within 5 percent of one another and all θi within 5 percent of one another. Solve with and without the same input error as in Step 4. Find the maximum position error and error magnification factor. Compare the conditioning of the GPS problem when the satellites are tightly or loosely bunched.

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Tightly Grouped Satellites - Change in position (Δx, Δy, Δz): 0.01 meters
Tightly Grouped Satellites - Error Magnification Factor (EMF): 2.00
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Loosely Grouped Satellites - Change in position (Δx, Δy, Δz): 0.01 meters
Loosely Grouped Satellites - Error Magnification Factor (EMF): 2.00
EMF comparison: Tightly Grouped = 2.00, Loosely Grouped = 2.00
```