CPRE 488 Homework 4 Spring 2024

PID Control
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GitHub Page

Problem 1

Sensor Basics. Review the follow resources:

Accelerometer Intro:
https://web.archive.org/web/20230329223955/https://www.hobbytronics.co.uk/accelerometer-info

Gyroscope Intro:
https://web.archive.org/web/20230220084525/http://www.hobbytronics.co.uk/gyro-info

Trade-offs:
https://web.archive.org/web/20230220083746/http://www.hobbytronics.co.uk/accelerometer-gyro

Problem 1a

Given a generic 3-axis accelerometer, show the math to derive the Roll and Pitch angle of the sensor. Simplifying assumption: assume the sensor will only be rotated about a single axis (X, Y, or Z), and that the sensor is static when the Roll or Pitch is calculated.

The force of gravity is (assumed to be) constant. Because of this we can assume that at 0 tilts we would have the full g force in the negative z direction.

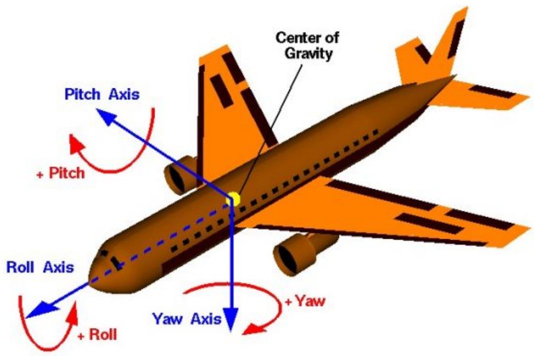


Figure 1: Rotations definitions[1].

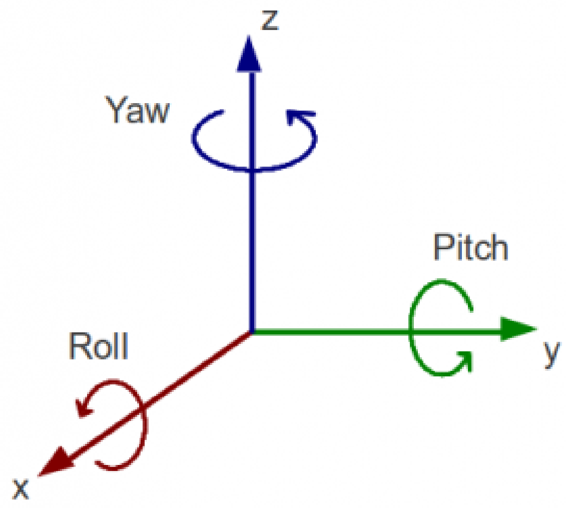


Figure 2: Roll, Pitch and Yaw and XYZ axis[2].

To determine the Roll we can use the amount of the force has shifted to the different axis.

Using the figure we can see that the roll is determined by the X axis.
$Roll = sin^{-1}(x/z)$

This can be easily checked by using g as z and 0 as x.

To determine the pitch we do the same thing but with the Y axis instead.
$Pitch = sin^{-1}(y/z)$

Problem 1b

> Repeat a) for a generic 3-axis gyroscope. Simplifying assumption: assume the sensor begins at Roll, Pitch, and Yaw orientation of (0, 0, 0) degrees, and is then rotated about a single axis to its final orientation.

The gyroscope already measures change in rotation so we can integrate that to get the current rotations.

$Roll = \int \omega_y \, dt$

$Pitch = \int \omega_x \, dt$

Problem 2

> PID Control. Next, review the following resources:
> Wikipedia

PID-demo

Problem 2a

In terms an average eighth grader could understand, explain how the P, D, and I components of a PID controller's correction output moves an object from an initial location to its goal location.

There is error which is the difference between your desired value and the actual value.

P corrects for the current error.
D corrects for the average error over time.
I corrects for the error in the rate of change.

These allow to not overshoot as much when correcting for the existing error.

Problem 2b

Provide pseudo-code for implementing the discrete version of the PID control algorithm.

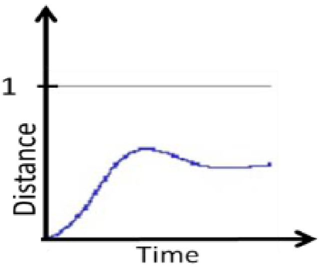
```
PID(expected value, actual value){

last error = error;
error = expected value - actual value;
sum = sum+ error;

u = P*error + I*(sum) + D*(error - last error);

return u

}
```

Problem 2c

Demonstrate that you can reason about the P, I, and D components of a PID controller. In the examples on the following page, a PID controller provides a corrective force to a ball that is being moved from point 'a' to point 'b' on a 45 degree slope. The first plot shows the response of the ball moving from a height of 0m to 1m under the control of a of properly tuned PID controller. For each of the remaining plots, the P, I, and/or D constant of a PID controller has not been tuned properly. A statement has been made for each plot. Indicate if the statement is True or False, and defend your answer.

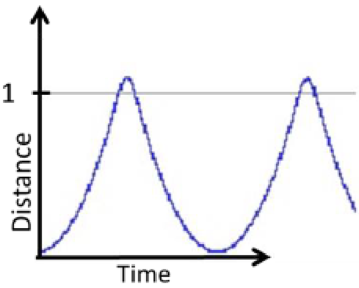
Properly tuned PID

i) P constant is too large

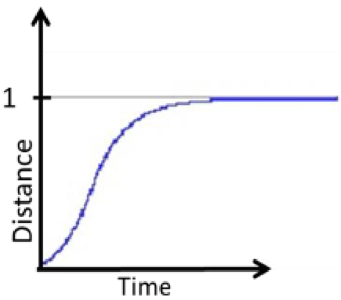


i) P constant is too large

This is false, a large P would cause the value to overshoot not undershoot. It is not correcting to the position.



iv) I constant is too small



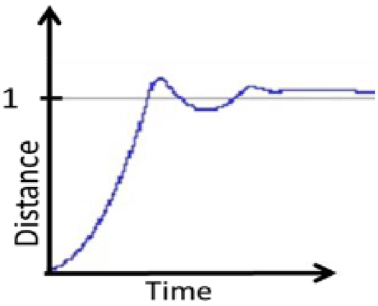
v) P constant too large

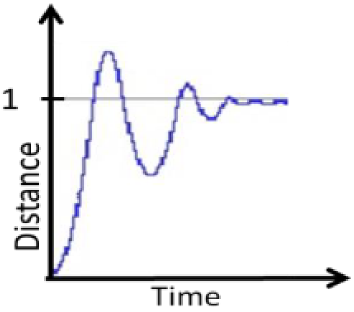
ii) D constant is too large

This is false, D constant being large would cause it to settle slower not oscillate.

iii) D constant is too large

This is true, the slow speed of change is controlled by the D.





iv) I constant is too small
False, a small I value would not cause oscillation. Increasing I would only create worse oscillations.

v) P constant too large
This is true. The overshooting is a sign that it is underdamped and is over correcting for the positional error.

References

[1] NASA: Aircraft Rotations

[2] Roll, Pitch and Yaw