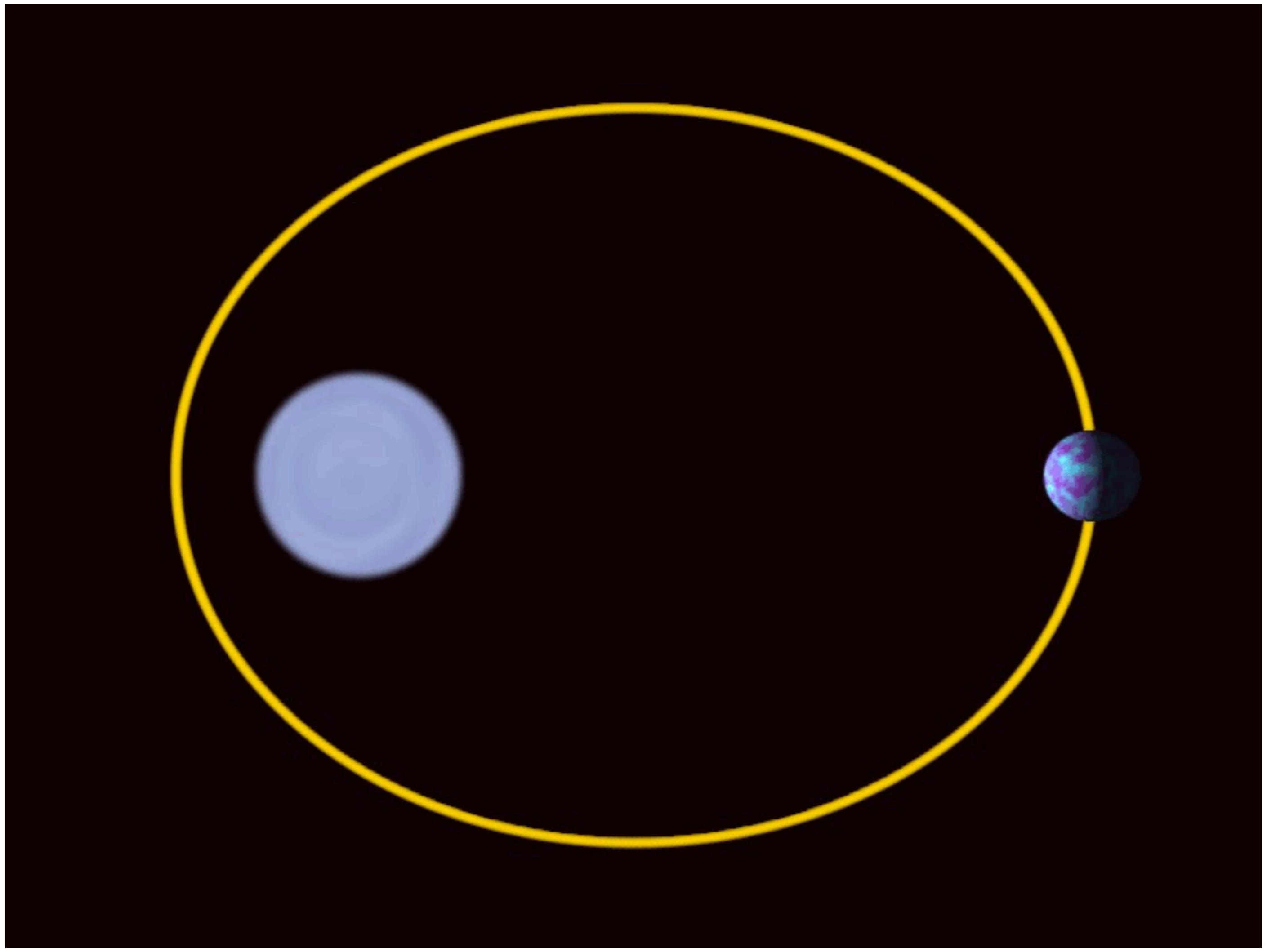
Orbital motion



PHYS 246 class 3

https://jnoronhahostler.github.io/IntroductionToComputationalPhysics/intro.html

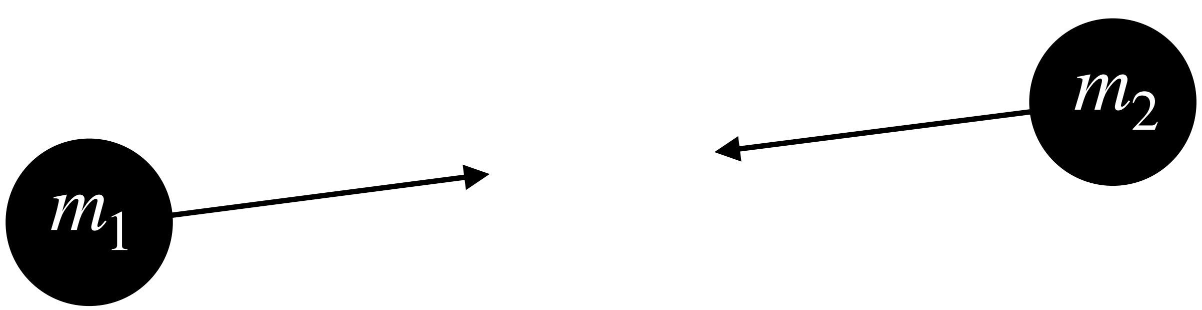
Announcements/notes

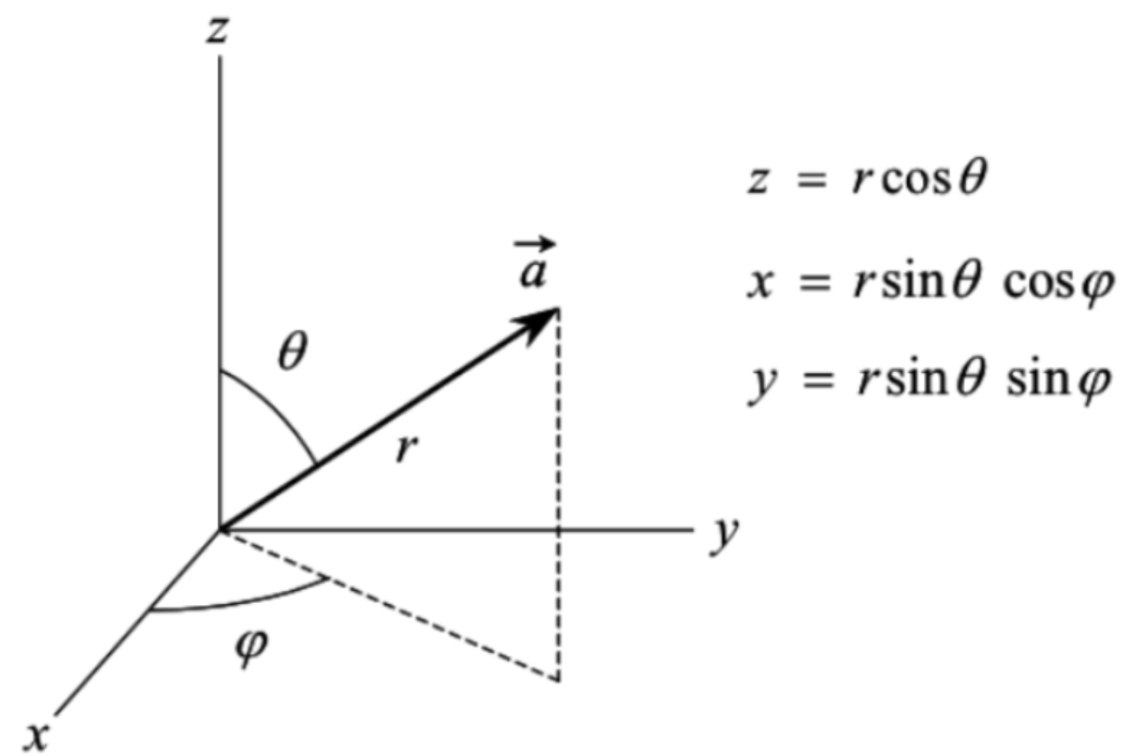
- "Dynamics" is due tonight on Gradescope
- PDF uploads are required to not be counted late
- Make sure to link all relevant pages for the appropriate rubric problem (including answer box, output, plots)
- Limit your comment length so it doesn't get cut off my the answer box dimensions
- If your output includes result, make it clear and descriptive. Don't just print out numbers for us to interpret what they correspond to.

Force of gravity

For Newtonian gravity $\overrightarrow{F}(\vec{r}) = m\vec{a}$ that we can rewrite in radius coordinates, integrating over the angles (since it's radially symmetric)

$$F(r) = -\frac{Gm_1m_2}{r^2}\hat{r}$$





How can we study orbital dynamics of the Earth and another body?

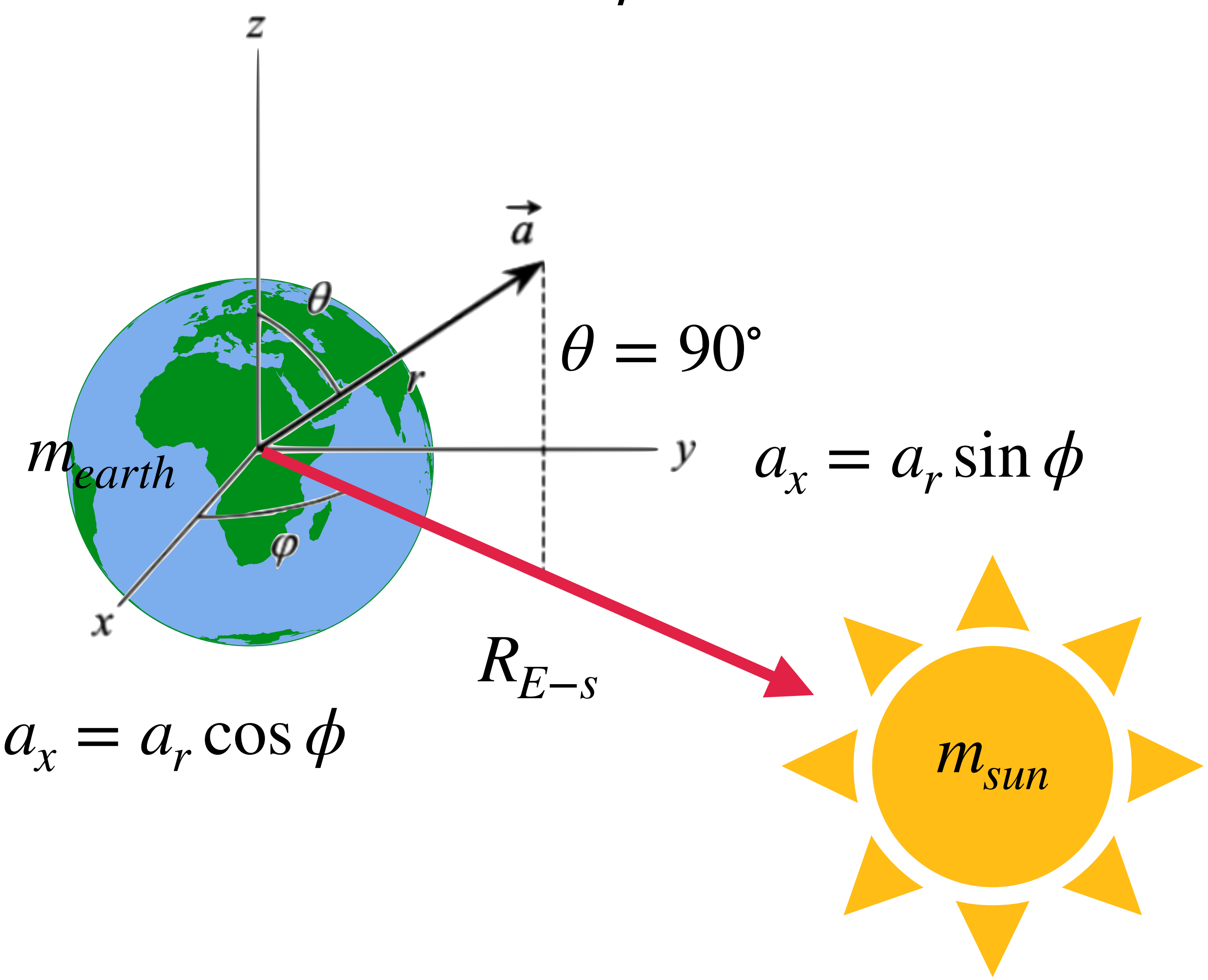
Earth+Sun

Obviously not to scale...

How much does the Earth accelerate?

$$m_{earth} a_r = -\frac{G m_{earth} m_{sun}}{r^2}$$

$$a_r = \frac{G m_{sun}}{r^2}$$



$$a_x = a_r \sin\phi$$

$$a_x = a_r \cos\phi$$

$$a_x = \frac{G m_{sun} x}{\left(x^2 + y^2\right)^{3/2}}$$

$$a_y = \frac{G m_{sun} y}{\left(x^2 + y^2\right)^{3/2}}$$

Where do I start?

2D dynamics

- Recall your last assignment, you already have solved 2D dynamics!
- Your acceleration just looked different this time around…

$$v_x(t_{i+1}) - v_x(t_i) = \int_{t_i}^{t_{i+1}} a_x dt$$

$$v_y(t_{i+1}) - v_y(t_i) = \int_{t_i}^{t_{i+1}} a_y dt$$

Sanity Check

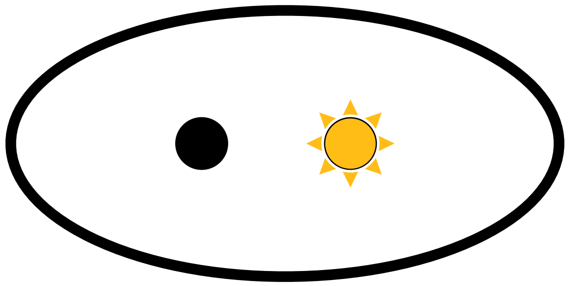
Limits, analytical solutions etc

- How do we know our code is correct?
- How do we test codes? What can we use for Earth orbital dynamics?
- How long does it take Earth to orbit the sun?

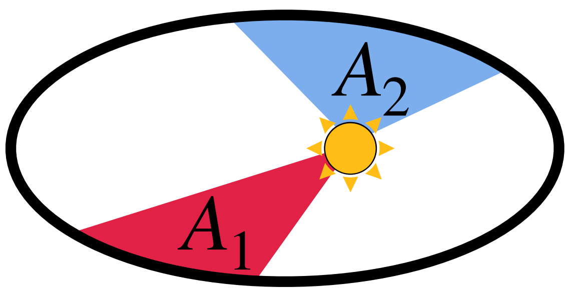
Kepler's laws

3 laws of orbital dynamics

- Law 1: The orbit of a planet is an ellipse with the Sun at one of two foci

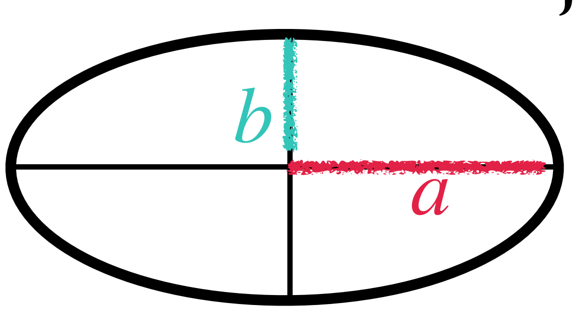


- Law 2: A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time



$$A_1 = A_2$$
$$t_1 = t_2$$

- Law 3: The square of the orbital period, T, of a planet is directly proportional to the cube of the semi-major axis, a, of its orbit.

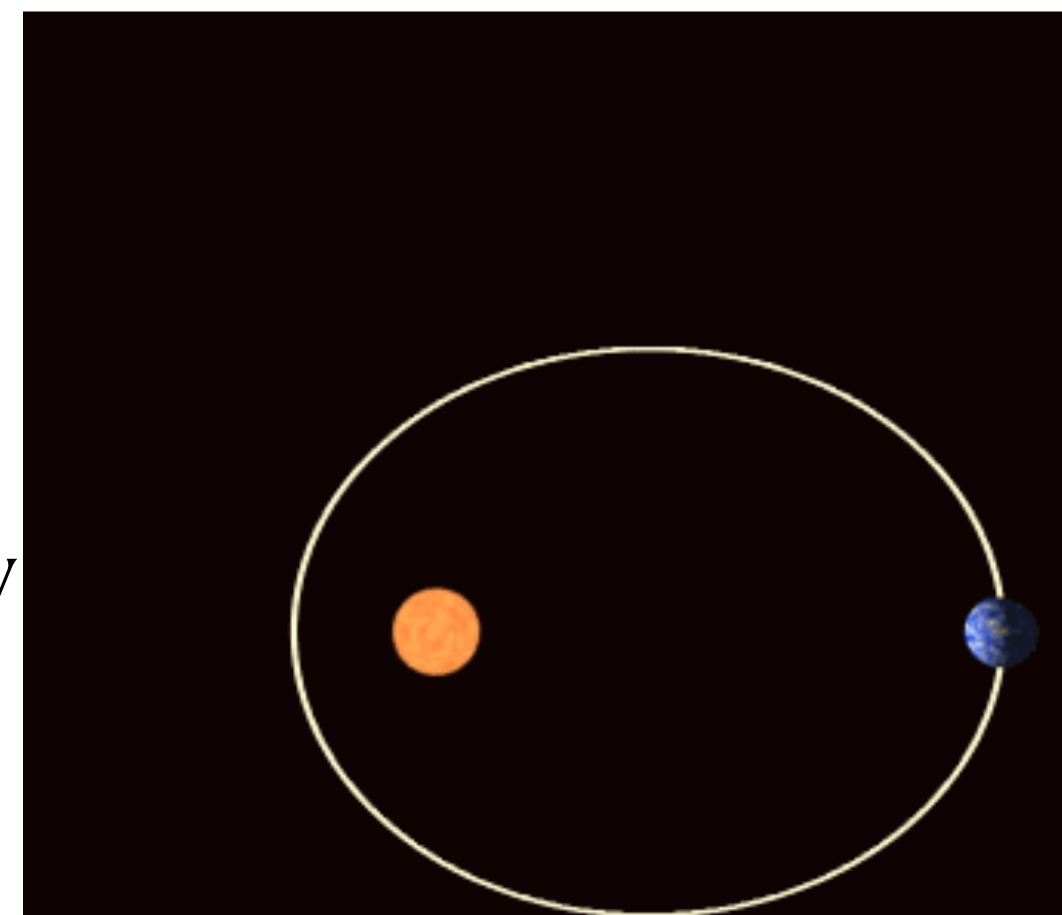


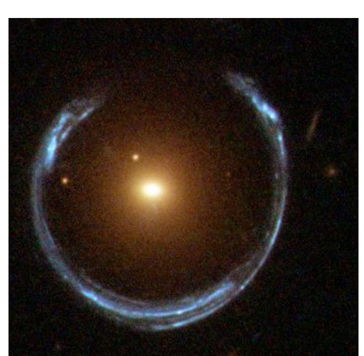
$$T^2 = a^3$$

From Newton to Einstein

Finding general relativity

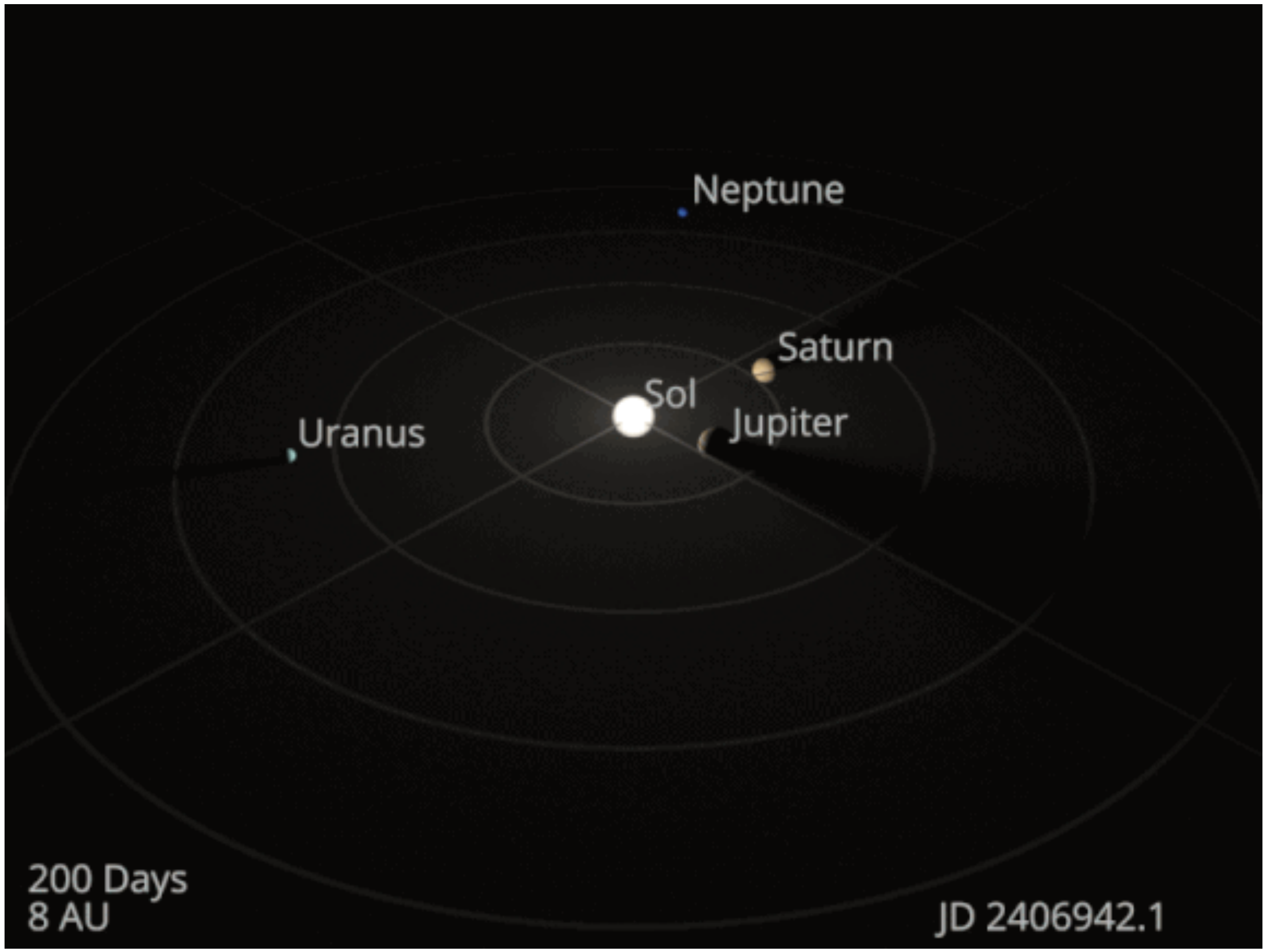
- 1800's, the universe was thought to be a "solved problem" that followed Newton's laws
- Scientists were busy computing the behavior of the solar system based on Newtonian gravity.
- Newtonian gravity couldn't explain precession of the perihelion
- Gravitational lensing effects under predicted in Newtonian gravity





Finding Neptune

Uranus was too slow!



1600's Neptune was originally thought to be a star, very stationary

1821 Alex Bouvard observed Uranus' orbit, too slow compared to predictions!

~1840's independently John Couch Adams (British) and Urbain Le Verrier (French) worked on calculations of a new planet (Neptune).

French/British rivalry on credit → 1988 "Neptune Papers"

Precession of Mercury

- Mercury precesses 565 arcseconds/100 years.

- Total due to the planets: 526.7 arcseconds/100 years

- Took years to do all the calculations!

- Ultimately resolved by general relativity in 1915.

- Accurate calculation made the detailed “laws” valuable (and falsified them).

When does Newtonian Gravity break down?

The rise of General Relativity

Two cases with Newtonian gravity breaks down:

1. Gravitational potential ϕ too large i.e. $\phi/c^2 \to 1$
   Fix: General Relativity!

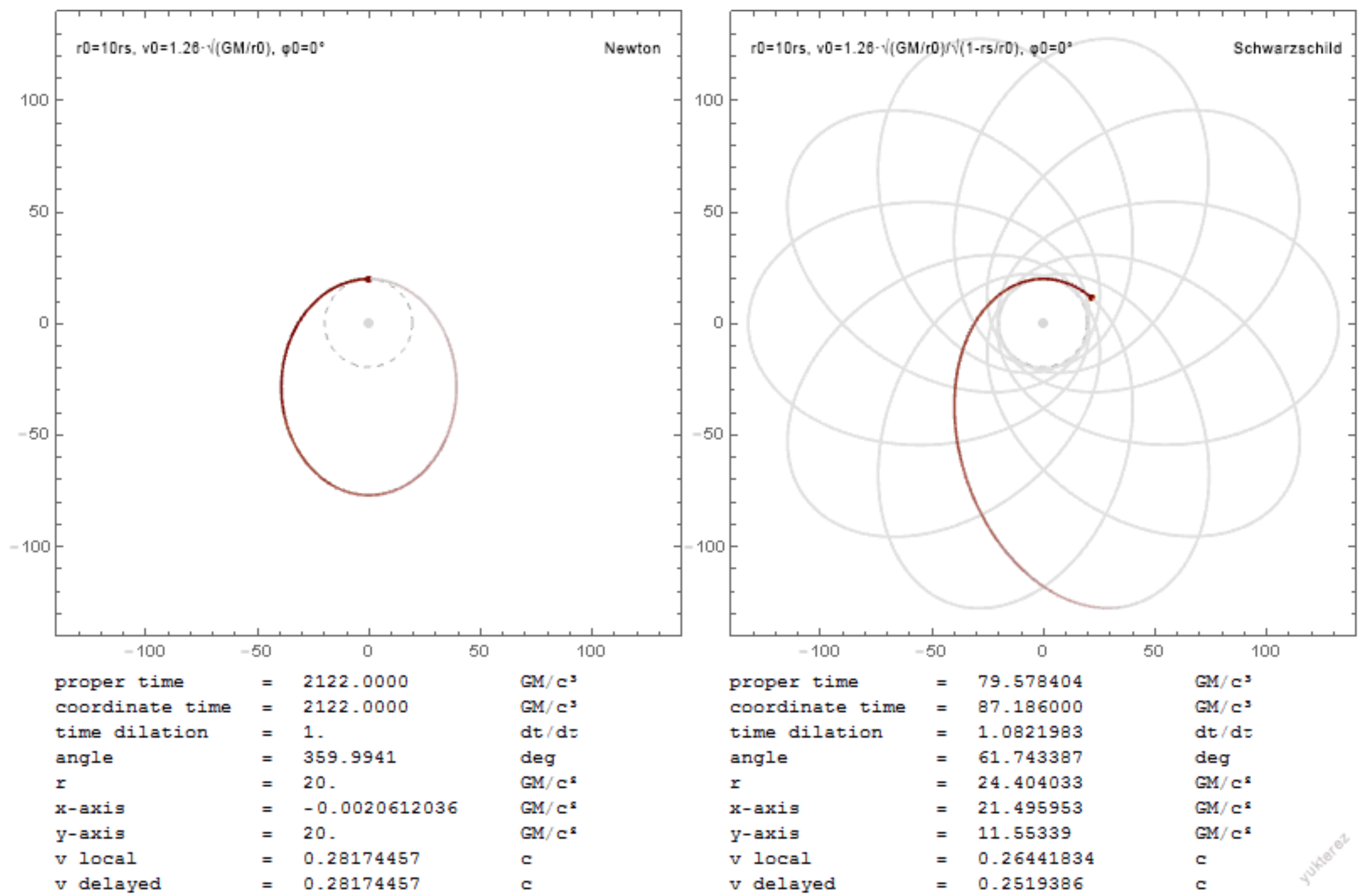
$$F = -\nabla\phi$$

2. Fast velocity i.e. $(v/c)^2 \to 1$
   Fix: Special Relativity!

Two options for solving GR:
1. Series expansion or
2. Large-scale numerical calculations (Maxwell works on this)!

GR corrections to Precession

Let's ignore all other planets



Our objective: find the correction to the precession due to general relativity.

Deviation of the GR correction Term

Curvature of space

$$\frac{(1-\frac{r_s}{4R})^2}{(1+\frac{r_s}{4R})^2}dt^2-(1+\frac{r_s}{4R})^2(dx^2+dy^2+dz^2)$$

Effective force (approximate)

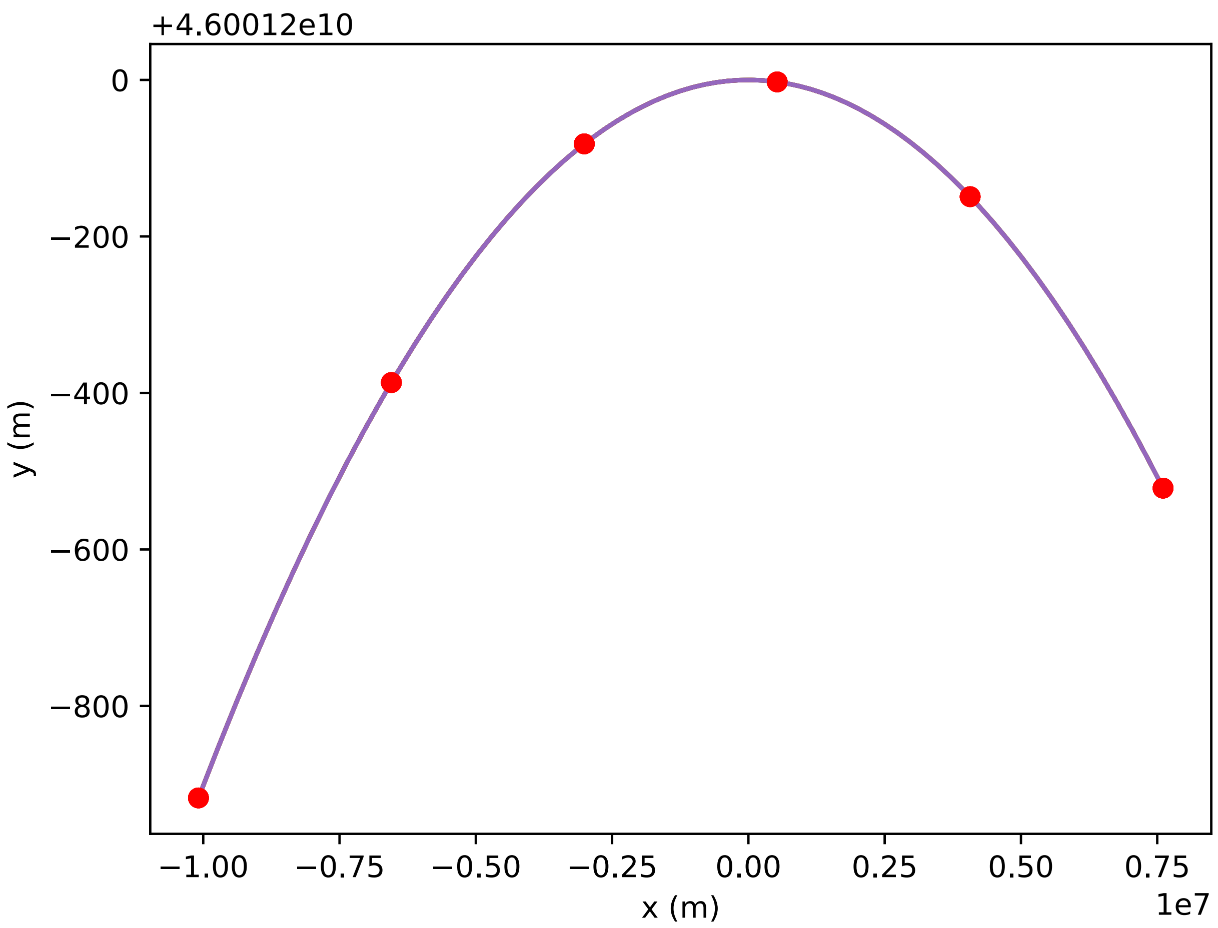
$$F/m=\left(-\frac{GM_\odot}{r^2}-\frac{3r_s\left(v_{\text{perihelion}}r_{\text{perihelion}}\right)^2}{2r^4}\right)\frac{\vec{r}}{r}$$

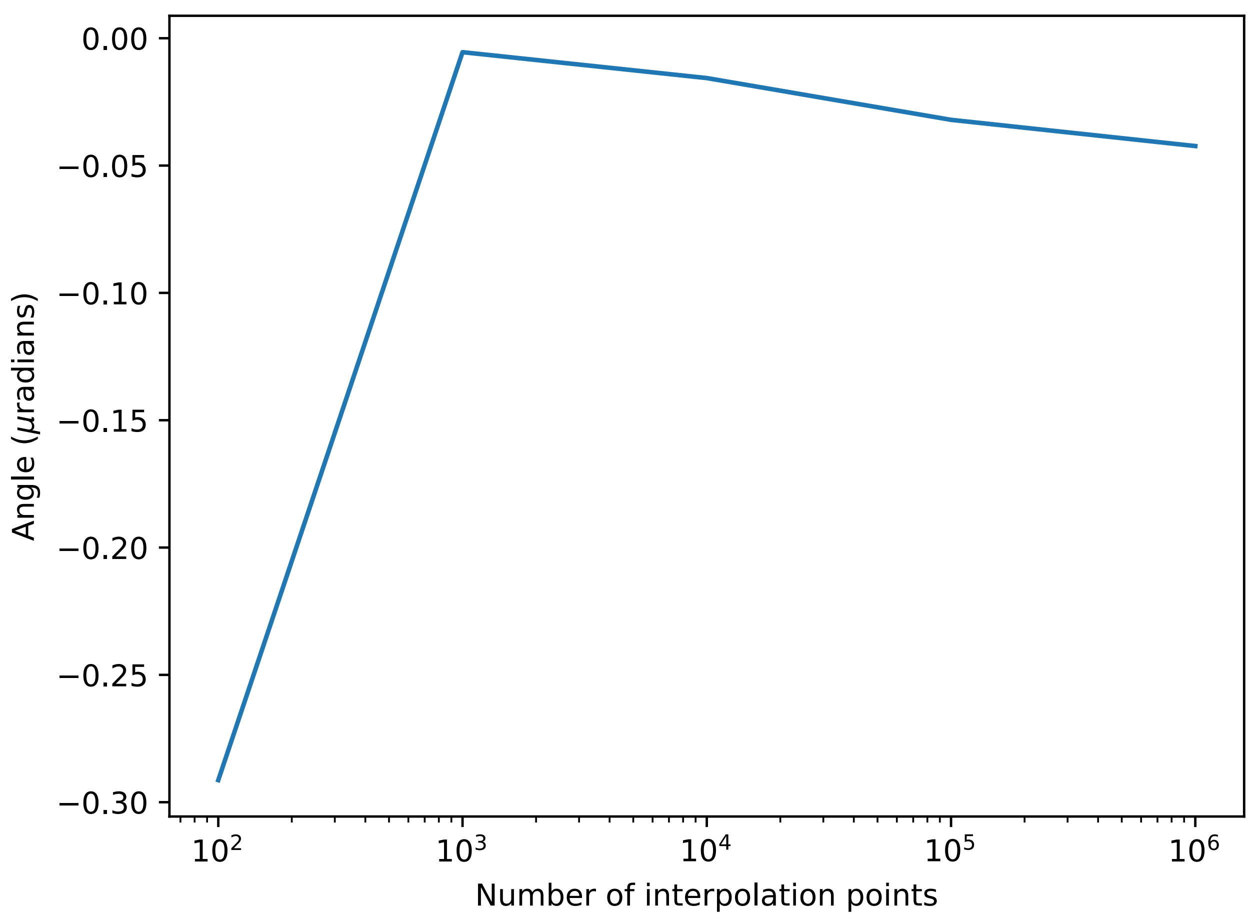
Karl Schwarzschild

https://en.wikipedia.org/wiki/Two-body_problem_in_general_relativity

Interpolation: use 10^4 vs 10^5 points

When is enough, enough?

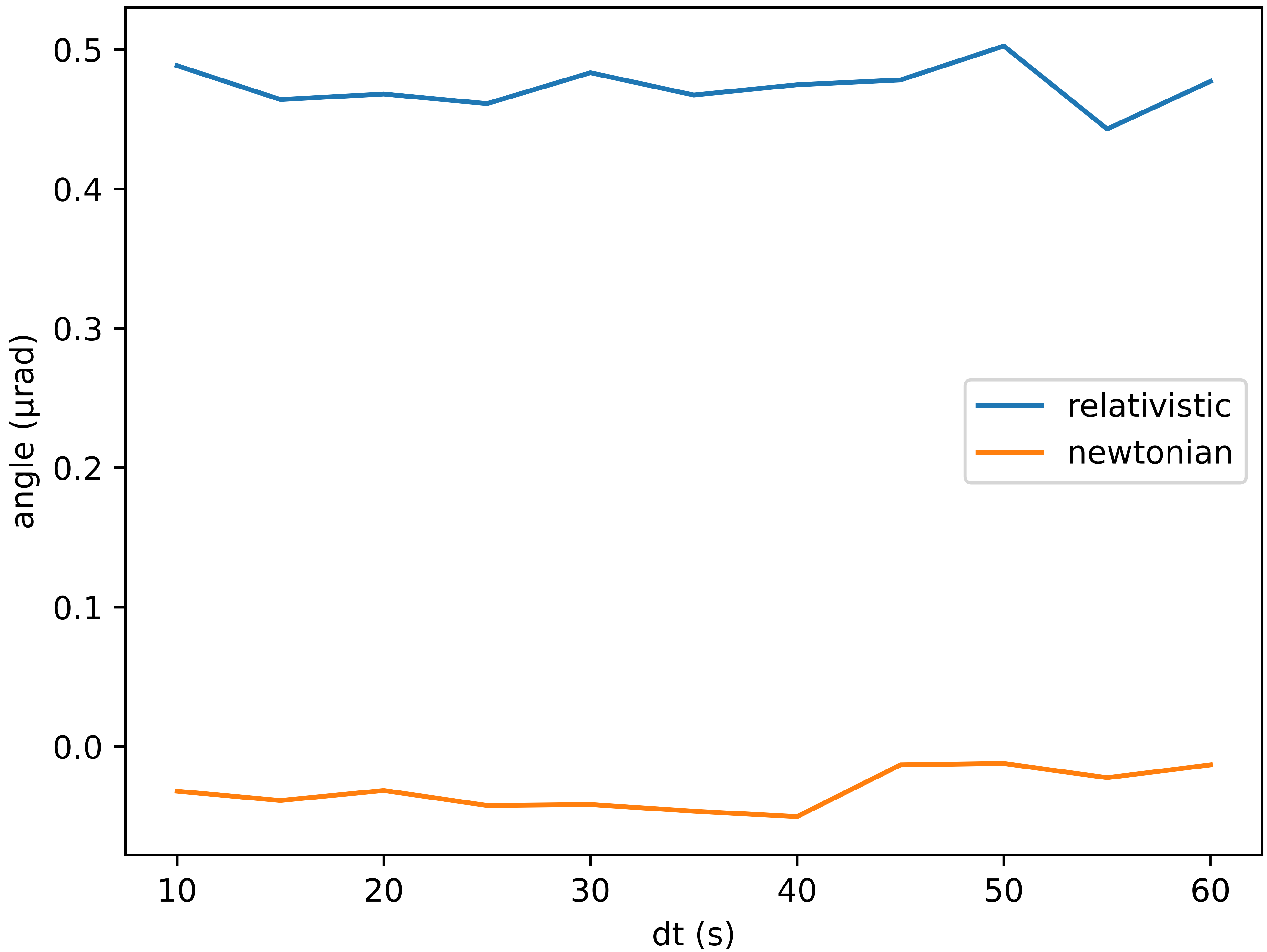




Convergence tests: we want the result to not change when we vary something that is a numerical variable that has nothing to do with the problem.

dt convergence

Pick $dt = 60$ seconds, results mostly independent of dt



ADD slides on animation

https://matplotlib.org/stable/users/explain/animations/animations.html

ADD slides on stopping codes

Never-ending codes...