TODO!!!

Definice 0.1 (Dot product on the space of matrices)

$$\mathbb{A} : \mathbb{B} = \mathrm{tr}(\mathbb{A}\mathbb{B}^T).$$

Definice 0.2 (Norm of matrix)

$$|\mathbb{A}| = (\mathbb{A} : \mathbb{A})^{\frac{1}{2}}.$$

Příklad

$$(\mathbf{a} \otimes \mathbf{b})^T = \mathbf{b} \otimes \mathbf{a}.$$

Důkaz

$$\mathbf{u} \cdot (\mathbf{a} \otimes \mathbf{b})^T \mathbf{v} = (\mathbf{a} \otimes \mathbf{b})\mathbf{u} \cdot \mathbf{v} = (\mathbf{a}(\mathbf{b} \cdot \mathbf{u}))\mathbf{v} = (\mathbf{b} \cdot \mathbf{u})(\mathbf{a} \cdot \mathbf{v}) = \mathbf{u} \cdot (\mathbf{b}(\mathbf{a} \cdot \mathbf{v})) = \mathbf{u} \cdot (\mathbf{b} \otimes \mathbf{a})\mathbf{v}.$$

□

Příklad

$$\det(e^{\mathbb{A}}) = e^{\mathrm{tr}\,\mathbb{A}}.$$

Důkaz

$$e^{\mathbb{A}} = \lim \left(\mathbb{I} + \frac{\mathbb{A}}{n}\right)^n.$$

$$\det e^{\mathbb{A}} = \lim_{n\to\infty} \left(\det\left(\mathbb{I} + \frac{\mathbb{A}}{n}\right)^n\right) = \lim_{n\to\infty}\left(\det\left(\mathbb{I} + \frac{\mathbb{A}}{n}\right)\right)^n =?$$

Subtask: Is there an approximation for $\det(\mathbb{I}+\mathbb{S})$, where $\mathbb{S}$ is a „small“ matrix. Yes, we did it (KontinuumDU1.pdf) for $\mathbb{S} \in \mathbb{R}^{3\times 3}$:

$$\det(\mathbb{I}+\mathbb{S}) = \det\mathbb{I}+\operatorname{tr}(\mathbb{I}\operatorname{cof}\mathbb{S})+\operatorname{tr}(\mathbb{S}^T\operatorname{cof}\mathbb{I})+\det\mathbb{S} \approx 1+\operatorname{tr}(\mathbb{S}^T\operatorname{cof}\mathbb{I})+o(\mathbb{S}^2) = 1+\operatorname{tr}(S)+o(\mathbb{S}^2).$$

And for $\mathbb{S} \in \mathbb{R}^{n\times n}$, one can see that:

$$\det(\mathbb{I}+\mathbb{S}) = \begin{pmatrix} 1+s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & 1+s_{22} & \dots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \dots & 1+s_{nn} \end{pmatrix} = (1+s_{11})(1+s_{22})\cdot\ldots\cdot(1+s_{nn})+o(\mathbb{S}^2) =$$

$$= 1+s_{11}+s_{22}+\ldots+s_{nn}+o(\mathbb{S}^2) = 1+\operatorname{tr}\mathbb{S}+o(\mathbb{S}^2).$$

$$? = \lim_{n\to\infty}\left(1+\frac{\operatorname{tr}\mathbb{A}}{n}+\ldots\right)^n = e^{\operatorname{tr}\mathbb{A}}.$$

□

Tvrzení 0.1

$$\det(\mathbb{I}+\mathbb{S}) = 1+\operatorname{tr}\mathbb{S}+\ldots$$

Definice 0.3 (Gateaux derivative)

$$D\mathbf{f}(\mathbf{x})[\mathbf{y}] := \frac{d}{d\tau}\mathbf{f}(\mathbf{x}+\tau\mathbf{y})|_{\tau=0}.$$

Definice 0.4 (Fréchet derivative)

$\mathbf{f}: U \to V$:

$$\lim_{\|\mathbf{y}\|_U\to 0} \frac{\|\mathbf{f}(\mathbf{x}+\mathbf{y})-\mathbf{f}(\mathbf{x})-D\mathbf{f}(\mathbf{x})[\mathbf{y}]\|_V}{\|\mathbf{y}\|_V} = 0.$$

Poznámka

Sometimes we write $\nabla f(\mathbf{x})\cdot\mathbf{y}$ instead of $Df(\mathbf{x})[\mathbf{y}]$ (from Riesz representation theorem).

For matrices ($\varphi : \mathbb{A} \in \mathbb{R}^{3\times 3} \to \mathbb{R}$):

$$\frac{\|\varphi(\mathbb{A}+\mathbb{B})-\varphi(\mathbb{A})-D\varphi(\mathbb{A})[\mathbb{B}]\|_{\mathbb{R}}}{\|\mathbb{B}\|_{\mathbb{R}^{3\times 3}}}.$$

Poznámka
We write $\frac{\partial\varphi}{\partial\mathbb{A}}(\mathbb{A}):\mathbb{B}$ instead of $D\varphi(\mathbb{A})[\mathbb{B}]$, where $\frac{\partial\varphi}{\partial\mathbb{A}}(\mathbb{A})$ is right matrix. Warning $\frac{\partial\varphi}{\partial\mathbb{A}}(\mathbb{A})\neq D\varphi(\mathbb{A})$, because of transposition $(\mathbb{A}:\mathbb{B}=\operatorname{tr}(\mathbb{A}\mathbb{B}^T)=\operatorname{tr}(\mathbb{A}^T\mathbb{B}))$.

Příklad

$$\frac{\partial \operatorname{tr}\mathbb{A}}{\partial\mathbb{A}}(\mathbb{A})[\mathbb{B}]=\frac{d}{d\tau}(\operatorname{tr}(\mathbb{A}+\tau\mathbb{B}))|_{\tau=0}=\frac{d}{d\tau}\left(\operatorname{tr}\mathbb{A}+\tau\operatorname{tr}\mathbb{B}\right)|_{\tau=0}=\operatorname{tr}\mathbb{B}=\mathbb{I}:\mathbb{B}.$$

So $\frac{\partial\operatorname{tr}\mathbb{A}}{\partial\mathbb{A}}=\mathbb{I}$.

Příklad

$$\frac{\partial\det\mathbb{A}}{\partial\mathbb{A}}(\mathbb{A})[\mathbb{B}]=\frac{d}{d\tau}(\det(\mathbb{A}+\tau\mathbb{B}))|_{\tau=0}=\frac{d}{d\tau}\left(\det(\mathbb{A})\cdot\det\left(\mathbb{I}+\tau\mathbb{A}^{-1}\mathbb{B}\right)\right)|_{\tau=0}=$$
$$=\frac{d}{d\tau}\left((\det\mathbb{A})\cdot\left(1+\tau\operatorname{tr}(\mathbb{A}^{-1}\mathbb{B})+o(\tau^2)\right)\right)|_{\tau=0}=(\det\mathbb{A})\operatorname{tr}\left(\mathbb{A}^{-1}\mathbb{B}\right)=$$
$$=(\det\mathbb{A})\operatorname{tr}\left(\left(\mathbb{A}^{-T}\right)^T\mathbb{B}\right)=\left((\det\mathbb{A})\mathbb{A}^{-T}\right):\mathbb{B}.$$

So $\frac{\partial\det\mathbb{A}}{\partial\mathbb{A}}=(\det\mathbb{A})\mathbb{A}^{-T}=\operatorname{cof}(\mathbb{A})$.

Příklad
$\mathbb{A}:\mathbb{R}\to\mathbb{R}^{3\times3}$.

$$\frac{d}{dt}(\det\mathbb{A}(t))=(\det\mathbb{A}(t))\operatorname{tr}\left(\mathbb{A}(t)^{-1}\frac{d\mathbb{A}(t)}{dt}\right).$$

Příklad
$\mathbb{F}:\mathbb{A}\in\mathbb{R}^{3\times3}\to\mathbb{F}(\mathbb{A})\in\mathbb{R}^{3\times3}$. $\mathbb{F}(\mathbb{A})=\mathbb{A}^{-1}$. (We know $\frac{1}{1+x}=1-x+\ldots$.)

$$\frac{\partial\mathbb{F}(\mathbb{A})}{\partial\mathbb{A}}(\mathbb{A})[\mathbb{B}]=\frac{d}{d\tau}\left((\mathbb{A}+\tau\mathbb{B})^{-1}\right)|_{\tau=0}=\frac{d}{d\tau}\left(\left(\mathbb{A}\left(\mathbb{I}+\tau\mathbb{A}^{-1}\mathbb{B}\right)\right)^{-1}\right)|_{\tau=0}=$$
$$=\frac{d}{d\tau}\left(\left(\mathbb{I}+\tau\mathbb{A}^{-1}\mathbb{B}\right)^{-1}\mathbb{A}^{-1}\right)|_{\tau=0}=\frac{d}{d\tau}\left(\left(\mathbb{I}-\tau\mathbb{A}^{-1}\mathbb{B}+\ldots\right)\mathbb{A}^{-1}\right)|_{\tau=0}=-\mathbb{A}^{-1}\mathbb{B}\mathbb{A}^{-1}.$$

So we have $\frac{\partial(\mathbb{A}^{-1})_{ij}}{\partial(\mathbb{A})_{kl}}(\mathbb{B})_{kl}$.

From chain rule (but this is easily solvable by differentiating $\mathbb{A}^{-1}(t)\mathbb{A}(t)=\mathbb{I}$):

$$\frac{d}{dt}\left(\mathbb{A}^{-1}\right)=-\mathbb{A}^{-1}\frac{d\mathbb{A}}{dt}\mathbb{A}^{-1}.$$

Příklad
$\mathbb{F}(\mathbb{A}) = e^{\mathbb{A}}$

$$\frac{\partial e^{\mathbb{A}}}{\partial \mathbb{A}}[\mathbb{B}] = \frac{d}{d\tau}(e^{\mathbb{A}+\tau\mathbb{B}})|_{\tau=0} = \frac{d}{d\tau}\left(\mathbb{I} + \frac{\mathbb{A}+\tau\mathbb{B}}{1!} + \frac{(\mathbb{A}+\tau\mathbb{B})^2}{2!}\right)|_{\tau=0}.$$

Věta 0.2 (Daleckii–Krein)

$\mathbb{A}$ real symmetric matrix, $\mathbb{A} \in \mathbb{R}^{k\times k}$, $\mathbb{A} = \sum_{i=1}^{k} \lambda_i \mathbf{v}_i \otimes \mathbf{v}_i$, where λ_i are eigenvalues and $\mathbf{v}_i$ are normalised orthogonal ($\mathbf{v}_i \cdot \mathbf{v}_j = \delta_{ij}$) eigenvectors.

f continuously differentiable real function defined on open set containing the spectrum of $\mathbb{A}$

$$\mathbb{F}(\mathbb{A}) := \sum_{i=1}^{k} f(\lambda_i)\mathbf{v}_i \otimes \mathbf{v}_i =: \sum_{i=1}^{k} f(\lambda_i)\mathbb{P}_i.$$

Then the formula for the Gateaux derivative of f at point $\mathbb{A}$ in direction $\mathbb{X}$ reads

$$D\mathbb{F}(\mathbb{A})[\mathbb{X}] = \frac{\partial \mathbb{F}}{\partial \mathbb{A}}[\mathbb{X}] = \sum_{i=1}^{k} \frac{df}{d\lambda}|_{\lambda=\lambda_i}\mathbb{P}_i\mathbb{X}\mathbb{P}_i + \sum_{i=1}^{k}\sum_{j=1, j\neq i}^{k} \frac{f(\lambda_i) - f(\lambda_j)}{\lambda_i - \lambda_j}\mathbb{P}_i\mathbb{X}\mathbb{P}_j.$$

Sometimes we write $D\mathbb{F}(\mathbb{A})[\mathbb{X}] = f^{[1]}(\mathbb{A}) \bigcirc \mathbb{X}$ (Schur product of matrices, it is point-wise multiplication). Then

$$[f^{[1]}(\mathbb{A})]_{ij} = \begin{cases} \frac{df}{d\lambda}|_{\lambda=\lambda_i}, & i = j, \\ \frac{f(\lambda_i)-f(\lambda_j)}{\lambda_i-\lambda_j}, & i \neq j. \end{cases}$$

Důkaz
No summation conventions, all sums are stated explicitly!

$$\mathbb{F}(\mathbb{A}) = \sum_{i=1}^{k} f(\lambda_i)\mathbf{v}_i \otimes \mathbf{v}_i =$$

$$= \sum_{i=1}^{k} f(\lambda_i(a_{11}, a_{12}, \ldots, a_{21}, \ldots))\mathbf{v}_i(a_{11}, a_{12}, \ldots, a_{21}, \ldots) \otimes \mathbf{v}_i(a_{11}, a_{12}, \ldots, a_{21}, \ldots).$$

$$\frac{\partial \mathbb{F}(\mathbb{A})}{\partial \mathbb{A}} = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \mathbb{A}} \mathbf{v}_i \otimes \mathbf{v}_i + f(\lambda_i) \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \otimes \mathbf{v}_i + f(\lambda_i)\mathbf{v}_i \otimes \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \right) =?.$$

We derivate $\mathbb{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$:

$$\frac{\partial \mathbb{A}}{\partial \mathbb{A}} \mathbf{v}_i + \mathbb{A} \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} = \frac{\partial \lambda_i}{\partial \mathbb{A}} \mathbf{v}_i + \lambda_i \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}}.$$

We multiply (with dot product) it by $\mathbf{v}_i$:

$$\mathbb{P}_i + \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \cdot \mathbb{A}^T \mathbf{v}_i = \frac{\partial \lambda_i}{\partial \mathbb{A}} \cdot 1 + \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \mathbb{A} \cdot \mathbf{v}_i.$$

$$\frac{\partial \lambda_i}{\partial \mathbb{A}} = \mathbb{P}_i = \mathbf{v}_i \otimes \mathbf{v}_i.$$

We again multiply derivative of $\mathbb{A}|\mathbf{v}_i = \lambda \mathbf{v}_i$, but this time by $\mathbf{v}_j$:

$$\mathbf{v}_j \otimes \mathbf{v}_i + \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \cdot \lambda_j \mathbf{v}_j = 0 + \lambda_i \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \cdot \mathbf{v}_j.$$

$$(\lambda_j - \lambda_i) \frac{\partial \mathbf{v}_i}{\partial \mathbb{A}} \cdot \mathbf{v}_j = -\mathbf{v}_j \otimes \mathbf{v}_i.$$

We also need $(\mathbf{v}_j \otimes \mathbf{v}_i)\mathbb{X}_{ij} = \ldots = \mathbb{P}_i \mathbb{X} \mathbb{P}_j$:

$$\ldots = (\mathbf{v}_j \otimes \mathbf{v}_i)(\mathbf{v}_i \cdot \mathbb{X}\mathbf{v}_j) = (\mathbf{v}_j \otimes \mathbf{v}_i)\mathbb{X}(\mathbf{v}_j \otimes \mathbf{v}_j).$$

□

TODO!!!

1 Kinematics

Definice 1.1

We have some abstract body with point P. We can look at it in reference configuration (some point in past), where $K_0(P) = \mathbf{X}$ (K_0 = placer), $t = t_0$. Or in current configuration (how it is situated now), where $K_t(P) = \mathbf{x}$.

The change of configuration, χ in $\mathbf{x} = \chi(\mathbf{X}, t)$ is called deformation (but it contains translation and rotation too!).

Definice 1.2

Let us consider quantity θ that describes the given material point. We can describe it by:

- $\theta(P, t)$;
- $\hat{\theta}(\mathbf{X}, t)$ (referential/Lagrangian description, commonly used for solids because deformation is with respect to reference configuration);
- $\tilde{\theta}(\mathbf{x}, t)$ (spatial/Eulerian description, commonly used for fluids because velocity is time-local property).

But people write those functions without ˆ or ˜

Poznámka

$$\tilde{\theta}(\mathbf{x}, t)|_{\mathbf{x}=\chi(\mathbf{X},t)} = \hat{\theta}(\mathbf{X}, t).$$

Definice 1.3 (Deformation gradient)

$$d\mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1 = \chi(\mathbf{X}_2, t) - \chi(\mathbf{X}_1, t) =$$
$$= \chi(\mathbf{X}_1 + d\mathbf{X}, t) - \chi(\mathbf{X}_1, t) = \chi(\mathbf{X}_1, t) + \frac{\partial \chi}{\mathbf{X}}(\mathbf{X}_1, t)d\mathbf{X} + \ldots - \chi(\mathbf{X}_1, t) = \frac{\partial \chi}{\mathbf{X}}(\mathbf{X}_1, t)d\mathbf{X}.$$

$$\mathbb{F}(\mathbf{X}, t) := \frac{\partial \chi}{\mathbf{X}}(\mathbf{X}_1, t)d\mathbf{X}. \qquad d\mathbf{x} = \mathbb{F}d\mathbf{X}$$

Poznámka
It can be derived by derivatives on curves (see lecture).

Důsledek
Transformation of infinitesimal line segment: $d\mathbf{x} = \mathbb{F}d\mathbf{X}$.

Transformation of infinitesimal surface elements: $d\mathbf{s} = (\det \mathbb{F})\mathbb{F}^{-T}d\mathbf{S} = \operatorname{cof} \mathbb{F} d\mathbf{S}$.

Transformation of infinitesimal volume: $dv = (\det \mathbb{F})dV$.

Důsledek (In tangent spaces)

$$F(\mathbf{X}, t_0) = f(\chi(\mathbf{X}, t), t).$$

Representation theorem:

$$(GradF)\mathbf{W} = \mathbf{U}_{GradF} \cdot \mathbf{W}$$

$$(Gradf)\mathbf{w} = \mathbf{u}_{Gradf} \cdot \mathbf{w}$$

$f(\chi(\mathbf{X}, t), t) = F(\mathbf{X}, t_0)$

$$Gradf(\mathbf{x}, t)|_{\mathbf{x}=\chi(\mathbf{X},t)} = GradF(\mathbf{X}, t_0)$$

$$\mathbf{U}_{GradF} \cdot \mathbf{W} = (GradF)\mathbf{W} = (Gradf)\mathbb{F}\mathbf{W} = (gradf)(\mathbb{F}\mathbf{W}) = \mathbf{u}_{gradf} \cdot \mathbb{F}\mathbf{W} = \mathbb{F}^T \mathbf{u}_{Gradf} \cdot \mathbf{W}.$$

$\mathbf{u}_{gradf} = \mathbb{F}^{-T}\mathbf{U}_{GradF}.$

Příklad (Hollow cylinder)
$r = f(R)$, $\varphi = \Phi$, $z = Z$.

Řešení

$$\mathbb{F} = \frac{\partial \chi_i}{\partial x_j}\mathbf{e}_i \otimes \mathbf{E}_j$$

$$X_1 = R\cos\Phi, \qquad X_2 = R\sin\Phi, x_1 = r\cos\Phi, x_2 = r\sin\Phi.$$

$$x_1 = \chi_1(X_1, X_2, t), \qquad x_2 = \chi(x_1, x_2, t), x_i = \chi_i(X_j, t).$$

By chain rule:

$$\frac{\partial x_1}{\partial X_2} = \frac{\partial r\cos\Phi}{\partial\partial X_2} = \frac{\partial}{\partial X_2}f(R)\cos\Phi.$$

$$\mathbb{F} = F_{rR}\mathbf{e}_r \otimes \mathbf{E}_R + F_{r\Phi}\mathbf{e}_r \otimes \mathbf{E}_\Phi + \dots$$

Řešení

From image:

$$\mathbf{E}_R \xrightarrow{\mathbb{F}} F_{rR}\mathbf{e}_r.$$

$$\mathbf{E}_\Phi \xrightarrow{\mathbb{F}} F_{\varphi\Phi}\mathbf{e}_\varphi$$

So $\mathbb{F} = \begin{pmatrix} F_{rR} & 0 \\ 0 & F_{\varphi\Phi} \end{pmatrix}$

TODO? (Solution by curve)

Poznámka
How to differentiate in time tensorial quantities related to the current configuration?

Upper convected derivative:

$$\overset{\triangledown}{\mathbb{A}}(\mathbf{x},t)|_{\mathbf{x}=\chi(\mathbf{X},t)} = \det \mathbb{F}(\mathbf{X},t)\left[\frac{d}{dt}\left(\mathbb{F}^{-1}(\mathbf{X},t)\mathbb{A}(\chi(\mathbf{X},t),t)\mathbb{F}^{-T}(\mathbf{X},t)\right)\right]\mathbb{F}^{T}(\mathbf{X},t).$$

1.1 Derivatives

Definice 1.4 (Lagragian velocity)

$$\mathbf{V}(\mathbf{X},t) = \frac{d\chi(\mathbf{X},t)}{dt}.$$

$$\mathbf{v}(\mathbf{x},t)|_{\mathbf{x}=\chi(\mathbf{X},t)}$$

Definice 1.5 (Eulerian velocity)

$$\mathbf{v}(\mathbf{x},t) = \mathbf{V}(\mathbf{X},t)|_{\mathbf{X}=\chi^{-1}(\mathbf{x},t)}.$$

Definice 1.6 (Material time derivative)

$\frac{d}{dt}$ = keep $\mathbf{X}$ fixed, and differentiate with respect to time.

$$\psi(\mathbf{X},t) \to \frac{d}{dt}\psi(\mathbf{X},t) = \frac{\partial\psi}{\partial t}(\mathbf{X},t)$$

$$\begin{aligned}\psi(\mathbf{x},t) \to \frac{d}{dt}\psi(\chi(\mathbf{X},t),t) &= \frac{\partial\psi}{\partial t}|_{\mathbf{x}=\chi(\mathbf{X},t)} + \frac{\partial\psi}{\partial x_i}(\mathbf{x},t)|_{\mathbf{x}=\chi(\mathbf{X},t)}\frac{d\chi_i}{dt}(\mathbf{X},t) = \\ &= \left(\frac{\partial\psi}{\partial t}(\mathbf{x},t)|_{\mathbf{x}=\chi(\mathbf{X},t)} + V_i(\mathbf{X},t)\frac{\partial\psi}{\partial x_i}(\mathbf{x},t)|_{\mathbf{x}=\chi(\mathbf{X},t)}\right) = \\ &= \left(\frac{\partial\psi}{\partial t}(\mathbf{x},t) + v_i(\mathbf{x},t)\frac{\partial\psi}{\partial x_i}(\mathbf{x},t)\right)|_{\mathbf{x}=\chi(\mathbf{x},t)}\end{aligned}$$

$$\frac{d}{dt}\psi(\mathbf{x},t) = \frac{\partial\psi}{\partial t}(\mathbf{x},t) + (\mathbf{v}(\mathbf{x},t)\cdot\nabla)\psi(\mathbf{x},t).$$

Definice 1.7 (Time derivative of deformation gradient $\mathbb{F}$)

$$\frac{d}{dt}\mathbb{F}(\mathbf{X},t) = \frac{d}{dt}\left(\frac{\partial\chi(\mathbf{X},t)}{\partial\mathbf{X}}\right) = \frac{\partial}{\partial\mathbf{X}}\frac{d\chi(\mathbf{X},t)}{dt} = \frac{\partial}{\partial\mathbf{X}}\mathbf{V}(\mathbf{X},t) =$$

$$= \frac{\partial}{\partial \mathbf{X}} \mathbf{v}(\chi(\mathbf{X}, t), t) = \frac{\partial \mathbf{v}}{\partial \mathbf{x}}(\mathbf{x}, t)|_{\mathbf{x}=\chi(\mathbf{X},t)} \frac{\partial \chi}{\partial \mathbf{X}}(\mathbf{X}, t) = \frac{\partial \mathbf{v}}{\partial x}|_{\mathbf{x}=\chi(\mathbf{X},t)} \mathbb{F}(\mathbf{X}, t).$$

$$\mathbb{L}(\mathbf{x}, t) := \nabla \mathbf{V}(\mathbf{x}, t) = \frac{\partial \mathbf{v}}{\partial \mathbf{x}}(\mathbf{x}, t).$$

Důsledek

$$\frac{d\mathbb{F}}{dt} = \mathbb{L}\mathbb{F}$$

Důsledek

$$\overset{\triangledown}{\mathbb{A}} = \frac{d\mathbb{A}}{dt} - \mathbb{L}\mathbb{A} - \mathbb{A}\mathbb{L}^T$$