TODO!!!

Definice 0.1 (WLOG)

$$D := U(0,1), \qquad T = \partial D.$$

TODO!!!

Definice 0.2

$f \in \mathcal{H}(D)$. We say that the boundary T is a natural boundary of f if $R_f = \emptyset$.

Například
$f(z) = \sum_{n=0}^{\infty} z^{2^n}$. Radius of convergence is equal to 1 and f has natural boundary.

Důkaz
$K = \left\{\exp\left(\frac{2\pi ik}{n}\right) | k, n \in \mathbb{N}\right\}$ is dense in T. f is "diverges on" this set, because $f(z^{2^N}) = f(z) - \sum_{n=1}^{N}$. For $\alpha \in (0,1)$ we have parametrization of one "line" $\alpha \cdot \exp\left(\frac{2k\pi i}{2^n}\right)$ (for k, n fixed).

$$f\left(\alpha^{2^N}\right) = f\left(\alpha \exp\left(\frac{2k\pi i}{2^N}\right)\right) + p\left(\alpha \exp\left(\frac{2k\pi i}{2^N}\right)\right).$$

□

For every domain $\Omega \subseteq \mathbb{C}$, there exists $f \in \mathcal{H}(\Omega)$ such that $\partial\Omega$ is natural boundary of f.

Důkaz
We use theorem (15.11 from Rudin or TODO from lecture). □