1 Σ_1^1 sets and trees on ω

Poznámka (Notation)

- $\mathbb{S} := \omega^{<\omega}$;
- $\nu|_k = (\nu(0), \dots, \nu(k-1))$, $\nu \in \mathbb{S} \cup \omega^\omega$ ($\nu|_0 = \varnothing$, empty sequence);
- $t \prec s \equiv \exists s' \in \mathbb{S} \cup \mathcal{N} : s = t^\wedge s'$ ($t \in \mathbb{S}, s \in \mathbb{S} \cup \mathcal{N}$);
- $\mathcal{N} := \omega^\omega$;
- $|s|$ is the length of s, $s \in \mathbb{S}$ ($s = (s(0), \dots, s(k-1)) \implies |s| = k$);
- $s \in \mathbb{S}$, $\nu \in \mathbb{S} \cup \mathcal{N}$: $s^\wedge \nu = (s(0), \dots, s(|s|-1), \nu(0), \dots)$.

Definice 1.1 (Souslin set (on TP space))

X topological space. We say $S \subset X$ be Souslin $\exists (F_s)_{s \in S}$ Souslin scheme of closed subset of X such that $S = \mathcal{A}_s(F_s) = \bigcup_{\sigma \in \mathcal{N}} \bigcap_{n \in \omega} F_{\sigma|_n}$.

Poznámka

a) P Polish topological space, then $A \in \Sigma_1^1 \Leftrightarrow A$ Souslin in P. (We already know.)

b) P topological space, then $A \subset P$ Souslin $\Leftrightarrow \exists F \in \Pi_1^0(\mathcal{N} \times P) : A = \Pi_P(F)$. (Difficult.)

c) We will assume only regular Souslin scheme (RSS): $F_{s^\wedge t} \subset F_s$, $s, t \in \mathbb{S}$ and $F_\varnothing = P$.

1.1 Souslin operation and trees

Definice 1.2 (Trees on ω, infinite branch, ill-founded trees, well-founded trees)

We define set of trees $\mathcal{T}$ by $\mathcal{T} := \{T \in \mathcal{P}(\mathbb{S}) | \forall s \in T, t \in T : t \prec s \implies t \in T\}$.

$T \in \mathcal{T}$ has infinite branch $\equiv \exists \sigma \in \mathcal{N} \forall n \in \omega : \sigma|_n \in T$ (i.e. $\sigma \in [T]$) (i.e. $[T] \neq \varnothing$).

Trees with infinite branches are called ill-founded (IF). The set of IF trees is denoted by $\mathcal{T}_I$. Trees without infinite branches are called well-founded (WF). The set of WF trees is denoted by $\mathcal{T}_W$.

$\mathcal{T}_s := \{T \in \mathcal{T} | s \in T\}$ are all trees containing $s \in \mathbb{S}$.