

ST227 Symbols (forked from DenysMelnyk6/ST227table)

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Note: need to change descriptions and equations to use set notation.

ST227 Definitions

Name	Symbol	Equation	Description
State Space	S	-	Possible values taken by the Markov chain.
Transition matrix	P	-	A matrix containing the probabilities of going between state spaces, rows being "old" and columns being "new" spaces, all 1-step.
Transition probability	P_{ij}^n	$\mathbb{P}(X_n = j \mid X_0 = i)$ $= \mathbb{P}_i(X_n = j)$	A probability of Markov Chain going from state i to state j in n -steps.
Initial distribution	λ	-	A vector containing the distribution of the first state of the chain.
Initial probability of state i	λ_i	$\mathbb{P}(X_0 = i)$	i^{th} entry of λ , probability of state i being the first state of the Markov's chain.
Period of a state	d_i	$\gcd\{n \in \mathbb{N} \mid (P^n)_{ii} > 0\}$	Greatest Common divisor of all loops leading from state i to i .
First hitting time	H_j/H_A	$\min\{n \in \mathbb{N}_0 \mid X_n = j\}$	Number of steps until state j is hit. Is 0 if the start point is state j or the set is \emptyset . (Similarly for set A)
Hitting probability	h_{ij}/h_{iA}	$\sum_{n=1}^{\infty} \mathbb{P}_i(H_A = n)$	Probability of reaching state j from state i . (Similarly for set A)
Expected hitting time	η_{ij}/η_{iA}	$\mathbb{E}_i[H_A]$ $= \sum_{n=1}^{\infty} n \mathbb{P}_i(H_A = n)$	Expected number of steps until state j is reached from state i . (Similarly for set A)
First return time	T_i	$\min\{n \in \mathbb{N} \mid X_n = i\}$	Number of steps until state i loops on itself.
Return probability	f_i	$\mathbb{P}_i(T_i < \infty)$ $= \sum_{j \in S} P_{ij} h_{ji}$	h_{ii} , probability of chain returning back to state i , from i .

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Name	Symbol	Equation	Description
Expected return time	m_i	$\mathbb{E}_i[T_i]$	η_{ii} , expected number of steps until state i loops on itself.
Number of visits	V_j/V_A	$\sum_{n=0}^{\infty} \mathbb{1}_{\{X_n=i\}}$	Number of visits to state j /set A .
Expected number of visits	γ_j^i	$\mathbb{E}_i \left[\sum_{n=0}^{T_j-1} \mathbb{1}_{\{X_n=j\}} \right]$	Expected number of visits to state j before chain returns to state i
Set of transient states	\mathcal{T}	-	Subset of S , containing all transient states.
Invariant measure	μ	-	A vector of positive entries, describing the measure, or "weight" of each state at every step of the chain. Has a property of $\mu P = \mu$.
Invariant distribution	π	-	A special case of invariant measure, distribution describing the distribution of the chain at every step of the chain, has a property of $\pi P = \pi$ and $\sum_{i \in S} \pi_i = 1$. Not every chain has a π .
Long term transition matrix	Π	$\lim_{n \rightarrow \infty} P^n$	All rows are equal to π . Describes the behaviour of the chain after arbitrarily many steps. Not applicable to every Markov Chain.
Graph	$G = (V, E)$	-	Graph G , where V is a set of vertices and $E \subset S \times S$, set of all edges. All graphs in this module are connected and weighted.