ST227 Symbols (forked from DenysMelnyk6/ST227table)

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To-do: Need to change descriptions and unify set notation.

Notation

Note that the following table uses the abbreviation:

$$\mathbb{P}_i(\cdot) = \mathbb{P}(\cdot \mid X_0 = i)$$
$$\mathbb{E}_i(\cdot) = \mathbb{E}(\cdot \mid X_0 = i)$$

ST227 Definitions

Name	Symbol	Equation	Description
State Space	S	–	Possible values taken by the Markov chain.
Transition matrix	P	–	A matrix containing the probabilities of going between state spaces, rows being "old" and columns being "new" spaces, all 1-step.
Transition probability	P_{ij}^n	$\mathbb{P}(X_n = j \mid X_0 = i)$ $=\mathbb{P}_i(X_n = j)$	A probability of Markov Chain going from state i to state j in n-steps.
Initial distribution	λ	–	A vector containing the distribution of the first state of the chain.
Initial probability of state i	λ_i	$\mathbb{P}(X_0 = i)$	i^{th} entry of λ, probability of state i being the first state of the Markov's chain.
Period of a state	d_i	$\gcd\{n \in \mathbb{N} \mid (P^n)_{ii} > 0\}$	Greatest Common divisor of all loops leading from state i to i.
First hitting time	H_j/H_A	$\min\{n \in \mathbb{N}_0 \mid X_n = j\}$	Number of steps until state j is hit. Is 0 if the start point is state j or the set is $\varnothing$. (Similarly for set A)
Hitting probability	h_{ij}/h_{iA}	$\sum_{n=1}^{\infty} \mathbb{P}_i(H_A = n)$	Probability of reaching state j from state i. (Similarly for set A)

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Name	Symbol	Equation	Description
Expected hitting time	η_{ij}/η_{iA}	$\mathbb{E}_i[H_A] = \sum_{n=1}^{\infty} n\mathbb{P}_i(H_A = n)$	Expected number of steps until state j is reached from state i. (Similarly for set A)
First return time	T_i	$\min\{n \in \mathbb{N} \mid X_n = i\}$	Number of steps until state i loops on itself.
Return probability	f_i	$\mathbb{P}_i(T_i < \infty) = \sum_{j\in S} P_{ij}h_{ji}$	h_{ii}, probability of chain returning back to state i, from i.
Expected return time	m_i	$\mathbb{E}_i[T_i]$	η_{ii}, expected number of steps until state i loops on itself.
Number of visits	V_j/V_A	$\sum_{n=0}^{\infty} \mathbb{1}_{\{X_n=i\}}$	Number of visits to state j/set A.
Expected number of visits	γ_j^i	$\mathbb{E}_i\left[\sum_{n=0}^{T_j-1} \mathbb{1}(X_n = j)\right]$	Expected number of visits to state j before chain returns to state i.
Set of transient states	$\mathcal{T}$	–	Subset of S, containing all transient states.
Invariant measure	μ	–	A vector of positive entries, describing the measure, or "weight" of each state at every step of the chain. Has a property of $\mu P = \mu$.
Invariant distribution	π	–	A special case of invariant measure, distribution describing the distribution of the chain at every step of the chain, has a property of $\pi P = \pi$ and $\sum_{i\in S} \pi_i = 1$. Not every chain has a π.
Long-term transition matrix	Π	$\lim_{n\to\infty} P^n$	All rows are equal to π. Describes the behaviour of the chain after arbitrarily many steps. Not applicable to every Markov Chain.
Graph	G	(V, E)	Graph G, where V is a set of vertices and $E \subset S \times S$, set of all edges. All graphs in this module are connected and weighted.

Note: A trick to memorize the **E**xpected **h**itting time η is that it looks like a deformed "h".
Similarly for the **E**xpected number of **v**isits γ with "v".