HYPOTHESIS TESTING

INTRODUCTION

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- many problems in different fields or discipline require that it has to be decided whether to accept or reject a statement or claim about some parameter

- the statement is called **hypothesis**, and the decision-making procedure about the hypothesis is called **hypothesis testing**

- this is an area of statistical inference in which one evaluates a statement about some characteristic of the parent population based upon the information contained in the random sample

Examples:

1. A scientist might want to know whether the planet is warming up.

2. A medical researcher may decide based on experimental evidence whether staying lean increases life span.

3. A teacher might want to see whether inclusion of a laboratory session in a course improves the performance of students.

4. A municipal officer might wish to verify whether imposition of fines improves compliance of fishers to fishery rules.

BASIC CONCEPTS IN STATISTICAL HYPOTHESIS TESTING

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A **statistical hypothesis** is an assertion or conjecture concerning one or more populations.

Note: The truth or falsity of a statistical hypothesis is never known with absolute certainty unless the entire population is examined which of course, would be impractical in most situations. Instead, a random sample is taken from the population of interest and use the data contained in this sample to provide evidence that either supports or does not support the hypothesis.

STATISTICAL HYPOTHESIS

The **null hypothesis** (H_o) is the hypothesis that is being tested; it represents what the experimenter doubts to be true.

The **alternative hypothesis** (H_a) is the operational statement of the theory that the experimenter believes to be true and wishes to prove. It is the contradiction of the null hypothesis.

Example:

Let p = true (population) proportion of UPV students who graduate on time (within the specified number of years by the program)

Null Hypothesis H_0: P = 0.90
Alternative Hypothesis H_1: P ≠ 0.90

STATISTICAL HYPOTHESIS

Notes:
- null hypothesis is one which asserts the absence of any effect claimed for a certain action or treatment

- alternative hypothesis is that which one is interested in

- **test is done in the null hypothesis and its rejection means that the null hypothesis is false and the alternative hypothesis is true**, whereas the inability to reject the null hypothesis simply means that one has insufficient evidence to reject it and does not necessarily imply that it is true.

📌 ONE-TAILED AND TWO-TAILED TEST OF HYPOTHESIS

🔍 A **one-tailed test of hypothesis** is a test where the alternative hypothesis specifies a one-directional difference for the parameter of interest.

a. Ho: $\mu = 14$ vs. Ha: $\mu > 14$

b. Ho: $\mu = 14$ vs. Ha: $\mu < 14$

c. Ho: $\mu_1 - \mu_2 = 0$ vs. Ha: $\mu_1 - \mu_2 > 0$

d. Ho: $\mu_1 - \mu_2 = 0$ vs. Ha: $\mu_1 - \mu_2 < 0$

🔍 A **two-tailed test of hypothesis** is a test where the alternative hypothesis does not specify a directional difference for the parameter of interest.

a. Ho: $\mu = 14$ vs.   Ha: $\mu \neq 14$

b. Ho: $\mu_1 - \mu_2 = 0$ vs. Ha: $\mu_1 - \mu_2 \neq 0$

📌 ONE-TAILED AND TWO-TAILED TEST OF HYPOTHESIS

Notes:

- In any test of hypothesis, the null hypothesis must contain the condition of equality while the alternative hypothesis will be stated involving only one of the following: $\neq$, $>$, $<$.

- Although the definitions of null and alternative hypotheses given use the word parameter, these definitions can be extended to include other terms such as distributions and independence.

📌 ONE-TAILED AND TWO-TAILED TEST OF HYPOTHESIS

Example: Stating the null hypothesis and alternative hypothesis

1. A manufacturer of a certain brand of toothpaste claims that the average fluoride content smaller than 1500ppm. State the null and alternative hypotheses to be used in testing this claim.
2. A medical researcher is interested in finding out whether staying lean will increase lifespan of mice. The researcher, in particular, is concerned with the life span of mice weighing 200 grams. Can we say that the length of life of 200-gram mice is 3 years on the average? State the null and alternative hypotheses.

Hypothesis Testing Common Phrases

=	≠	>	<
Is equal to	Is not equal to	Is greater than	Is less than
Is the same as	Is different from	Is above	Is below
Has not changed from	Has changed from	Is higher than	Is lower than
	Is not the same as	Is longer than	Is shorter
		Is bigger than	Is smaller
		Is increased	Is decreased
			Is reduced

A **test statistic** is a statistic whose value is calculated from sample measurements and on which the statistical decision will be based.

The **critical region** or **rejection region** is the set of values of the test statistic for which the null hypothesis will be rejected.

The **acceptance region** is the set of values of the test statistic for which the null hypothesis will not be rejected.

The acceptance and rejection regions are separated by a **critical value** of the test statistic.

CRITICAL REGION AND ACCEPTANCE REGION

Notes:
- The location of the critical region can be determined only after the alternative hypothesis has been stated.

- If H_1 is stated involving the "<" symbol, then the critical region is on the left of the critical value.

- If H_1 involves the ">" symbol, the critical is found on the right of the critical value.

- However, if the alternative hypothesis contains "≠" symbol, the critical region is on both sides of the critical value.

TYPE I ERROR AND TYPE II ERROR

🔍 The **Type I error** is the error made by rejecting the null hypothesis when it is true.

🔍 The **Type II error** is the error made by accepting (not rejecting) the null hypothesis when it is false.

TYPE I ERROR AND TYPE II ERROR

Decision	Null Hypothesis	
	True	False
Reject Ho	Type I error	Correct decision
Accept Ho	Correct decision	Type II error

➤ *The probability of a Type I error is denoted by α.*

➤ *The probability of a Type II error is denoted by β.*

➤ *For a fixed sample size n, a decrease in the probability of one error will always result in an increase in the probability of the other error.*

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LEVEL OF SIGNIFICANCE

The **level of significance**, α, is the maximum probability of Type I error (Rejecting a true null hypothesis) the researcher is willing to commit.

📌 LEVEL OF SIGNIFICANCE

Notes:

- The level of significance α is the size of the critical region. It can be any level, depending on the seriousness of the type I error.

- Researchers and statisticians usually set the level of significance to 0.10, 0.05, or 0.01

Notes:

- If the cost or risk of erroneously rejecting a true null hypothesis is too high, it may be as low as 0.001. That is, if the null hypothesis is rejected, the probability of a type I error will be 10%, 5%, or 1%, depending on which level of significance is used. Equivalently, when $\alpha = 0.05$, there is a 5% chance of rejecting a true null hypothesis.

- The type I error and type II error are related. A decrease in the probability of one generally results in an increase in the probability of the other.

Notes:
- An increase in the sample size n will reduce α and β simultaneously.

The **power of a test** is the probability of rejecting the null hypothesis when it is false.

Power may be defined also as 1 - β, where β is the probability of accepting a false null hypothesis.

📌 STEPS IN HYPOTHESIS TESTING (TRADITIONAL METHOD)

1. *State the null hypothesis (H_o) and the alternative hypothesis (H_a).*

3. Select the appropriate test statistic and establish the critical region.

5. *Make the decision!*

2. *Choose the level of significance α.*

4. *Collect the data and compute the value of the test statistic from the sample data.*

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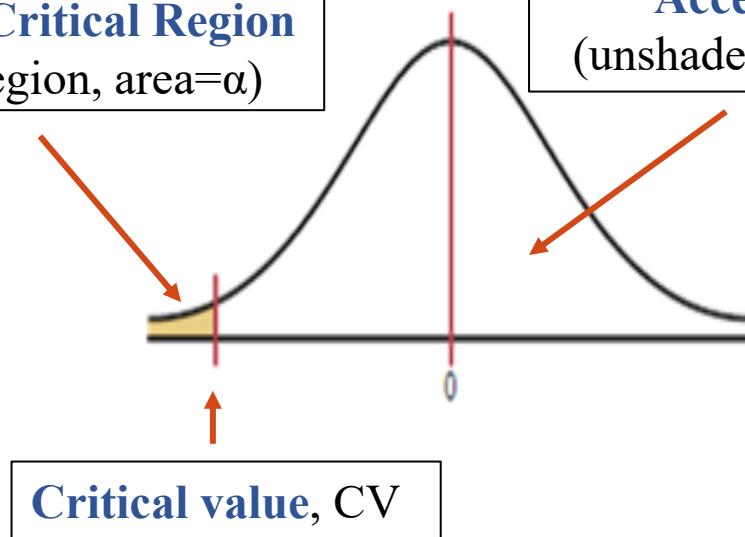
HYPOTHESIS TESTING PROCEDURE: One-tailed (Left)

Let θ be the parameter of interest and θ_0 any specified value.

H_0: $\theta = \theta_0$
H_1: $\theta < \theta_0$

Rejection/Critical Region
(shaded region, area=α)

Acceptance Region
(unshaded region, area=$1-\alpha$)

Critical value, CV
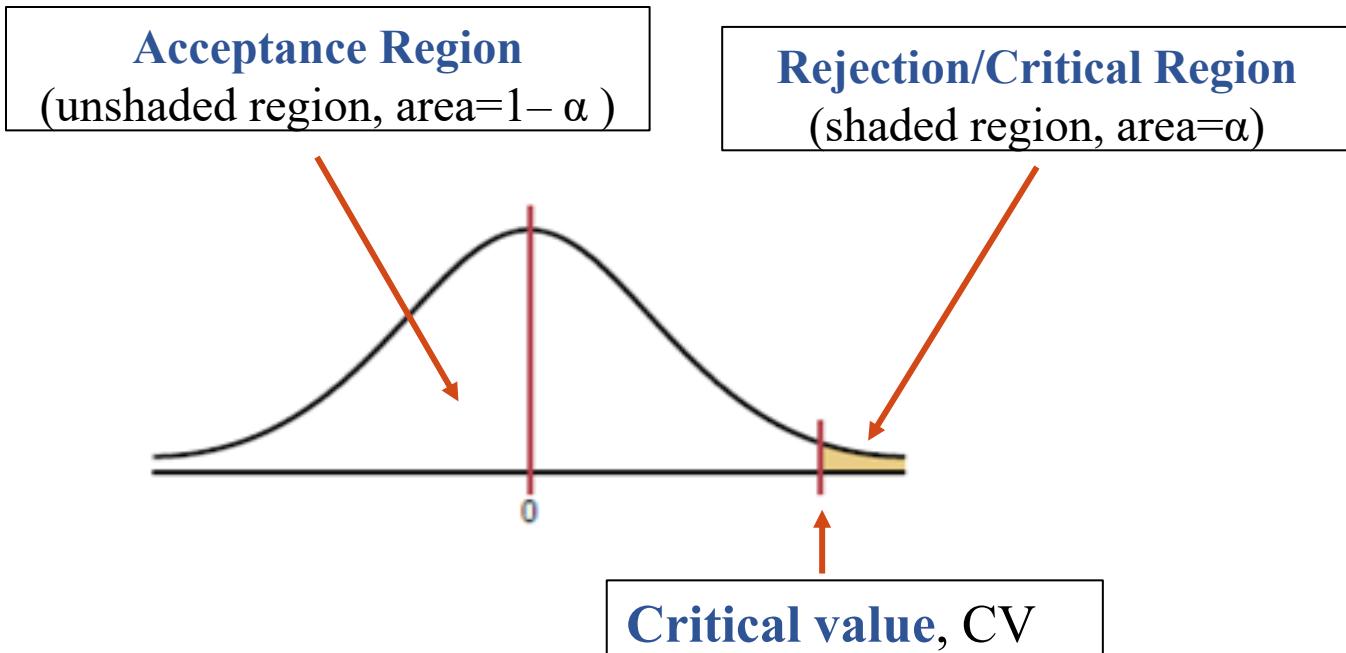
⊹ HYPOTHESIS TESTING PROCEDURE: One-tailed(Right)

Let θ be the parameter of interest and θ_0 any specified value.

H_0: $\theta = \theta_0$
H_1: $\theta > \theta_0$

Acceptance Region
(unshaded region, area=$1 - \alpha$)

Rejection/Critical Region
(shaded region, area=α)

Critical value, CV

📌 HYPOTHESIS TESTING PROCEDURE: Two-tailed

Let θ be the parameter of interest and θ_0 any specified value.
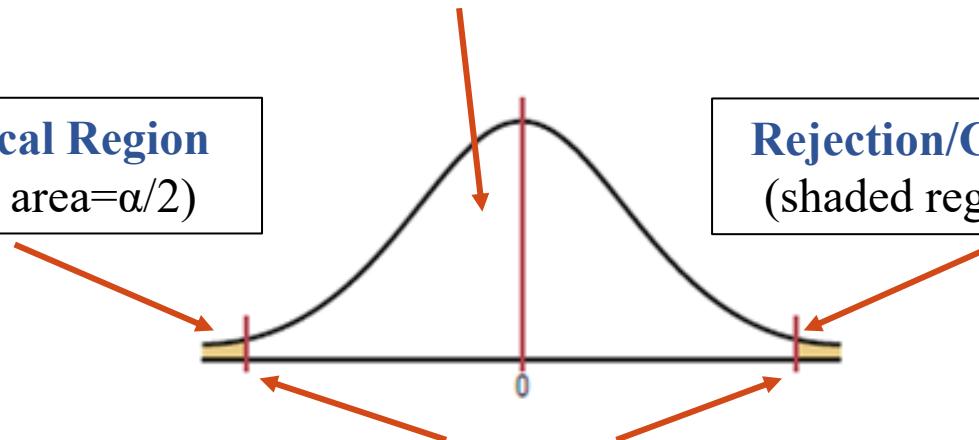
H_0: $\theta = \theta_0$

H_1: $\theta \neq \theta_0$

Acceptance Region
(unshaded region, area=$1-\alpha$)
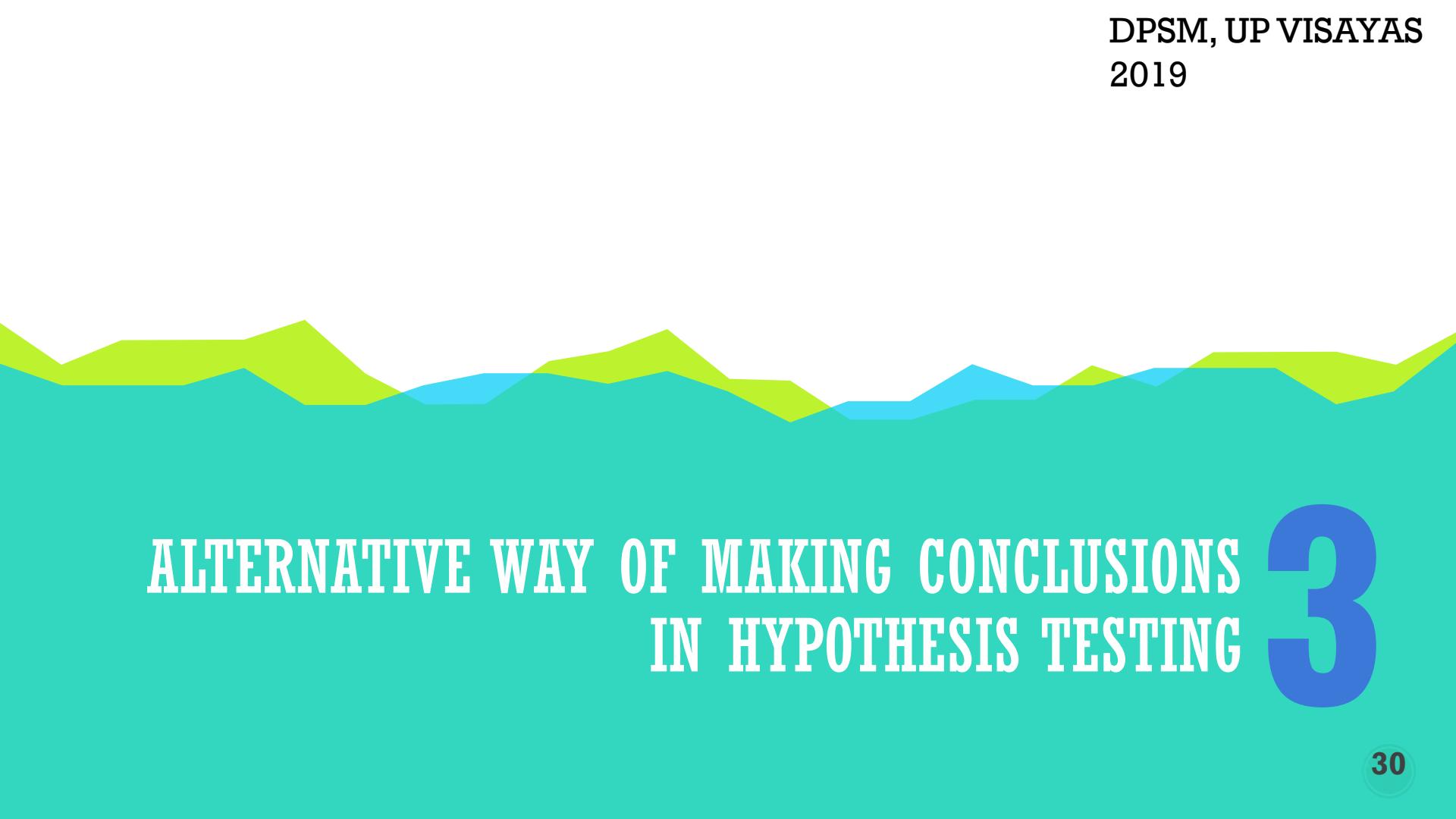
Rejection/Critical Region
(shaded region, area=$\alpha/2$)

Rejection/Critical Region
(shaded region, area=$\alpha/2$)

Critical values, CV

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Notes:
- The critical value is a value that will be obtained from the appropriate table.
- For the same data set, as α increases the size of the critical region also increases.
- If H_0 is rejected at α – level of significance the H_0 will also be rejected at a higher level of significance using the same data. For example, if H_0 is rejected at α = 0.05 then testing at α = 0.10 will also lead to the rejection of H_0. However, H_0 will not necessarily be rejected at α = 0.01.

ALTERNATIVE WAY OF MAKING CONCLUSIONS IN HYPOTHESIS TESTING

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- one may want to make a decision by obtaining the **p-value**
- **p-value** is the smallest value of α for which Ho will be rejected based on sample information
- the smaller the p-value, the more evidence the data have.
- reporting the p-value will allow the reader of the published research to evaluate the extent to which the data disagree with H_0

p-value	Interpretation
$p < 0.01$	Very strong evidence against H_0; test is highly statistically significant
$0.01 \leq p < 0.05$	Moderate evidence against H_0; test is statistically significant
$0.05 \leq p < 0.10$	Suggestive evidence against H_0
$p \geq 0.10$	Little or no real evidence against H_0

ALTERNATIVE WAY OF MAKING CONCLUSIONS IN HYPOTHESIS TESTING

- P-values can be used to also make a decision on whether to reject or not the null hypothesis H_0 in combination with the level of significance α.
- In particular, it enables researcher to choose his personal value of α.
- If the α level is selected in advance, it should be used in making the decision.
- **If the p-value ≤ α, then H_0 is rejected, otherwise, H_0 is not rejected.** The test is said to be **statistically significant** if it rejects H_0 at 5% and **highly significant** if rejects H_0 at 1%.

STEPS IN HYPOTHESIS TESTING (P-VALUE METHOD)

1. *State the null hypothesis (H_o) and the alternative hypothesis (H_a).*

3. *Collect and evaluate the data. Compute the value of the test statistic from the sample data*

5. *Make the decision!*

Find the p-value

2. *Choose the level of significance α.*

Note:

In hypothesis testing, it should be noted that that one should distinguish between **statistical significance** and **practical significance**. When the null hypothesis is rejected at a given level of significance, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However the results may not have any practical significance in the real world.

TESTING A HYPOTHESIS ON A POPULATION MEAN 4

(ONE SAMPLE CASE)
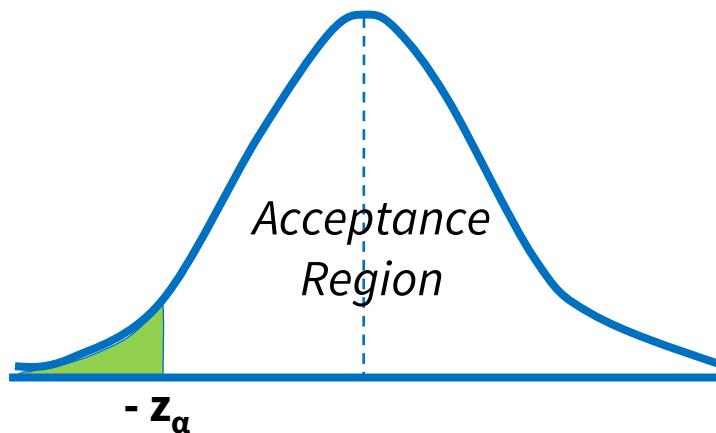
TEST CONCERNING A POPULATION MEAN

Ho	Test Statistic	Ha	Critical Region
a. σ known , popn is normal regardless of n			
$\mu = \mu_o$	$Z = \dfrac{\overline{X} - \mu_o}{\sigma/\sqrt{n}}$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \neq \mu_o$	$z < -z_\alpha$ $z > z_\alpha$ $\lvert z \rvert > z_{\alpha/2}$
b. σ unknown,,popn is normal, n < 30			
$\mu = \mu_o$	$t = \dfrac{\overline{X} - \mu_o}{S/\sqrt{n}}$ $\upsilon = n - 1$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \neq \mu_o$	$t < -t_\alpha$ $t > t_\alpha$ $\lvert t \rvert > t_{\alpha/2}$
c. σ unknown, n ≥ 30			
$\mu = \mu_o$	$Z = \dfrac{\overline{X} - \mu_o}{S/\sqrt{n}}$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \neq \mu_o$	$z < -z_\alpha$ $z > z_\alpha$ $\lvert z \rvert > z_{\alpha/2}$

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TEST CONCERNING A POPULATION MEAN

Acceptance Region

$-z_\alpha$

$H_o : \mu = \mu_0$

$H_a: \mu < \mu_0$

Critical region: $z < -z_\alpha$

(Case I and Case III)

Acceptance Region

$-t_\alpha$

$H_o : \mu = \mu_0$

$H_a: \mu < \mu_0$

Critical region: $t < -t_\alpha$

(Case II)

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TEST CONCERNING A POPULATION MEAN

Acceptance Region

z_α

$H_o : \mu = \mu_0$

$H_a: \mu > \mu_0$

Critical region: $z > z_\alpha$

(Case I and Case III)

Acceptance Region

t_α

$H_o : \mu = \mu_0$

$H_a: \mu > \mu_0$

Critical region: $t > t_\alpha$

(Case II)

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TEST CONCERNING A POPULATION MEAN

Acceptance Region

$- z_{\alpha/2}$ $z_{\alpha/2}$

$H_o : \mu = \mu_0$

$H_a : \mu \neq \mu_0$
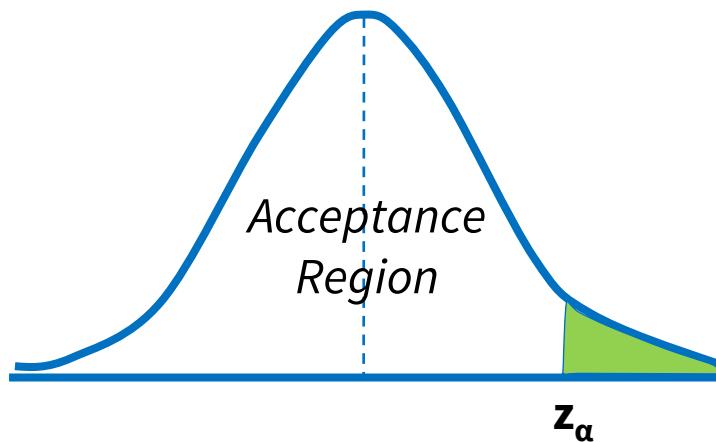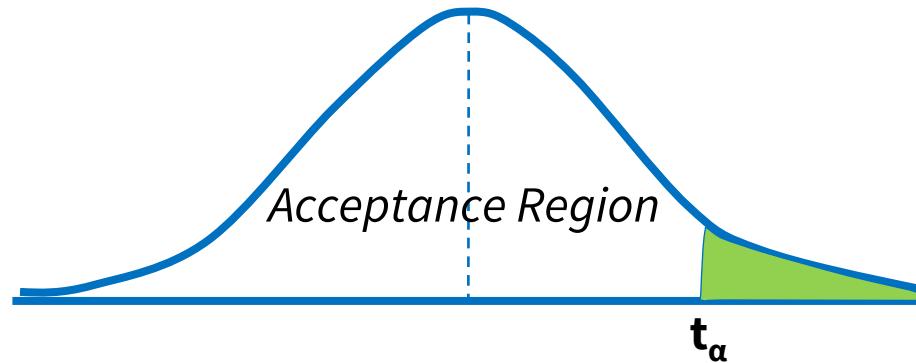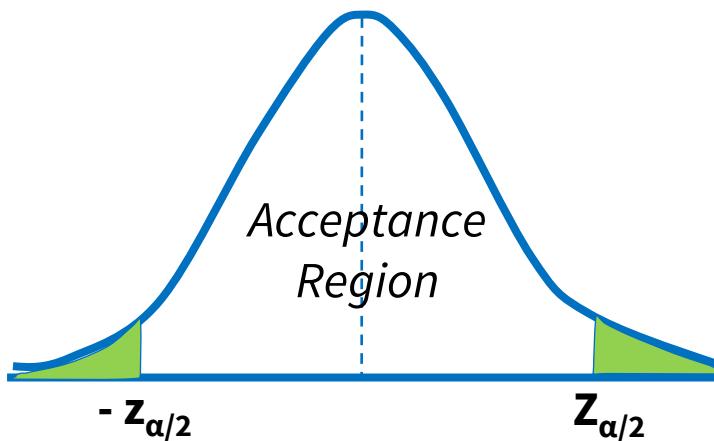
Critical region: $|z| > z_{\alpha/2}$

(Case I and Case III)

Acceptance Region

$- t_{\alpha/2}$ $t_{\alpha/2}$

$H_o : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

Critical region: $|t| > t_{\alpha/2}$

(Case II)

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Example 1

A policeman in the City Police District claims that the number of car accidents along Lacson Street is 15 per week, on the average. To check his claim, car accidents were monitored for 36 randomly selected weeks, resulting to a mean of 17 accidents and variance 9. Does the sample evidence contradict the policeman's claim at 5% level of significance?

Example 1

Given:

$\mu_0 = 15$ n=36 $\bar{x} = 17$ $s^2 = 9$

Required:

Test the hypothesis at $\alpha = 0.05$

Let μ be the true average number of car accidents along Lacson Street

1. $H_0 : \mu = 15$ vs $H_a : \mu \neq 15$
2. Level of Significance: α=0.05

3. Test Statistic: $z = \dfrac{\bar{X} - \mu_0}{s / \sqrt{n}}$ (since σ is unknown and n is large)

Since this is a two-sided test of hypothesis, the critical region is $|z| > z_{\alpha/2}$.

$|z| > z_{0.05/2}$

$|z| > z_{0.025}$

$|z| > 1.96$

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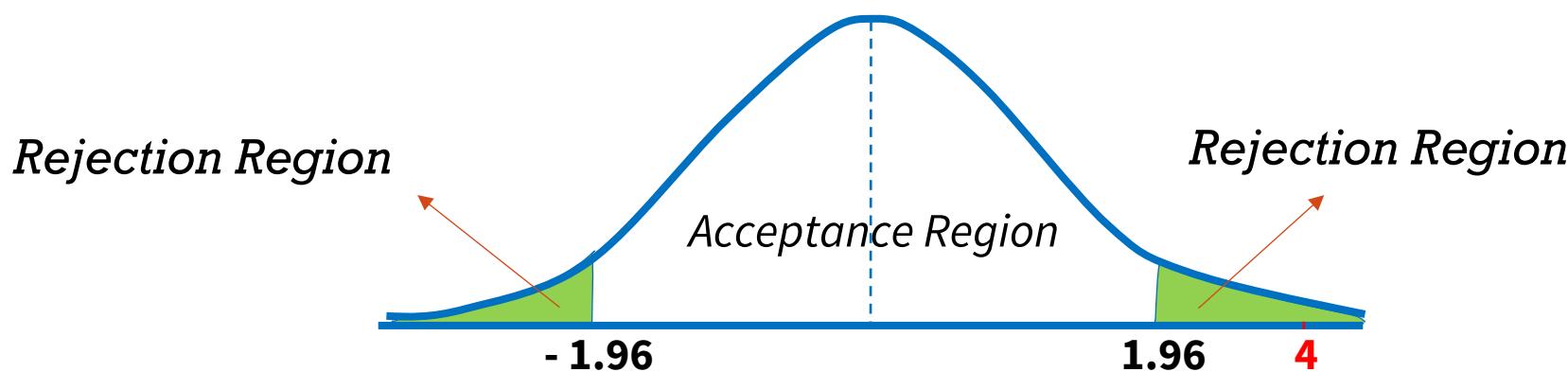
TEST CONCERNING A POPULATION MEAN

4. Computation:

$$z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{17 - 15}{3/\sqrt{36}} = 4$$

Critical Region: $|z| > 1.96$

Rejection Region

Rejection Region

Acceptance Region

- 1.96   1.96   **4**

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5. Decision: Since $|z| = 4 > 1.96$, H_0 is rejected at 5% level of significance.

Conclusion: We have sufficient evidence to believe that the mean number of accidents along Lacson Street is not 15 per week, at 5% level of significance.

Example 2

A sociologist wishes to test the null hypothesis that the mean age of gang members in a large city is 14 years against the alternative that the mean is less than 14 years at level of significance $\alpha = 0.10$. A sample of 25 ages from the police gang unit records in the city found that $\bar{x} = 13.1$ years and s = 2.5 years. What conclusion can you draw? Assume that the distribution of ages is approximately normal.

Example 2

Given:

$\mu_0 = 14$ n=25 $\bar{x} = 13.1$ s = 2.5

Required:

Test the hypothesis at $\alpha = 0.10$

Let μ be the population average age of gang members in a large city

1. $H_0 : \mu = 14$ vs $H_a : \mu < 14$
2. Level of Significance: α=0.10

3. Test Statistic: $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$ with n-1 degrees of freedom

Since this is a left-tailed test , the critical region is $t < - t_\alpha$.

       $t < - t_\alpha^{(n-1)}$
       $t < - t_{0.10}^{(24)}$
       $t < - 1.318$

4. Computation:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{13.1 - 14}{2.5/\sqrt{25}} = -1.8 \qquad \text{Critical region: } t < -1.318$$

Rejection Region

Acceptance Region

- **1.8**   **- 1.318**

5. Decision: Since t= -1.8 < -1.318, H_0 is rejected at 5% level of significance.

Conclusion: We have sufficient evidence to believe that the mean age of the gang members is less than 14 years.

Example 3

National student exam scores are scaled so that the population of scores is normal with a mean of 500 and a standard deviation of 80. In a particular class of 35 students, the mean score was 520. Wondering if this is a class of above average performance, the instructor wants to test using 5% level of significance. What is his verdict?

Example 3

Given:
$\mu_0 = 500$ $\sigma = 80$ $n = 35$ $\bar{x} = 520$

Required:
Test the hypothesis at $\alpha = 0.05$

Let μ be the true average exam score of the class

1. $H_0 : \mu = 500$ vs $H_a : \mu > 500$
2. Level of Significance: α=0.05
3. Test Statistic: $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

Since this is a right-tailed test , the critical region is $z > z_\alpha$.

      $z > z_{0.05}$
      $z > 1.645$
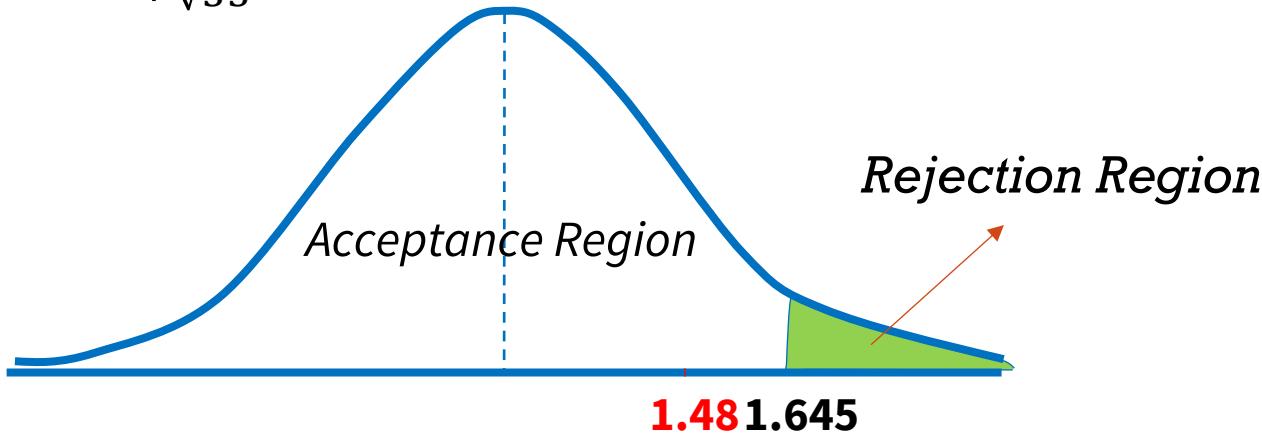
4. Computation:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{520 - 500}{80/\sqrt{35}} = 1.48 \quad \text{Critical region: } z > 1.645$$

Acceptance Region

Rejection Region

1.48 1.645

5. Decision: Since z=1.48 < 1.645, H_0 is not rejected at 5% level of significance.

Conclusion: We don't have sufficient evidence to support the claim that the class has above average performance.

Using Jamovi to Perform the One-Sample T-test:
 From the menus choose:

1.   **Analyses > T- Tests>One-Sample t Test…**
2.   Select one or more variables to be tested against the same hypothesized value. Choose Student's under the **Test option**
3.   Enter a numeric test value against which each sample mean is compared under the **Hypothesis option** and choose the appropriate alternative hypothesis.
4.   Tick Normality under **Assumption Checks option.**
5.   Optionally, click **Descriptives**

Example:

The following data represent a sample of the number of home fires started by candles for the past several years. (Data are from the National Fire Protection Association.) Can you conclude that the mean number of home fires started by candles each year is more than 5500 at 10% level of significance?   Perform an appropriate test of hypothesis using the p-value method.

5460   5900   6090   6310   7160   8440   9930

Using Jamovi:
First, check the data for normality

Tests of Normality

		statistic	p
Number of Home fires	Shapiro-Wilk	0.883	0.239
	Kolmogorov-Smirnov	0.247	0.704
	Anderson-Darling	NaN	

Note. Additional results provided by *moretests*

Let μ = mean number of home fires started by candles each year

1. H_o: μ = 5500
   H_1: μ > 5500 (claim)

2. α=0.05

TEST CONCERNING A POPULATION MEAN

Descriptives

	N	Mean	Median	SD	SE
Number of Home fires	7	7041	6310	1610	609

One Sample T-Test

		Statistic	df	p
Number of Home fires	Student's t	2.53	6.00	0.022

Note. H_a population mean > 5500

TEST CONCERNING A POPULATION MEAN
One Sample T-Test

		Statistic	**df**	**p**
Number of Home fires	Student's t	2.53	6.00	0.022

Note. H_a population mean > 5500

- Since p-value of 0.022 is less than 0.05, H_o is rejected at 10% level of significance.
- That is, the population mean number of home fires started by candles each year is more than 5500.
- Also, the difference between the sample mean (7041.43) and the hypothesized mean (5500) is 1541.43 (positive).
- Thus, on the average, the number of home fires started by candles is more than 5500 each year.

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS

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(BASED ON TWO INDEPENDENT SAMPLES)

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Ho	Test Statistic	Ha	Critical region
a. σ_1^2 and σ_2^2 known, regardless of n_1 and n_2, populations are normally distributed			
$\mu_1 - \mu_2 = d_o$	$Z = \dfrac{(\overline{X}_1 - \overline{X}_2) - d_o}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$	$\mu_1 - \mu_2 < d_o$ $\mu_1 - \mu_2 > d_o$ $\mu_1 - \mu_2 \neq d_o$	$z < -z_\alpha$ $z > z_\alpha$ $\lvert z \rvert > z_{\alpha/2}$
b. $\sigma_1^2 = \sigma_2^2$ but unknown, $n_1 < 30$ or $n_2 < 30$ or both, and populations are normally distributed			
$\mu_1 - \mu_2 = d_o$	$t = \dfrac{(\overline{X}_1 - \overline{X}_2) - d_o}{S_p \sqrt{(1/n_1) + (1/n_2)}}$ $\upsilon = n_1 + n_2 - 2$ $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$	$\mu_1 - \mu_2 < d_o$ $\mu_1 - \mu_2 > d_o$ $\mu_1 - \mu_2 \neq d_o$	$t < -t_\alpha$ $t > t_\alpha$ $\lvert t \rvert > t_{\alpha/2}$

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Ho	Test Statistic	Ha	Critical region
c. $\sigma_1^2 \neq \sigma_2^2$ and unknown, $n_1 < 30$ or $n_2 < 30$ or both, and populations are normally distributed			
$\mu_1 - \mu_2 = d_o$	$t = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}$ $\upsilon = \dfrac{(S_1^2/n_1 + S_2^2/n_2)^2}{\dfrac{(S_1^2/n_1)^2}{n_1 - 1} + \dfrac{(S_2^2/n_2)^2}{n_2 - 1}}$	$\mu_1 - \mu_2 < d_o$ $\mu_1 - \mu_2 > d_o$ $\mu_1 - \mu_2 \neq d_o$	$t < -t_\alpha$ $t > t_\alpha$ $\lvert t \rvert > t_{\alpha/2}$
d. σ_1^2 and σ_2^2 are unknown but $n_1 \geq 30$ and $n_2 \geq 30$,			
$\mu_1 - \mu_2 = d_o$	$Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}$	$\mu_1 - \mu_2 < d_o$ $\mu_1 - \mu_2 > d_o$ $\mu_1 - \mu_2 \neq d_o$	$z < -z_\alpha$ $z > z_\alpha$ $\lvert z \rvert > z_{\alpha/2}$

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Example 1:

To test the hypothesis that 12-year old girls are, on the average, at least an inch taller than 12-year old boys, two random samples of boys and girls from a large school are measured and their heights recorded. Sixteen boys average 49.8 inches in height with a standard deviation of 5.1 inches and fifteen girls average 53.9 inches with a standard deviation of 4.7 inches. Test the hypothesis at the 0.05 level of significance.

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Solution:

Let μ_1 = average height of 12-year old girls
        μ_2 = average height of 12-year old boys

$$\overline{X}_1 = 53.9 \qquad s_1 = 4.7 \qquad n_1 = 15$$
$$\overline{X}_2 = 49.8 \qquad s_2 = 5.1 \qquad n_2 = 16$$

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

1. H_0: $\mu_1 - \mu_2 = 1$

   H_a: $\mu_1 - \mu_2 > 1$

2. $\alpha = 0.05$

3. test statistic: $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{(S_1^2 / n_1) + (S_2^2 / n_2)}}$

   $$\upsilon = \dfrac{(S_1^2 / n_1 + S_2^2 / n_2)^2}{\dfrac{(S_1^2 / n_1)^2}{n_1 - 1} + \dfrac{(S_2^2 / n_2)^2}{n_2 - 1}}$$

   Critical Region: $t > t_\alpha$ with df v

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

4. $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}} = \dfrac{(53.9 - 49.8) - 1}{\sqrt{(4.7^2/15) + (5.1^2/16)}} = 1.7611$

$\upsilon = \dfrac{(S_1^2/n_1 + S_2^2/n_2)^2}{\dfrac{(S_1^2/n_1)^2}{n_1 - 1} + \dfrac{(S_2^2/n_2)^2}{n_2 - 1}} = \dfrac{(4.7^2/15 + 5.1^2/16)^2}{\dfrac{(4.7^2/15)^2}{15 - 1} + \dfrac{(5.1^2/16)^2}{16 - 1}} = 28.99 = 29$

Critical Region: $t > t_\alpha$

$t > t_{0.05}(29)$

$t > 1.699$

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

5. Since t = 1.7611 > $t_{0.05}$ (29) = 1.699, thus, we reject H_0 at 5% level of significance and conclude that on the average, 12-year old girls are at least an inch taller than 12-year old boys.

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Example 2:

A social worker wishes to test whether or not the average amount spent on food for a family of four is the same in Iloilo City as it is in Bacolod City. She asks a random sample of 40 families of four in each city to keep track of their expenditure for food in a day. The mean amounts spent for food per family is P126.50 with a standard deviation of P14.40 in Iloilo City and P119.75 with a standard deviation of P13.65 in Bacolod City. Can we conclude that there is a significant difference in the mean amount spent on food for a family of four in the two cities? Use 0.02 level of significance.

📌 TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Solution:

Let μ_1 = average amount spent on food for a family of 4 in Iloilo City

μ_2 = average amount spent on food for a family of 4 in Bacolod City

$\overline{X}_1 = 126.50$ $s_1 = 14.40$ $n_1 = n_2 = 40$

$\overline{X}_2 = 119.75$ $s_2 = 13.65$

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

1. H_0: $\mu_1 - \mu_2 = 0$

   H_a: $\mu_1 - \mu_2 \neq 0$

2. $\alpha = 0.02$

3. test statistic: $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}$

Critical Region: Reject H_o if $|z| > z_{\alpha/2}$

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

4. $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_o}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}} = \dfrac{(126.5 - 119.75) - 0}{\sqrt{(14.40^2/40) + (13.65^2/40)}} = 2.1516$

Critical Region:

$|z| > z_{\alpha/2}$

$|z| > z_{0.02/2}$

$|z| > z_{0.01}$

$|z| > 2.33$

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TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

5. Since $| z | = | 2.1516 | = 2.1516 < z_{0.02/2} = 2.33$, thus, we do not reject H_0 at 2% level of significance. Hence, we can conclude that at 2% level of significance, we do not have sufficient evidence to prove that there is a significant difference in the mean amount spent on food for a family of four in the two cities.

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TESTING HYPOTHESIS ON DIFFERENCE BETWEEN TWO POPOULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Using Jamovi to Perform the One-Sample T-test:

From the menus choose:

1. **Analyses > T- Tests>Independent Samples T-Test…**
2. Select the dependent variable and drag it to the Dependent box. Select the grouping variable and drag it to the Grouping variable box. Choose Student's or Welch under the **Test option**
3. Under the **Hypothesis option,** choose the appropriate alternative hypothesis.
4. Tick Homogeneity test and Normality test under **Assumption Checks option.**
5. Optionally, click **Descriptives**

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TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Example:

The CEO of an airport hypothesizes that the mean number of passengers is lower for American airports than foreign airports. Is there enough evidence to support the hypothesis at 2.5% level of significance? The data in millions of passengers per year are shown for randomly selected airports. Assume the variable is normally distributed and homogenous in the two populations.

American airports		Foreign airports	
36.8	50.2	60.7	51.2
42.3	61.2	72.7	78.6
60.5	40.1		

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Using Jamovi, first check the Normality Assumption :

Tests of Normality

		statistic	p
Number of passengers in millions	Shapiro-Wilk	0.896	0.197
	Kolmogorov-Smirnov	0.185	0.822
	Anderson-Darling	0.419	0.261

Note. Additional results provided by *moretests*

Thus, the number of passengers (in millions) is assumed to be coming from Normal populations since p-value = 0.197 is greater than 0.025 level of significance.

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Next, check the Homogeneity of Variances Assumption

Homogeneity of Variances Tests

		F	df	df2	p
Number of passengers in millions	Levene's	0.142	1	8	0.716
	Variance ratio	0.738	5	3	0.713

Note. Additional results provided by *moretests*

Using Levene's test, the population variances of number of passengers (in millions) are assumed to be equal for both airports since p-value = 0.716 is greater than 0.025.

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Now, showing some Descriptives :

Group Descriptives

	Group	N	Mean	Median	SD	SE
Number of passengers in millions	American	6	48.5	46.3	10.5	4.30
	Foreign	4	65.8	66.7	12.3	6.13

A sample of 6 American airports showed to have an average of 48.5 million passengers while a sample of 4 foreign airports have an average of 65.8 million passengers.

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TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Now, showing the result of the T-test :

Independent Samples T-Test

		Statistic	df	p
Number of passengers in millions	Student's t	-2.39	8.00	0.022
	Welch's t	-2.31	5.83	0.031

Note. H_a American < Foreign

Using Student's t – test (due to equal variances assumption is satisfied), we reject Ho since the p-value = 0.022 is less than 0.025 level of significance.

At 0.05 level of significance, there is sufficient evidence to say that the true mean number of passengers for American airports is less than the true mean number of passengers for foreign airports, (p-value =0.022).

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Solution using Jamovi Outputs :

Assumptions: (see outputs from previous slides)
- Normality is assumed because p-value = 0.197 > α = 0.025.
- Homogeneity of variances is also met (i.e. Assume $\sigma_A^2 = \sigma_F^2$) since p –value =0.716 > α = 0.025

Note: Given the assumptions and the sample sizes are less than 30, this is an example of case (b), so use Student's t results for this hypothesis testing.

Given:

$n_A = 6$ $\bar{x}_A = 48.5$ $s = 10.5$
$n_F = 4$ $\bar{x}_F = 65.8$ $s = 12.3$

Let μ_A= true mean number of passengers (in millions) in American airports
    μ_F = true mean number of passengers (in millions) in Foreign airpotrts

1. $H_0: \mu_A = \mu_F$ **vs.** $H_a: \mu_A < \mu_F$
2. Level of significance: $\alpha = 0.025$

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TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Solution using Jamovi Outputs :

Let μ_A = true mean number of passengers (in millions) in American airports

μ_F = true mean number of passengers (in millions) in Foreign airpotrts

3. Test Statistic: -2.39
4. Decision Rule: Reject H_0 if $\text{p} - \text{value} \leq \alpha = 0.025$
5. p - value: 0.022
6. Decision: Reject H_0 since $\text{p} - \text{value} = 0.022 < \alpha = 0.025$
7. Conclusion: At 0.025 level of significance, there is sufficient evidence to say that the true mean number of passengers for American airports is less than the true mean number of passengers for foreign airports.

TESTING HYPOTHESIS ON THE DIFFERENCE BETWEEN TWO POPULATION MEANS (BASED ON TWO INDEPENDENT SAMPLES)

Exercise :

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. Is there enough evidence to support the claim at 0.05 level of confidence?

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5