

MultilayerGraphs.jl: Multilayer Network Science in Julia

Claudio Moroni ^{1,2*} and Pietro Monticone ^{1,2*}

¹ University of Turin, Italy ² Interdisciplinary Physics Team, Italy * These authors contributed equally.

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Summary

MultilayerGraphs.jl is a Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs.

A multilayer graph consists of multiple subgraphs called *layers* which can be interconnected through [bipartite graphs](#) called *interlayers* composed of the vertex sets of two different layers and the edges between them. The vertices in each layer represent a single set of nodes, although not all nodes have to be represented in every layer.

Formally, a multilayer graph can be defined as a triple $G = (V, E, L)$, where:

- V is the set of vertices;
- E is the set of edges, pairs of nodes (u, v) representing a connection, relationship or interaction between the nodes u and v ;
- L is a set of layers, which are subsets of V and E encoding the nodes and edges within each layer.

Each layer ℓ in L is a tuple (V_ℓ, E_ℓ) , where V_ℓ is a subset of V that represents the vertices within that layer, and E_ℓ is a subset of E that represents the edges within that layer.

MultilayerGraphs.jl is an integral part of the [JuliaGraphs](#) ecosystem extending Graphs.jl ([Fairbanks et al., 2021](#)) so all the methods and metrics exported by Graphs.jl work for multilayer graphs, but due to the special nature of multilayer graphs the package features a peculiar implementation that maps a standard integer-labelled vertex representation to a more user-friendly framework exporting all the objects an experienced practitioner would expect such as nodes, vertices, layers, interlayers, etc.

MultilayerGraphs.jl features specific methods and metrics including the global clustering coefficient, the overlay clustering coefficient, the multilayer eigenvector centrality, the multilayer modularity and the Von Neumann entropy.

Finally, MultilayerGraphs.jl has been integrated within the [JuliaDynamics](#) ecosystem so that any Multilayer(Di)Graph can be utilised as an argument to the GraphSpace constructor in Agents.jl ([Datseris et al., 2022](#)).

Statement of Need

Several theoretical frameworks have been proposed to formally subsume all instances of multilayer graphs ([Aleta & Moreno, 2019](#); [Artime et al., 2022](#); [Bianconi, 2018](#); [Boccaletti et al., 2014](#); [Cozzo et al., 2018](#); [M. D. Domenico et al., 2013](#); [M. D. Domenico, 2022](#); [Kivela et al., 2014](#); [Lee et al., 2015](#)).

Multilayer graphs have been adopted to model the structure and dynamics of a wide spectrum of high-dimensional, non-linear, multi-scale, time-dependent complex systems including physi-

cal, chemical, biological, neuronal, socio-technical, epidemiological, ecological and economic networks (Aleta et al., 2022, 2020; Amato et al., 2017; Arruda et al., 2017; Azimi-Tafreshi, 2016; Baggio et al., 2016; Buldú & Porter, 2018; Cozzo et al., 2013; M. D. Domenico, 2017; M. D. Domenico et al., 2016; Estrada & Gómez-Gardeñes, 2014; Gosak et al., 2018; Granell et al., 2013; Lim et al., 2019; Mangioni et al., 2020; Massaro & Bagnoli, 2014; Pilosof et al., 2017; Soriano-Paños et al., 2018; Timóteo et al., 2018).

At the best of our knowledge there are currently no software packages dedicated to the creation, manipulation and analysis of multilayer graphs implemented in the Julia language (Bezanson et al., 2017) apart from MultilayerGraphs.jl itself (Moroni & Monticone, 2022).

Main Features

The two main data structures are collections of layers connected through interlayers called MultilayerGraph and MultilayerDiGraph.

The **vertices** of a multilayer graph are representations of one set of distinct objects called Nodes. Each layer may represent all the node set or just a subset of it. The vertices of Multilayer(Di)Graph are implemented via the MultilayerVertex custom type. Each MultilayerVertex encodes information about the node it represents, the layer it belongs to and its metadata.

Both the **intra-layer** and **inter-layer edges** are embedded in the MultilayerEdge struct, whose arguments are the two connected multilayer vertices, the edge weight and its metadata. It's important to highlight that Multilayer(Di)Graphs are weighted and able to store metadata by default (i.e. they have been assigned the IsWeighted and IsMeta traits from SimpleTraits.jl).

The **layers** are implemented via the Layer struct composed of an underlying graph and a mapping from its integer-labelled vertices to the collection of MultilayerVertices the layer represents. **Interlayers** are similarly implemented via the Interlayer mutable struct, and they are generally constructed by providing the two connected layers, the (multilayer) edge list between them and a graph. This usage of underlying graphs allows for an easier debugging procedure during construction and a more intuitive analysis afterwards allowing the package to leverage all the features of the JuliaGraphs ecosystem so that it can be effectively considered as a real proving ground of its internal consistency.

The Multilayer(Di)Graph structs are weighted and endowed with the functionality to store both vertex-level and edge-level metadata by default so that at any moment the user may add or remove a Layer or specify an Interlayer and since different layers and interlayers could be better represented by graphs that are weighted or unweighted and with or without metadata, it was crucial for us to provide the most general and adaptable structure. A Multilayer(Di)Graph is instantiated by providing the ordered list of layers and the list of interlayers to the constructor. The latter are automatically specified, so there is no need to instantiate all of them.

Alternatively, it is possible to construct a Multilayer(Di)Graph making use of a graph generator-like signature allowing the user to set the degree distribution or the degree sequence and employs graph realisation methods such as the Havel-Hakimi algorithm for undirected graphs (Hakimi, 1962) and the Kleitman-Wang algorithm for directed ones (Kleitman & Wang, 1973).

Multilayer(Di)Graphs structure may be represented via dedicated WeightTensor, MetadataTensor and SupraWeightMatrix structs, all of which support indexing with MultilayerVertices. Once a Multilayer(Di)Graph has been instantiated, its layers and interlayers can be accessed as their properties.

For a more comprehensive exploration of the package features and functionalities we strongly recommend consulting the package [documentation](#).

87 Installation and Usage

88 To install MultilayerGraphs.jl it is sufficient to activate the pkg mode by pressing] in the Julia
89 REPL and then run the following command:

```
pkg> add MultilayerGraphs
```

90 In the following code chunk we synthetically illustrate:

- 91 ▪ how to define **layers** and **interlayers** with a variety of constructors and underlying graphs;
- 92 ▪ how to construct a **directed multilayer graph** with those layers and interlayers;
- 93 ▪ how to add nodes, vertices and edges to the multilayer graph;
- 94 ▪ how to compute some multilayer metrics as defined in M. D. Domenico et al. (2013).

```
# Import necessary dependencies
using Distributions, Graphs, SimpleValueGraphs
using MultilayerGraphs

# Set relevant constants
n_nodes = 100
node_list = [Node("node_$(i)") for i in 1:n_nodes]

# Simple directed layer
n_vertices = rand(1:100)
layer_simple_directed = layer_simpLEDigraph(
    :layer_simple_directed,
    sample(node_list, n_vertices; replace=false),
    Truncated(Normal(5, 5), 0, 20),
    Truncated(Normal(5, 5), 0, 20)
)

# Simple weighted layer
n_vertices = rand(1:n_nodes)
n_edges = rand(n_vertices:(n_vertices * (n_vertices - 1) - 1))
layer_simple_directed_weighted = layer_simpleweighteddigraph(
    :layer_simple_directed_weighted,
    sample(node_list, n_vertices; replace=false),
    n_edges,
    default_edge_weight=(src, dst) -> rand()
)

# Simple directed value layer
n_vertices = rand(1:n_nodes)
n_edges = rand(n_vertices:(n_vertices * (n_vertices - 1) - 1))
default_vertex_metadata = v -> ("vertex_$(v)_metadata")
default_edge_metadata = (s, d) -> (rand(),)
layer_simple_directed_value = Layer(
    :layer_simple_directed_value,
    sample(node_list, n_vertices; replace=false),
    n_edges,
    ValDiGraph(
        SimpleDiGraph{Int64}();
        vertexval_types=(String,),
        vertexval_init=default_vertex_metadata,
        edgeval_types=(Float64,),
        edgeval_init=default_edge_metadata,
    ),
)
```

```

Float64;
default_vertex_metadata=default_vertex_metadata,
default_edge_metadata=default_edge_metadata
)

# List of layers
layers = [layer_simple_directed, layer_simple_directed_weighted, layer_simple_directed_v

# Simple directed interlayer
n_vertices_1 = nv(layer_simple_directed)
n_vertices_2 = nv(layer_simple_directed_weighted)
n_edges = rand(1:(n_vertices_1 * n_vertices_2 - 1))
interlayer_simple_directed = interlayer_simpdigraph(
    layer_simple_directed,
    layer_simple_directed_weighted,
    n_edges
)

# Simple directed meta interlayer
n_vertices_1 = nv(layer_simple_directed_weighted)
n_vertices_2 = nv(layer_simple_directed_value)
n_edges = rand(1:(n_vertices_1 * n_vertices_2 - 1))
interlayer_simple_directed_meta = interlayer_metadigraph(
    layer_simple_directed_weighted,
    layer_simple_directed_value,
    n_edges;
    default_edge_metadata=(src, dst) ->
        (edge_metadata="metadata_of_edge_from_$(src)_to_$(dst)"),
    transfer_vertex_metadata=true
)

# List of interlayers
interlayers = [interlayer_simple_directed, interlayer_simple_directed_meta]

# Directed multilayer graph
multilayerdigraph = MultilayerDiGraph(
    layers,
    interlayers;
    default_interlayers_structure="multiplex"
)

# Create node
new_node_1 = Node("new_node_1")
# Add node to multilayer
add_node!(multilayerdigraph, new_node_1)
# Create vertex representing the node
new_vertex_1 = MV(
    new_node_1,
    :layer_simple_directed_value,
    ("new_metadata")
)
# Add vertex
add_vertex!(
    multilayerdigraph,
    new_vertex_1

```

```
)

# Create node in another layer
new_node_2 = Node("new_node_2")
# Create vertex representing the new node
new_vertex_2 = MV(new_node_2, :layer_simple_directed)
# Add the new vertex
add_vertex!(
    multilayerdigraph,
    new_vertex_2;
    add_node=true
)
# Create an edge
new_edge = MultilayerEdge(
    new_vertex_1,
    new_vertex_2,
    ("some_edge_metadata")
)
# Add the edge
add_edge!(
    multilayerdigraph,
    new_edge
)

# Metrics
multilayer_global_clustering_coefficient(multilayerdigraph)
overlay_clustering_coefficient(multilayerdigraph)
eigenvector_centrality(multilayerdigraph)
modularity(
    multilayerdigraph,
    rand([1, 2, 3, 4], length(nodes(multilayerdigraph)),
        length(multilayerdigraph.layers))
)
)
```

Related Packages

R

Here is a list of related software packages implemented in the [R language](#):

- [muxViz](#) implements functions to perform multilayer correlation analysis, multilayer centrality analysis, multilayer community structure detection, multilayer structural reducibility, multilayer motifs analysis and utilities to statically and dynamically visualise multilayer graphs ([D. Domenico et al., 2014](#));
- [multinet](#) implements functions to import, export, create and manipulate multilayer graphs, several state-of-the-art multiplex graph analysis algorithms for centrality measures, layer comparison, community detection and visualization ([Magnani et al., 2021](#));
- [mully](#) implements functions to import, export, create, manipulate and merge multilayer graphs and utilities to visualise multilayer graphs in 2D and 3D ([Hammoud & Kramer, 2018](#));
- [multinets](#) implements functions to import, export, create, manipulate multilayer graphs and utilities to visualise multilayer graphs ([Lazega et al., 2008](#)).

Python

Here is a list of related software packages implemented in the [Python language](#):

- [MultiNetX](#) implements methods to create undirected networks with weighted or unweighted links, to analyse the spectral properties of adjacency or Laplacian matrices and to visualise multilayer graphs and dynamical processes by coloring the nodes and links accordingly;
- [PyMNet](#) implements data structures for multilayer graphs and multiplex graphs, methods to import, export, create, manipulate multilayer graphs and for the rule-based generation and lazy-evaluation of coupling edges and utilities to visualise multilayer graphs ([Kivela et al., 2014](#)).

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References

- Aleta, A., Martín-Corral, D., Bakker, M. A., Piontti, A. P. y, Ajelli, M., Litvinova, M., Chinazzi, M., Dean, N. E., Halloran, M. E., Longini, I. M., Pentland, A., Vespignani, A., Moreno, Y., & Moro, E. (2022). Quantifying the importance and location of SARS-CoV-2 transmission events in large metropolitan areas. *Proceedings of the National Academy of Sciences*, 119(26). <https://doi.org/10.1073/pnas.2112182119>
- Aleta, A., Martín-Corral, D., Piontti, A. P. y, Ajelli, M., Litvinova, M., Chinazzi, M., Dean, N. E., Halloran, M. E., Jr, I. M. L., Merler, S., Pentland, A., Vespignani, A., Moro, E., & Moreno, Y. (2020). Modelling the impact of testing, contact tracing and household quarantine on second waves of COVID-19. *Nature Human Behaviour*, 4(9), 964–971. <https://doi.org/10.1038/s41562-020-0931-9>
- Aleta, A., & Moreno, Y. (2019). Multilayer networks in a nutshell. *Annual Review of Condensed Matter Physics*, 10(1), 45–62. <https://doi.org/10.1146/annurev-conmatphys-031218-013259>
- Amato, R., Díaz-Guilera, A., & Kleineberg, K.-K. (2017). Interplay between social influence and competitive strategical games in multiplex networks. *Scientific Reports*, 7(1). <https://doi.org/10.1038/s41598-017-06933-2>
- Arruda, G. F. de, Cozzo, E., Peixoto, T. P., Rodrigues, F. A., & Moreno, Y. (2017). Disease localization in multilayer networks. *Physical Review X*, 7(1). <https://doi.org/10.1103/physrevx.7.011014>
- Artime, O., Benigni, B., Bertagnolli, G., dAndrea, V., Gallotti, R., Ghavasieh, A., Raimondo, S., & Domenico, M. D. (2022). *Multilayer network science*. Cambridge University Press. <https://doi.org/10.1017/9781009085809>
- Azimi-Tafreshi, N. (2016). Cooperative epidemics on multiplex networks. *Physical Review E*, 93(4). <https://doi.org/10.1103/physreve.93.042303>
- Baggio, J. A., BurnSilver, S. B., Arenas, A., Magdanz, J. S., Kofinas, G. P., & Domenico, M. D. (2016). Multiplex social ecological network analysis reveals how social changes affect community robustness more than resource depletion. *Proceedings of the National Academy of Sciences*, 113(48), 13708–13713. <https://doi.org/10.1073/pnas.1604401113>
- Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. (2017). Julia: A fresh approach to numerical computing. *SIAM Review*, 59(1), 65–98. <https://doi.org/10.1137/141000671>

- 153 Bianconi, G. (2018). *Multilayer networks*. Oxford University Press. [https://doi.org/10.1093/](https://doi.org/10.1093/oso/9780198753919.001.0001)
154 [oso/9780198753919.001.0001](https://doi.org/10.1093/oso/9780198753919.001.0001)
- 155 Boccaletti, S., Bianconi, G., Criado, R., Genio, C. I. del, Gómez-Gardeñes, J., Romance, M.,
156 Sendiña-Nadal, I., Wang, Z., & Zanin, M. (2014). The structure and dynamics of multilayer
157 networks. *Physics Reports*, 544(1), 1–122. <https://doi.org/10.1016/j.physrep.2014.07.001>
- 158 Buldú, J. M., & Porter, M. A. (2018). Frequency-based brain networks: From a multiplex
159 framework to a full multilayer description. *Network Neuroscience*, 2(4), 418–441. https://doi.org/10.1162/netn_a_00033
160
- 161 Cozzo, E., Arruda, G. F. de, Rodrigues, F. A., & Moreno, Y. (2018). *Multiplex networks*.
162 Springer International Publishing. <https://doi.org/10.1007/978-3-319-92255-3>
- 163 Cozzo, E., Baños, R. A., Meloni, S., & Moreno, Y. (2013). Contact-based social contagion in
164 multiplex networks. *Physical Review E*, 88(5). <https://doi.org/10.1103/physreve.88.050801>
- 165 Datseris, G., Vahdati, A. R., & DuBois, T. C. (2022). Agents.jl: A performant and
166 feature-full agent-based modeling software of minimal code complexity. *SIMULATION*,
167 003754972110688. <https://doi.org/10.1177/00375497211068820>
- 168 Domenico, D., Porter, & Arenas. (2014). MuxViz: A tool for multilayer analysis and
169 visualization of networks. *Journal of Complex Networks*, 3(2), 159–176. <https://doi.org/10.1093/comnet/cnu038>
170
- 171 Domenico, M. D. (2017). Multilayer modeling and analysis of human brain networks. *Giga-*
172 *Science*, 6(5). <https://doi.org/10.1093/gigascience/gix004>
- 173 Domenico, M. D. (2022). *Multilayer networks: Analysis and visualization*. Springer International
174 Publishing. <https://doi.org/10.1007/978-3-030-75718-2>
- 175 Domenico, M. D., Granell, C., Porter, M. A., & Arenas, A. (2016). The physics of spreading
176 processes in multilayer networks. *Nature Physics*, 12(10), 901–906. <https://doi.org/10.1038/nphys3865>
177
- 178 Domenico, M. D., Solé-Ribalta, A., Cozzo, E., Kivelä, M., Moreno, Y., Porter, M. A., Gómez,
179 S., & Arenas, A. (2013). Mathematical formulation of multilayer networks. *Physical Review*
180 *X*, 3(4). <https://doi.org/10.1103/physrevx.3.041022>
- 181 Estrada, E., & Gómez-Gardeñes, J. (2014). Communicability reveals a transition to coordinated
182 behavior in multiplex networks. *Physical Review E*, 89(4). [https://doi.org/10.1103/](https://doi.org/10.1103/physreve.89.042819)
183 [physreve.89.042819](https://doi.org/10.1103/physreve.89.042819)
- 184 Fairbanks, J., Besançon, M., Simon, S., Hoffiman, J., Eubank, N., & Karpinski, S. (2021).
185 *JuliaGraphs/graphs.jl: An optimized graphs package for the julia programming language*.
186 [https://github.com/JuliaGraphs/Graphs.jl/](https://github.com/JuliaGraphs/Graphs.jl)
- 187 Gosak, M., Markovič, R., Dolensšek, J., Rupnik, M. S., Marhl, M., Stožer, A., & Perc, M.
188 (2018). Network science of biological systems at different scales: A review. *Physics of Life*
189 *Reviews*, 24, 118–135. <https://doi.org/10.1016/j.plrev.2017.11.003>
- 190 Granell, C., Gómez, S., & Arenas, A. (2013). Dynamical interplay between awareness and
191 epidemic spreading in multiplex networks. *Physical Review Letters*, 111(12). <https://doi.org/10.1103/physrevlett.111.128701>
192
- 193 Hakimi, S. L. (1962). On realizability of a set of integers as degrees of the vertices of a linear
194 graph. i. *Journal of the Society for Industrial and Applied Mathematics*, 10(3), 496–506.
195 <https://doi.org/10.1137/0110037>
- 196 Hammoud, Z., & Kramer, F. (2018). Mully: An r package to create, modify and visualize
197 multilayered graphs. *Genes*, 9(11), 519. <https://doi.org/10.3390/genes9110519>

- 198 Kivela, M., Arenas, A., Barthelemy, M., Gleeson, J. P., Moreno, Y., & Porter, M. A. (2014).
 199 Multilayer networks. *Journal of Complex Networks*, 2(3), 203–271. [https://doi.org/10.](https://doi.org/10.1093/comnet/cnu016)
 200 [1093/comnet/cnu016](https://doi.org/10.1093/comnet/cnu016)
- 201 Kleitman, D. J., & Wang, D. L. (1973). Algorithms for constructing graphs and digraphs with
 202 given valences and factors. *Discrete Mathematics*, 6(1), 79–88. [https://doi.org/10.1016/](https://doi.org/10.1016/0012-365x(73)90037-x)
 203 [0012-365x\(73\)90037-x](https://doi.org/10.1016/0012-365x(73)90037-x)
- 204 Lazega, E., Jourda, M.-T., Mounier, L., & Stofer, R. (2008). Catching up with big fish in
 205 the big pond? Multi-level network analysis through linked design. *Social Networks*, 30(2),
 206 159–176. <https://doi.org/10.1016/j.socnet.2008.02.001>
- 207 Lee, K.-M., Min, B., & Goh, K.-I. (2015). Towards real-world complexity: An introduction to
 208 multiplex networks. *The European Physical Journal B*, 88(2). [https://doi.org/10.1140/](https://doi.org/10.1140/epjb/e2015-50742-1)
 209 [epjb/e2015-50742-1](https://doi.org/10.1140/epjb/e2015-50742-1)
- 210 Lim, S., Radicchi, F., Heuvel, M. P. van den, & Sporns, O. (2019). Discordant attributes of
 211 structural and functional brain connectivity in a two-layer multiplex network. *Scientific*
 212 *Reports*, 9(1). <https://doi.org/10.1038/s41598-019-39243-w>
- 213 Magnani, M., Rossi, L., & Vega, D. (2021). Analysis of multiplex social networks with r.
 214 *Journal of Statistical Software*, 98(8). <https://doi.org/10.18637/jss.v098.i08>
- 215 Mangioni, G., Jurman, G., & Domenico, M. D. (2020). Multilayer flows in molecular networks
 216 identify biological modules in the human proteome. *IEEE Transactions on Network Science*
 217 *and Engineering*, 7(1), 411–420. <https://doi.org/10.1109/tNSE.2018.2871726>
- 218 Massaro, E., & Bagnoli, F. (2014). Epidemic spreading and risk perception in multiplex
 219 networks: A self-organized percolation method. *Physical Review E*, 90(5). [https://doi.](https://doi.org/10.1103/PhysRevE.90.052817)
 220 [org/10.1103/PhysRevE.90.052817](https://doi.org/10.1103/PhysRevE.90.052817)
- 221 Moroni, C., & Monticone, P. (2022). *MultilayerGraphs.jl: A julia package for the creation,*
 222 *manipulation and analysis of the structure, dynamics and functions of multilayer graphs.*
 223 University of Turin (UniTO); Interdisciplinary Physics Team (InPhyT). [https://doi.org/10.](https://doi.org/10.5281/zenodo.7009172)
 224 [5281/zenodo.7009172](https://doi.org/10.5281/zenodo.7009172)
- 225 Pilosof, S., Porter, M. A., Pascual, M., & Kéfi, S. (2017). The multilayer nature of eco-
 226 logical networks. *Nature Ecology & Evolution*, 1(4). [https://doi.org/10.1038/](https://doi.org/10.1038/s41559-017-0101)
 227 [s41559-017-0101](https://doi.org/10.1038/s41559-017-0101)
- 228 Soriano-Paños, D., Lotero, L., Arenas, A., & Gómez-Gardeñes, J. (2018). Spreading processes
 229 in multiplex metapopulations containing different mobility networks. *Physical Review X*,
 230 8(3). <https://doi.org/10.1103/PhysRevX.8.031039>
- 231 Timóteo, S., Correia, M., Rodríguez-Echeverría, S., Freitas, H., & Heleno, R. (2018). Multilayer
 232 networks reveal the spatial structure of seed-dispersal interactions across the great rift
 233 landscapes. *Nature Communications*, 9(1). <https://doi.org/10.1038/s41467-017-02658-y>