JOSS
The Journal of Open Source Software

MultilayerGraphs.jl: Multilayer Network Science in Julia

Claudio Moroni [1,2*] and **Pietro Monticone** [1,2*]

1 University of Turin, Italy **2** Interdisciplinary Physics Team, Italy * These authors contributed equally.

DOI: 10.xxxxxx/draft

Software

- Review
- Repository
- Archive

Editor: Open Journals

Reviewers:

- @openjournals

Submitted: 01 January 1970
Published: unpublished

Summary

MultilayerGraphs.jl is a Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs.

A multilayer graph consists of multiple subgraphs called *layers* which can be interconnected through bipartite graphs called *interlayers* composed of the vertex sets of two different layers and the edges between them. The vertices in each layer represent a single set of nodes, although not all nodes have to be represented in every layer.

Formally, a multilayer graph can be defined as a triple $G = (V, E, L)$, where:

- V is the set of vertices;
- E is the set of edges, pairs of nodes (u, v) representing a connection, relationship or interaction between the nodes u and v;
- L is a set of layers, which are subsets of V and E encoding the nodes and edges within each layer.

Each layer ℓ in L is a tuple (V_ℓ, E_ℓ), where V_ℓ is a subset of V that represents the vertices within that layer, and E_ℓ is a subset of E that represents the edges within that layer.

MultilayerGraphs.jl is an integral part of the JuliaGraphs ecosystem extending Graphs.jl (Fairbanks et al., 2021) so all the methods and metrics exported by Graphs.jl work for multilayer graphs, but due to the special nature of multilayer graphs the package features a peculiar implementation that maps a standard integer-labelled vertex representation to a more user-friendly framework exporting all the objects an experienced practitioner would expect such as nodes, vertices, layers, interlayers, etc.

MultilayerGraphs.jl features specific methods and metrics including the global clustering coefficient, the overlay clustering coefficient, the multilayer eigenvector centrality, the multilayer modularity and the Von Neumann entropy.

Finally, MultilayerGraphs.jl has been integrated within the JuliaDynamics ecosystem so that any `Multilayer(Di)Graph` can be utilised as an argument to the `GraphSpace` constructor in Agents.jl (Datseris et al., 2022).

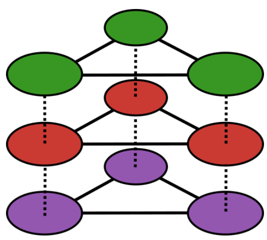

Figure 1: Logo of MultilayerGraphs.jl.

Statement of Need

Several theoretical frameworks have been proposed to formally subsume all instances of multilayer graphs (Aleta & Moreno, 2019; Artime et al., 2022; Bianconi, 2018; Boccaletti et al., 2014; Cozzo et al., 2018; M. D. Domenico et al., 2013; M. D. Domenico, 2022; Kivela et al., 2014; Lee et al., 2015).

Multilayer graphs have been adopted to model the structure and dynamics of a wide spectrum of high-dimensional, non-linear, multi-scale, time-dependent complex systems including physical, chemical, biological, neuronal, socio-technical, epidemiological, ecological and economic networks (Aleta et al., 2022, 2020; Amato et al., 2017; Arruda et al., 2017; Azimi-Tafreshi, 2016; Baggio et al., 2016; Buldú & Porter, 2018; Cozzo et al., 2013; M. D. Domenico, 2017; M. D. Domenico et al., 2016; Estrada & Gómez-Gardeñes, 2014; Gosak et al., 2018; Granell et al., 2013; Lim et al., 2019; Mangioni et al., 2020; Massaro & Bagnoli, 2014; Pilosof et al., 2017; Soriano-Paños et al., 2018; Timóteo et al., 2018).

At the best of our knowledge there are currently no software packages dedicated to the creation, manipulation and analysis of multilayer graphs implemented in the Julia language (Bezanson et al., 2017) apart from MultilayerGraphs.jl itself (Moroni & Monticone, 2022).

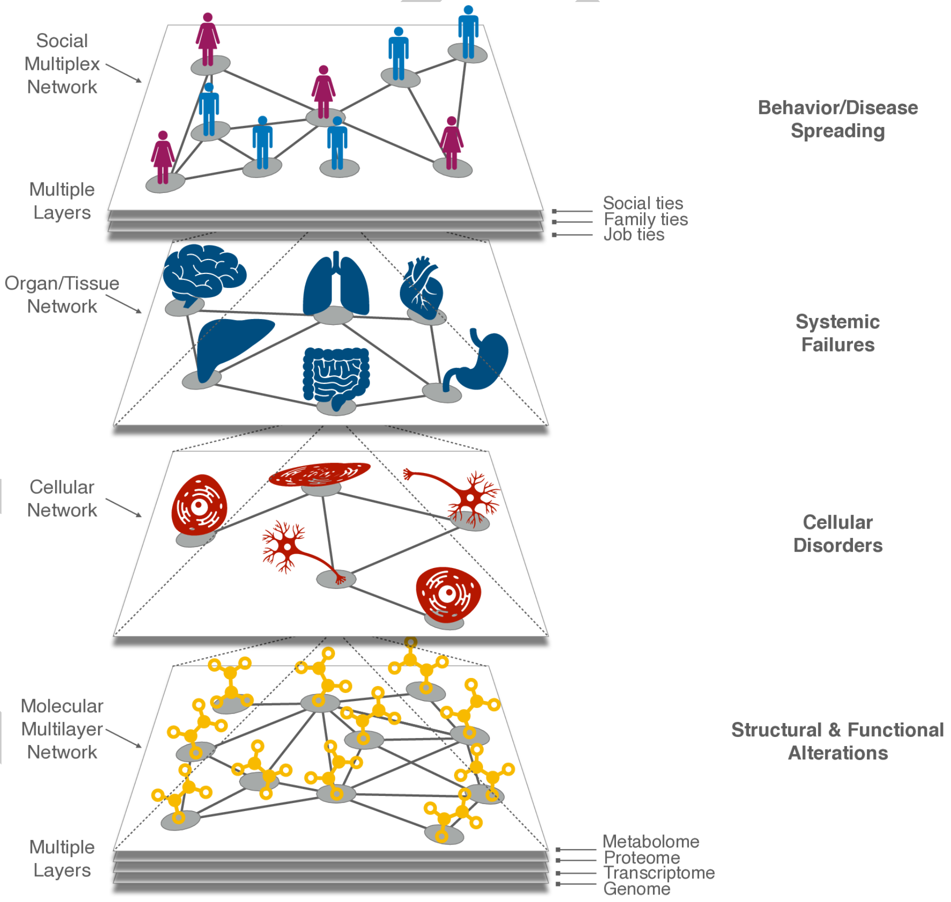

Figure 2: Diagram illustrating a multi-scale multilayer graph applied to model social systems from the molecular to the population level (De Domenico, 2022).

Main Features

The two main data structures are collections of layers connected through interlayers called `MultilayerGraph` and `MultilayerDiGraph`.

The **vertices** of a multilayer graph are representations of one set of distinct objects called `Nodes`. Each layer may represent all the node set or just a subset of it. The vertices of `Multilayer(Di)Graph` are implemented via the `MultilayerVertex` custom type. Each `MultilayerVertex` encodes information about the node it represents, the layer it belongs to and its metadata.

Both the **intra-layer** and **inter-layer edges** are embedded in the `MultilayerEdge` struct, whose arguments are the two connected multilayer vertices, the edge weight and its metadata. It's important to highlight that `Multilayer(Di)Graphs` are weighted and able to store metadata by default (i.e. they have been assigned the `IsWeighted` and `IsMeta` traits from SimpleTraits.jl).

The **layers** are implemented via the `Layer` struct composed of an underlying graph and a mapping from its integer-labelled vertices to the collection of `MultilayerVertexs` the layer represents. **Interlayers** are similarly implemented via the `Interlayer` mutable struct, and they are generally constructed by providing the two connected layers, the (multilayer) edge list between them and a graph. This usage of underlying graphs allows for an easier debugging procedure during construction and a more intuitive analysis afterwards allowing the package to leverage all the features of the JuliaGraphs ecosystem so that it can be effectively considered as a real proving ground of its internal consistency.

The `Multilayer(Di)Graph` structs are weighted and endowed with the functionality to store both vertex-level and edge-level metadata by default so that at any moment the user may add or remove a `Layer` or specify an `Interlayer` and since different layers and interlayers could be better represented by graphs that are weighted or unweighted and with or without metadata, it was crucial for us to provide the most general and adaptable structure. A `Multilayer(Di)Graph` is instantiated by providing the ordered list of layers and the list of interlayers to the constructor. The latter are automatically specified, so there is no need to instantiate all of them.

Alternatively, it is possible to construct a `Multilayer(Di)Graph` making use of a graph generator-like signature allowing the user to set the degree distribution or the degree sequence and employs graph realisation methods such as the Havel-Hakimi algorithm for undirected graphs (Hakimi, 1962) and the Kleitman-Wang algorithm for directed ones (Kleitman & Wang, 1973).

`Multilayer(Di)Graphs` structure may be represented via dedicated `WeightTensor`, `MetadataTensor` and `SupraWeightMatrix` structs, all of which support indexing with `MultilayerVertexs`. Once a `Multilayer(Di)Graph` has been instantiated, its layers and interlayers can be accessed as their properties.

For a more comprehensive exploration of the package features and functionalities we strongly recommend consulting the package README and documentation.

Installation and Usage

We invite the user to read the documented example and run the associated script we have designed to synthetically illustrate:

- how to **install** the package;
- how to define **layers** and **interlayers** with a variety of constructors and underlying graphs;
- how to construct a **directed multilayer graph** with those layers and interlayers;
- how to add nodes, vertices and edges to the multilayer graph;
- how to compute some multilayer metrics as defined in M. D. Domenico et al. (2013).

Related Packages

R

Here is a list of related software packages implemented in the R language:

- `muxViz` implements functions to perform multilayer correlation analysis, multilayer centrality analysis, multilayer community structure detection, multilayer structural reducibility, multilayer motifs analysis and utilities to statically and dynamically visualise multilayer graphs (D. Domenico et al., 2014);
- `multinet` implements functions to import, export, create and manipulate multilayer graphs, several state-of-the-art multiplex graph analysis algorithms for centrality measures, layer comparison, community detection and visualization (Magnani et al., 2021);
- `mully` implements functions to import, export, create, manipulate and merge multilayer graphs and utilities to visualise multilayer graphs in 2D and 3D (Hammoud & Kramer, 2018);
- `multinets` implements functions to import, export, create, manipulate multilayer graphs and utilities to visualise multilayer graphs (Lazega et al., 2008).

Python

Here is a list of related software packages implemented in the Python language:

- `MultiNetX` implements methods to create undirected networks with weighted or unweighted links, to analyse the spectral properties of adjacency or Laplacian matrices and to visualise multilayer graphs and dynamical processes by coloring the nodes and links accordingly;
- `PyMNet` implements data structures for multilayer graphs and multiplex graphs, methods to import, export, create, manipulate multilayer graphs and for the rule-based generation and lazy-evaluation of coupling edges and utilities to visualise multilayer graphs (Kivela et al., 2014).

Acknowledgements

This open-source research software project received no financial support.

References

Aleta, A., Martín-Corral, D., Bakker, M. A., Piontti, A. P. y, Ajelli, M., Litvinova, M., Chinazzi, M., Dean, N. E., Halloran, M. E., Longini, I. M., Pentland, A., Vespignani, A., Moreno, Y., & Moro, E. (2022). Quantifying the importance and location of SARS-CoV-2 transmission events in large metropolitan areas. *Proceedings of the National Academy of Sciences*, *119*(26). https://doi.org/10.1073/pnas.2112182119

Aleta, A., Martín-Corral, D., Piontti, A. P. y, Ajelli, M., Litvinova, M., Chinazzi, M., Dean, N. E., Halloran, M. E., Jr, I. M. L., Merler, S., Pentland, A., Vespignani, A., Moro, E., & Moreno, Y. (2020). Modelling the impact of testing, contact tracing and household quarantine on second waves of COVID-19. *Nature Human Behaviour*, *4*(9), 964–971. https://doi.org/10.1038/s41562-020-0931-9

Aleta, A., & Moreno, Y. (2019). Multilayer networks in a nutshell. *Annual Review of Condensed Matter Physics*, *10*(1), 45–62. https://doi.org/10.1146/annurev-conmatphys-031218-013259

Amato, R., Díaz-Guilera, A., & Kleineberg, K.-K. (2017). Interplay between social influence and competitive strategical games in multiplex networks. *Scientific Reports*, *7*(1). https:

//doi.org/10.1038/s41598-017-06933-2

Arruda, G. F. de, Cozzo, E., Peixoto, T. P., Rodrigues, F. A., & Moreno, Y. (2017). Disease localization in multilayer networks. *Physical Review X*, *7*(1). https://doi.org/10.1103/physrevx.7.011014

Artime, O., Benigni, B., Bertagnolli, G., dAndrea, V., Gallotti, R., Ghavasieh, A., Raimondo, S., & Domenico, M. D. (2022). *Multilayer network science*. Cambridge University Press. https://doi.org/10.1017/9781009085809

Azimi-Tafreshi, N. (2016). Cooperative epidemics on multiplex networks. *Physical Review E*, *93*(4). https://doi.org/10.1103/physreve.93.042303

Baggio, J. A., BurnSilver, S. B., Arenas, A., Magdanz, J. S., Kofinas, G. P., & Domenico, M. D. (2016). Multiplex social ecological network analysis reveals how social changes affect community robustness more than resource depletion. *Proceedings of the National Academy of Sciences*, *113*(48), 13708–13713. https://doi.org/10.1073/pnas.1604401113

Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. (2017). Julia: A fresh approach to numerical computing. *SIAM Review*, *59*(1), 65–98. https://doi.org/10.1137/141000671

Bianconi, G. (2018). *Multilayer networks*. Oxford University Press. https://doi.org/10.1093/oso/9780198753919.001.0001

Boccaletti, S., Bianconi, G., Criado, R., Genio, C. I. del, Gómez-Gardeñes, J., Romance, M., Sendiña-Nadal, I., Wang, Z., & Zanin, M. (2014). The structure and dynamics of multilayer networks. *Physics Reports*, *544*(1), 1–122. https://doi.org/10.1016/j.physrep.2014.07.001

Buldú, J. M., & Porter, M. A. (2018). Frequency-based brain networks: From a multiplex framework to a full multilayer description. *Network Neuroscience*, *2*(4), 418–441. https://doi.org/10.1162/netn_a_00033

Cozzo, E., Arruda, G. F. de, Rodrigues, F. A., & Moreno, Y. (2018). *Multiplex networks*. Springer International Publishing. https://doi.org/10.1007/978-3-319-92255-3

Cozzo, E., Baños, R. A., Meloni, S., & Moreno, Y. (2013). Contact-based social contagion in multiplex networks. *Physical Review E*, *88*(5). https://doi.org/10.1103/physreve.88.050801

Datseris, G., Vahdati, A. R., & DuBois, T. C. (2022). Agents.jl: A performant and feature-full agent-based modeling software of minimal code complexity. *SIMULATION*, 003754972110688. https://doi.org/10.1177/00375497211068820

De Domenico, M. (2022). *Multilayer networks illustrated*. https://doi.org/10.17605/OSF.IO/GY53K

Domenico, D., Porter, & Arenas. (2014). MuxViz: A tool for multilayer analysis and visualization of networks. *Journal of Complex Networks*, *3*(2), 159–176. https://doi.org/10.1093/comnet/cnu038

Domenico, M. D. (2017). Multilayer modeling and analysis of human brain networks. *GigaScience*, *6*(5). https://doi.org/10.1093/gigascience/gix004

Domenico, M. D. (2022). *Multilayer networks: Analysis and visualization*. Springer International Publishing. https://doi.org/10.1007/978-3-030-75718-2

Domenico, M. D., Granell, C., Porter, M. A., & Arenas, A. (2016). The physics of spreading processes in multilayer networks. *Nature Physics*, *12*(10), 901–906. https://doi.org/10.1038/nphys3865

Domenico, M. D., Solé-Ribalta, A., Cozzo, E., Kivelä, M., Moreno, Y., Porter, M. A., Gómez, S., & Arenas, A. (2013). Mathematical formulation of multilayer networks. *Physical Review X*, *3*(4). https://doi.org/10.1103/physrevx.3.041022

Estrada, E., & Gómez-Gardeñes, J. (2014). Communicability reveals a transition to coordinated behavior in multiplex networks. *Physical Review E*, *89*(4). https://doi.org/10.1103/physreve.89.042819

Fairbanks, J., Besançon, M., Simon, S., Hoffiman, J., Eubank, N., & Karpinski, S. (2021). *JuliaGraphs/graphs.jl: An optimized graphs package for the julia programming language*. https://github.com/JuliaGraphs/Graphs.jl/

Gosak, M., Markovič, R., Dolenšek, J., Rupnik, M. S., Marhl, M., Stožer, A., & Perc, M. (2018). Network science of biological systems at different scales: A review. *Physics of Life Reviews*, *24*, 118–135. https://doi.org/10.1016/j.plrev.2017.11.003

Granell, C., Gómez, S., & Arenas, A. (2013). Dynamical interplay between awareness and epidemic spreading in multiplex networks. *Physical Review Letters*, *111*(12). https://doi.org/10.1103/physrevlett.111.128701

Hakimi, S. L. (1962). On realizability of a set of integers as degrees of the vertices of a linear graph. i. *Journal of the Society for Industrial and Applied Mathematics*, *10*(3), 496–506. https://doi.org/10.1137/0110037

Hammoud, Z., & Kramer, F. (2018). Mully: An r package to create, modify and visualize multilayered graphs. *Genes*, *9*(11), 519. https://doi.org/10.3390/genes9110519

Kivela, M., Arenas, A., Barthelemy, M., Gleeson, J. P., Moreno, Y., & Porter, M. A. (2014). Multilayer networks. *Journal of Complex Networks*, *2*(3), 203–271. https://doi.org/10.1093/comnet/cnu016

Kleitman, D. J., & Wang, D. L. (1973). Algorithms for constructing graphs and digraphs with given valences and factors. *Discrete Mathematics*, *6*(1), 79–88. https://doi.org/10.1016/0012-365x(73)90037-x

Lazega, E., Jourda, M.-T., Mounier, L., & Stofer, R. (2008). Catching up with big fish in the big pond? Multi-level network analysis through linked design. *Social Networks*, *30*(2), 159–176. https://doi.org/10.1016/j.socnet.2008.02.001

Lee, K.-M., Min, B., & Goh, K.-I. (2015). Towards real-world complexity: An introduction to multiplex networks. *The European Physical Journal B*, *88*(2). https://doi.org/10.1140/epjb/e2015-50742-1

Lim, S., Radicchi, F., Heuvel, M. P. van den, & Sporns, O. (2019). Discordant attributes of structural and functional brain connectivity in a two-layer multiplex network. *Scientific Reports*, *9*(1). https://doi.org/10.1038/s41598-019-39243-w

Magnani, M., Rossi, L., & Vega, D. (2021). Analysis of multiplex social networks with r. *Journal of Statistical Software*, *98*(8). https://doi.org/10.18637/jss.v098.i08

Mangioni, G., Jurman, G., & Domenico, M. D. (2020). Multilayer flows in molecular networks identify biological modules in the human proteome. *IEEE Transactions on Network Science and Engineering*, *7*(1), 411–420. https://doi.org/10.1109/tnse.2018.2871726

Massaro, E., & Bagnoli, F. (2014). Epidemic spreading and risk perception in multiplex networks: A self-organized percolation method. *Physical Review E*, *90*(5). https://doi.org/10.1103/physreve.90.052817

Moroni, C., & Monticone, P. (2022). *MultilayerGraphs.jl: A julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs*. University of Turin (UniTO); Interdisciplinary Physics Team (InPhyT). https://doi.org/10.5281/zenodo.7009172

Pilosof, S., Porter, M. A., Pascual, M., & Kéfi, S. (2017). The multilayer nature of ecological networks. *Nature Ecology &Amp; Evolution*, *1*(4). https://doi.org/10.1038/s41559-017-0101

Soriano-Paños, D., Lotero, L., Arenas, A., & Gómez-Gardeñes, J. (2018). Spreading processes in multiplex metapopulations containing different mobility networks. *Physical Review X*, *8*(3). https://doi.org/10.1103/physrevx.8.031039

Timóteo, S., Correia, M., Rodríguez-Echeverría, S., Freitas, H., & Heleno, R. (2018). Multilayer networks reveal the spatial structure of seed-dispersal interactions across the great rift landscapes. *Nature Communications*, *9*(1). https://doi.org/10.1038/s41467-017-02658-y