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MultilayerGraphs.jl: Multilayer Network Science in Julia

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Summary

MultilayerGraphs.jl is a Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs.

A multilayer graph is a graph consisting of multiple standard subgraphs called *layers* which can be interconnected through bipartite graphs called *interlayers* composed of the vertex sets of two different layers and the edges between them. The vertices in each layer represent a single set of nodes, although not all nodes have to be represented in every layer.

Formally, a multilayer graph can be defined as a triple $G = (V, E, L)$, where:

- V is the set of vertices;
- E is the set of edges, pairs of nodes (u, v) representing a connection, relationship or interaction between the nodes u and v;
- L is a set of layers, which are subsets of V and E encoding the nodes and edges within each layer.

Each layer ℓ in L is a tuple (V_ℓ, E_ℓ), where V_ℓ is a subset of V that represents the vertices within that layer, and E_ℓ is a subset of E that represents the edges within that layer.

MultilayerGraphs.jl is an integral part of the JuliaGraphs ecosystem extending Graphs.jl (Fairbanks et al., 2021) so all the methods and metrics exported by Graphs.jl work for multilayer graphs, but due to the special nature of multilayer graphs the package features a peculiar implementation that maps a standard integer-labelled vertex representation to a more user-friendly framework exporting all the objects an experienced practitioner would expect such as nodes (`Node`), vertices (`MultilayerVertex`), layers (`Layer`), interlayers (`Interlayer`), etc.

MultilayerGraphs.jl features multilayer-specific methods and metrics including the global clustering coefficient, the overlay clustering coefficient, the multilayer eigenvector centrality, the multilayer modularity and the Von Neumann entropy.

Finally, MultilayerGraphs.jl has been integrated within the JuliaDynamics ecosystem so that any `Multilayer(Di)Graph` can be utilised as an argument to the `GraphSpace` constructor in Agents.jl (Datseris et al., 2022).

Statement of Need

Several theoretical frameworks have been proposed to formally subsume all instances of multilayer graphs (Aleta & Moreno, 2019; Artime et al., 2022; Bianconi, 2018; Boccaletti et al., 2014; Cozzo et al., 2018; M. D. Domenico et al., 2013; M. D. Domenico, 2022; Kivela et al., 2014; Lee et al., 2015).

Multilayer graphs have been adopted to model the structure and dynamics of a wide spectrum of high-dimensional, non-linear, multi-scale, time-dependent complex systems including physical, chemical, biological, neuronal, socio-technical, epidemiological, ecological and economic networks (Aleta et al., 2022, 2020; Amato et al., 2017; Arruda et al., 2017; Azimi-Tafreshi, 2016; Baggio et al., 2016; Buldú & Porter, 2018; Cozzo et al., 2013; M. D. Domenico, 2017; M. D. Domenico et al., 2016; Estrada & Gómez-Gardeñes, 2014; Gosak et al., 2018; Granell et al., 2013; Lim et al., 2019; Mangioni et al., 2020; Massaro & Bagnoli, 2014; Pilosof et al., 2017; Soriano-Paños et al., 2018; Timóteo et al., 2018).

We have chosen the Julia language for this software package because it is a modern, open-source, high-level, high-performance dynamic language for technical computing (Bezanson et al., 2017). At the best of our knowledge there are currently no software packages dedicated to the creation, manipulation and analysis of multilayer graphs implemented in the Julia language apart from MultilayerGraphs.jl itself (Moroni & Monticone, 2022).

Main Features

The two main data structures are `MultilayerGraph` and `MultilayerDiGraph`: collections of layers connected through interlayers.

The **vertices** of a multilayer graph are representations of one set of distinct objects called `Nodes`. Each layer may represent all the node set or just a subset of it. The vertices of `Multilayer(Di)Graph` are implemented via the `MultilayerVertex` custom type. Each `MultilayerVertex` encodes information about the node it represents, the layer it belongs to and its metadata.

Both the **intra-layer** and **inter-layer edges** are embedded in the `MultilayerEdge` struct, whose arguments are the two connected multilayer vertices, the edge weight and its metadata. It's important to highlight that `Multilayer(Di)Graph`s are weighted and able to store metadata by default (i.e. they have been assigned the `IsWeighted` and `IsMeta` traits from SimpleTraits.jl).

The **layers** are implemented via the `Layer` struct composed of an underlying graph and a mapping from its integer-labelled vertices to the collection of `MultilayerVertex`s the layer represents. **Interlayers** are similarly implemented via the `Interlayer` mutable struct, and they are generally constructed by providing the two connected layers, the (multilayer) edge list between them and a graph. This usage of underlying graphs allows for an easier debugging procedure during construction and a more intuitive analysis afterwards allowing the package to leverage all the features of the JuliaGraphs ecosystem so that it can be effectively considered as a real proving ground of its internal consistency.

The `Multilayer(Di)Graph` structs are weighted and endowed with the functionality to store both vertex-level and edge-level metadata by default so that at any moment the user may add or remove a `Layer` or specify an `Interlayer` and since different layers and interlayers could be better represented by graphs that are weighted or unweighted and with or without metadata, it was crucial for us to provide the most general and adaptable structure. A `Multilayer(Di)Graph` is instantiated by providing the ordered list of layers and the list of interlayers to the constructor. The latter are automatically specified, so there is no need to instantiate all of them.

Alternatively, it is possible to construct a `Multilayer(Di)Graph` making use of a graph generator-like signature allowing the user to set the degree distribution or the degree sequence and employs graph realisation methods such as the Havel-Hakimi algorithm for undirected graphs (Hakimi, 1962) and the Kleitman-Wang algorithm for directed ones (Kleitman & Wang, 1973).

`Multilayer(Di)Graphs` structure may be represented via dedicated `WeightTensor`, `MetadataTensor` and `SupraWeightMatrix` structs, all of which support indexing with `MultilayerVertexs`. Once a `Multilayer(Di)Graph` has been instantiated, its layers and interlayers can be accessed as their properties.

Installation and Usage

To install MultilayerGraphs.jl it is sufficient to activate the `pkg` mode by pressing `]` in the Julia REPL and then run the following command:

```
pkg> add MultilayerGraphs
```

In the following code chunks we synthetically illustrate how to define, handle and analyse a `MultilayerGraph` in order to showcase some of the main features outlined in the previous section.

First of all we need to import the necessary dependencies and set a few relevant constants:

```julia
using Revise
using StatsBase, Distributions
using Graphs, SimpleWeightedGraphs, MetaGraphs, SimpleValueGraphs
using MultilayerGraphs

const vertextype   = Int64
const _weighttype  = Float64
const min_vertices = 5
const max_vertices = 7
const min_edges    = 1
const max_edges    = max_vertices*(max_vertices-1)
const n_nodes      = max_vertices
```

[...]

Then we define a multilayer graph by specifying its layers and interlayers:

```julia
multilayergraph = MultilayerGraph(  layers,
                                    interlayers;
                                    default_interlayers_null_graph = SimpleGraph{vertext
                                    default_interlayers_structure = "multiplex"
)
```

[...]

For a more comprehensive exploration of the package features and functionalities we strongly recommend consulting the tutorial included in the package documentation.

Related Packages

R

Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the R language:

- `muxViz` implements functions to perform multilayer correlation analysis, multilayer centrality analysis, multilayer community structure detection, multilayer structural reducibility, multilayer motifs analysis and utilities to statically and dynamically visualise multilayer graphs (D. Domenico et al., 2014);
- `multinet` implements functions to import, export, create and manipulate multilayer graphs, several state-of-the-art multiplex graph analysis algorithms for centrality measures, layer comparison, community detection and visualization (Magnani et al., 2021);
- `mully` implements functions to import, export, create, manipulate and merge multilayer graphs and utilities to visualise multilayer graphs in 2D and 3D (Hammoud & Kramer, 2018);

- multinets implements functions to import, export, create, manipulate multilayer graphs and utilities to visualise multilayer graphs (Lazega et al., 2008).

Python

Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the Python language:

- MultiNetX implements methods to create undirected networks with weighted or unweighted links, to analyse the spectral properties of adjacency or Laplacian matrices and to visualise multilayer graphs and dynamical processes by coloring the nodes and links accordingly;
- PyMNet implements data structures for multilayer graphs and multiplex graphs, methods to import, export, create, manipulate multilayer graphs and for the rule-based generation and lazy-evaluation of coupling edges and utilities to visualise multilayer graphs (Kivela et al., 2014).

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