Final Report: Portfolio Optimization Strategies Analysis

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May 16, 2025

1 Introduction

This report evaluates two factor-based long-short portfolio optimization strategies, utilizing the Fama-French 3-factor model (Momentum, Value, Size), over the period from March 2007 to December 2024. The strategies are analyzed across estimation windows of 40, 90, and 180 days and risk parameters $\lambda = 0.1, 0.5, 1.0$, with a focus on performance during the subprime crisis (2008), COVID-19 period (mid-2020 to mid-2021), and non-crisis periods. We compare the strategies against the S&P 500 (SPY ETF) benchmark, assessing sensitivity to estimator look-back periods and market conditions.

2 Methodology

2.1 Notations and Models

The portfolio consists of n assets with weights $\omega \in \mathbb{R}^n$, expected returns ρ, and covariance matrix Σ derived from the Fama-French 3-factor model. The model for asset i's return is:

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{SMB} + b_i^v r_{HML} + \alpha_i + \varepsilon_i,$$

where r_f is the risk-free rate, r_M is the market return, r_{SMB} and r_{HML} are Size and Value factors, and ε_i is the idiosyncratic error with $\mathbb{E}(\varepsilon_i) = 0$. The expected return is:

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{SMB} + b_i^v \rho_{HML} + \alpha_i.$$

The portfolio's beta is $\beta_p^m = \sum_{i=1}^n \beta_i^m \omega_i$, where $\beta_i^m = \frac{\text{cov}(r_i, r_M)}{\sigma^2(r_M)}$. The covariance matrix is:

$$\text{cov}(R_t) = \mathbf{B}\Omega_f\mathbf{B}' + D,$$

where $\mathbf{B}$ is the matrix of factor loadings, Ω_f is the factor covariance matrix, and $D = \text{diag}(\sigma_1^2, \ldots, \sigma_n^2)$ is the idiosyncratic variance matrix.

2.2 Investment Strategies

Our report implements the following strategies:

$$\textbf{(Strategy I)} \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \rho^T\omega - \lambda\sqrt{\omega^T\Sigma\omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \ -2 \leq \omega_i \leq 2, \end{cases}$$

and

$$
\textbf{(Strategy II)} \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega - r_{spy}}{TEV(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \ -2 \leq \omega_i \leq 2, \end{cases}
$$

where Σ is the covariance matrix between the securities returns, the Beta of security S_i, denoted by

$$
\beta_i^m = \frac{\text{cov}(r_i, r_m)}{\sigma^2(r_m)}
$$

is the as defined in the CAPM Modol so that $\beta_P^m = \sigma_{i=1}^n \beta_i^m \omega_i$ is the Beta of the portfolio. Finally, we have the Track Error Volatility (TEV), which is given by

$$
\text{TEV}(\omega) = \sigma(r_p(\omega) - r_{\text{SPY}})
$$

Strategy I maximizes the expected return minus a risk penalty based on the portfolio volatility. We enforce a tight beta constraint (-0.5 to 0.5), which keeps the portfolio mostly uncorrelated with the market. This strategy would benefit risk-averse investors, such as pension funds, insurance companies, and investors seeking capital preservation or market-neutral strategies.

On the other hand, Strategy II optimizes the Information Ratio (excess return over SPY per unit of the TEV), adjusted by a volatility penalty. The strategy allows a wide range of beta exposure (-2 to 2), allowing for leveraged bets on or against the market. Investors who would prefer this strategy are risk-tolerant or benchmark-aware investors, such as Hedge funds, Active portfolio managers, or investors seeking alpha generation relative to a market index. Strategy II is also useful in trending or recovering markets, where deviations from the benchmark can be profitable.

2.3 Investment Universe

The portfolio comprises 12 ETFs: FXE, EWJ, GLD, QQQ, SPY, SHV, DBA, USO, XBI, ILF, EPP, FEZ, sourced from Yahoo Finance. The S&P 500 (SPY) serves as the benchmark.

2.4 Analysis Setup

The analysis spans from March 2007 to December 2024 and is divided into five sub-periods to capture different market regimes: (1) the pre-subprime crisis period, (2) the subprime crisis of 2008, (3) the post-subprime recovery, (4) the COVID-19 crisis (mid-2020 to mid-2021), and (5) the post-COVID expansion. This segmentation enables us to evaluate the resilience and adaptability of the portfolio strategies under varying market stress conditions.

To ensure robustness, all portfolio allocations are conducted under a non-anticipative backtesting framework. Each weekly rebalancing relies solely on historical data available up to the rebalancing date, mimicking real-world trading constraints. Portfolios are re-optimized weekly using rolling estimation windows for expected returns and covariance matrices.

We evaluate three look-back configurations to relect the different time horizons:

- Short-Term (ST): 40-day window
- Mid-Term (MT): 90-day window
- Long-Term (LT): 180-day window

Each configuration is tested under three levels of risk aversion captured by $\lambda \in \{0.1, 0.5, 1.0\}$. For notation, we define S_C^R as the strategy using C days for covariance estimation and R days for expected return estimation. For example,S_{40}^{90} refers to a strategy with a 40-day look-back for risk and a 90-day look-back for return.

3 Results

3.1 Performance Metrics

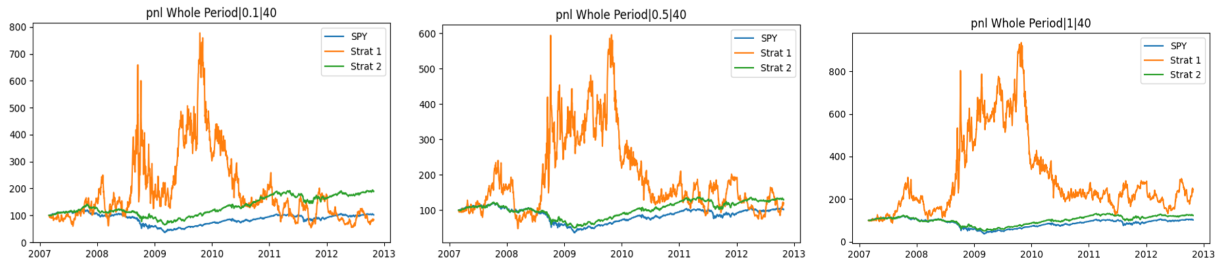
Table 1 summarizes key metrics for both strategies and SPY over the entire period, focusing on the 180-day estimation window for brevity (full results in Appendix A).

Table 1: Performance Metrics (180-day Estimation, Whole Period)

Strategy	λ	Cum. Return	Ann. Vol.	Sharpe Ratio	Max Drawdown	VaR (95%)	CVaR (95%)
Strategy I	0.1	-0.945	1.089	-0.009	-0.999	-0.106	-0.154
Strategy I	0.5	0.432	0.709	0.002	-0.970	-0.067	-0.105
Strategy I	1.0	8.732	0.510	0.016	-0.856	-0.050	-0.076
Strategy II	0.1	8.081	0.171	0.046	-0.294	-0.017	-0.026
Strategy II	0.5	6.430	0.159	0.045	-0.274	-0.016	-0.024
Strategy II	1.0	5.997	0.156	0.044	-0.268	-0.016	-0.024
SPY	-	4.963	0.198	0.032	-0.552	-0.019	-0.031

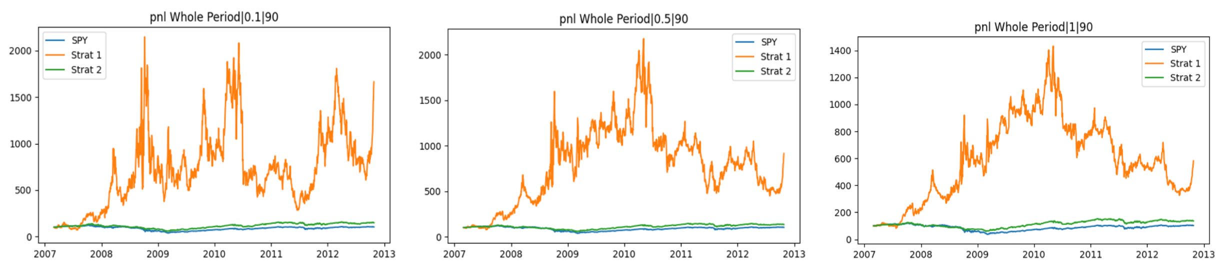
3.2 Cumulative Returns

Figures 1, 2, and 3 show cumulative returns for Strategy II across $\lambda = 0.1, 0.5, 1.0$ for 40, 90, and 180-day estimation windows, respectively, compared to SPY, assuming a $100 initial investment.



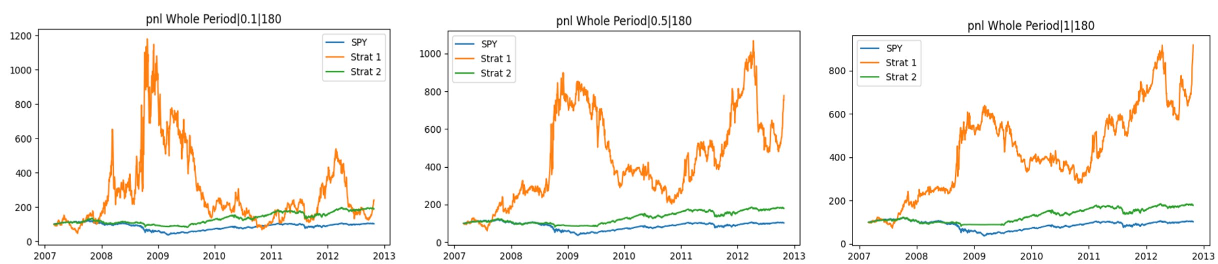
(a) $\lambda = 0.1$ (b) $\lambda = 0.5$ (c) $\lambda = 1.0$

Figure 1: Cumulative Returns (Strategy II, 40-day Estimation vs. SPY)



(a) $\lambda = 0.1$ (b) $\lambda = 0.5$ (c) $\lambda = 1.0$

Figure 2: Cumulative Returns (Strategy II, 90-day Estimation vs. SPY)

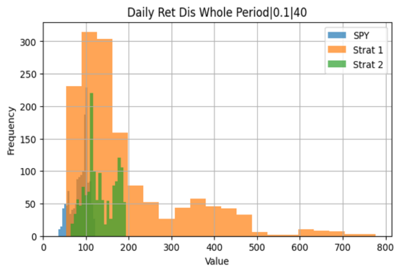


(a) $\lambda = 0.1$ (b) $\lambda = 0.5$ (c) $\lambda = 1.0$

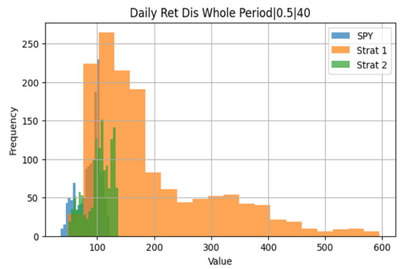
Figure 3: Cumulative Returns (Strategy II, 180-day Estimation vs. SPY)

3.3 Return Distributions

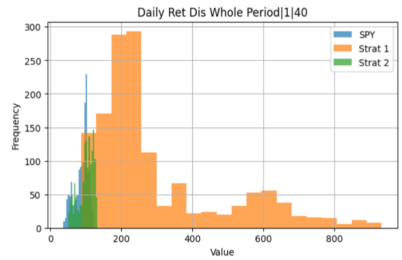
Figures 4, 5, and 6 illustrate daily return distributions for Strategy II across $\lambda = 0.1, 0.5, 1.0$ for 40, 90, and 180-day estimation windows, respectively, highlighting negative skewness and high kurtosis.



(a) $\lambda = 0.1$

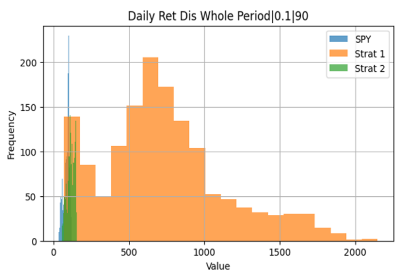


(b) $\lambda = 0.5$

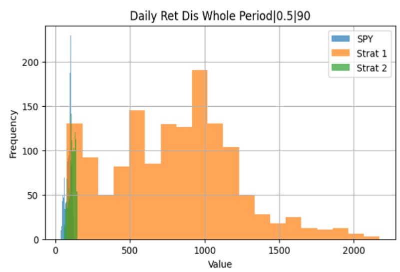


(c) $\lambda = 1.0$

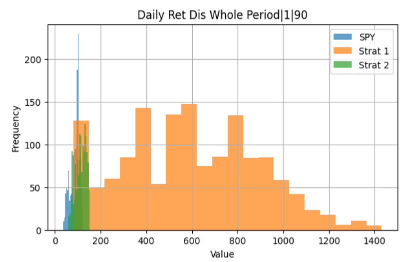
Figure 4: Daily Return Distributions (Strategy II, 40-day Estimation)



(a) $\lambda = 0.1$

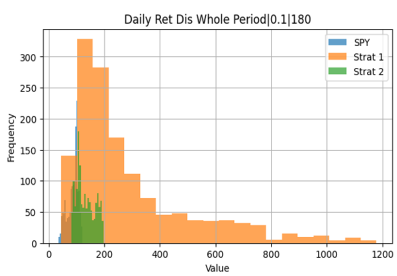


(b) $\lambda = 0.5$

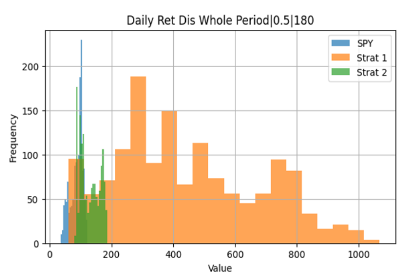


(c) $\lambda = 1.0$

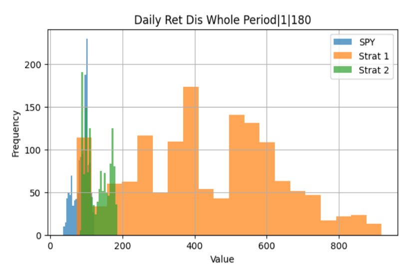
Figure 5: Daily Return Distributions (Strategy II, 90-day Estimation)



(a) $\lambda = 0.1$



(b) $\lambda = 0.5$



(c) $\lambda = 1.0$

Figure 6: Daily Return Distributions (Strategy II, 180-day Estimation)

4 Discussion

4.1 Estimator Sensitivity

Strategy II outperforms Strategy I and SPY across all estimation windows and λ values, with the 180-day window and $\lambda = 0.1$ yielding the highest Sharpe ratio (0.046) and cumulative return (8.081). Strategy I shows high volatility (e.g., 1.089 for 180-day, $\lambda = 0.1$) and negative returns in some configurations, likely due to its tight beta constraint ($[-0.5, 0.5]$). The 40-day estimator increases volatility and drawdowns (e.g., -0.467 for Strategy II, $\lambda = 0.1$), particularly during volatile periods, while the 180-day estimator reduces drawdowns (e.g., -0.294) and stabilizes performance, as seen in Figures 1–3.

4.2 Market Conditions

During the subprime crisis, Strategy II with 180-day estimation maintained lower volatility and drawdowns compared to 40-day estimation, which was sensitive to market shocks (see Figure 1). In the COVID period, Strategy II's wider beta range ($[-2, 2]$) capitalized on market rebounds, outperforming SPY, particularly with $\lambda = 0.1$ (Figure 3). Non-crisis periods favored $\lambda = 0.5$, balancing risk and return, as evident in Figure 2. Return distributions (Figures 4–6) show consistent negative skewness and high kurtosis, indicating fat-tailed risks across all configurations.

5 Recommendations

1. **Primary Configuration**: Use Strategy II with a 180-day estimation window and $\lambda = 0.1$ for maximum returns or $\lambda = 0.5$ for balanced risk-return profiles.

6 Conclusion

Strategy II with a 180-day estimation window offers superior performance, balancing responsiveness and stability. The choice of λ allows tailoring to investor risk preferences, with $\lambda = 0.1$ maximizing returns and $\lambda = 0.5$ optimizing risk-adjusted returns, as supported by the comprehensive analysis in Figures 1–6.

A Full Performance Metrics

Table 2: Complete Performance Metrics (Whole Period)

Estimation	λ	Strategy	Cum. Return	Ann. Vol.	Sharpe	Max DD	Skewness	Kurtosis	VaR (95%)	CVaR (95%)
S40/60	0.1	I	-0.901	1.160	-0.007	-0.998	0.254	3.022	-0.111	-0.157
S40/60	0.1	II	9.393	0.184	0.045	-0.467	-0.496	8.048	-0.018	-0.028
S90/60	0.1	I	61.120	1.132	0.013	-0.962	0.187	3.260	-0.111	-0.157
S90/60	0.1	II	7.081	0.178	0.042	-0.497	-0.422	8.211	-0.018	-0.027
S180/60	0.1	I	-0.945	1.089	-0.009	-0.999	0.061	3.859	-0.106	-0.154
S180/60	0.1	II	8.081	0.171	0.046	-0.294	-0.249	7.655	-0.017	-0.026
S40/60	0.5	I	1.559	0.979	0.003	-0.986	0.292	4.101	-0.094	-0.133
S40/60	0.5	II	5.213	0.170	0.038	-0.508	-0.523	8.891	-0.017	-0.026
S90/60	0.5	I	88.703	0.875	0.018	-0.908	0.139	5.533	-0.080	-0.125
S90/60	0.5	II	5.037	0.164	0.039	-0.479	-0.427	9.863	-0.017	-0.025
S180/60	0.5	I	0.432	0.709	0.002	-0.970	-0.206	5.516	-0.067	-0.105
S180/60	0.5	II	6.430	0.159	0.045	-0.274	-0.239	9.341	-0.016	-0.024
S40/60	1.0	I	0.291	0.830	0.001	-0.967	0.288	6.358	-0.078	-0.116
S40/60	1.0	II	4.642	0.168	0.036	-0.510	-0.541	9.019	-0.017	-0.026
S90/60	1.0	I	66.597	0.671	0.022	-0.790	0.213	9.552	-0.062	-0.096
S90/60	1.0	II	4.760	0.161	0.038	-0.449	-0.432	10.250	-0.016	-0.025
S180/60	1.0	I	8.732	0.510	0.016	-0.856	-0.219	5.352	-0.050	-0.076
S180/60	1.0	II	5.997	0.156	0.044	-0.268	-0.225	9.328	-0.016	-0.024
S180/60	-	SPY	4.963	0.198	0.032	-0.552	-0.074	14.037	-0.019	-0.031