Indian Institute of Technology Gandhinagar

Mechanical Engineering Lab-I
ME 351

Experiment Report

Experiment Title: **The Bernoulli's Apparatus**

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Objective Of The Experiment

- To show the pressure variation along a converging-diverging pipe section.
- The goal is to experimentally validate Bernoulli's assumptions and a theorem when it is applied to a steady flow of water through a tapered duct and observe potential discrepancies between theoretical analysis and observed data.

Essential Background

Bernoulli's equation forms the basis for solving a wide variety of fluid flow problems such as jets issuing from an orifice, jet trajectory, flow under a gate and over a weir, flow metering by obstruction meters, flow around submerged objects, flows associated with pumps and turbines, etc.

Bernoulli equation is a linear momentum-based relation between pressure, velocity, and elevation in a frictionless flow. It states that when incompressible fluid flows in the duct, the fluid's static pressure will decrease with the increase in flow velocity. Inversely, with the decrease of flow velocity, the static pressure of the fluid will increase. To use the Bernoulli equation correctly, one must confine it to regions of the flow that are nearly frictionless.

It was derived using the following assumptions and these assumptions are based on the laws of conservation of mass and energy:

(a) Steady flow ($\partial V/\partial t = 0$)
(b) Incompressible flow (ρ = constant)
(c) Frictionless flow
(d) Flow along a single streamline: Different streamlines may have unique "Bernoulli constants." In most cases, a frictionless flow region is irrotational ; that is, curl(V) = 0. For irrotational flow, the Bernoulli constant is the equivalent all over.

$$\int_1^2 \frac{\partial V}{\partial t} ds + \int_1^2 \frac{dp}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0 \tag{1}$$

Eq. (1) is Bernoulli's equation for unsteady frictionless flow along a streamline. It is in differential form and can be integrated between any two points.

The pressure, speed and height at two points in a steady flowing, non-viscous, incompressible fluid are related by the equation:

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = constant \tag{2}$$

The changes from 1 to 2 in Eq. (2) represent reversible pressure work, kinetic energy change, and potential energy change. The fact that the total remains the same means that there is no energy exchange due to viscous dissipation, heat transfer, or shaft work.

In many incompressible-flow Bernoulli analyses, elevation changes are negligible. Thus, Eq. (2) reduces the balance between pressure and kinetic energy. We can write this as:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 = \text{ constant} \tag{3}$$

(2) Several modifications to Bernoulli's equation have been proposed to circumvent the assumptions posed. Report them.

All functional fluids are viscous and offer resistance to fluid flow. So that there are some losses in fluid flow between the two sections. Bernoulli's equation was derived from the assumption that fluid is non-viscous i.,e. frictionless, which is not applicable to practical fluid; hence, Bernoulli's equation is modified by considering losses. The Eqs. (1), (2) and (3) does not account for possible energy exchange due to heat or work.

(a) Modified Bernoulli's equation for real fluid:

$$\frac{\mathrm{P_1}}{\gamma} + \frac{\mathrm{V_1}^2}{2g} + \mathrm{z_1} = \frac{\mathrm{P_2}}{\gamma} + \frac{\mathrm{V_2}^2}{2g} + \mathrm{z_2} + \mathrm{h_L} \tag{4}$$

Where, h_L = loss of energy between point 1 and 2

(b) Modified Bernoulli's equation for incompressible flow with one inlet and one outlet:

$$\frac{P_{in}}{\boldsymbol{\gamma}} + \frac{V_{in}^2}{2g} + z_{in} = \frac{P_{out}}{\boldsymbol{\gamma}} + \frac{V_{out}^2}{2g} + z_{out} + h_{friction} - h_{pump} + h_{turbine} \tag{5}$$

(c) Modified Bernoulli's equation when kinetic energy correction factor 'α' considered:

$$\frac{P_{in}}{\boldsymbol{\gamma}} + \alpha\, \frac{V_{in}^2}{2g} + z_{in} = \frac{P_{out}}{\boldsymbol{\gamma}} + \alpha\, \frac{V_{out}^2}{2g} + z_{out} + h_{friction} - h_{pump} + h_{turbine} \tag{6}$$

(3) What is head loss in a pipe flow? Why is it important? Can you quantify head loss?

Head loss refers to a measurement of the energy scattered in a fluid system because of friction along the length of a pipe or hydraulic system and those due to fittings, valves, and other system structures. The total head loss of a fluid as it moves through a fluid system is the sum of the elevation head, velocity head, and pressure head losses.

There are two types of head loss:

1. Major head loss, which is due to friction in pipes and ducts.
2. Minor head loss, which is due to components such as valves, fittings, bends, elbow and tees.

The head loss is given in terms of wall shear stress:

$$h_f = (z_1 - z_2) + \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) = \Delta z + \frac{\Delta p}{\rho g} = \frac{2\tau_w}{\rho g}\frac{L}{R} = \frac{4\tau_w}{\rho g}\frac{L}{d} \tag{7}$$

Head loss can be quantified as:

$$h_f = f\,\frac{L}{d}\,\frac{V^2}{2g} \text{ where } f = \text{fcn}\left(\text{Re}_d, \frac{\varepsilon}{d}, \text{duct shape}\right) \tag{8}$$

where dimensionless parameter f is called the Darcy friction factor. The quantity ε is the wall roughness height, which is important in turbulent (but not laminar) pipe flow.

For a round pipe of diameter d with fully developed laminar flow, head loss is:

$$h_f = \frac{32\mu LV}{\rho g d^2} = \frac{128\mu LQ}{\pi\rho g d^4} \tag{9}$$

(4) What is friction factor and what does it depend on?

The friction factor represents the loss of pressure of a fluid in a pipe due to the interactions in between the fluid and the pipe.

- The friction factor for fluids in laminar flow (Re_d < 2300) depends only on Reynolds number. It is calculated as $f = \frac{64}{\text{Re}_d}$.
- The friction factor for fluids in turbulent flow (Re_d > 4000) not only depends on the Reynolds number but also the relative roughness of the pipe and duct shape. The relation between them is $f = \frac{0.316}{Re_d^{\frac{1}{4}}}$
- Between Re 2300 to 4000 the relationship between f and Re reverses and this flow is called as Transitional Flow (Critical Flow). Here the flow is neither wholly laminar nor wholly turbulent. It is a combination of the two flow condition.
- When flow rate increases the friction factor decreases irrespective of flowing fluid.

(5) How do you evaluate the friction factor for a given flow?

In laminar flow, the pipe friction factor decreases inversely with Reynolds number. These formulas are valid whenever the pipe Reynolds number, $\text{Re}_d = \frac{\rho V d}{\mu}$ is less than about 2300.

$$f_{\text{lam}} = \frac{8T_{w,\text{lam}}}{\rho V^2} = \frac{8\left(\frac{8\mu V}{d}\right)}{\rho V^2} = \frac{64}{\frac{\rho V d}{\mu}} = \frac{64}{\text{Re}_d}$$

(10)

where, w = shear stress on the wall
V = velocity
d = diameter of pipe
μ = viscosity of fluid

In turbulent pipe flow, the pipe friction factor depends on the Reynolds number, the relative roughness of the pipe and duct shape

$$\frac{1}{f^{1/2}} = -2 \log \left(\frac{\epsilon l d}{3.7} + \frac{2.51}{\text{Re}_d f^{1/2}} \right)$$

(11)

Apparatus

Figure 1 : Bernoulli's Apparatus

Figure 2 : Piezometer to calculate the head

The experimental setup consists of a convergent divergent pipe with pressure recordings at different cross sections (Figure 2). The water is supplied to an above tank by a centrifugal pump. The flow rates in the pipe can be adjusted with the control valve and bypass valve. The cross sectional area of the divergent pipe is as follows:

Sr. no	Cross-Sectional Area $A * 10^{-4}$ (m^2)	Distance from the Reference point S (m)
1	6.1575	0.030
2	4.9088	0.059

3	3.4636	0.088
4	2.4053	0.117
5	1.5000	0.146
6	2.4053	0.175
7	3.4636	0.204
8	4.9088	0.233
9	6.1575	0.262

Table 1 : Parameters

Procedure

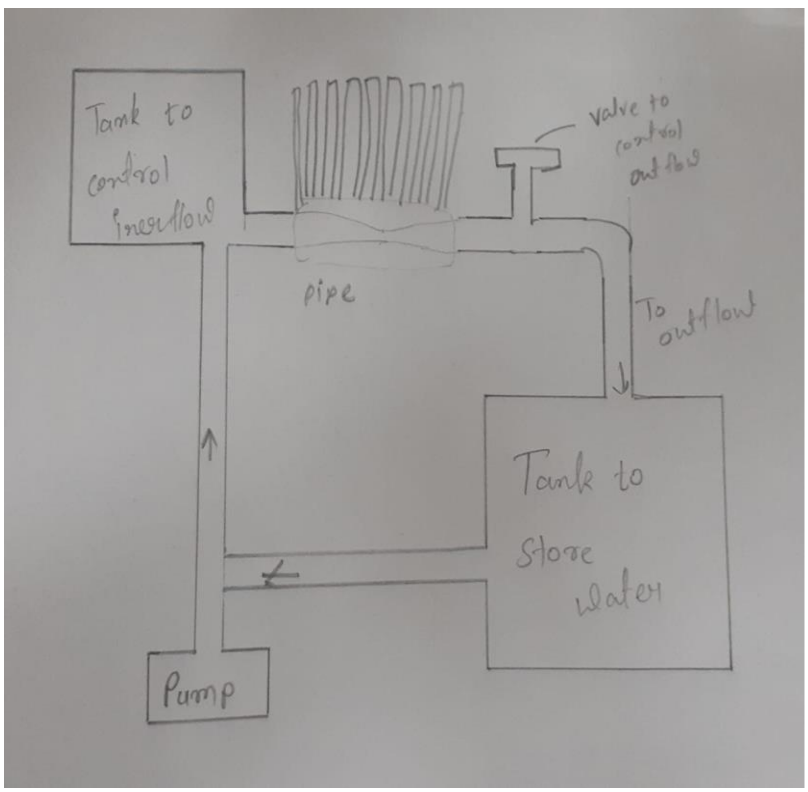

Figure 3 : Schematic diagram of the apparatus

Procedure followed in performing the experiment:

(1) All piezometer tubes are checked and appropriately associated with the corresponding pressure taps are air-bubble free.

(2) The discharged valve is adjusted to high measurable flow rate.
(3) Regulate flow of water through pipe with the help of given flow control valve.
(4) Make sure that there are no water bubbles in the tube for measuring pressure heads.
(5) After the level is stabilized, the water flow rate id measured using the volumetric method.
(6) Measure the flow rate using measuring tank and calculate the time.
(7) Calculate the velocity at each section using continuity equation.
(8) Repeat the with different flow rates.

Observation and Theoretical Analysis

(1) Observed result

(a) Measuring volumetric flow rate in the converging diverging pipe. Observed result are mentioned in Table (2).

Cross sectional area of container = 40 X 25 cm^2
Height of water level in container = 8 cm

Sr. No.	Height of water (m)	Time Taken (sec)	Volumetric flow rate Q $* 10^{-6}$ (m^3/s)
1	0.01	28.38	35.23608175
2	0.01	40.31	24.80774001
3	0.01	26.78	37.34129948
4	0.01	23.45	42.64392324
5	0.01	17.23	58.03830528
6	0.01	14.01	71.37758744

Table 2 : Measured volumetric Flows

We have calculated six volumetric flows, but we take only the first 3 to get a closer agreement between the theoretical prediction and the observations.

(b) Measuring velocity at different cross-sectional area of the convergent divergent pipe using volumetric flow rate. Observed results are mentioned below

Sr. no	Cross Sectional Area A * 10^{-4} (m^2)	Distance from a reference point (m)	Velocity (m/s)
1	6.1575	0.003	0.0572246557
2	4.9088	0.059	0.07178145728
3	3.4636	0.088	0.1017325377
4	2.4053	0.117	0.1464935008
5	1.5000	0.146	0.2349072117
6	2.4053	0.175	0.1464935008
7	3.4636	0.204	0.1017325377
8	4.9088	0.233	0.07178145728
9	6.1575	0.262	0.0572246557

Table 3 : Velocity Profile for Volumetric Flow rate = 35.23608175 X 10^{-6} m^3 /s

Sr. no	Cross Sectional Area A * 10^{-4} (m^2)	Distance from a reference point (m)	Velocity (m/s)
1	6.1575	0.030	0.04028865613
2	4.9088	0.059	0.05053728002
3	3.4636	0.088	0.07162414833
4	2.4053	0.117	0.10313782070
5	1.5000	0.146	0.16538493340

6	2.4053	0.175	0.10313782070
7	3.4636	0.204	0.07162414833
8	4.9088	0.233	0.05053728002
9	6.1575	0.262	0.04028865613

Table 4 : Velocity Profile for Volumetric Flow rate = 24.80774001 X $10^{-6}\ m^3$ /s

Sr. no	Cross Sectional Area $A * 10^{-4}$ (m^2)	Distance from a reference point (m)	Velocity (m/s)
1	6.1575	0.030	0.06064360451
2	4.9088	0.059	0.07607011791
3	3.4636	0.088	0.1078106579
4	2.4053	0.117	0.1552459131
5	1.5000	0.146	0.2489419965
6	2.4053	0.175	0.1552459131
7	3.4636	0.204	0.1078106579
8	4.9088	0.233	0.07607011791
9	6.1575	0.262	0.06064360451

Table 5 : Velocity Profile for Volumetric Flow rate = 37.34129948 X $10^{-6}\ m^3$ /s

(c) Calculating Bernoulli's constant using pressure and velocity head. Observed results are mentioned in Table (6), (7) and (8).

Pressure head, P = ρ x H x g

Where,

ρ = density of water
H = height of water
g = 9.8 m/s^2

$$\text{Velocity Head} = \frac{1}{2} * \rho * V^2$$

V = velocity of water at different cross section

Bernoulli's constant = Pressure head + Velocity head

We have not added the atmospheric pressure in pressure head as it will not affect the Bernoulli's constant (Pressure head + Velocity head). Also, height is same throughout the pipe. So potential energy term will cancel out.

Bernoulli equation for pipe:

$$P_1 + \frac{1}{2}\,\rho V_1^2 + \rho h_1 g = P_2 + \frac{1}{2}\rho V_2^2 + \rho h_2 g \qquad (13)$$

Here, $h_1 = h_2$ (pipe is horizontal) So, equation reduces to:

$$\mathrm{P}_1 + \frac{1}{2}\,\rho V_1^2 = \mathrm{P}_2 + \frac{1}{2}\rho V_2^2 = \text{constant} \qquad (14)$$

Sr. no	Cross Sectional Area $A * 10^{-4}$ (m^2)	Velocity (m/s)	Height of Water (cm)	Pressure head (m)	Velocity head (m)	Bernoulli's Constant
1	6.1575	0.0572246557	20.7	0.207	0.0001669042416	0.2071669042
2	4.9088	0.07178145728	20.6	0.206	0.0002626186345	0.2062626186
3	3.4636	0.1017325377	20.5	0.205	0.0005274979216	0.2055274979
4	2.4053	0.1464935008	20.3	0.203	0.001093799479	0.2040937995
5	1.5000	0.2349072117	19.5	0.195	0.002812507548	0.1978125075
6	2.4053	0.1464935008	19.4	0.194	0.001093799479	0.1950937995
7	3.4636	0.1017325377	19.6	0.196	0.0005274979216	0.1965274979
8	4.9088	0.07178145728	19.8	0.198	0.0002626186345	0.1982626186
9	6.1575	0.0572246557	20.0	0.200	0.0001669042416	0.2001669042

Table 6 : Bernoulli's constant for Volumetric Flow rate = 35.23608175 X 10^{-6} m^3 /s

Sr. no	Cross Sectional Area A * 10^{-4} (m^2)	Velocity (m/s)	Height of Water (cm)	Pressure head (m)	Velocity head (m)	Bernoulli's Constant
1	6.1575	0.04028865613	17.5	0.175	0.00008273067345	0.1750827307
2	4.9088	0.05053728002	17.3	0.173	0.0001301741423	0.1731301741
3	3.4636	0.07162414833	17.3	0.173	0.0002614688391	0.1732614688
4	2.4053	0.1031378207	17.1	0.171	0.0005421717666	0.1715421718
5	1.5000	0.1653849334	16.7	0.167	0.0013940966470	0.1683940966
6	2.4053	0.1031378207	16.4	0.164	0.0005421717666	0.1645421718
7	3.4636	0.07162414833	16.7	0.167	0.0002614688391	0.1672614688
8	4.9088	0.05053728002	16.8	0.168	0.0001301741423	0.1681301741
9	6.1575	0.04028865613	16.9	0.169	0.00008273067345	0.1690827307

Table 7 : Bernoulli’s constant for Volumetric Flow rate = 24.80774001X 10^{-6} m^3 /s

Sr. no	Cross Sectional Area A * 10^{-4} (m^2)	Velocity (m/s)	Height of Water (cm)	Pressure head (m)	Velocity head (m)	Bernoulli's Constant
1	6.1575	0.06064360451	18.4	0.184	0.00018744377	0.1841874438
2	4.9088	0.07607011791	18.2	0.182	0.0002949369438	0.1822949369
3	3.4636	0.1078106579	18.2	0.182	0.0005924127401	0.1825924127
4	2.4053	0.1552459131	18.2	0.182	0.001228404359	0.1832284044
5	1.5000	0.2489419965	17.6	0.176	0.003158619655	0.1791586197
6	2.4053	0.1552459131	17.6	0.176	0.001228404359	0.1772284044
7	3.4636	0.1078106579	17.9	0.179	0.0005924127401	0.1795924127
8	4.9088	0.07607011791	18.0	0.180	0.0002949369438	0.1802949369
9	6.1575	0.06064360451	18.1	0.181	0.000187443770	0.1811874438

Table 8 : Bernoulli’s constant for Volumetric Flow rate = 37.34129948 X 10^{-6} m^3 /s

(2) Factors causing a mismatch between theory and experimental data

As we can observe from table (6), (7) and (8) that Bernoulli's equation is not satisfied. There are some uncertainties in Bernoulli's constant. These uncertainties can arise due to a lot of reason. Some of them are listed below:

(a) We have not considered the head loss due to friction in the pipe.
(b) Inflow from the Pump id not study.
(c) There are leaks in the constant head inlet tank.
(d) Errors in measuring the height of water level,i.e, errors in calculating pressure head.
(e) Inflow and outflow rate of water from converging / diverging pipe was not exactly equal which makes the water level in pipes (for measuring pressure) unstable.
(f) Errors in calculating volumetric flow rate.

(3) Try to quantify as many of these "factors" as possible

We can add up the factors such as friction loss to improve the results, converge the errors, and use the modified Bernoulli's Equation to acquire the best results. We can certainly also look at the heat loss from the set-up and try to put it up with the acquired answers.

Conclusion

As Bernoulli states, high velocity of fluid flow results in low pressure, and based on the continuity equation, smaller areas result in high velocity. Thus, the result proved that both equations could be utilized to decide fluid flow velocity in a piezometer. Despite the value not being the same, the pattern of increasing and decreasing at the converging and diverging portions is the same. So that, as the velocity increases, the total head pressure increases for both convergent and divergent flow.