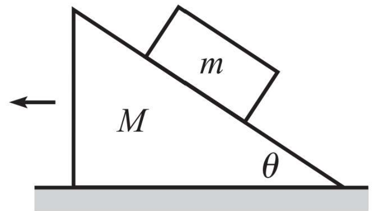
Developing the Equations of Motion

We first find the energy equations.



We define:

→ Position of block: x_{block} where → is +

→ " " plane: x_{plane} " " " "

$$U = mg\overbrace{\underbrace{(x_{block} - x_{plane})}_{\text{accounts for plane sliding}}\tan\theta}^{\text{Height of block}}$$

$$T = \frac{1}{2}M\dot{x}_{plane}^2 + \frac{1}{2}m\overbrace{\Big(\underbrace{\dot{x}_{block}^2}_{\text{Horizontal}} + \underbrace{[(\dot{x}_{block} - \dot{x}_{plane})\tan\theta]^2}_{\text{Vertical}}\Big)}^{\text{Break velocities into components}}$$

$$\Rightarrow \mathcal{L} = T - U = \frac{1}{2}M\dot{x}_{plane}^2 + \frac{1}{2}m\left(\dot{x}_{block}^2 + (\dot{x}_{block} - \dot{x}_{plane})^2\tan^2\theta\right) - mg(x_{block} - x_{plane})\tan\theta$$

Block EoM:

$$\frac{d\mathcal{L}}{dx_{block}} - \frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}_{block}}\right) = 0$$

$$\Rightarrow -mg\tan\theta - \frac{d}{dt}\left(m\dot{x}_{block} + 2(\dot{x}_{block} - \dot{x}_{plane})\tan^2\theta\right) = 0$$

$$\Rightarrow -mg\tan\theta = m\ddot{x}_{block} + 2(\ddot{x}_{block} - \ddot{x}_{plane})\tan^2\theta$$

Plane EoM

$$\frac{d\mathcal{L}}{dx_{plane}} - \frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}_{plane}}\right) = 0$$

$$\Rightarrow mg\tan\theta - \frac{d}{dt}\left(M\dot{x}_{plane} - 2(\dot{x}_{block} - \dot{x}_{plane})\tan^2\theta\right) = 0$$

$$\Rightarrow mg\tan\theta = M\ddot{x}_{plane} - 2(\ddot{x}_{block} - \ddot{x}_{plane})\tan^2\theta$$