Logical Reasoning

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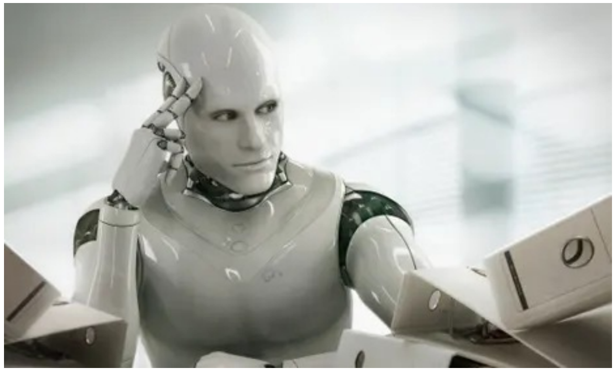
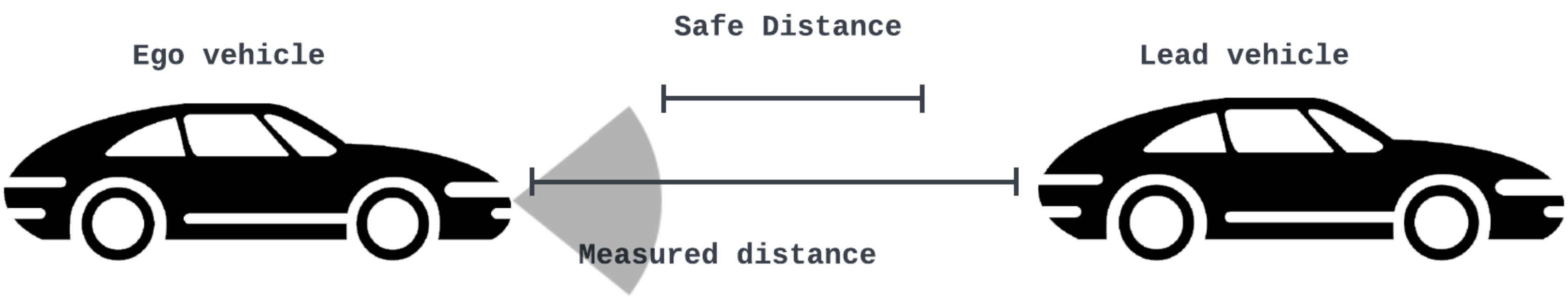
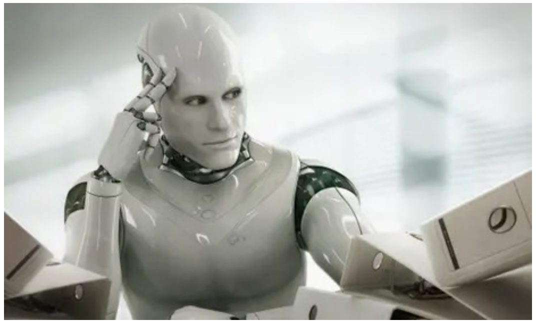
Scenario: Adaptive Cruise Control



Is it having an accident?



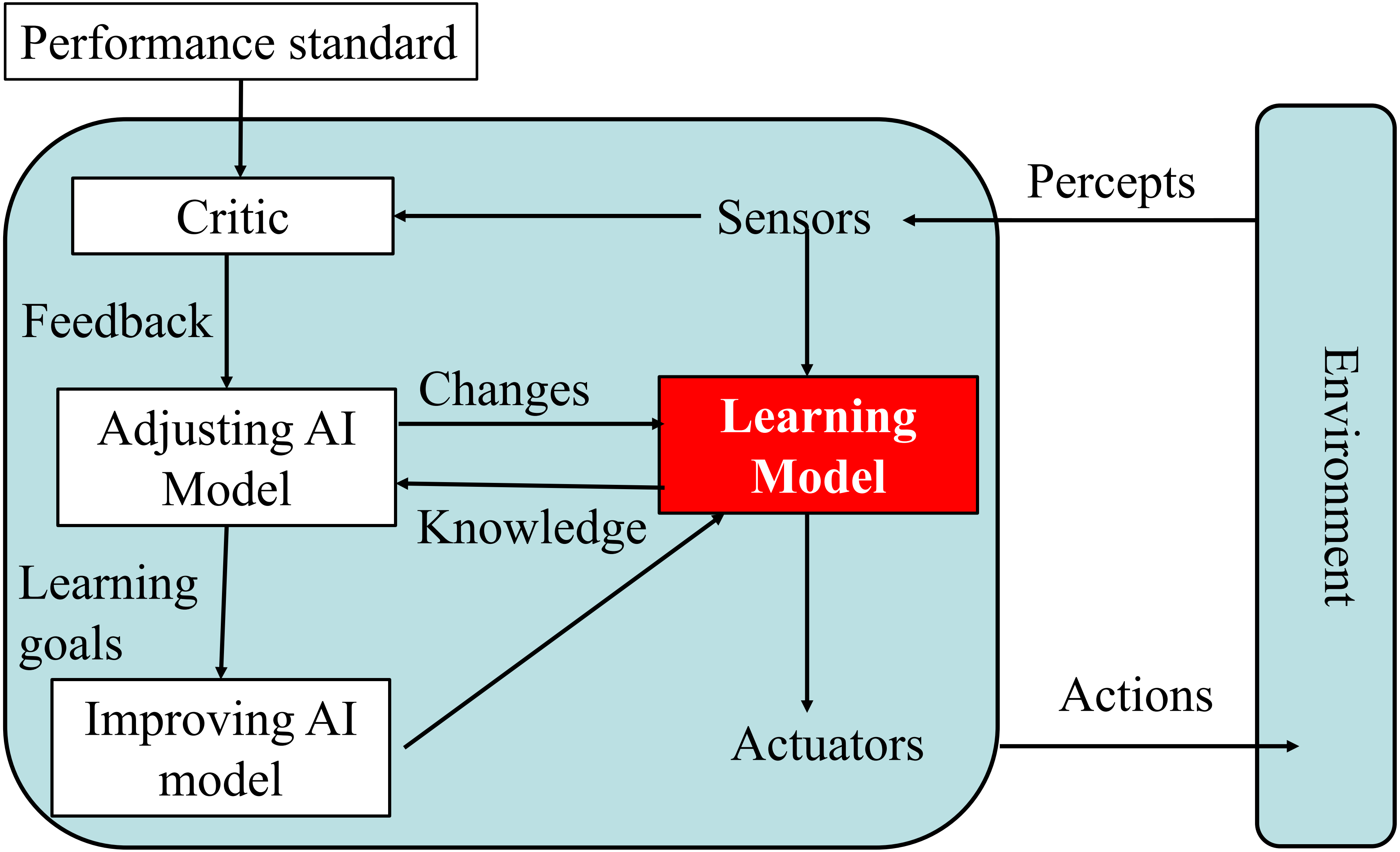


Is it having an accident?

- Humans know things and that helps them to do things

- Agent use a process of reasoning over internal representation of knowledge to decide on actions to take

Learning Agents

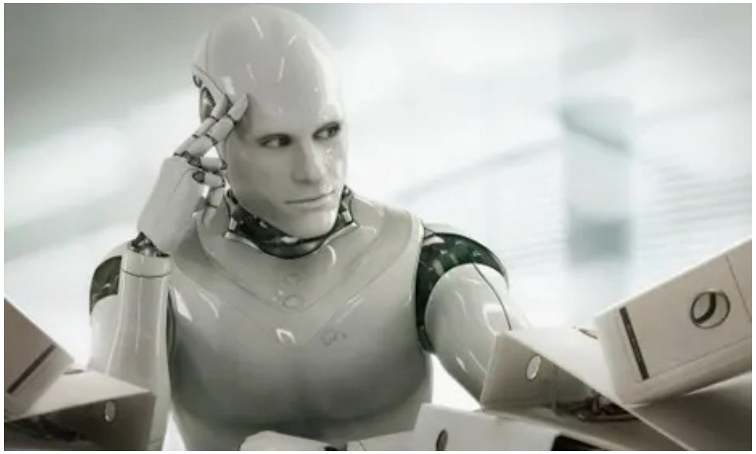


- Agent uses a process of reasoning over internal representation of knowledge to decide on actions to take

TELL (KB, makePercept(t))

Action <-- ASK (KB, whichAction(t))

TELL (KB, Action(action,t))



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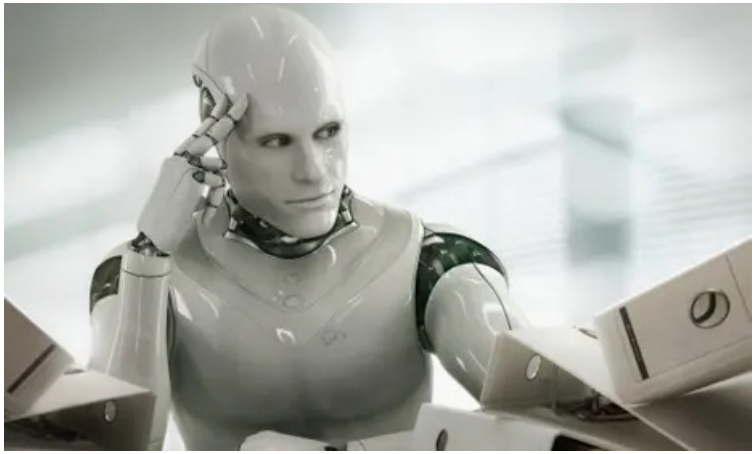


We need to represent the knowledge base: a set of sentences expressed according to the syntax of a logical language.

A language is a mechanism for expression

Ingredients of Logic

- Syntax: a set of valid formulas

  Example: CRUISECONTROL ∧ SNOW

- Semantic: specify a set of models/assignments

CRUISECONTROL	SNOW	CRUISECONTROL ∧ SNOW
1	1	1
1	0	0
0	1	0
0	0	0

- Inference: given f , what new formula g can be added?

  Example: Given CRUISECONTROL ∧ SNOW derive ACCIDENT

Syntax vs Semantic

- Syntax: valid expression
- Semantic: meaning of the expression
- For example: 1+2 and 2+1 have different syntaxes but same languages

Simple knowledge base

A set of the perceptions of the world

- SNOW is false

Rules

- DISTANCELIMIT is false if SNOW
- DISTANCELIMIT is TRUE if CRUISECONTROL
- ACCIDENT if DISTANCELIMIT is FALSE
- CRUISECONTROL is false if ATTACK

Logical agents

- Propositional logic
- First-order logic
- Etc.

Propositional Logic

- Propositional symbols (formulas): f and g
- Logical connective: $\neg \ \wedge \vee \rightarrow \ \leftrightarrow$

- If f and g are formulas so does:
- Negation: $\neg f$
- Conjunction $f \wedge g$
- Disjunction: $f \vee g$
- Implication: $f \rightarrow g$
- Biordinal: $f \leftrightarrow g$

Use of Propositional Logic

- DISTANCELIMIT is false if SNOW
- DISTANCELIMIT is TRUE if CRUISECONTROL
- ACCIDENT if DISTANCELIMIT is FALSE
- CRUISECONTROL is false if ATTACK
- SNOW is false

- $SNOW \rightarrow \neg DISTANCELIMIT$
- $CRUISECONTROL \rightarrow DISTANCELIMIT$
- $\neg DISTANCELIMIT \rightarrow ACCIDENT$
- $ATTACK \rightarrow \neg CRUISECONTROL$
- $\neg SNOW$

Logical agents

Let α and β be two sentences

$\alpha \models \beta \; iff \; M(\alpha) \subseteq \mathrm{M}(\beta)$

. β follows logical from α and α entails β

. All world configurations in α are also in β

Example: KB $\models$ σ: accident

The agent concludes from the knowledge base that there is accident

Model

- A model is an assignment of truth values to symbols

- The models for *CRUISECONTROL, SNOW, ACCIDENT* are

  {*CRUISECONTROL:0, SNOW:0, ACCIDENT:0* }

  {*CRUISECONTROL:0, SNOW:1, ACCIDENT:1* }

  {*CRUISECONTROL:1, SNOW:0, ACCIDENT: 0* }

  {CRUISECONTROL:1, SNOW:1, ACCIDENT: 1 }

- Indicates possible combinations of truth values to the symbols in the world

Interpretation of Proposition

Valid models are valid assignments

DISTANCELIMIT	ACCIDENT	¬DISTANCELIMIT	¬DISTANCELIMIT → ACCIDENT
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

We have done entailment by model checking:
enumerating models and checking the validity of the formula.

Interpretation of Proposition

- Interpreting proposition ***f***: **C**RUISECONTROL ∧ **S**NOW → **A**CCIDENT (Is *p* true?)
- Model w = {CRUISECONTROL:1, SNOW: 1, ACCIDENT: 0 }

w is not a model for p

$I(C \land S \rightarrow A, w) = 0$

I is for interpretation

$I(C \land S), w) = 1$

$I(A, w) = 0$

$I(C, w) = 1$

$I(S, w) = 1$

This is entailment by model checking: enumerating models and checking the validity of the formula.

Probabilistic generation

- $P(A/(C \land S))$: Probability of accident given Cruise Control and Snow

- $P(A/(C \land S)) = P(f{=}true) = \frac{\sum_{w \in W} I(f = true)}{|W|}$

W: space of w

Review: Logical Agent

- Syntax: a set of valid formula

  Example: CRUISECONTROL $\wedge$ SNOW

- Semantic: specify a set of models/assignment

- Inference: given f, what new formula g can be added?

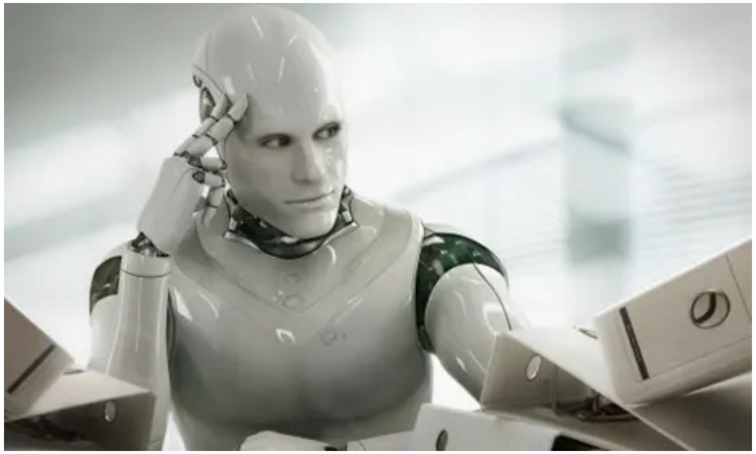
  Example: CRUISECONTROL $\wedge$ SNOW derive ACCIDENT

Propositional Theorem Proving

TELL (KB, makePercept(t))

Action <-- ASK (KB, whichAction(t))

TELL (KB, Action(action,t))



=> $KB = KB \cup \{f\}$

Propositional Theorem Proving

- We have done entailment by model checking: enumerating models and checking the validity of the formula.
- An alternative is to use theorem proving: applying rules of inference to the sentences of the knowledge base.
- Inference rules can be applied to derive proof.
  - Proof is a chain of conclusions that leads to desired goal.

Inference Rule 1: Moduls Ponens

- $\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$ Whenever any sentences of the form $\alpha \Rightarrow \beta$ and α are given then sentence β could be inferred

- Example: $\frac{\neg D \Rightarrow A, \neg D}{A}$. That is given that NOT DISTANCELIMIT implies ACCIDENT and NOT DISTANCELIMIT is TRUE than ACCIDENT is TRUE

Exercise: Propositional Logic

- CRUISECONTROL → DISTANCELIMIT
- ¬ DISTANCELIMIT → ACCIDENT
- SNOW → ¬ DISTANCELIMIT
- ¬ CRUISECONTROL → ATTACK
- SNOW

- ?Accident

$$\frac{\alpha \implies \beta, \quad \alpha}{\beta}$$

Propositional Logic

- CRUISECONTROL → DISTANCELIMIT
- ¬ DISTANCELIMIT → ACCIDENT
- SNOW → ¬ DISTANCELIMIT
- ¬ CRUISECONTROL → ATTACK
- SNOW

$$\frac{\alpha \;\Rightarrow\; \beta, \;\; \alpha}{\beta}$$

- SNOW
- SNOW → ¬ DISTANCELIMIT
- ¬ DISTANCELIMIT
- ¬ DISTANCELIMIT → ACCIDENT
- ACCIDENT

Propositional Logic

- CRUISECONTROL $\rightarrow$ DISTANCELIMIT
- $\neg$ DISTANCELIMIT $\rightarrow$ ACCIDENT
- SNOW $\rightarrow$ $\neg$ DISTANCELIMIT
- $\neg$ CRUISECONTROL $\rightarrow$ ATTACK
- $\neg$ SNOW

- ?Accident

$$\frac{\alpha \implies \beta, \quad \alpha}{\beta}$$

Inference Rule 2: And Elimination

- $\frac{\alpha \ \wedge \beta, \alpha}{\beta}$ Whenever any sentences of the form $\alpha \ \wedge \ \beta$ then β could be inferred. (if $\alpha \ \wedge \ \beta$ *is true and* α *is true then* β *is also true*)

- Example
  $\frac{\neg D \ \wedge S, \neg D}{S}$ Given NOT DISTANCELIMIT and SNOW true

then infer NOT DISTANCELIMIT true and SNOW is true

Agents Based on Proposition Logic

- Logical agents operate by deducing what to do from the knowledge base of sentences about the world.

- Knowledge base is composed of Axioms and Percepts.

- Axioms : General knowledge about how the world operates.
- Percept: Sentences obtained from agent experiences

Exercise

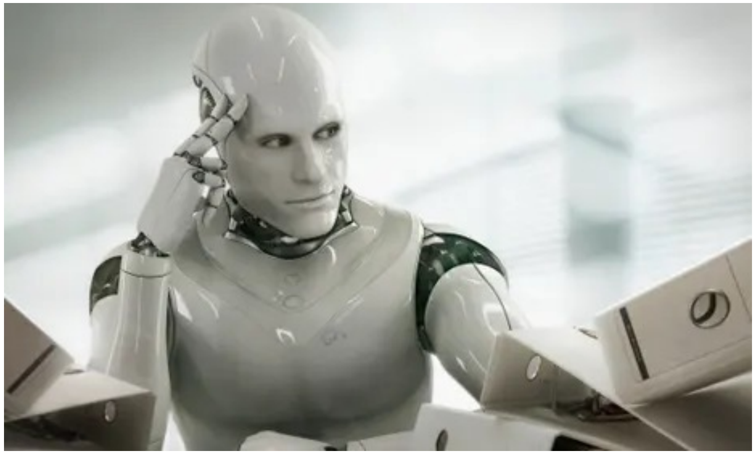
Logical agents use percepts and axioms to reason about the actions to take. Give two examples or more of percepts and axioms of an AI agent and point to the main difference between the two concepts.

Agent State

TELL (KB, makePercept(t))

Action <-- ASK (KB, whichAction(t))

TELL (KB, Action(action,t))



- The agent may change its state by performing actions
- State of the world should use sequence index to differentiate states at different time steps.
- We note $ACCIDENT^{t.}$ To imply accident at time t

Limitations of Propositional Logic

Goal: Represent all the students who took CSCE5214 know AI

Recall: Propositional logic is about facts

JohnTookCSCE5214 → JohnKnowsAI

LalithaTookCSCE5214 → LalithaKnowsAI

VishalTookCSCE5214 → VishalKnowsAI

We have about 120 students, which implies we must state 120 formulas ☹

Can we do better?

Limitations of Propositional Logic

The limitations include:

- Each sentences applies to a specific entity. You need a separate statement for each members of a group.
- You can use only Boolean operators to combine sentences
- You cannot use quantifier operators (for all and there exist)
- The terms can only have Boolean values, e.g., not numerical values

First-Order Logic

Goal: Represent all the students who took CSCE524 know AI

Solution:

- Student(John, CSCE5214) → Knows(John, AI)
- Student (x, CSCE5214) → Knows (x, AI)

Syntax of First-Order Logic

A term is a logical expression that refers to an object, e.g., John.

A function refers to some function on the object, e.g., *f*

A function is formulated using predicate symbol, e.g., LeftLeg()

Atomic sentence is formed by a predicate symbol followed optionally by terms to state facts.

e.g. Brother(Richard, John)

Syntax of First-Order Logic

- A complex sentence uses logical connectives to combine atomic and complex sentences.

  E.g. $\text{King(Richard)} \vee \text{King(John)}$

  $\neg$King (Richard) $\Rightarrow$ King(John)

- Quantifiers express properties of the entire collection of objects
  - Universal quantifier ($\forall$)
    - All kings are persons: $\forall\ king(\mathrm{x}) \Rightarrow person(x)$

      If *x* is a *king,* then *x* is a *person*

Semantic First-Order Logic

- Existential quantifier (∃): $\exists\, Crown(\mathrm{x})\ \wedge OnHead(x, John) \Longrightarrow King(John)$

Which of the following is correct

1. Richard is crown and Richard is on John’s head
2. King John is a crown and King John is on John’s head
3. The crown is a crown, and the crown is on John’s head

Semantic First-Order Logic

Connection between universal and existential quantification

$\forall x\ Likes(x, iceCream)$ is equivalent to $\neg\exists x\ \neg Likes(x, iceCream)$

- The De Morgan rules for quantifiers are:
  - $\neg\exists x\ P \ \equiv \forall x \neg\ P$
  - $\neg\forall x\ P \ \equiv \exists x \neg\ P$
  - $\forall x\ P \ \equiv \neg\exists x \neg\ P$
  - $\exists\ x\ P \ \equiv \neg\forall x \neg\ P$

Use of First-Order-Logic in Knowledge-base

- Add assertion to the KB through TELL(KB, sentence)
  - TELL(KB, King(John))
  - TELL(KB, Person(Richard))

- Ask the KB a question
  - ASK(KB, King(John)) → returns true or false
  - ASK(KB, $\exists$x Person(x)) → return true or false

Exercise: Use of First-Order-Logic in Knowledge-base

- $American(x) \wedge Weapon(y) \wedge Hostile(z) \wedge selles(x, y, z) \Longrightarrow criminal(x)$

- $Enemy(Nono, America)$
- $American(west)$
- $Missile(x) \wedge Owns(nano, x) \Longrightarrow Sells(West, x, Nono)$

ASK (Criminal(West))?

Question

What role can first order logic play in smart agent?

Exercise

How do companies implement first order logic in autonomous systems like autonomous cars?

Formal Languages for Logics

- Propositional logic represents and allows to reason about facts. The agents believes about facts are: true/false/unknown values.
- First-order logic represents and allows to reason about facts, objects, and relations. The agents believes about facts are: true/false/unknown values.
- Temporal logic represents and allows to reason about facts, objects, relations, and time. The agents believes about facts are: true/false/unknown values.
- Probability theory represents facts. The agents has degrees of believe about the facts in [0,1]
- Fuzzy logic represents facts with degree of truth in [0,1]. The agents believes about facts known interval values.

Summary

- Logical Agent uses a process of reasoning over internal representation of knowledge to decide on actions to take.
- Propositional logic allows to represent and reason about facts.
- First-order logic allows to represent and reason about facts, objects, and relations.
- Logical agent operates by deducing what to do from the knowledge-base of sentences about the world.
- The Knowledge-base is composed of Axioms and Percepts.
  - Axiom: General knowledge about how the world operates.
  - Percept: Sentences obtained from agent experiences

Thank you

Any Question?