Chapter 13

Sinusoidal Alternating Waveforms

RMS OR EFFECTIVE VALUE
AVERAGE OR MEAN VALUE

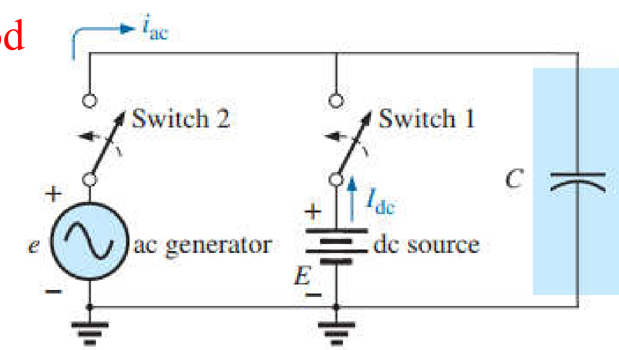
Average Value or Mean Value

Average Value: The **average value** of an alternating current is expressed by that dc current which transfers across any circuit the same ***charge as*** is transferred by that alternating current during the same time.

In case of symmetrical waveform, the average value over a full cycle is zero. Thus, **the average value is calculated over half-cycle for symmetrical waveform**. But **for asymmetrical waveform, the average value is calculated over a full cycle**.

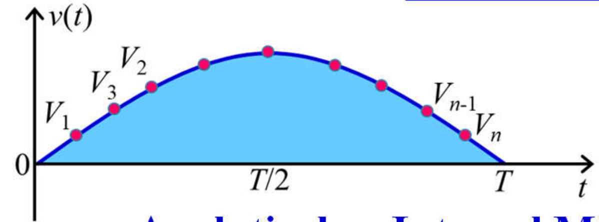
Average value can be calculated by the following methods:

- Graphical Method
- Analytical or Integral Method



Graphical Method:

$$V_{ave} = \frac{V_1 + V_2 + V_3 + \dots + V_n}{n}$$

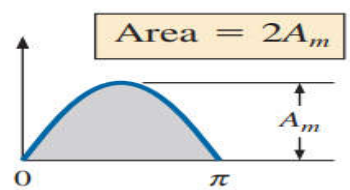


Analytical or Integral Method:

For asymmetrical wave:

$$\text{Average Value} = \frac{\text{Area under the curve in one cycle}}{\text{Duration of one cycle}}$$

$$I_{ave} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{2\pi}\int_0^{2\pi} i(\theta)d\theta$$



For symmetrical wave:

$$\text{Average Value} = \frac{\text{Area under the curve in } half\text{-cycle}}{\text{Duration of } half\text{-cycle}}$$

$$I_{ave} = \frac{1}{T/2}\int_0^{T/2} i(t)dt = \frac{1}{\pi}\int_0^{\pi} i(\theta)d\theta$$

For Sine and Cosine Waves:

$$Average\ Value = \frac{2}{\pi}(Peak\ Value) = 0.637 \times (Peak\ Value)$$

For Symmetrical Triangular waveforms :

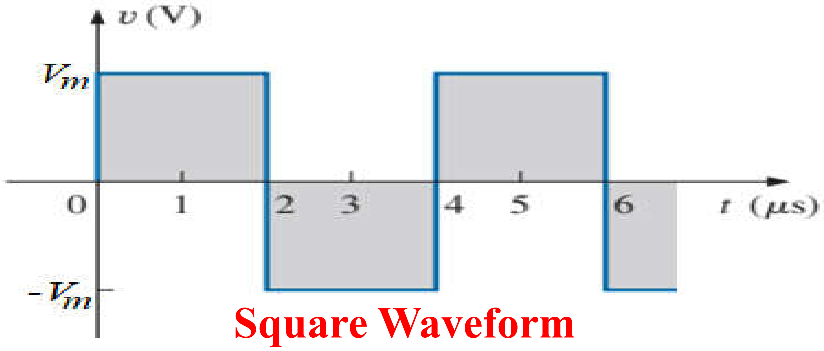
$$\text{Average Value} = \frac{1}{2}(\text{Peak Value}) = 0.5 \times (\text{Peak Value})$$

For Symmetrical Sawtoth waveforms :

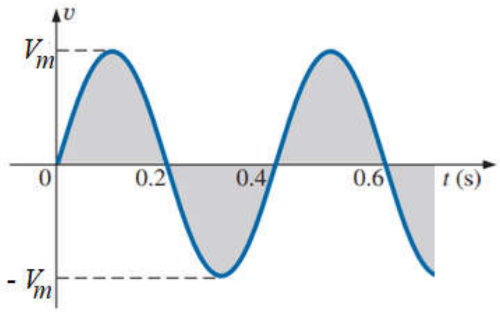
$$\text{Average Value} = \frac{1}{2}(\text{Peak Value}) = 0.5 \times (\text{Peak Value})$$

For Square or Symmtrical Rectangular Waveforms :

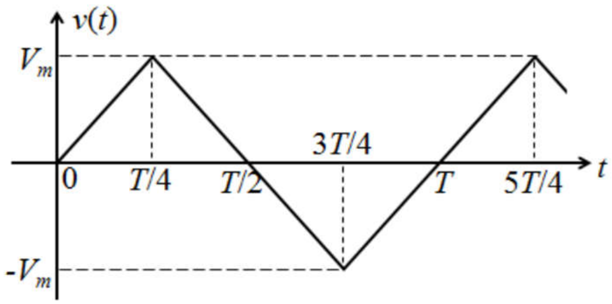
$$\text{Average Value} = \text{Peak Value}$$



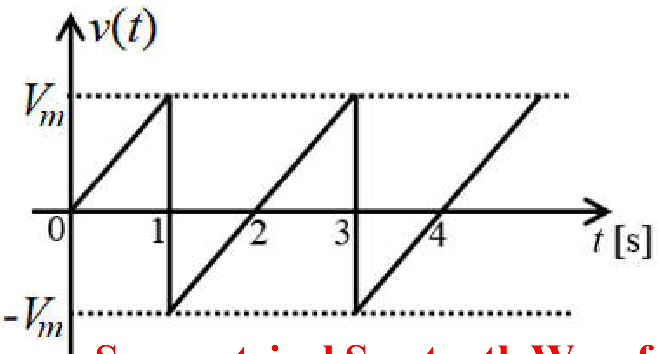
Square Waveform



Sine Waveform



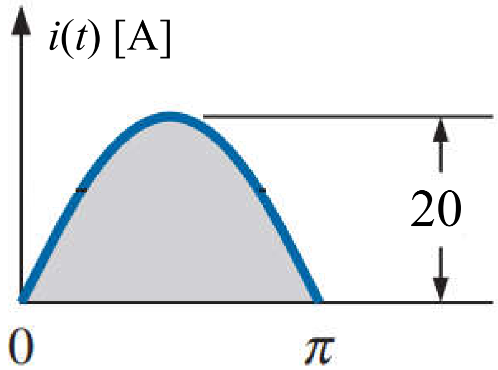
Symmetrical Triangular Waveform



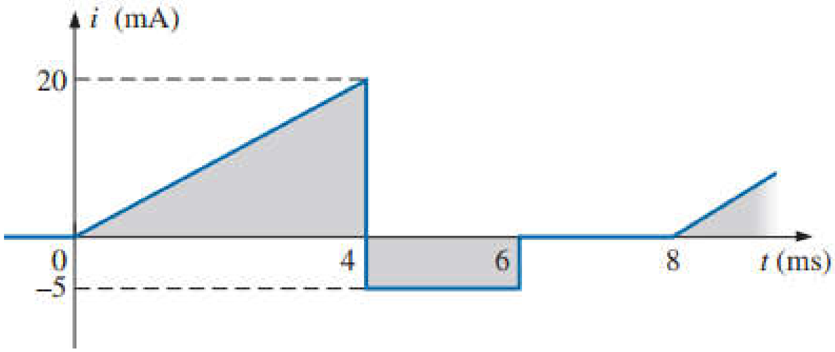
Symmetrical Sawtooth Waveform

EXAMPLE 13.14.1 Determine the average value for the following waveforms.

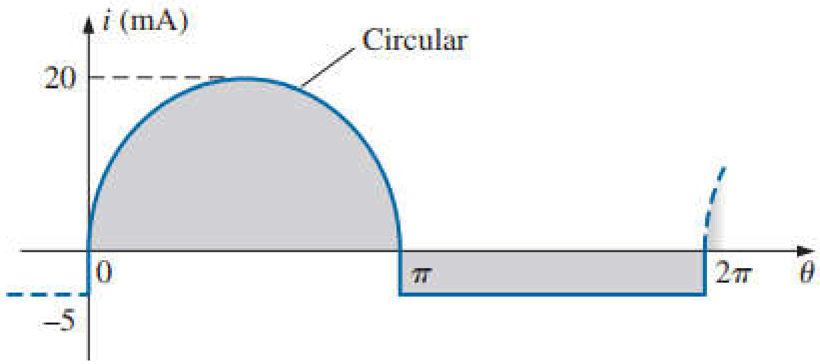
We know that the area of half-cycle of sine wave = 2 × Peak Value



$$V_{ave} = \frac{Area}{Duration} = \frac{2\times 20}{\pi} = \mathbf{12.74\,A}$$



$$V_{ave} = \frac{(1/2)\times 20\times 4 + (-5)\times 2}{8} = \mathbf{3.87\,mA}$$



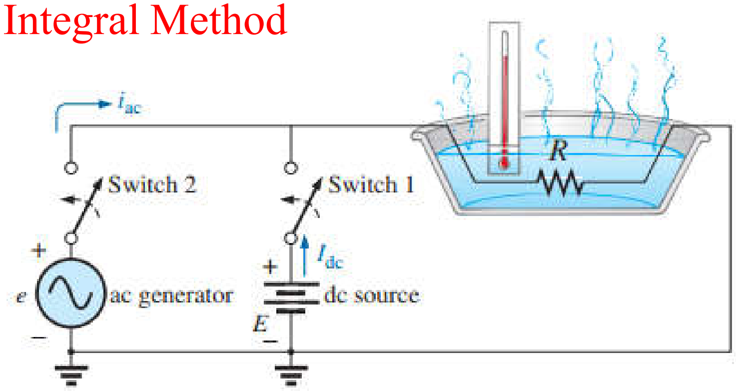
$$V_{ave} = \frac{2\times 20 + (-5)\times \pi}{2\pi} = \mathbf{3.87\,mA}$$

Root Mean Square (RMS) or Effective Value

RMS or Effective Value: The **effective or RMS value** of an alternating current is given by that dc current which, when flowing through a given circuit for a given time, produces the same amount of **heat** as produced by the alternating current, when flowing through the same circuit for the same time.

RMS value can be calculated by the following methods:

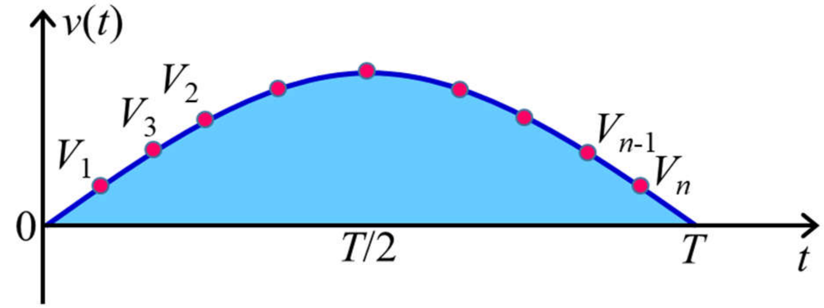
- ❑ Graphical Method
- ❑ Analytical or Integral Method



$$P_{dc} = I_{dc}^2 R = \frac{V_{dc}^2}{R}$$

$$P_{ac} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

$$P_{ac} = P_{dc}$$



Graphical Method:

$$V = V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \ldots + V_n^2}{n}}$$

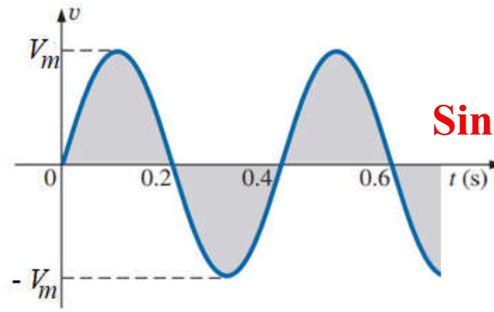
Analytical or Integral Method:

$$I_{\text{rms}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}} \qquad \textbf{(13.31)}$$

$$I_{\text{rms}} = \sqrt{\frac{\text{area } (i^2(t))}{T}} \qquad \textbf{(13.32)}$$

For Sine or Cosine Waveforms:

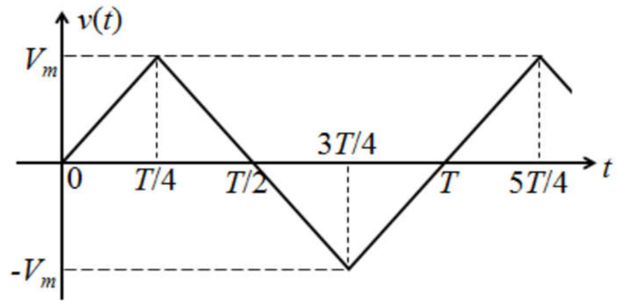
$$\text{RMS or Effective Value} = \frac{1}{\sqrt{2}}(\text{Peak Value}) = 0.707 \times (\text{Peak Value})$$



Sine Waveform

For Symmetrical Triangular waveforms:

$$\text{RMS Value} = \frac{1}{\sqrt{3}}(\text{Peak Value}) = 0.577 \times (\text{Peak Value})$$



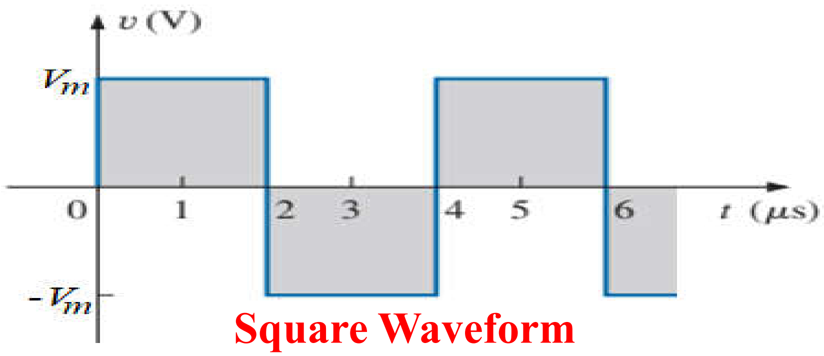
Symmetrical Triangular Waveform

For Symmetrical Sawtoth waveforms:

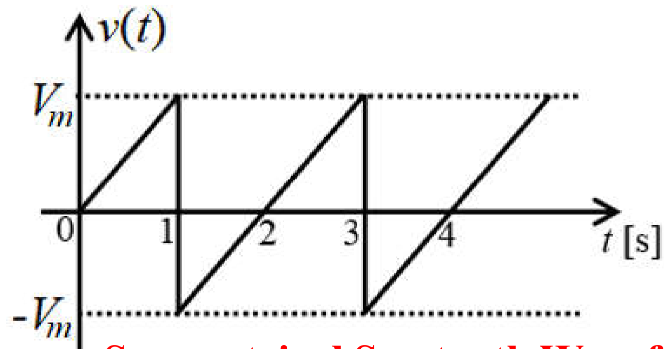
$$\text{RMS Value} = \frac{1}{\sqrt{3}}(\text{Peak Value}) = 0.577 \times (\text{Peak Value})$$

For Square or Symmtrical Rectangular Waveforms:

$$\text{Effective or RMS Value} = \text{Peak Value}$$

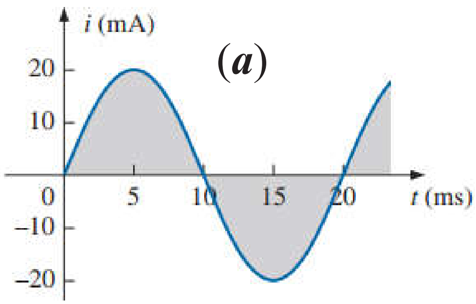


Square Waveform



Symmetrical Sawtooth Waveform

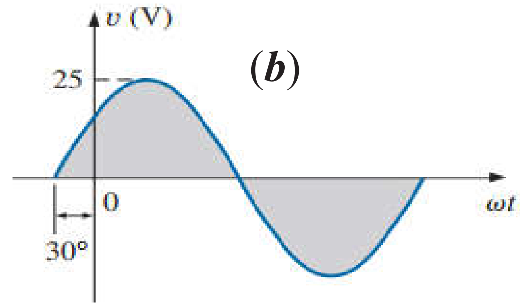
EXAMPLE 13.20 Find the average value and rms values for the following sinusoidal waveforms.



Here, $I_m = 20$ mA

$$I_{ave} = 0.637 \times 20\text{mA} = \mathbf{12.74\ mA}$$

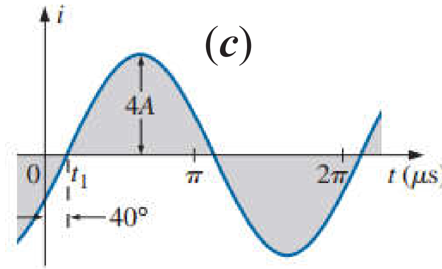
$$I = I_{rms} = 0.707 \times 20\text{mA} = \mathbf{14.14\ mA}$$



Here, $V_m = 25$ V

$$V_{ave} = 0.637 \times 25\text{V} = \mathbf{15.93\ V}$$

$$V = V_{rms} = 0.707 \times 25\text{V} = \mathbf{17.68\ V}$$



Here, $I_m = 4$ A

$$I_{ave} = 0.637 \times 4\text{A} = \mathbf{2.55\ A}$$

$$I = I_{rms} = 0.707 \times 4\text{A} = \mathbf{2.83\ A}$$

EXAMPLE 13.21 The 120 V dc source in Fig. 13.59(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13.59(b)] is to deliver the same power to the load.

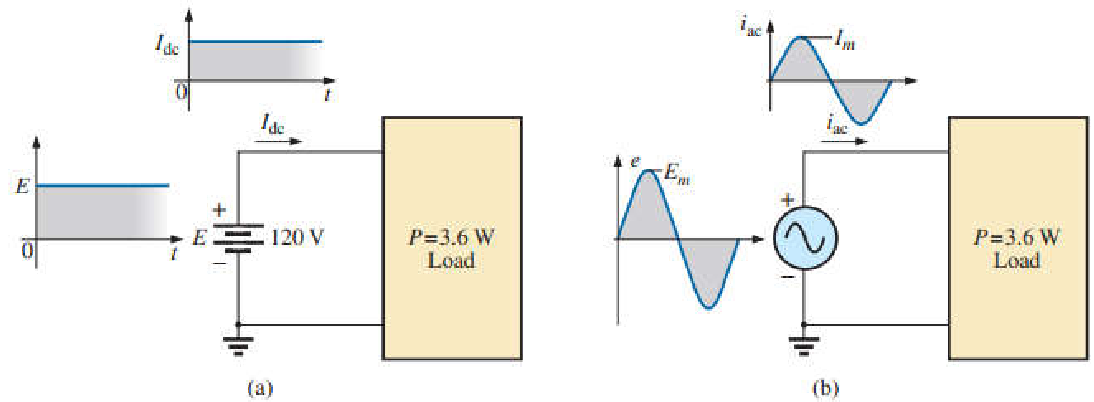


FIG. 13.59 *Example 13.21.*

Solution: $P_{dc} = V_{dc} I_{dc}$

and $$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2} I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$

$$E_m = \sqrt{2} E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

EXAMPLE 13.20.1 Find the average value and rms values for the following sinusoidal waveforms:

(***a***) $i(t) = 10\sin(\omega t + 30°)$ A

(***b***) $v(t) = 150\cos(\omega t - 60°)$ A

(***c***) $i(t) = -12\cos(\omega t + 80°)$ μA

(***d***) $v(t) = -200\sin(\omega t - 120°)$ mV

(***a***) $I_{ave} = 0.637 \times 10\text{A} = \mathbf{6.37\ A}$

$I = I_{rms} = 0.707 \times 10\text{A} = \mathbf{7.07\ A}$

(***b***) $V_{ave} = 0.637 \times 150\text{V} = \mathbf{95.55\ V}$

$V = V_{rms} = 0.707 \times 150\text{V} = \mathbf{106.05\ V}$

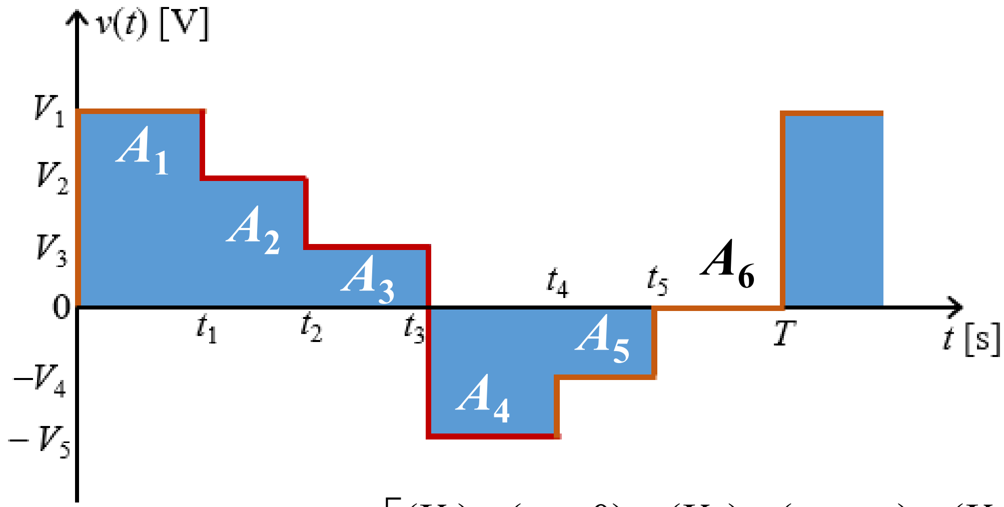
(***c***) $I_{ave} = 0.637 \times 12\mu\text{A} = \mathbf{7.64}\ \mu\mathbf{A}$

$I = I_{rms} = 0.707 \times 12\mu\text{A} = \mathbf{8.48}\ \mu\mathbf{A}$

(***d***) $V_{ave} = 0.637 \times 200\text{mV} = \mathbf{127.4\ mV}$

$V = V_{rms} = 0.707 \times 200\text{mV} = \mathbf{141.1\ mV}$

Average Value and RMS Value for Rectangular Waveform



$$A_1 = (V_1) \times (t_1 - 0) \qquad A_2 = (V_2) \times (t_2 - t_1)$$

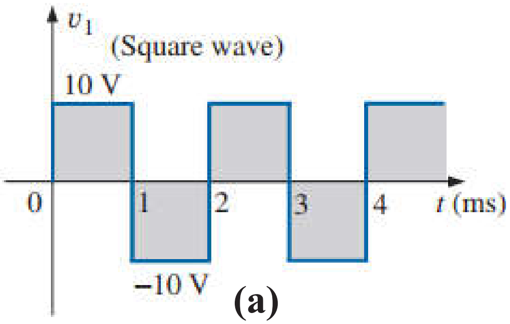
$$A_3 = (V_3) \times (t_3 - t_2) \qquad A_4 = (-V_5) \times (t_4 - t_3)$$

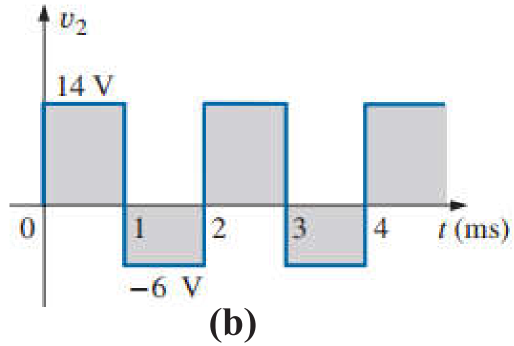
$$A_5 = (-V_4) \times (t_5 - t_4) \qquad A_6 = (0) \times (T - t_5)$$

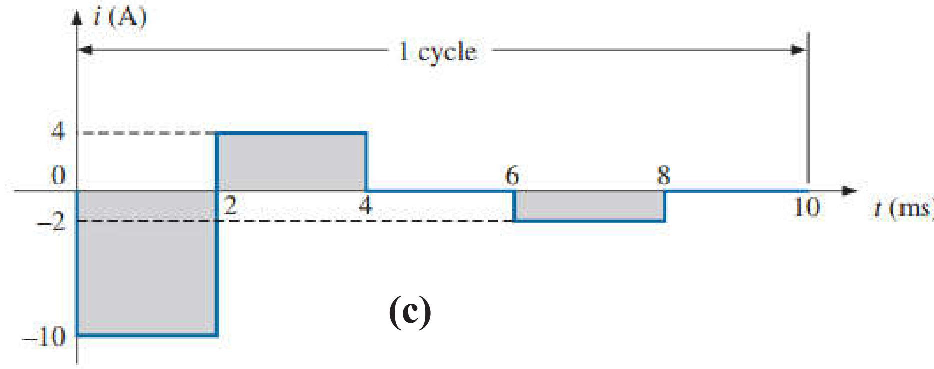
$$V_{ave} = \frac{\begin{bmatrix} (V_1)\times(t_1 - 0) + (V_2)\times(t_2 - t_1) + (V_3)\times(t_3 - t_2) + (-V_5)\times(t_4 - t_3) \\ + (-V_4)\times(t_5 - t_4) + (0)\times(T - t_5) \end{bmatrix}}{T}$$

$$V_{rms} = \sqrt{\frac{\begin{bmatrix} (V_1)^2 \times(t_1 - 0) + (V_2)^2 \times(t_2 - t_1) + (V_3)^2 \times(t_3 - t_2) + (-V_5)^2 \times(t_4 - t_3) \\ + (-V_4)^2 \times(t_5 - t_4) + (0)^2 \times(T - t_5) \end{bmatrix}}{T}}$$

EXAMPLE 13.14 Determine the average value, the rms value for the following waveforms.







$$\text{(a) } V_{ave} = \frac{(10)\times(1-0)+(-10)(2-1)}{2} = 0$$

$$V_{rms} = \sqrt{\frac{(10)^2\times(1-0)+(-10)^2(2-1)}{2}} = \mathbf{10\ V}$$

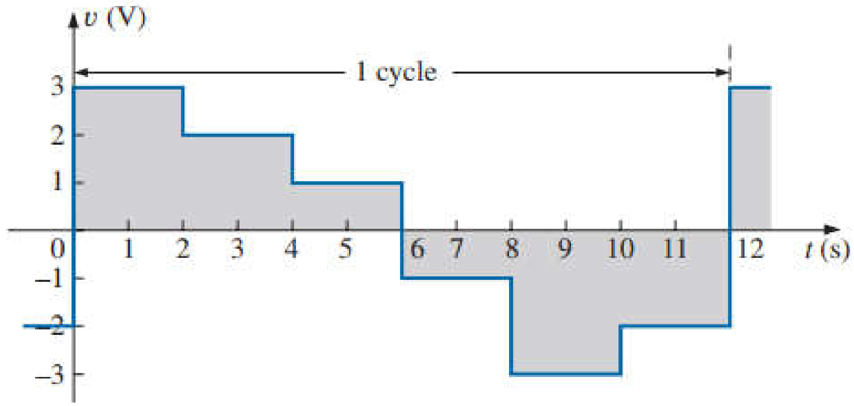
$$\text{(b) } V_{ave} = \frac{(14)\times(1-0)+(-6)(2-1)}{2} = \mathbf{4\,V}$$

$$V_{rms} = \sqrt{\frac{(14)^2\times(1-0)+(-6)^2(2-1)}{2}} = \mathbf{10.77\ V}$$

$$\text{(c) } I_{ave} = \frac{(-10)\times 2+(4)\times 2+(-2)\times 2}{10} = \mathbf{-1.6\,A}$$

$$I_{rms} = \sqrt{\frac{(-10)^2\times 2+(4)^2\times 2+(-2)^2\times 2}{10}} = \mathbf{4.9\,A}$$

EXAMPLE 13.14.1 Determine the average value, the rms value for the following waveforms. Also, determine the average power consumption if the voltage applied across 10 ohm resistance.



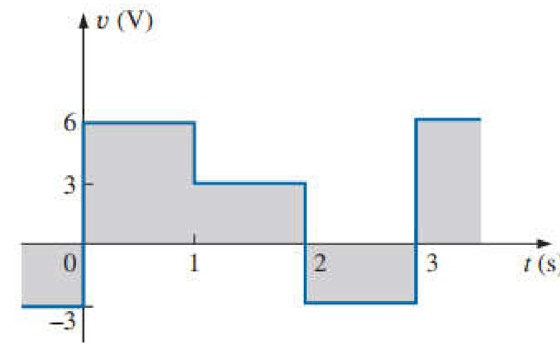
$$V_{ave} = \frac{(3)\times 2+(2)\times 2+(1)\times 2+(-1)\times 2+(-3)\times 2+(-2)\times 2}{12}$$

$$= \mathbf{0\,V}$$

$$V_{rms} = \sqrt{\frac{(3)^2\times 2+(2)^2\times 2+(1)^2\times 2+(-1)^2\times 2+(-3)^2\times 2+(-2)^2\times 2}{12}}$$

$$= \mathbf{2.16\,V}$$

$$P_{ave} = \frac{V_{rms}^2}{R} = \frac{(2.16)^2}{10} = \mathbf{0.466\,W}$$



$$V_{ave} = \frac{(6)\times 1+(3)\times 1+(-3)\times 1}{3} = \mathbf{2\,V}$$

$$V_{rms} = \sqrt{\frac{(6)^2\times 1+(3)^2\times 1+(-3)^2\times 1}{3}}$$

$$= \mathbf{4.24\,V}$$

$$P_{ave} = \frac{V_{rms}^2}{R} = \frac{(4.24)^2}{10} = \mathbf{1.78\,W}$$

Practice Problems 37 ~ 46 [Ch. 13]

Chapter 14

The Basic Elements and Phasors

Phasor Algebra/Complex Number

Vector Quantities Represent by Complex Number :

1. Magnitude **2.** Direction

Phasor Quantities Represent by Complex Number:

1. Magnitude (RMS value for voltage and current)
2. Direction (Phase angle)
3. Continuously change with respect to time [such as sine and cosine waves)

Complex Number can be represented by three different ways:

1. Polar or Phasor form
2. Cartesian or Rectangular form
3. Exponential form

14.7 RECTANGULAR FORM:

$$\boxed{C = X + jY \qquad (14.17)}$$

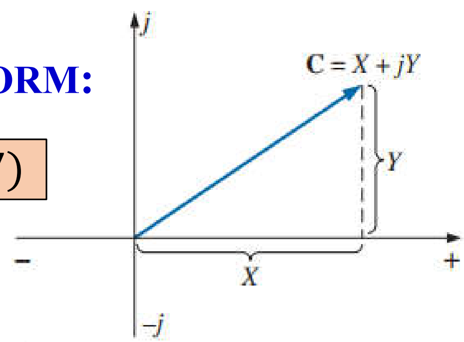


FIG. 14.39 *Defining the rectangular form.*

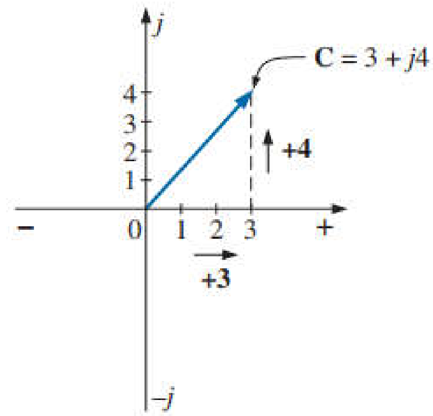
$$\boxed{j = \sqrt{-1}} \qquad (14.24)$$

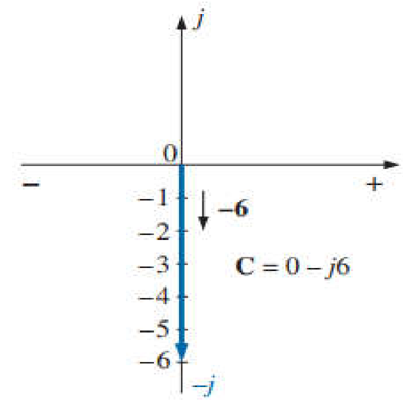
$$\boxed{j^2 = -1} \qquad (14.25)$$

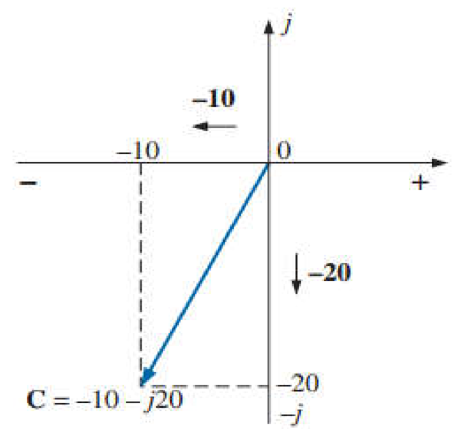
$$\boxed{\frac{1}{j} = -j} \qquad (14.26)$$

EXAMPLE 14.13 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 3 + j4$ b. $\mathbf{C} = 0 - j6$ c. $\mathbf{C} = -10 - j20$







14.8 POLAR OR PHASOR FORM:

$$\boxed{\boldsymbol{C} = Z\angle\theta \qquad (\mathbf{14.18})}$$

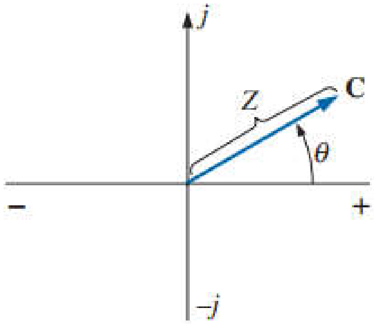


FIG. 14.43 *Defining the polar form.*

$$\boxed{\boldsymbol{C} = -Z\angle\theta = Z\angle\theta \pm 180° \ (\mathbf{14.19})}$$

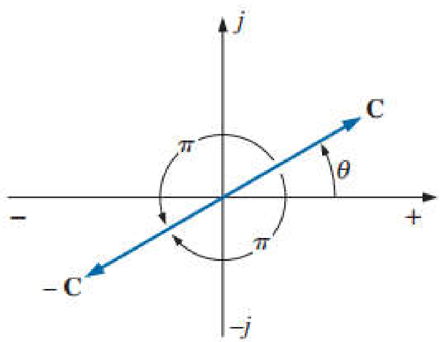
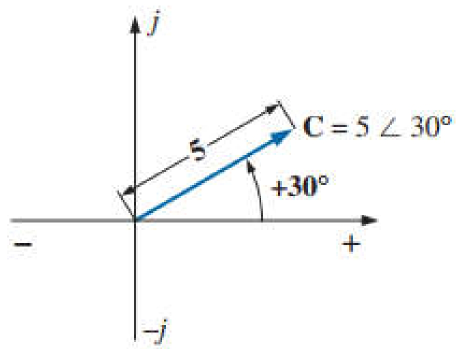
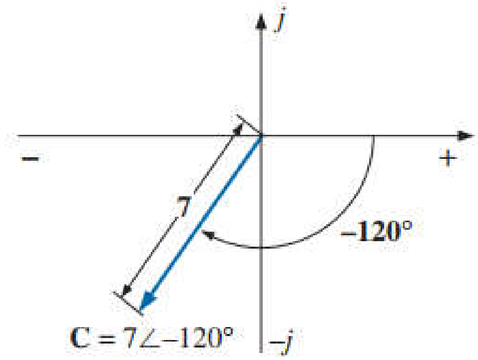


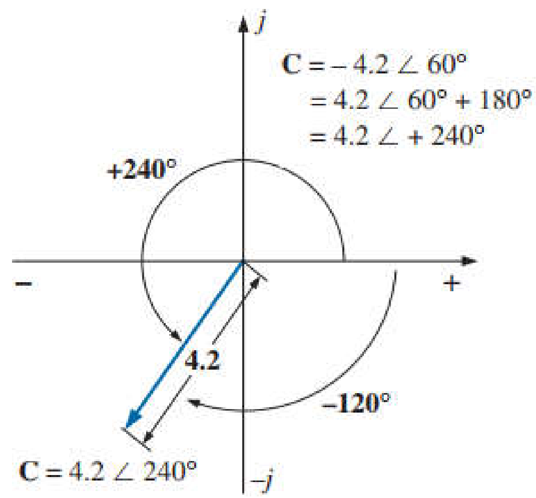
FIG. 14.44
Demonstrating the effect of a negative sign on the polar form.

EXAMPLE 14.14 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 5\ \angle 30°$ b. $\mathbf{C} = 7\ \angle -120°$ c. $\mathbf{C} = -4.2\ \angle 60°$







14.9 CONVERSION BETWEEN FORMS

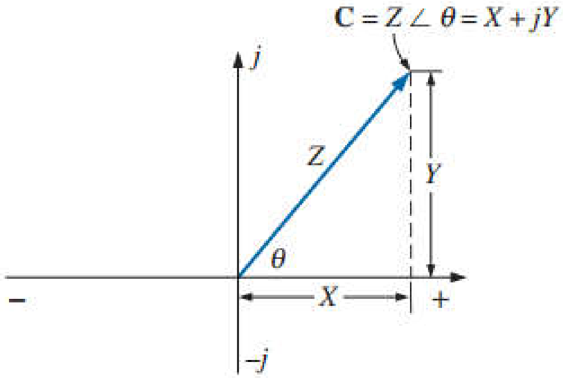


FIG. 14.48 *Conversion between forms.*

Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}} \quad \textbf{(14.20)}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}} \quad \textbf{(14.21)}$$

Polar to Rectangular

$$\boxed{X = Z\cos\theta} \quad \textbf{(14.22)}$$

$$\boxed{Y = Z\sin\theta} \quad \textbf{(14.23)}$$

EXAMPLE 14.15 Convert the following from rectangular to polar form:

$$\mathbf{C} = 3 + j4 \qquad \text{(Fig. 14.49)}$$

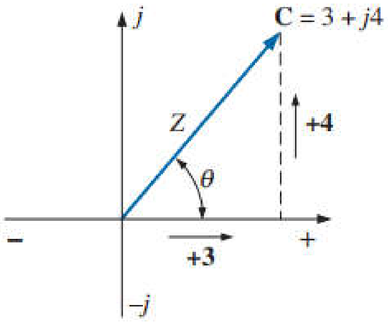


FIG. 14.49 *Example 14.15.*

Solution:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

and $\mathbf{C = 5\angle 53.13^\circ}$

EXAMPLE 14.16 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10\angle 45^\circ \qquad \text{(Fig. 14.50)}$$

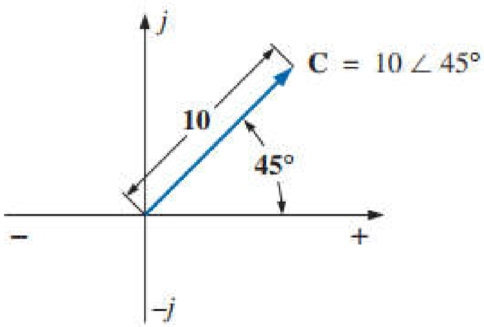


FIG. 14.50 *Example 14.16.*

Solution:

$$X = 10\cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10\sin 45^\circ = (10)(0.707) = 7.07$$

and $\mathbf{C = 7.07 + j7.07}$

14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

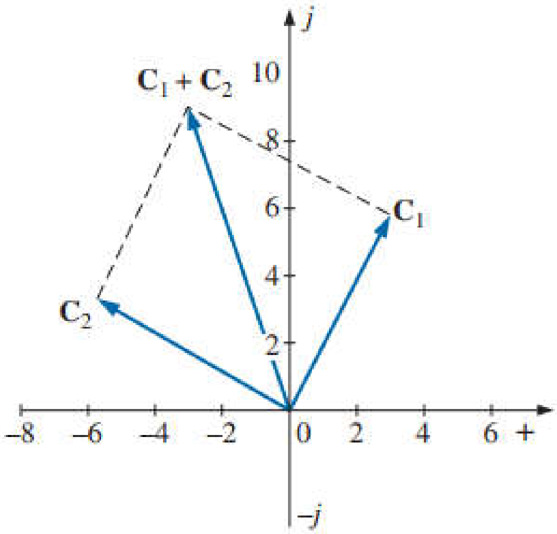
Addition

$\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)} \qquad \textbf{(14.27)}$$

EXAMPLE 14.19 Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions: $\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$



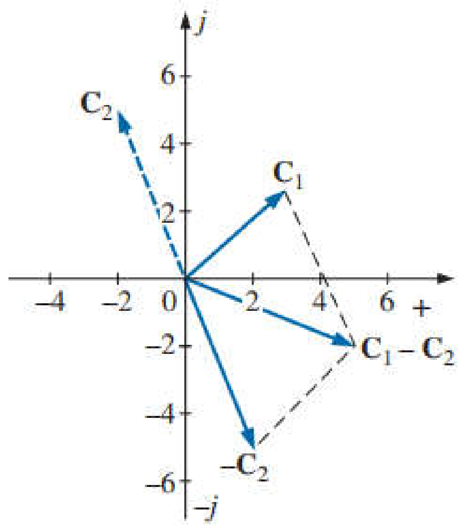
Subtraction

$\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

$$\boxed{\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_1 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]} \qquad \textbf{(14.28)}$$

EXAMPLE 14.20 Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

Solutions: $\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$



Multiplication

$\mathbf{C}_1 = X_1 + jY_1$ and $\mathbf{C}_2 = X_2 + jY_2$

then $\mathbf{C}_1 \cdot \mathbf{C}_2$:

$$
\begin{array}{r}
X_1 + jY_1 \\
\underline{X_2 + jY_2} \\
X_1X_2 + jY_1X_2 \\
\underline{+ jX_1Y_2 + j^2Y_1Y_2} \\
X_1X_2 + j(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1)
\end{array}
$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)} \qquad \textbf{(14.29)}$$

Division

$\mathbf{C}_1 = X_1 + jY_1$ and $\mathbf{C}_2 = X_2 + jY_2$

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)}$$

$$= \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}} \qquad \textbf{(14.31)}$$

Multiplication in Polar Form:

$\mathbf{C}_1 = Z_1 \angle\theta_1$ and $\mathbf{C}_2 = Z_2 \angle\theta_2$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1Z_2 \underline{/\theta_1 + \theta_2}} \qquad \textbf{(14.30)}$$

Division in Polar Form:

$\mathbf{C}_1 = Z_1 \angle\theta_1$ and $\mathbf{C}_2 = Z_2 \angle\theta_2$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2}\underline{/\theta_1 - \theta_2}} \qquad \textbf{(14.32)}$$

EXAMPLE 14.23

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 5 \angle 20°$ and $\mathbf{C}_2 = 10 \angle 30°$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 2 \angle -40°$ and $\mathbf{C}_2 = 7 \angle +120°$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20°)(10 \angle 30°) = (5)(10) \underline{/20° + 30°} = \mathbf{50 \angle 50°}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40°)(7 \angle +120°) = (2)(7) \underline{/-40° + 120°}$
$= \mathbf{14 \angle +80°}$

EXAMPLE 14.25

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10°$ and $\mathbf{C}_2 = 2 \angle 7°$.
b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120°$ and $\mathbf{C}_2 = 16 \angle -50°$.

Practice Problem 39 ~ 49 [Ch. 14]

Solutions:

a. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{15 \angle 10°}{2 \angle 7°} = \frac{15}{2} \underline{/10° - 7°} = \mathbf{7.5 \angle 3°}$$

b. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{8 \angle 120°}{16 \angle -50°} = \frac{8}{16} \underline{/120° - (-50°)} = \mathbf{0.5 \angle 170°}$$

APPLICATION OF COMPLEX NUMBERS IN AC CIRCUIT

Instantaneous Form (Time Domain) Equation:

$$e(t) = E_m sin(\omega t \pm \theta_e)\text{ V}$$
$$v(t) = V_m sin(\omega t \pm \theta_v)\text{ V}$$
$$i(t) = I_m sin(\omega t \pm \theta_i)\text{ A}$$

Phasor Form (Polar Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_{rms}\angle\theta_e = E\angle\pm\theta_e\text{ V}$$
$$\boldsymbol{V} = \vec{V} = V_{rms}\angle\theta_v = V\angle\pm\theta_v\text{ V}$$
$$\boldsymbol{I} = \vec{I} = I_{rms}\angle\theta_i = I\angle\pm\theta_i\text{ A}$$

Rectangular Form (Cartesian Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_r \pm jE_i\ \text{ V}$$
$$\boldsymbol{V} = \vec{V} = V_r \pm jV_i\ \text{ V}$$
$$\boldsymbol{I} = \vec{I} = I_r \pm jI_i\ \text{ A}$$

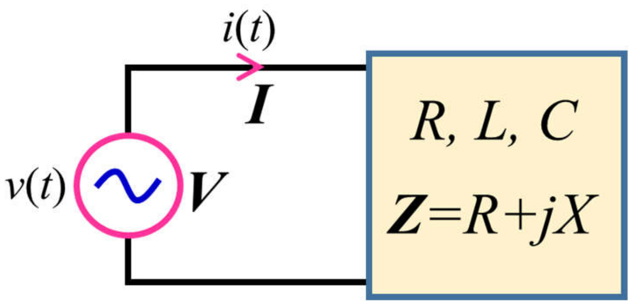
EXAMPLE 14.27 Convert the following from the time to (***i***) the phasor domain, and (***ii***) the rectangular domain.

Time Domain	Phasor Domain	Rectangular Domain
(***a***) $v(t) = 70.7\sin(\omega t - 60^o)$ V	$\vec{V} = (0.707\times70.7)\text{V}\angle-60^o = \mathbf{50V\angle-60^o}$	$\boldsymbol{V} = 25 - j43.3$ V
(***b***) $i(t) = 21.21\cos(\omega t + 20^o)$ A $= 21.21\sin(\omega t + 110^o)$ A	$\vec{I} = (0.707\times21.21)\text{A}\angle110^o = \mathbf{15A\angle110^o}$	$\boldsymbol{I} = 15[\cos(110^o)+j\sin(110^o)]$ $= -5.13 + j14.1$ A
(***c***) $e(t) = -\ 200\cos\omega t$ V $= 200\sin(\omega t - 90^o)$ V	$\vec{E} = (0.707\times200)\text{V}\angle-90^o = \mathbf{141.42V\angle-90^o}$	$\boldsymbol{E} = 141.42[\cos(-90^o)+j\sin(-90^o)]$ $= 0 - j141.42$ V
(***d***) $i(t) = -4.5\sin(\omega t + 30^o)$ A $= 4.5\sin(\omega t - 150^o)$ A $= 4.5\sin(\omega t + 210^o)$ A	$\vec{I} = (0.707\times4.5)\text{A}\angle-150^o = \mathbf{3.18A\angle-150^o}$ $\vec{I} = (0.707\times4.5)\text{A}\angle210^o = \mathbf{3.18A\angle210^o}$	$\boldsymbol{I} = 3.18[\cos(210^o)+j\sin(210^o)]$ $= -2.75 - j1.59$ A

EXAMPLE 14.27.1 Convert the following from Cartesian form to (***i***) the phasor domain, and (***ii***) the instantaneous form for 50 Hz.

Rectangular Form	Phasor Form	Instantaneous Form
(***a***) $\vec{V} = 25 - j43.3$ V RMS value: 50 V Phase Angle: –60º Peak Value: 70.7 V	$V = \sqrt{25^2 + (-43.3)^2} = 50$ V $\theta_v = \tan^{-1}\left[\frac{-43.3}{25}\right] = -60^{\circ}$ $\boldsymbol{V = 50V\angle{-60^{\circ}}}$	$\omega = 2\pi \times 50 = 314$ rad/s $v(t) = (\sqrt{2}) \times 50\sin(314t - 60^{\circ})$ V $= \mathbf{70.7\sin(314\mathit{t} - 60^{\circ})\ V}$
(***b***) $\vec{E} = j150$ V RMS value: 150 V Phase Angle: 90º Peak Value: 212.13 V	$E = \sqrt{0^2 + 150^2} = 150$ V $\theta_e = \tan^{-1}\left[\frac{150}{0}\right] = 90^{\circ}$ $\boldsymbol{E = 150V\angle 90^{\circ}}$	$e(t) = (\sqrt{2}) \times 150\sin(314t + 90^{\circ})$ V $= \mathbf{212.13\sin(314\mathit{t} + 90^{\circ})\ V}$ $= \mathbf{212.13\cos 314\mathit{t}\ V}$
(***d***) $\vec{I} = -j5$ A RMS value: 5 A Phase Angle: –90º Peak Value: 7.07 A	$I = \sqrt{0^2 + (-5)^2} = 5$ A $\theta_i = \tan^{-1}\left[\frac{-5}{0}\right] = -90^{\circ}$ $\boldsymbol{I = 5A\angle{-90^{\circ}}}$	$i(t) = (\sqrt{2}) \times 5\sin(314t - 90^{\circ})$ A $= \mathbf{7.07\sin(314\mathit{t} - 90^{\circ})\ A}$ $= \mathbf{-\ 7.07\cos 314\mathit{t}\ A}$
(***e***) $\vec{V} = -100$ V RMS value: 100 V	$\boldsymbol{V = 100V\angle \pm 180^{\circ}}$ Phase Angle: ± 180º Peak Value: 141.42 V	$v(t) = (\sqrt{2}) \times 100\sin(314t \pm 180^{\circ})$ V $= \mathbf{141.42\sin(314\mathit{t} \pm 180^{\circ})\ V}$ $= \mathbf{-\ 141.42\sin 314\mathit{t}\ V}$

IMPEDANCE (Z)
ADMITTANCE (Y)



IMPEDANCE

Impedance: Impedance is the ratio of **voltage** to **current**.

Impedance opposes the flow of current.

Impedance represent by ***Z***. Its unit is **ohm** (Ω).

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{V_{rms}\angle\theta_v}{I_{rms}\angle\theta_i} = \frac{V\angle\theta_v}{I\angle\theta_i} = \frac{V}{I}\angle(\theta_v - \theta_i) = Z\angle\theta_z = R + jX \ \ \Omega$$

Magnitude of Impedance: $Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}} = \frac{V}{I} \quad \Omega$

Angle of Impedance: $\theta_z = \theta_v - \theta_i$

Resistance (Real Part of Impedance): $R = Z\cos\theta_z \quad \Omega$

Reactance (Imaginary Part of Impedance): $X = Z\sin\theta_z \quad \Omega$

Practically, $-90^o \leq \theta_z \leq 90^o$

Reactance is the property of inductor and capacitor to oppose the flow of current. The are two reactance in electrical circuit: (***i***) **inductive reactance** (X_L), and (***ii***) **capacitive reactance** (X_C).

Inductive Reactance :

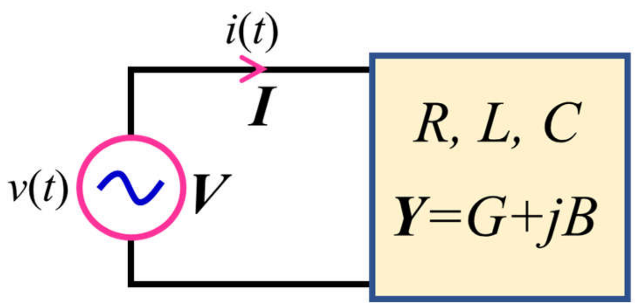
$$X_L = \omega L = 2\pi f L \ [\Omega] \qquad X_L \propto f$$

Capacitive Reactance :

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \ [\Omega] \qquad X_C \propto \frac{1}{f}$$

Impedance (***Z***) is not a phasor quantity because for a circuit it is constant. That means impedance does not change with respect to time.

ADMITTANCE



Admittance (*Y*) is also not a phasor quantity.

Admittance: Admittance is the ratio of **current** to **voltage**. ***Admittance*** is **reciprocal of impedance**.
Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit.
Admittance represent by ***Y***. Its unit is **Siemens** (S).

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{I_{rms}\angle\theta_i}{V_{rms}\angle\theta_v} = \frac{I\angle\theta_i}{V\angle\theta_v} = \frac{I}{V}\angle(\theta_i - \theta_v)$$
$$= Y\angle\theta_y = G + jB \text{ S}$$

Magnitude of Admittance: $Y = \frac{1}{Z} = \frac{I_m}{V_m} = \frac{I_{rms}}{V_{rms}} = \frac{I}{V} \quad \text{S}$

Angle of Admittance: $\theta_y = -\theta_z = \theta_i - \theta_v$

Conductance (Real Part of admittance):

$$G = \frac{1}{R} = Y cos\theta_y \qquad \text{S}$$

Susceptance (Imaginary Part of admittance):

$$B = \frac{1}{X} = Y sin\theta_y \qquad \text{S}$$

Susceptance is the property of inductor and capacitor to help the flow of current. The are two susceptance in electrical circuit: (***i***) **inductive susceptance** (B_L), and (***ii***) **capacitive susceptance** (B_C).

Inductive Susceptance:

$$B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L} \text{ [S]} \qquad B_L \propto \frac{1}{f}$$

Capacitive Susceptance:

$$B_C = \frac{1}{X_C} = \omega C = 2\pi f C \text{ [S]} \qquad B_C \propto f$$

EXAMPLE The supply voltage and current of a circuit are $v(t) = 100\sin 314t$ V and $i(t) = 15\cos(314t–120^o)$ A.

(***a***) Find (***i***) the magnitude of impedance, (***ii***) the angle of impedance, (***iii***) the value of resistance, and (***iv***) the value of reactance.

(***b***) Find (***i***) the magnitude of admittance, (***ii***) the angle of admittance, (***iii***) the value of conductance, and (***iv***) the value of susceptance.

(***c***) Write the impedance and admittance in both polar and cartesian or rectangular form.

Solution: Converting current from cosine to sine, we have: $i(t) = 15\sin(314t–30^o)$ A.

Now, $V_m = 100$ V, $I_m = 15$ A, $\theta_v = 0^o$ and $\theta_i = –30^o$

$$(\boldsymbol{a})\ (\boldsymbol{i})\ \ Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100\text{V}}{15\text{A}} = \mathbf{6.67\ \Omega}$$

$$(\boldsymbol{ii})\ \ \theta_Z = \theta_v - \theta_i = 0° - (-30°) = \mathbf{30°}$$

$$(\boldsymbol{iii})\ \ R = Z\cos\theta_Z = 6.67 \times \cos(30°) = \mathbf{5.78\ \ \Omega}$$

$$(\boldsymbol{iv})\ \ X = Z\sin\theta_Z = 6.67 \times \sin(30°) = \mathbf{3.34\ \ \Omega}$$

$$(\boldsymbol{b})\ (\boldsymbol{i})\ \ Y = \frac{1}{Z} = \frac{I}{V} = \frac{I_m}{V_m} = \frac{15\text{A}}{100\text{V}} = \mathbf{0.15\ S}\ \text{ or }\ \mathbf{150\,mS}$$

$$(\boldsymbol{ii})\ \ \theta_y = -\theta_Z = \theta_i - \theta_v = -30° - 0° = -3\mathbf{0}°$$

$$(\boldsymbol{iii})\ \ G = Y\cos\theta_y = 150 \times \cos(-30°) = \mathbf{129.9\ \ mS}$$

$$(\boldsymbol{iv})\ \ B = Y\sin\theta_y = 150 \times \sin(-30°) = \mathbf{-75\ \ mS}$$

$$(\boldsymbol{c})\ \boldsymbol{Z} = \vec{Z} = 6.67\Omega\angle 30°$$

$$\boldsymbol{Z} = \vec{Z} = 5.78 + j3.34\ \Omega$$

$$\boldsymbol{Y} = \vec{Y} = 150\text{mS}\angle -30°$$

$$\boldsymbol{Y} = \vec{Y} = 129.9 + j75\ \text{mS}$$

EXAMPLE The supply voltage and current of a circuit are $\boldsymbol{V} = 200\text{V}\angle 90^{\circ}$ and $\boldsymbol{I} = 10\text{A}\angle 30^{\circ}$.
(***a***) Find the impedance and admittance in both polar and cartesian or rectangular form.
(***b***) Find (***i***) the magnitude of impedance, (***ii***) the angle of impedance, (***iii***) the value of resistance, and (***iv***) the value of reactance.
(***c***) Find (***i***) the magnitude of admittance, (***ii***) the angle of admittance, (***iii***) the value of conductance, and (***iv***) the value of susceptance.

Solution:

$$(\boldsymbol{a})\ \boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{200\text{V}\angle 90^{\circ}}{10\text{V}\angle 30^{\circ}} = 20\Omega\angle 60^{\circ} = 10 + j17.32\ \ \Omega$$

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Y}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{10\text{V}\angle 30^{\circ}}{200\text{V}\angle 90^{\circ}} = 005\text{S}\angle -60^{\circ} = 0.025 + j0.0433\ \ \text{S} = 25 + j43.3\ \ \text{mS}$$

$$(\boldsymbol{b})\ (\boldsymbol{i})\ \ Z = 20\ \ \ \ \Omega; \qquad (\boldsymbol{ii})\ \ \theta_Z = 60^{\circ}; \qquad (\boldsymbol{iii})\ R = 10\ \Omega; \qquad (\boldsymbol{iv})\ X = 17.32\ \Omega$$

$$(\boldsymbol{c})\ (\boldsymbol{i})\ \ Y = 0.05\ \text{S}; \qquad (\boldsymbol{ii})\ \ \theta_y = -60^{\circ}; \qquad (\boldsymbol{iii})\ G = 25\ \text{mS}; \qquad (\boldsymbol{iv})\ B = 43.3\ \text{mS}$$

EXAMPLE The supply voltage and impedance of a circuit are $v(t) = 282.84\cos 314t$ V and $\boldsymbol{Z} = 20\Omega\angle 60^{\circ}$. Find the current $i(t)$.

Solution: Converting voltage from cosine to sine, we have: $v(t) = 282.84\sin(314t+90^{\circ})$ V.

Now, $V_m = 282.84$ V, $\theta_v = 90^{\circ}$ and $Z = 20\ \Omega$, $\theta_z = 60^{\circ}$

We know that: $Z = \dfrac{V_m}{I_m}$ $\qquad \theta_z = \theta_v - \theta_i$

$$I_m = \frac{V_m}{Z} = \frac{282.84}{20} = 14.142 \text{ A}$$

$$\theta_i = \theta_v - \theta_z = 90^{\circ} - 60^{\circ} = 30^{\circ}$$

Thus, $i(t) = 14.142\sin(314t + 30^{\circ})$ A

EXAMPLE The supply current and impedance of a circuit are $i(t) = 15\sin 377t$ V and $\boldsymbol{Z} = 17.32 + j10\ \Omega$. Find the voltage $v(t)$.

Solution: Converting impedance from Cartesian to Polar form:

$$\boldsymbol{Z} = 17.32 + j10\ \Omega = 20\Omega\angle 30^{\circ}$$

Now, $I_m = 15$ V, $\theta_i = 0^{\circ}$ and $Z = 20\ \Omega$, $\theta_z = 30^{\circ}$

We know that: $Z = \dfrac{V_m}{I_m}$ $\qquad \theta_z = \theta_v - \theta_i$

$$V_m = ZI_m = 20 \times 15 = 300 \text{ V}$$

$$\theta_v = \theta_i + \theta_z = 0^{\circ} + 30^{\circ} = 30^{\circ}$$

Thus, $v(t) = 300\sin(377t + 30^{\circ})$ V

Voltage Source Given in Different Ways

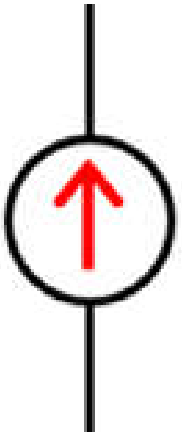
$v(t) = 100\sin(377t + 60°)\ \text{V}$

$\vec{V} = \boldsymbol{V} = 222\angle 40°\ \text{V}$

$\vec{V} = \boldsymbol{V} = 200 - j173.2\ \text{V}$

$220\ \text{V}$

Current Source Given in Different Ways



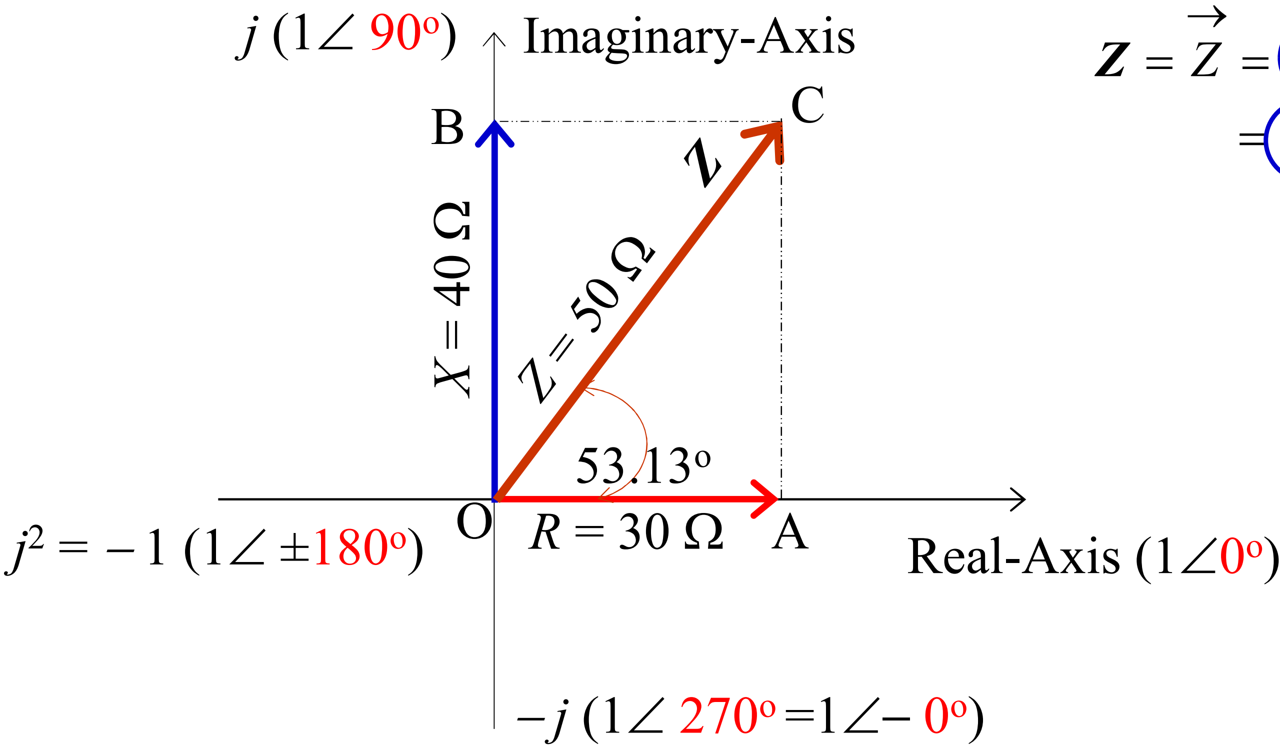
$i(t) = 10\sin(314t - 30°)\ \text{A}$

$\vec{I} = \boldsymbol{I} = 15\angle -50°\ \text{A}$

$\vec{I} = \boldsymbol{I} = 16 + j24\ \text{A}$

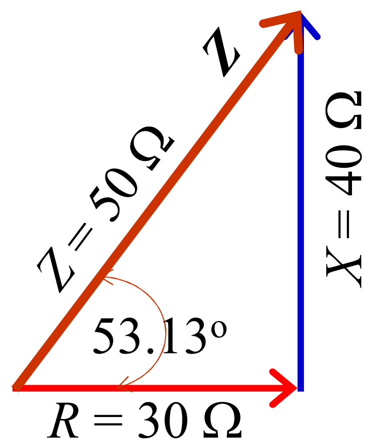
$18\ \text{A}$

Impedance Diagram



Parallelogram Method

$$\boldsymbol{Z} = \vec{Z} = 30 + j40\ \Omega$$
$$= 50\Omega \angle 53.13^\circ$$

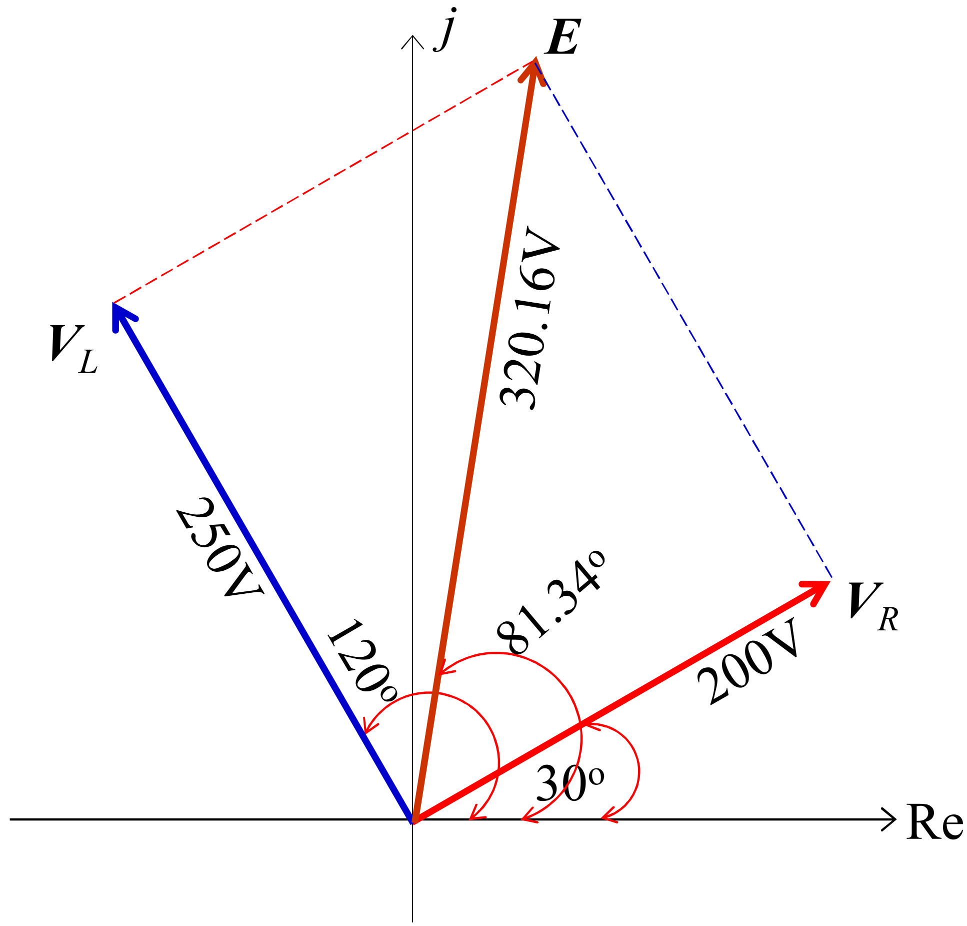


Moving Method

$$\boldsymbol{V}_R = 200\text{V}\angle 30°$$

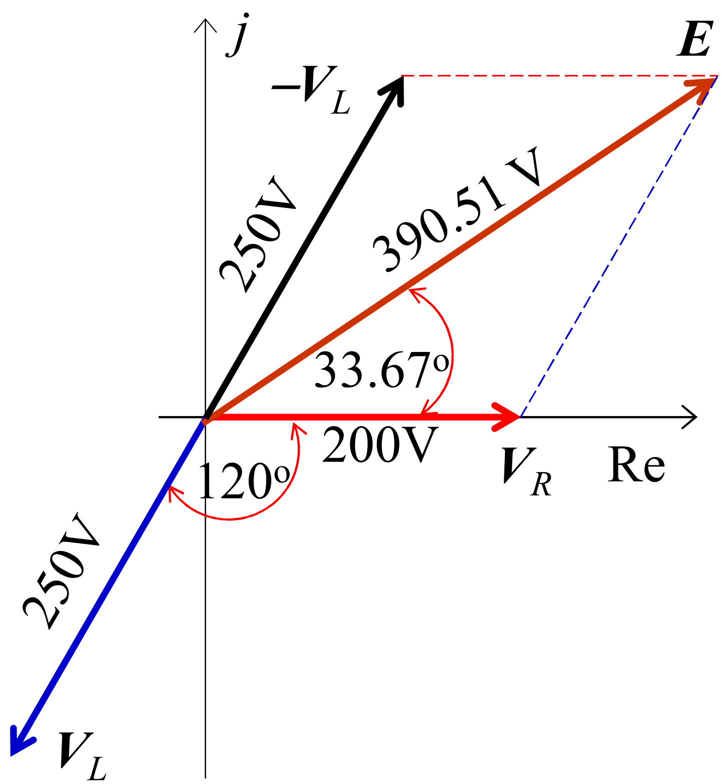
$$\boldsymbol{V}_L = 250\text{V}\angle 120°$$

$$\boldsymbol{E} = \boldsymbol{V}_R + \boldsymbol{V}_L = 320.16\text{V}\angle 81.34°$$



Phasor or Vector Diagram

Parallelogram Method



$$V_R = 200\text{V}\angle 0°$$

$$V_L = 250\text{V}\angle -120°$$

$$E = V_R - V_L$$
$$= 390.51\angle 33.67°$$

POWER CALCULATION IN AC CIRCUIT

Instantaneous Power [$p(t)$]

Let, instantaneous voltage and current are:

$$v(t) = V_m \sin(\omega t + \theta_v) \quad [\text{V}]$$

$$i(t) = I_m \sin(\omega t + \theta_i) \quad [\text{A}]$$

By shifting the angle θ_i the instantaneous voltage and current are:

$$v(t) = V_m \sin(\omega t + \theta) \quad [\text{V}]$$

$$i(t) = I_m \sin \omega t \quad [\text{A}]$$

where, $\theta = \theta_z = (\theta_v - \theta_i)$

and θ is called **Power Factor Angle**.

The instantaneous power is as follows:

$$p(t) = v(t)i(t) = V_m \sin(\omega t + \theta) I_m \sin \omega t \quad [\text{W}]$$

After simplification, the instantaneous power can be written as:

$$p(t) = \underbrace{P(1 - \cos 2\omega t)}_{\textbf{Active Power}} + \underbrace{Q \sin 2\omega t}_{\textbf{Reactive Power}}$$

where, $P = \dfrac{V_m I_m}{2} \cos\theta$ W $\quad Q = P_x = \dfrac{V_m I_m}{2} \sin\theta$ Var

$$P = V_{rms} I_{rms} \cos\theta = VI \cos\theta \quad \text{W} \qquad \textbf{(14.13)}$$

$$Q = V_{rms} I_{rms} \sin\theta = VI \sin\theta \quad \text{Var} \qquad \textbf{(19.12)}$$

$$\because \; V = \frac{V_m}{\sqrt{2}} \quad I = \frac{I_m}{\sqrt{2}}$$

The first term [$\boldsymbol{P(1 - \cos 2\omega t)}$] in the preceding equation is called **instantaneous real** [or **true** or **active** or **wattfull** or **useful**] **power**.
The unit of instantaneous real power is watt [**W**].

The second term [$\boldsymbol{Q \sin 2\omega t}$] in the preceding equation is called **instantaneous reactive volt-ampere** or **instantaneous reactive** [or **imaginary** or **wattless** or **useless** or **quadrature**] **power**.
The unit of instantaneous reactive power is volt-ampere reactive [**Var**].

Power (or Average or Real or Active or True or Wattfull or Usefull Power)

The average value can be obtained by:

$$P_{ave} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T [P(1-\cos 2\omega t) + Q\sin 2\omega t]dt = P = \frac{V_m I_m}{2}\cos\theta = VI\cos\theta \quad \text{W}$$

The average power is also called ***real/active/true/wattfull/usefull power or*** **simply** ***Power***. The **unit** of real power is **watt**. The real power is measured by **wattmeter**.

Real power converts from electrical energy to other form of energy. This is happened in resistive circuit.

Reactive or Imaginary or Quadrature or Wattless or Useless Power (or Reactive Volt-Ampere)

The **peak or maximum value of instantaneous reactive power** (or instantaneous reactive volt-ampere) is called the ***reactive/imaginary/quadrature/wattless/useless power*** (or ***reactive volt-ampere***). The **unit** of reactive power is called **var (reactive volt-ampere)**. The reactive power is measured by **varmeter**. It is given by:

$$Q = P_x = \frac{V_m I_m}{2}\sin\theta \quad [\text{var}] = VI\sin\theta \quad [\text{var}]$$

Reactive power is used for storing energy. This is happened in inductive and capacitive circuit.

Reactive power is **positive** (for **Inductive load**) or **negative** (for **Capacitive load**).

Apparent Power or Volt-Ampere

Apparent power is the product of the rms value of voltage and the rms value of current.

The **unit** of apparent power is called **VA (volt-ampere)**.

$$S = \sqrt{P^2 + Q^2} \quad [\text{VA}] \; = \frac{V_m I_m}{2} \quad [\text{VA}]$$

$$= V_{rms} I_{rms} = VI \quad [\text{VA}]$$

Power Factor

Cosine θ ($\cos\theta$) which is a factor, by which volt-amperes are multiplied to give power, is called power factor. Power factor is always **positive**. Power factor can be given by:

$$pf = F_p = \cos\theta = \cos\theta_z = \frac{P}{S} \qquad 0 \le pf \le 1$$

Unity Power Factor: If $\theta = \theta_z = \theta_v - \theta_i = 0^o$ the **power factor is 1** which is called **unity power factor**.

Lagging Power Factor: If $\theta = \theta_z = \theta_v - \theta_i > 0^o$ then **current lags** voltage which is called **lagging power factor**.

Leading Power Factor: If $\theta = \theta_z = \theta_v - \theta_i < 0^o$ then **current leads** voltage which is called **leading power factor**.

Reactive Factor

Sine θ ($\sin\theta$) which is a factor, by which volt-amperes are multiplied to give reactive power, is called reactive factor.

Reactive factor may be **positive** (for Inductive load) or **negative** (for Capacitive load). Reactive factor can be given by:

$$rf = F_q = \sin\theta = \sin\theta_z = \frac{Q}{S} \qquad -1 \le rf \le 1$$

Complex Power

Voltage and Current in Cartesian Form

$$\boldsymbol{V} = V\angle\theta_v = V\cos\theta_v + jV\sin\theta_v = V_r + jV_i$$

$$V_r = V\cos\theta_v; \qquad V_i = V\sin\theta_v$$

$$\boldsymbol{I} = I\angle\theta_i = I\cos\theta_i + jI\sin\theta_i = I_r + jI_i$$

$$I_r = I\cos\theta_i; \qquad I_i = I\sin\theta_i$$

Real or Active or Average Power

$$P = VI\cos\theta = VI\cos(\theta_v - \theta_i)$$
$$= VI\cos\theta_v\cos\theta_i + VI\sin\theta_v\sin\theta_i = V_rI_r + V_iI_i$$

Reactive or Imaginary or Quadrature Power

$$Q = VI\sin\theta = VI\sin(\theta_v - \theta_i)$$
$$= VI\sin\theta_v\cos\theta_i - VI\cos\theta_v\sin\theta_i = V_iI_r - V_rI_i$$

Complex Power by Conjugate Current

Using the previous equations of P and Q, the complex power can be written as follows:

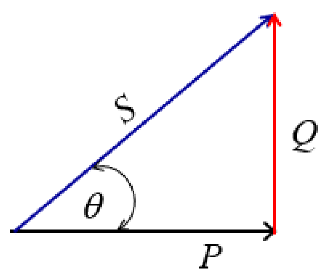
$$\boldsymbol{S} = P + jQ = (V_rI_r + V_iI_i) + j(V_iI_r - V_rI_i)$$

$$\boldsymbol{S} = P + jQ = (V_r + jV_i)(I_r - jI_i) = \boldsymbol{VI}^* = S\angle\theta_s$$

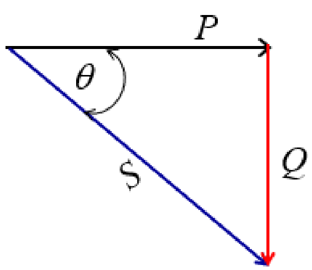
$$P = \mathrm{Re}[\boldsymbol{S}] = \mathrm{Re}[\boldsymbol{VI}^*]; \qquad Q = \mathrm{Im}[\boldsymbol{S}] = \mathrm{Im}[\boldsymbol{VI}^*]$$

Power Triangle

Graphical representation of **active power**, **reactive power**, and **apparent power** in a complex plane is called power triangle.



When Q is Positive



When Q is Negative

EXAMPLE Determine the power factor, the reactive factor and indicate whether power factor is leading or lagging for the following input voltage and current pairs of a network:

(*a*) $v(t) = 150\sin(377t + 70°)$ V
$i(t) = 3\sin(377t + 10°)$ A

(*b*) $v(t) = 100\sin(314t - 50°)$ V
$i(t) = 12\sin(314t + 40°)$ A

(*c*) $v(t) = 120\sin(157t + 30°)$ V
$i(t) = 8\cos(157t - 110°)$ A

(*d*) $v(t) = -80\cos(200t + 60°)$ V
$i(t) = 5\sin(200t - 30°)$ A

Solution: (*a*) Here, $\theta_v = 70^o$ and $\theta_i = 10^o$, thus

$$\theta = \theta_v - \theta_i = 70^o - 10^o = 60^o$$

$$pf = \cos\theta = \cos(60°) = \mathbf{0.5\ \ lagging}$$

$$rf = \sin\theta = \sin(60°) = \mathbf{0.866}$$

(*b*) Here, $\theta_v = -50^o$ and $\theta_i = 40^o$, thus

$$\theta = \theta_v - \theta_i = -50^o - (40^o) = -90^o$$

$$pf = \cos\theta = \cos(-90°) = \mathbf{0\ \ leading\ power\ factor}$$

$$rf = \sin\theta = \sin(-90°) = \mathbf{-1}$$

(*c*) $i(t) = 8\cos(157t - 110°) = 8\sin(157t + 90° - 150°)$ A
$i(t) = 8\sin(157t - 60°)$ A
Here, $\theta_v = 30^o$ and $\theta_i = -60^o$, thus

$$\theta = \theta_v - \theta_i = 30^o - (-60^o) = 90^o$$

$$pf = \cos\theta = \cos(90°) = \mathbf{0\ \ lagging\ power\ factor}$$

$$rf = \sin\theta = \sin(90°) = \mathbf{1}$$

(*d*) $v(t) = -80\cos(200t + 60°) = 80\sin(200t - 90° + 60°)$ V
$v(t) = 80\sin(200t - 30°)$ V
Here, $\theta_v = -30^o$ and $\theta_i = -30^o$, thus

$$\theta = \theta_v - \theta_i = -30^o - (-30^o) = 0^o$$

$$pf = \cos\theta = \cos(0°) = \mathbf{1\ \ unity\ power\ factor}$$

$$rf = \sin\theta = \sin(0°) = \mathbf{0}$$

EXAMPLE The supply voltage and current of a circuit are $v(t) = 100\sin(314t + 80°)$ V and $i(t) = 12\sin(377t + 50°)$ A.

(**a**) Calculate the power factor, the reactive factor and comment on the power factor.
(**b**) Calculate the power, the reactive power and the apparent power delivered by source.
(**c**) Write the instantaneous power equation.
(**d**) Draw the power triangle.

Solution: (***a***) Here, $\theta_v = 80^o$ and $\theta_i = 50^o$, thus

$$\theta = \theta_v - \theta_i = 80^o - 50^o = 30^o$$

$$pf = \cos\theta = \cos(30^o) = \mathbf{0.866}$$
$$rf = \sin\theta = \sin(30^o) = \mathbf{0.5}$$

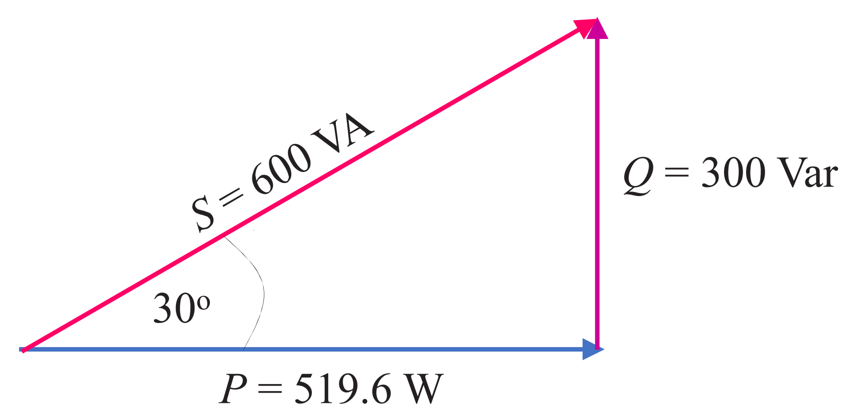
(**b**) $P = \frac{V_m I_m}{2} cos\theta = VI\cos\theta = \frac{100 \times 12}{2} \times 0.866 = \mathbf{519.6\ W}$

$$Q = \frac{V_m I_m}{2} sin\theta = VI\sin\theta = \frac{100 \times 12}{2} \times 0.5 = \mathbf{300\ Var}$$

$$S = \frac{V_m I_m}{2} = VI = \frac{100 \times 12}{2} = \mathbf{600\ VA}$$

(***c***) $p(t) = P(1 - \cos2\omega t) + Q\sin2\omega t$ W
$= 519.6(1-\cos754t) + 300\sin754t$ W

(***d***) Power triangle:



EXAMPLE The supply voltage and current of a circuit are $\boldsymbol{V} = 150\text{V}\angle 50°$ and $\boldsymbol{I} = 5\text{A}\angle 110°$.
(**a**) Calculate the complex power an represent it in both polar and cartesian forms.
(**b**) From the result of (a), find the real power, the reactive power and the apparent power.
(**c**) Calculate the power factor and the reactive factor and make commend on power factor.
(**d**) Write the instantaneous power equation for the 400 rad/s of source voltage.
(**e**) Draw the power triangle.

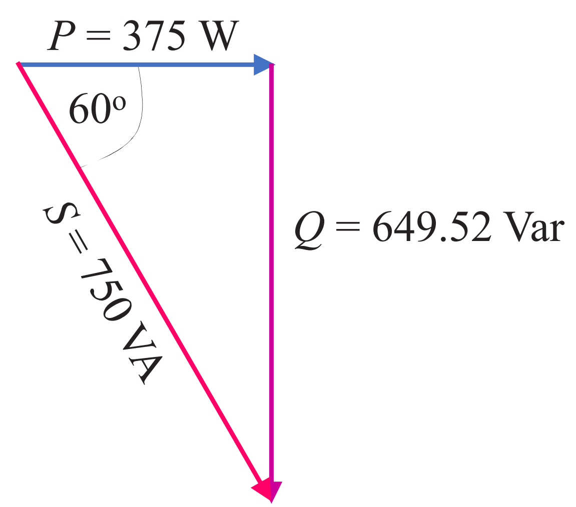
Solution: (***a***)
$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = (150\text{V}\angle 50°)(5\text{A}\angle 110°)^*$$
$$= (150\text{V}\angle 50°)(5\text{A}\angle -110°)$$
$$= 750\text{VA}\angle -60°$$
$$= 375 - j649.52 \text{ VA} = P + jQ$$

(***b***) From (a) we have:
$P = 375$ W, $Q = -649.52$ Var and $S = 750$ VA

(***c***)
$$pf = \frac{P}{S} = \frac{375\ W}{750\ VA} = \mathbf{0.5\ Leading}$$
$$rf = \frac{Q}{S} = \frac{-649.52\ Var}{750\ VA} = \mathbf{-0.866}$$

(***d***)
$$p(t) = P(1 - \cos 2\omega t) + Q\sin 2\omega t \text{ W}$$
$$= 375(1 - \cos 800t) - 649.52\sin 800t \text{ W}$$

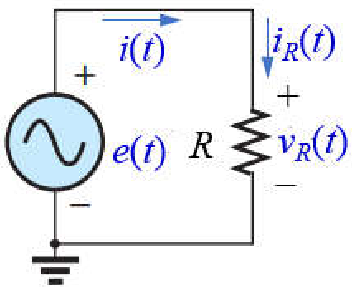
(***e***) Power triangle:



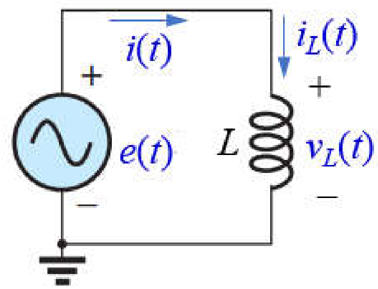
Pure Resistive, Pure Inductive and Pure Capacitive Circuits

Based on Instantaneous Equations

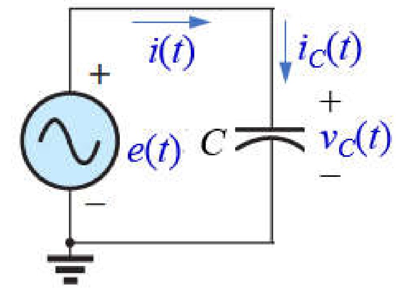
PURE RESISTIVE CIRCUIT



PURE INDUCTIVE CIRCUIT



PURE CAPACITIVE CIRCUIT



Instantaneous or Transient or Time-domain Voltage and Current Relation

$$v_R(t) = Ri_R(t) \qquad i_R(t) = \frac{v_R(t)}{R}$$

$$v_R(t) = e(t) \qquad i_R(t) = i(t)$$

$$v_L(t) = L\frac{di_L(t)}{dt} \qquad i_L(t) = \frac{1}{L}\int v_L(t)dt$$

$$v_L(t) = e(t) \qquad i_L(t) = i(t)$$

$$v_C(t) = \frac{1}{C}\int i_C(t)dt \qquad i_C(t) = C\frac{di_C(t)}{dt}$$

$$v_C(t) = e(t) \qquad i_C(t) = i(t)$$

Let, the input is $e(t) = E_m \sin(\omega t + \theta_e)$V; according to KVL and KCL, we have:

$$i(t) = \frac{E_m}{R}\sin(\omega t + \theta_e)$$

$$i(t) = \frac{E_m}{L}\int \sin(\omega t + \theta_e)dt$$

$$= \frac{E_m}{X_L}\sin(\omega t + \theta_e - 90°)$$

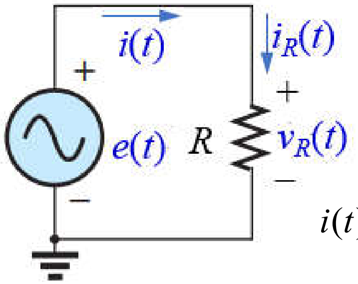
where, $X_L = \omega L = 2\pi f L \;\; \Omega$

$$i(t) = C\frac{d}{dt}[E_m \sin(\omega t + \theta_e)]$$

$$= \frac{E_m}{X_C}\sin(\omega t + \theta_e + 90°)$$

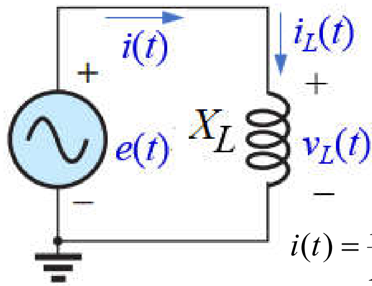
where, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} \;\; \Omega$

PURE RESISTIVE CIRCUIT



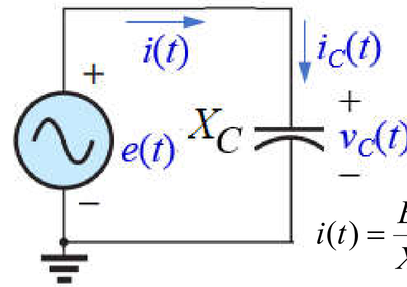
$$i(t) = \frac{E_m}{R}\sin(\omega t + \theta_e)$$

PURE INDUCTIVE CIRCUIT



$$i(t) = \frac{E_m}{X_L}\sin(\omega t + \theta_e - 90°)$$

PURE CAPACITIVE CIRCUIT



$$i(t) = \frac{E_m}{X_C}\sin(\omega t + \theta_e + 90°)$$

Compare the obtained current equation with general current equation of $i(t) = I_m \sin(\omega t + \theta_i)$ A, we have

Resistive:

$$I_m = \frac{E_m}{R} = \frac{V_{Rm}}{R}; \qquad \theta_i = \theta_e$$

$$E_m = V_{Rm} = RI_{Rm} = RI_m$$

$$\theta_{vR} = \theta_{iR}$$

V_{Rm}: Peak value of resistor voltage
I_{Rm}: Peak value of resistor current

Inductive:

$$I_m = \frac{E_m}{X_L}; \qquad \theta_i = \theta_e - 90°$$

$$E_m = V_{Lm} = X_L I_{Lm} = X_L I_m$$

$$\theta_e = \theta_i + 90° \qquad \theta_{vL} = \theta_{iL} + 90°$$

V_{Lm}: Peak value of inductor voltage
I_{Lm}: Peak value of inductor current

Capacitive:

$$I_m = \frac{E_m}{X_C}; \qquad \theta_i = \theta_e + 90°$$

$$E_m = V_{Cm} = X_C I_{Cm} = X_C I_m$$

$$\theta_e = \theta_i - 90° \qquad \theta_{vC} = \theta_{iC} + 90°$$

V_{Cm}: Peak value of capacitor voltage
I_{Cm}: Peak value of capacitor current

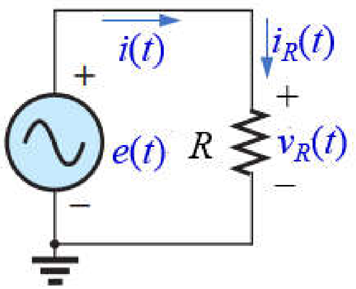
Impedance Magnitude and Impedance Angle

$$Z = \frac{E_m}{I_m} = R; \qquad \theta_z = \theta_e - \theta_i = 0°$$

$$Z = \frac{E_m}{I_m} = X_L; \qquad \theta_z = \theta_e - \theta_i = 90°$$

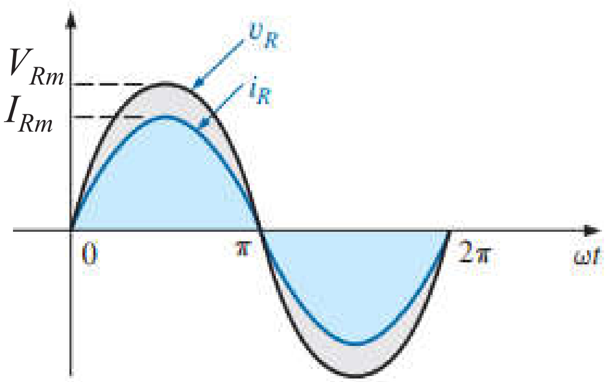
$$Z = \frac{E_m}{I_m} = X_C; \qquad \theta_z = \theta_e - \theta_i = -90°$$

PURE RESISTIVE CIRCUIT



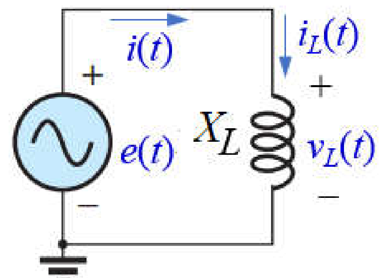
$Z = R$

$\theta_z = 0°$



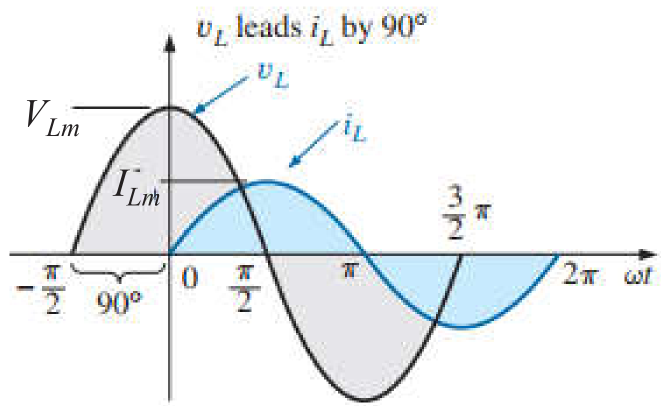
The phase difference $v_R(t)$ and $i_R(t)$ is **0º**. $v_R(t)$ and $i_R(t)$ are **in phase**.

PURE INDUCTIVE CIRCUIT



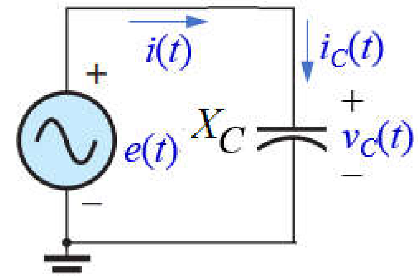
$Z = X_L$

$\theta_z = 90°$



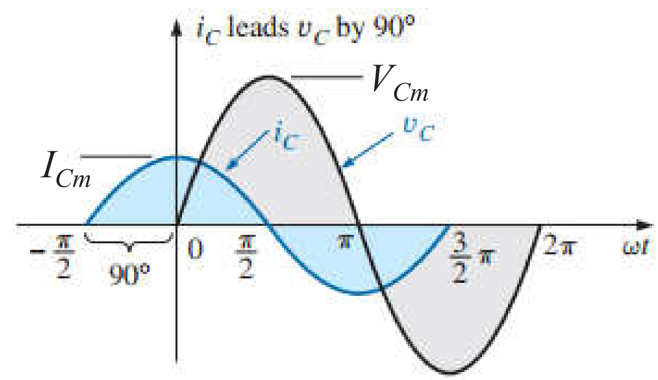
The phase difference $v_L(t)$ and $i_L(t)$ is **90º**. $v_L(t)$ **leads** and $i_L(t)$ or $i_L(t)$ **lags** $v_L(t)$.

PURE CAPACITIVE CIRCUIT



$Z = X_C$

$\theta_z = -90°$



The phase difference $v_C(t)$ and $i_C(t)$ is **90º**. $v_C(t)$ **lags** and $i_C(t)$ or $i_C(t)$ **leads** $v_C(t)$.

Phase Relation Between Voltage and Current

X_L VERSUS FREQUENCY (f) CURVE

Inductive reactance is **directly proportional** to frequency ($X_L \propto f$) so the inductive reactance versus frequency curve is a straight line with slope equal to $2\pi L$.

If frequency **decreases**, inductive reactance will be **decreases**.

f frequency **increases**, inductive reactance will be **increases**.

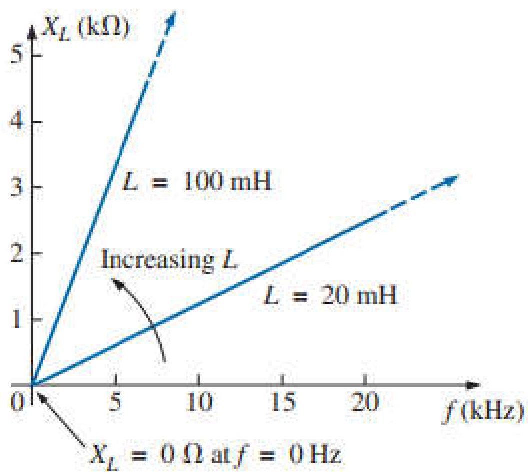


FIG. 14.20 X_L *versus frequency.*

Inductive Reactance With DC Supply

For DC voltage the **frequency is zero** so inductive reactance with DC supply is ***zero*** that means inductor behave as a ***short-circuit*** with DC input.

X_C VERSUS FREQUENCY (f) CURVE

Capacitive reactance is **inversely proportional** to frequency ($X_C \propto 1/f$) so the capacitive reactance versus frequency curve is a *rectangular hyperbola*.

If frequency **decreases**, capacitive reactance will be **increases**.

If frequency **increases**, capacitive reactance will be **decreases**.

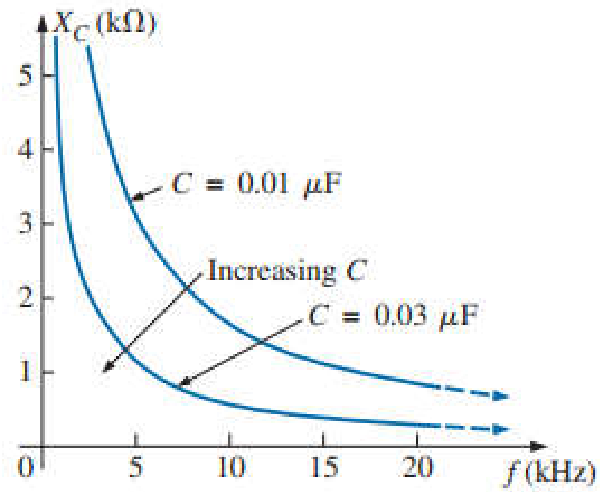


FIG. 14.22 X_C *versus frequency.*

Capacitive Reactance With DC Supply

For DC voltage the frequency is zero so capacitive reactance with DC supply is ***infinity*** that means capacitor behave as an ***open-circuit*** with DC input.

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for $v(t)$ and $i(t)$.

$$v(t) = 100\sin 377t \text{ V}$$

Solution: (a) Given, $V_m = 100$ V, $\theta_v = 0^o$, $\omega = 377$ rad/s and $R = 10\ \Omega$

For a resistive circuit, we know that

$$I_m = \frac{V_m}{R} \qquad \theta_i = \theta_v$$

thus $$I_m = \frac{100\text{V}}{10\Omega} = 10\text{ A} \qquad \theta_i = \theta_v = 0°$$

The sinusoidal expression of current is:

$$\boldsymbol{i(t) = 10\sin 377t \text{ A}}$$

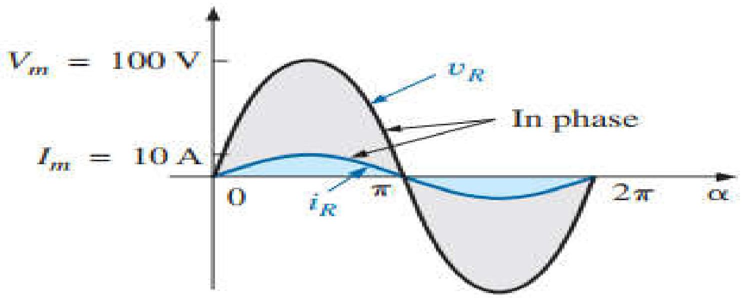


FIG. 14.13 *Example 14.1(a).*

EXAMPLE 14.2 The current through a 5 Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for:

$$i(t) = 40\sin(377t + 30°) \text{ A.}$$

Solution:

(a) Given, $I_m = 40$ A, $\theta_i = 30^o$, $\omega = 377$ rad/s and $R = 5\ \Omega$

We know that $I_m = \dfrac{V_m}{R}$; $\quad \theta_i = \theta_v$

thus $V_m = RI_m = (5\Omega)(40\text{ A}) = 200$ V

$$\theta_v = \theta_i = 30°$$

The sinusoidal expression of voltage is:

$$\boldsymbol{v(t) = 200\sin(377t + 30°) \text{ V}}$$

Practice Book Problems [Ch. 14] 4 and 5

EXAMPLE 14.3(a) The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil for $i(t)$ = 10sin377t A. Sketch the curves for $v(t)$ and $i(t)$.

Solution: (a) Given, I_m = 10 A, θ_i = 0°, ω = 377 rad/s and L = 0.1 H

We know that, $X_L = \omega L = (377\,\text{rad/s})(0.1\,\text{H}) = 37.3\ \Omega$

We know that $I_m = \dfrac{V_m}{X_L}$; $\quad \theta_i = \theta_v - 90°$

thus $V_m = X_L I_m = (37.7\,\Omega)(10\,\text{A}) = 377\ \text{A}$

$\theta_v = \theta_i + 90° = 90°$

The sinusoidal expression of voltage is:

$v(t)$ = 377sin(377t + 90°) V

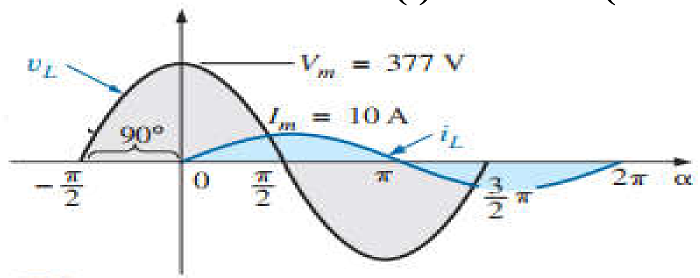


FIG. 14.15 *Example 14.3(a).*

EXAMPLE 14.4 The voltage across a 0.5 H coil is provided. Find the sinusoidal expression for the current through the coil for $v(t)$ = 100sin(20t + 30°) V.

Solution: (a) Given, V_m = 100 V, θ_v = 30°, and ω = 20 rad/s and L = 0.5 H

$X_L = \omega L = (20\,\text{rad/s})(0.5\,\text{H}) = 10\ \Omega$

We know that $I_m = \dfrac{V_m}{X_L}$; $\quad \theta_i = \theta_v - 90°$

thus $I_m = \dfrac{100\text{V}}{10\Omega} = 10\,\text{A} \qquad \theta_i = 30° - 90° = -60°$

The sinusoidal expression of current is:

$i(t)$ = 10sin(20t – 60°) A

Practice Book Problems [Ch. 14] 6 and 12

EXAMPLE 14.5 The voltage across a 1 μF capacitor is provided. Find the sinusoidal expression for the current through the capacitor for $v(t) = 30\sin 400t$ V. Sketch the curves for $v(t)$ and $i(t)$.

Solution: (a) Given, $V_m = 30$ V, $\theta_v = 0°$, $\omega = 400$ rad/s and $C = 1\ \mu\text{F} = 1\times10^{-6}$ F

We know that, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(400\,\text{rad/s})(1\times10^{-6}\,\text{F})} = \dfrac{10^6}{400}\,\Omega = 2500\,\Omega$

We know that $I_m = \dfrac{V_m}{X_C}$; $\quad \theta_i = \theta_v + 90°$

thus $I_m = \dfrac{30\text{V}}{2500\Omega} = 0.012\,\text{A} = 12\ \text{mA}$

$\theta_i = 0° + 90° = 90°$

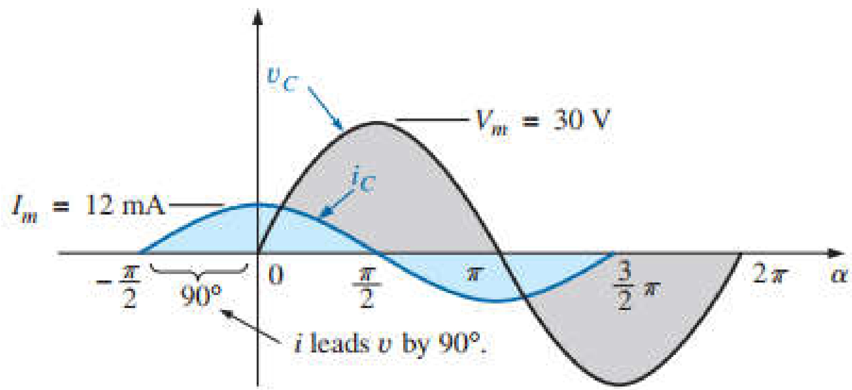


FIG. 14.17 *Example 14.5.*

The sinusoidal expression of current is:

$$\boldsymbol{i(t) = 12\sin(20t + 90°)\ \text{mA} = 12\times10^{-3}\sin(20t + 90°)\ \text{A}}$$

EXAMPLE 14.6 The current through a 100 μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor for $i(t) = 40\sin(500t + 60°)$ A. Sketch the curves for $v(t)$ and $i(t)$.

Solution: (a) Given, $I_m = 40$ A, $\theta_i = 60°$, $\omega = 500$ rad/s and $C = 100$ μF $= 100\times10^{-6}$ F

We know that, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(500\,\text{rad/s})(100\times10^{-6}\,\text{F})} = \dfrac{10^6}{5\times10^4}\,\Omega = \dfrac{10^2}{5}\,\Omega = 20\,\Omega$

We know that $I_m = \dfrac{V_m}{X_C}$; $\quad \theta_i = \theta_v + 90°$

thus $V_m = X_C I_m = (20\,\Omega)(40\,\text{A}) = 800\,\text{V}$

$$\theta_v = \theta_i - 90° = 60° - 90° = -30°$$

The sinusoidal expression of voltage is: $\boldsymbol{v(t) = 800\sin(500t - 30°)}$ **V**

Practice Book Problems [Ch. 14] 13 and 19

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C, L, or R if sufficient data are provided (Fig. 14.18):

a. $v = 100 \sin(\omega t + 40°)$
$i = 20 \sin(\omega t + 40°)$

b. $v = 1000 \sin(377t + 10°)$
$i = 5 \sin(377t - 80°)$

c. $v = 500 \sin(157t + 30°)$
$i = 1 \sin(157t + 120°)$

d. $v = 50 \cos(\omega t + 20°)$
$i = 5 \sin(\omega t + 110°)$

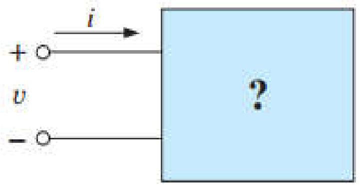


FIG. 14.18 *Example 14.7.*

Solutions:

a. Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{100\text{ V}}{20\text{ A}} = \mathbf{5\ \Omega}$$

b. Since v *leads* i by 90°, the element is an *inductor*, and

$$X_L = \frac{V_m}{I_m} = \frac{1000\text{ V}}{5\text{ A}} = 200\ \Omega$$

so that $X_L = \omega L = 200\ \Omega$ or

$$L = \frac{200\ \Omega}{\omega} = \frac{200\ \Omega}{377\text{ rad/s}} = \mathbf{0.53\ H}$$

c. Since i *leads* v by 90°, the element is a *capacitor*, and

$$X_C = \frac{V_m}{I_m} = \frac{500\text{ V}}{1\text{ A}} = 500\ \Omega$$

so that $X_C = \dfrac{1}{\omega C} = 500\ \Omega$ or

$$C = \frac{1}{\omega 500\ \Omega} = \frac{1}{(157\text{ rad/s})(500\ \Omega)} = \mathbf{12.74\ \mu F}$$

d. $v = 50 \cos(\omega t + 20°) = 50 \sin(\omega t + 20° + 90°)$
$= 50 \sin(\omega t + 110°)$

Since v and i are *in phase*, the element is a *resistor*, and

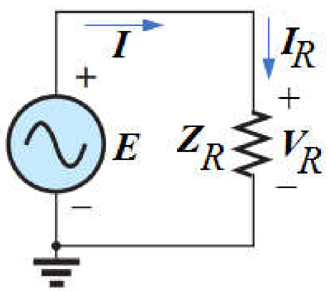
$$R = \frac{V_m}{I_m} = \frac{50\text{ V}}{5\text{ A}} = \mathbf{10\ \Omega}$$

Practice Book Problems [Ch. 14] 20 and 21

Pure Resistive, Pure Inductive and Pure Capacitive Circuits Based on Complex or Phasor Algebra

PURE RESISTIVE CIRCUIT

Circuit in Phasor Notation



$$V_R = E$$
$$I_R = I$$
$$Z = Z_R = \frac{V_R}{I_R}$$

$$V_R = V_R \angle \theta_{vR} \qquad I_R = I_R \angle \theta_{iR}$$

$$Z_R = Z_R \angle \theta_{zR} \qquad Z_R = \frac{V_R}{I_R} = R$$

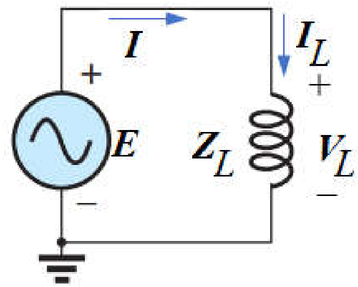
V_R: rms value of resistor voltage
I_R: rms value of resistor current

Impedance in Both Polar Form and Cartesian or Rectangular Form

$$Z = Z_R = \frac{V_R}{I_R} \ \Omega$$

$$Z = Z_R = R\angle 0^\circ \ \Omega = R + j0 \ \Omega$$

PURE INDUCTIVE CIRCUIT



$$V_L = E$$
$$I_L = I$$
$$Z = Z_L = \frac{V_L}{I_L}$$

$$V_L = V_L \angle \theta_{vL} \qquad I_L = I_L \angle \theta_{iL}$$

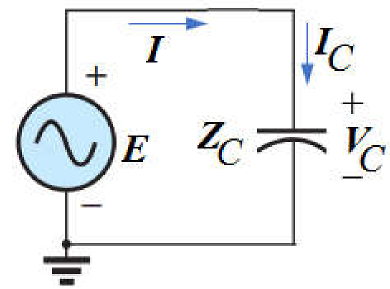
$$Z_L = Z_L \angle \theta_{zL} \qquad Z_L = \frac{V_L}{I_L} = X_L$$

V_L: rms value of inductor voltage
I_L: rms value of inductor current

$$Z = Z_L = \frac{V_L}{I_L} \ \Omega$$

$$Z = Z_L = X_L \angle 90^\circ \ \Omega = 0 + jX_L \ \Omega$$

PURE CAPACITIVE CIRCUIT



$$V_C = E$$
$$I_C = I$$
$$Z = Z_C = \frac{V_C}{I_C}$$

$$V_C = V_C \angle \theta_{vC} \qquad I_C = I_C \angle \theta_{iC}$$

$$Z_C = Z_C \angle \theta_{zC} \qquad Z_C = \frac{V_C}{I_C} = X_C$$

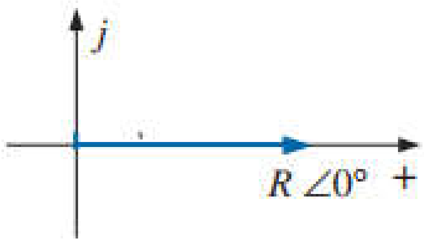
V_C: rms value of capacitor voltage
I_C: rms value of capacitor current

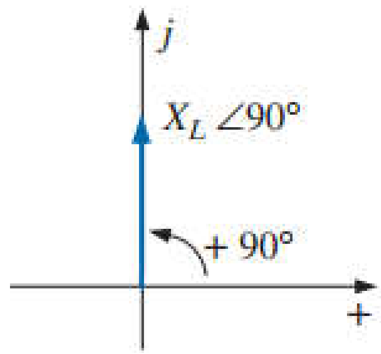
$$Z = Z_C = \frac{V_C}{I_C} \ \Omega$$

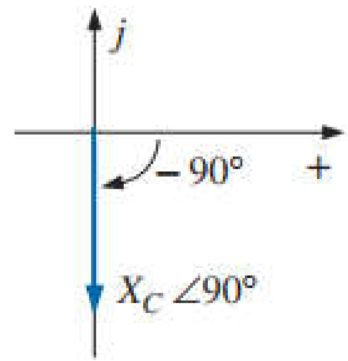
$$Z = Z_C = X_C \angle -90^\circ \ \Omega = 0 - jX_C \ \Omega$$

PURE RESISTIVE CIRCUIT	PURE INDUCTIVE CIRCUIT	PURE CAPACITIVE CIRCUIT

Impedance Diagram







Admittance in Both Polar Form and Cartesian or Rectangular Form

Pure resistive circuit:

$$\boldsymbol{Y} = \boldsymbol{Y}_R = \frac{\boldsymbol{I}_R}{\boldsymbol{V}_R} = \frac{1}{\boldsymbol{Z}_R}$$

$$\boldsymbol{Y} = \boldsymbol{Y}_R = G\angle 0° \ \text{S} = G + j0 \ \text{S}$$

where, $G = \dfrac{1}{R}$ S

Pure inductive circuit:

$$\boldsymbol{Y} = \boldsymbol{Y}_L = \frac{\boldsymbol{I}_L}{\boldsymbol{V}_L} = \frac{1}{\boldsymbol{Z}_L}$$

$$\boldsymbol{Y} = \boldsymbol{Y}_L = B_L\angle -90° \ \text{S} = 0 - jB_L \ \text{S}$$

where, $B_L = \dfrac{1}{X_L}$ S

Pure capacitive circuit:

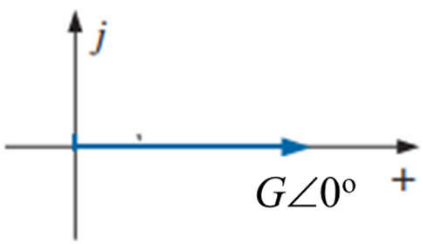
$$\boldsymbol{Y} = \boldsymbol{Y}_C = \frac{\boldsymbol{I}_C}{\boldsymbol{V}_C} = \frac{1}{\boldsymbol{Z}_C}$$

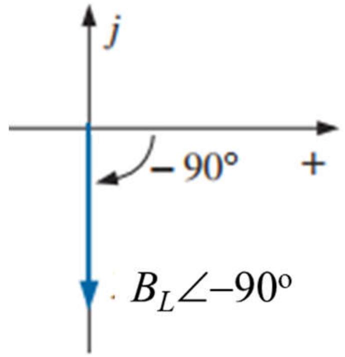
$$\boldsymbol{Y} = \boldsymbol{Y}_C = B_C\angle 90° \ \text{S} = 0 + jB_C \ \text{S}$$

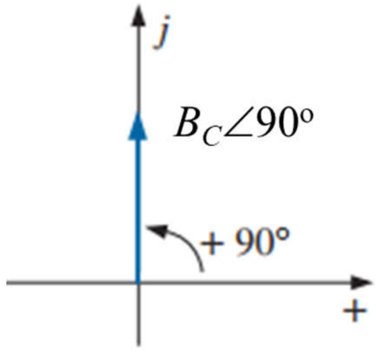
where, $B_C = \dfrac{1}{X_C}$ S

PURE RESISTIVE CIRCUIT	PURE INDUCTIVE CIRCUIT	PURE CAPACITIVE CIRCUIT

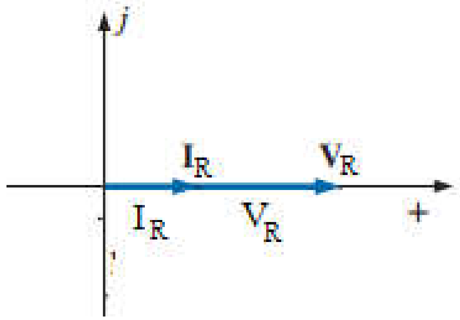
Admittance Diagram

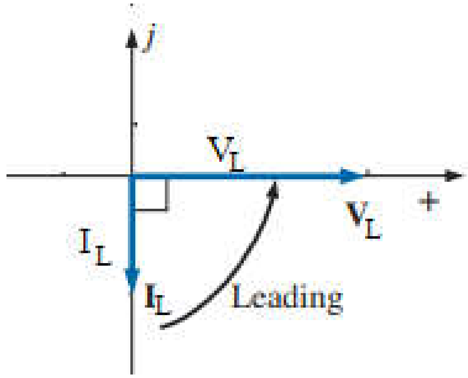


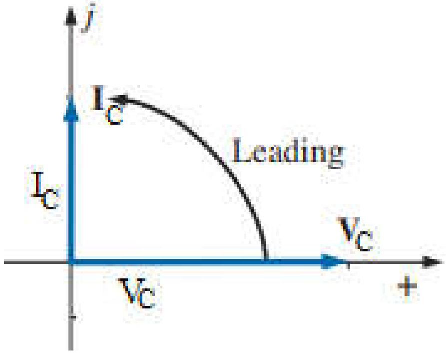




Phasor Diagram

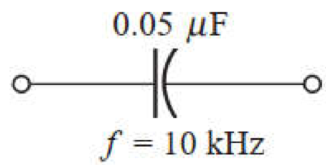






Problem 1 [Ch. 15] Express the impedances of following figure in both polar and rectangular forms.

$$\boldsymbol{Z} = 6.8\Omega\angle 0° = 6.8\ \Omega$$

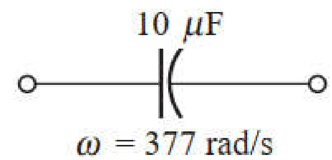


$$\omega = 2\pi \times 10 \times 10^3 = 62.8\ \text{krad/s}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(62.8 \times 10^3\ \text{rad/s}\right)\left(0.05 \times 10^{-6}\ \text{F}\right)}$$

$$= 318.47\ \Omega$$

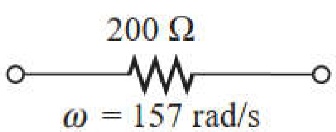
$$\boldsymbol{Z} = 318.47\Omega\angle -90° = -j318.47\ \Omega$$



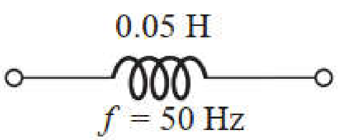
$$X_C = \frac{1}{\omega C} = \frac{1}{\left(377\text{rad/s}\right)\left(10 \times 10^{-6}\ \text{F}\right)}$$

$$= 265.25\ \Omega$$

$$\boldsymbol{Z} = 265.25\Omega\angle -90° = -j265.25\ \Omega$$



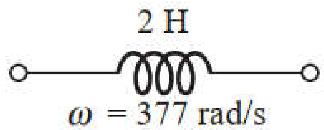
$$\boldsymbol{Z} = 200\Omega\angle 0° = 200\ \Omega$$



$$\omega = 2\pi \times 50 = 314\ \text{rad/s}$$

$$X_L = \omega L = \left(314\ \text{rad/s}\right)\left(0.05\ \text{H}\right) = 15.17\ \Omega$$

$$\boldsymbol{Z} = 15.17\Omega\angle 90° = j15.17\ \Omega$$

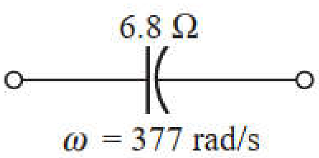


$$X_L = \left(377\ \text{rad/s}\right)\left(2\ \text{H}\right) = 754\ \Omega$$

$$\boldsymbol{Z} = 754\Omega\angle 90° = j754\ \Omega$$

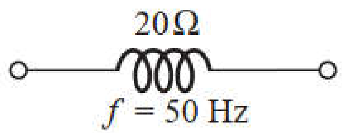
Problem 1 [Ch. 15]

(a) Express the impedances of following figure in both polar and rectangular forms.
(b) Calculate the value of inductor and capacitor



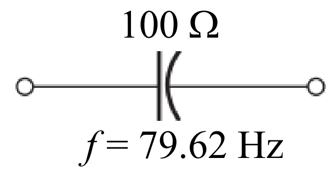
$$\boldsymbol{Z} = 6.8\Omega\angle -90^\circ = -j6.8\ \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{(377\text{rad/s})(6.8\Omega)}$$
$$= 390.1\ \mu\text{F}$$



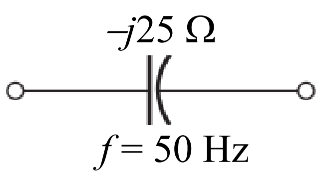
$$\boldsymbol{Z} = 20\Omega\angle 90^\circ = j20\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{20\Omega}{2\pi \times 50} = 63.69\ \text{mH}$$



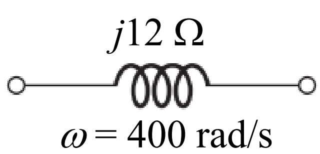
$$\boldsymbol{Z} = 100\Omega\angle -90^\circ = -j100\ \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi(79.62\,\text{Hz})(100\Omega)}$$
$$= 20\ \mu\text{F}$$



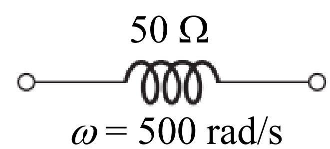
$$\boldsymbol{Z} = 25\Omega\angle -90^\circ = -j25\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times (50\,\text{Hz})(25\,\Omega)}$$
$$= 127.39\ \mu\text{F}$$



$$\boldsymbol{Z} = 12\Omega\angle 90^\circ = j12\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{12\Omega}{400} = 30\ \text{mH}$$



$$\boldsymbol{Z} = 50\Omega\angle 90^\circ = j50\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{50\Omega}{500} = 100\ \text{mH}$$

EXAMPLE 15.1 Using complex algebra, find the current i for the circuit of Fig. 15.2. Sketch the waveforms of v and i.

Solution: $v = 100 \sin \omega t \Rightarrow$ phasor form $\mathbf{V} = 70.71\text{ V}\angle 0°$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V\angle\theta}{R\angle 0°} = \frac{70.71\text{ V}\angle 0°}{5\ \Omega\angle 0°} = 14.14\text{ A}\angle 0°$$

$$i = \sqrt{2}(14.14)\sin \omega t = \mathbf{20\sin \omega t}$$

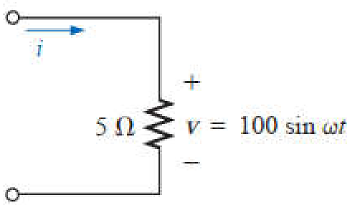


FIG. 15.2 *Example 15.1.*

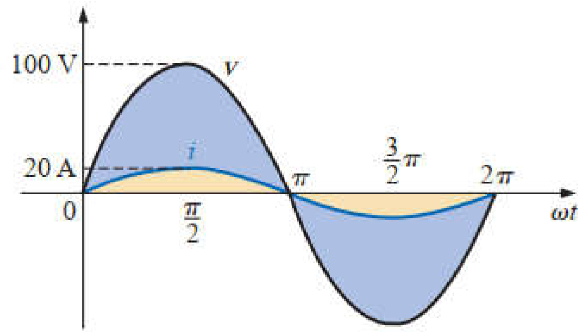
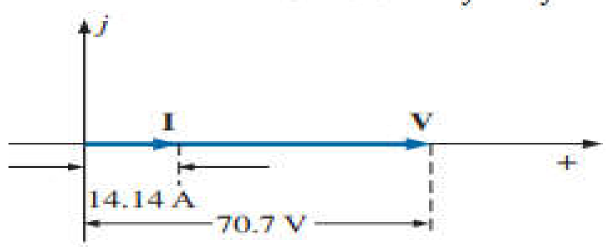


FIG. 15.3 *Waveforms for Example 15.1.*



Phasor diagrams

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit of Fig. 15.4. Sketch the waveforms of v and i.

Solution: $i = 4\sin(\omega t + 30°) \Rightarrow$ phasor form $\mathbf{I} = 2.828\text{ A}\angle 30°$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0°) = (2.828\text{ A}\angle 30°)(2\ \Omega\angle 0°)$$
$$= 5.656\text{ V}\angle 30°$$

and $v = \sqrt{2}(5.656)\sin(\omega t + 30°) = \mathbf{8.0\sin(\omega t + 30°)}$

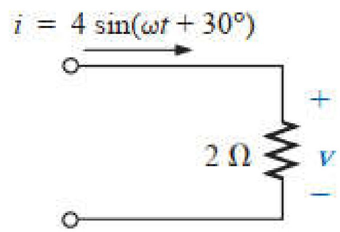


FIG. 15.4 *Example 15.2.*

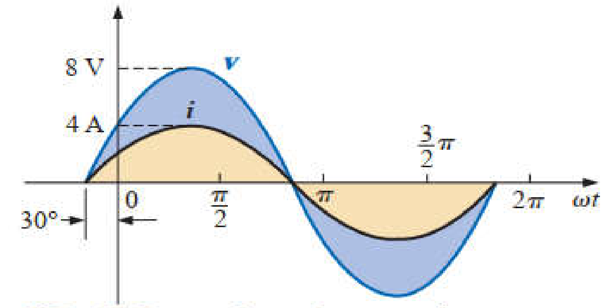
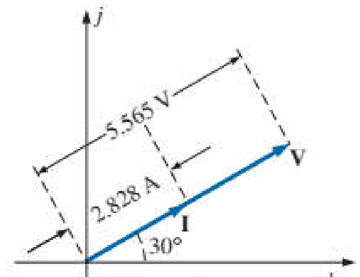


FIG. 15.5 *Waveforms for Example 15.2.*



Phasor diagrams

EXAMPLE 15.3 Using complex algebra, find the current i for the circuit in Fig. 15.8. Sketch the v and i curves.

Solution: Note Fig. 15.9:

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968\text{ V}\angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V\angle\theta}{X_L\angle 90^\circ} = \frac{16.968\text{ V}\angle 0^\circ}{3\ \Omega\angle 90^\circ} = 5.656\text{ A}\angle -90^\circ$$

and $\quad i = \sqrt{2}(5.656)\sin(\omega t - 90^\circ) = \mathbf{8.0\ sin(\omega t - 90^\circ)}$

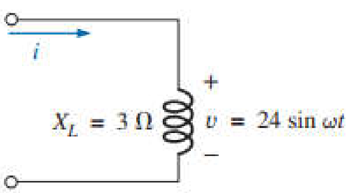


FIG. 15.8 *Example 15.3.*

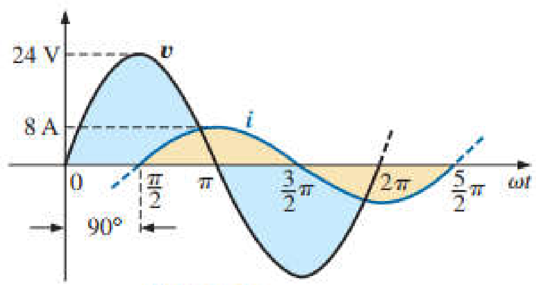
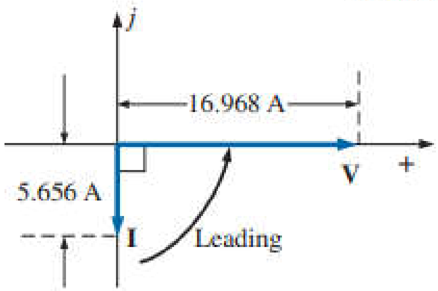


FIG. 15.9
Waveforms for Example 15.3.



Phasor diagrams

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit in Fig. 15.10. Sketch the v and i curves.

Solution: Note Fig. 15.11:

$$i = 5 \sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 3.535\text{ A}\angle 30^\circ$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (I\angle\theta)(X_L\angle 90^\circ) = (3.535\text{ A}\angle 30^\circ)(4\ \Omega\angle +90^\circ)$$
$$= 14.140\text{ V}\angle 120^\circ$$

and $\quad v = \sqrt{2}(14.140)\sin(\omega t + 120^\circ) = \mathbf{20\ sin(\omega t + 120^\circ)}$

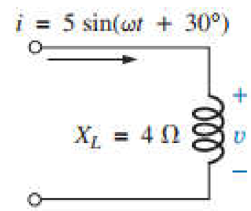
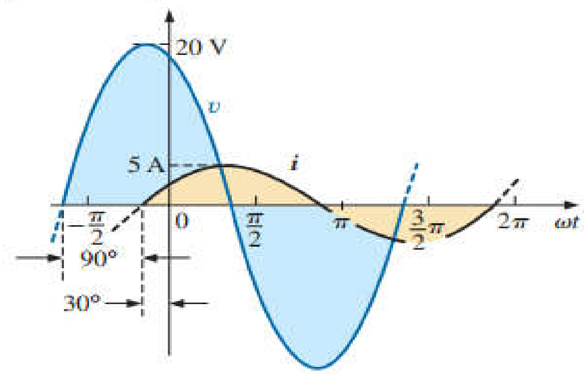
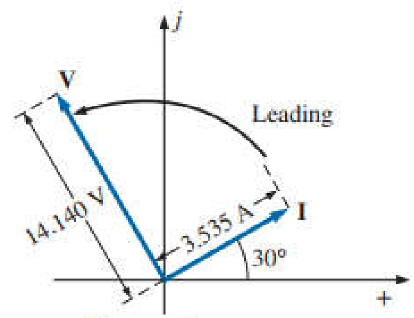


FIG. 15.10 *Example 15.4.*



5.11 *Waveforms for Example 15.4.*



Phasor diagrams

EXAMPLE 15.5 Using complex algebra, find the current i for the circuit in Fig. 15.14. Sketch the v and i curves.

Solution: Note Fig. 15.15:

$$v = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V} \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90^\circ} = \frac{10.605 \text{ V} \angle 0^\circ}{2\ \Omega \angle -90^\circ} = 5.303 \text{ A} \angle 90^\circ$$

and $\quad i = \sqrt{2}(5.303) \sin(\omega t + 90^\circ) = \mathbf{7.5 \sin(\omega t + 90^\circ)}$

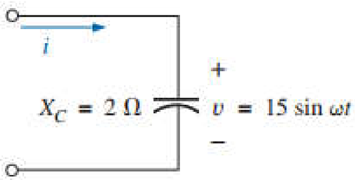


FIG. 15.14 *Example 15.5.*

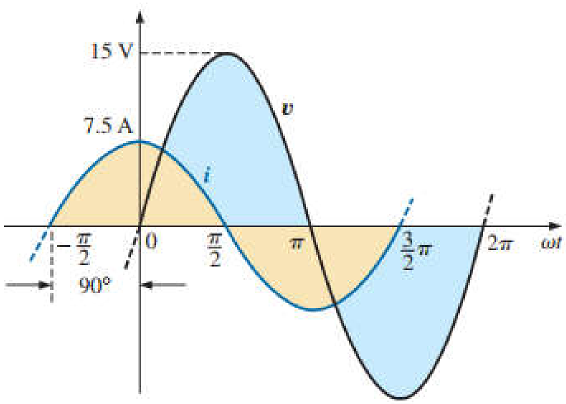
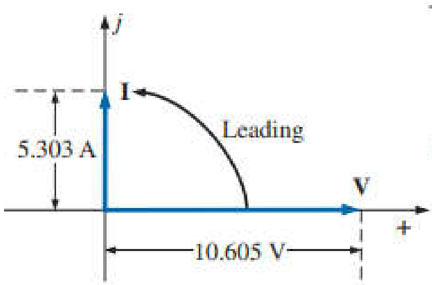


FIG. 15.15 *Waveforms for Example 15.5.*



Phasor diagrams

EXAMPLE 15.6 Using complex algebra, find the voltage v for the circuit in Fig. 15.16. Sketch the v and i curves.

Solution: Note Fig. 15.17:

$$i = 6 \sin(\omega t - 60^\circ) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A} \angle -60^\circ$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90^\circ) = (4.242 \text{ A} \angle -60^\circ)(0.5\ \Omega \angle -90^\circ)$$
$$= 2.121 \text{ V} \angle -150^\circ$$

and $\quad v = \sqrt{2}(2.121) \sin(\omega t - 150^\circ) = \mathbf{3.0 \sin(\omega t - 150^\circ)}$

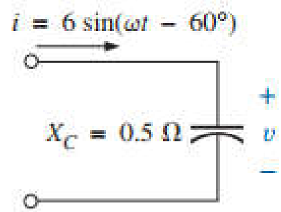
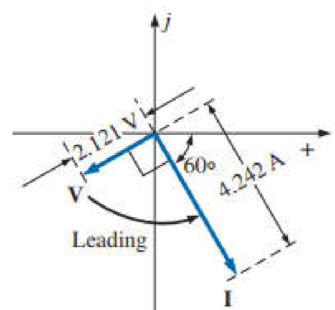


FIG. 15.16 *Example 15.6.*



Phasor diagrams

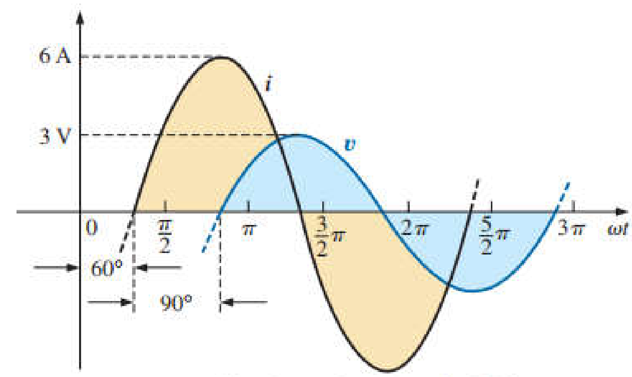


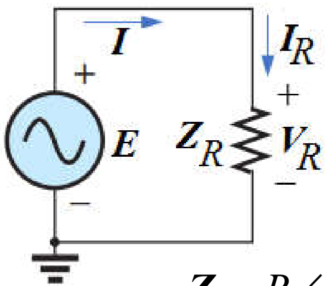
FIG. 15.17 *Waveforms for Example 15.6.*

Practice Book Problems [Ch. 15] 2 and 3

Power and Energy Related Theory for

Pure Resistive, Pure Inductive and Pure Capacitive Circuits

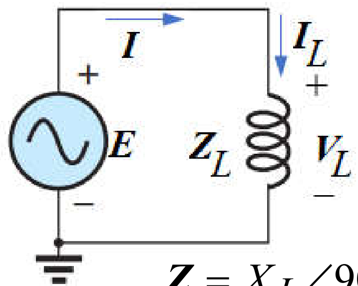
PURE RESISTIVE CIRCUIT



$$\boldsymbol{I}_R = \boldsymbol{I}$$
$$\boldsymbol{V}_R = \boldsymbol{E}$$
$$\boldsymbol{Z}_R = \boldsymbol{Z}$$

$$\boldsymbol{Z} = R\angle 0^\circ \ \Omega = R \ \Omega$$

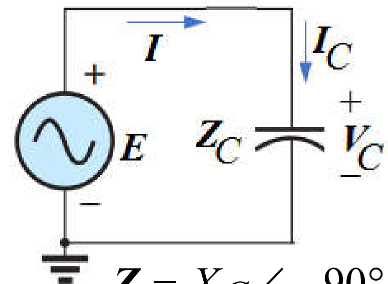
PURE INDUCTIVE CIRCUIT



$$\boldsymbol{I}_L = \boldsymbol{I}$$
$$\boldsymbol{V}_L = \boldsymbol{E}$$
$$\boldsymbol{Z}_L = \boldsymbol{Z}$$

$$\boldsymbol{Z} = X_L\angle 90^\circ \ \Omega = jX_L \ \Omega$$

PURE CAPACITIVE CIRCUIT



$$\boldsymbol{I}_C = \boldsymbol{I}$$
$$\boldsymbol{V}_C = \boldsymbol{E}$$
$$\boldsymbol{Z}_C = \boldsymbol{Z}$$

$$\boldsymbol{Z} = X_C\angle -90^\circ \ \Omega = -jX_C \ \Omega$$

Power Factor, reactive Factor, Real Power, Reactive Power, Apparent Power, Instantaneous power

Pure resistive circuit

$$pf = F_p = \cos\theta = \cos 0^\circ = 1$$
$$rf = F_q = \sin\theta = \sin 0^\circ = 0$$

Unity Power Factor

$$P = V_R I_R = I_R^2 R = \frac{V_R^2}{R}$$
$$Q = 0$$
$$S = V_R I_R$$
$$p_R(t) = V_R I_R\left(1 - \cos 2\omega t\right) \ \text{W}$$

Pure inductive circuit

$$pf = F_p = \cos\theta = \cos 90^\circ = 0$$
$$rf = F_q = \sin\theta = \sin 90^\circ = 1$$

Zero Lagging Power Factor

$$P = 0$$
$$Q_L = V_L I_L = I_L^2 X_L = \frac{V_L^2}{X_L}$$
$$S = V_R I_R$$
$$p_L(t) = V_L I_L \sin 2\omega t \ \text{W}$$

Pure capacitive circuit

$$pf = F_p = \cos\theta = \cos\left(-90^\circ\right) = 0$$
$$rf = F_q = \sin\theta = \sin\left(-90^\circ\right) = -1$$

Zero Leading Power Factor

$$P = 0$$
$$Q_C = -V_C I_C = -I_C^2 X_C = \frac{V_C^2}{X_C}$$
$$S = V_R I_R$$
$$p_C(t) = -V_C I_C \sin 2\omega t \ \text{W}$$

PURE RESISTIVE CIRCUIT

$$p_R(t) = V_R I_R (1 - \cos 2\omega t) \text{ W}$$

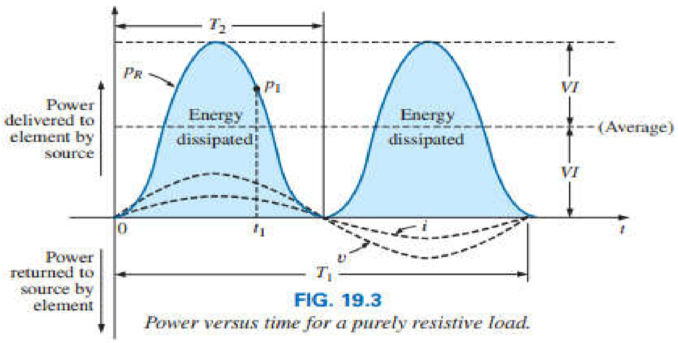


FIG. 19.3
Power versus time for a purely resistive load.

PURE INDUCTIVE CIRCUIT

$$p_L(t) = V_L I_L \sin 2\omega t \text{ W}$$

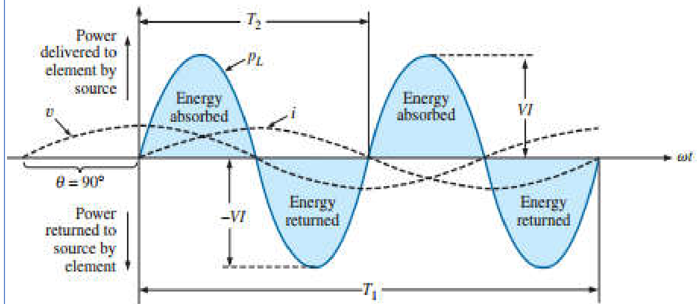


FIG. 19.9
The power curve for a purely inductive load.

PURE CAPACITIVE CIRCUIT

$$p_C(t) = -V_C I_C \sin 2\omega t \text{ W}$$

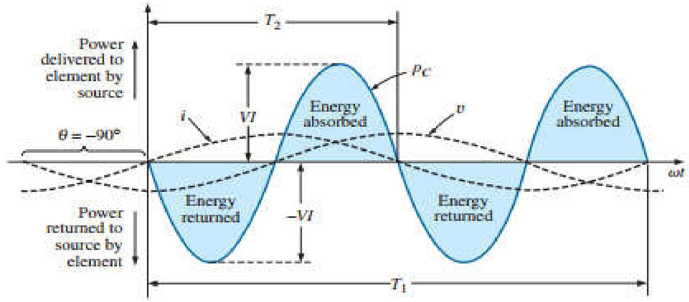


FIG. 19.13
The power curve for a purely capacitive load.

T_1 = Period of input voltage or current; T_2 = Period of power curve

Energy Dissipation and Energy Stored Calculation by using the Equation of: $W = Pt$ J

The energy **dissipated** by the resistor (W_R) over **one full cycle**:

$$W_R = (V_R I_R)T \quad [\text{J}] \qquad \textbf{(19.4)}$$

$$W_R = \frac{V_R I_R}{f} \quad [\text{J}] \qquad \textbf{(19.5)}$$

$$W_R = 2\pi \frac{V_R I_R}{\omega} \quad [\text{J}] \qquad \textbf{(19.5.1)}$$

The energy **stored** by the **inductor** (W_L) and **capacitor** (W_C) during the positive portion of the cycle (Fig. 19.9 and Fig. 19.13) is equal to that returned during the negative portion and can be determined **over half cycle** using the following equation:

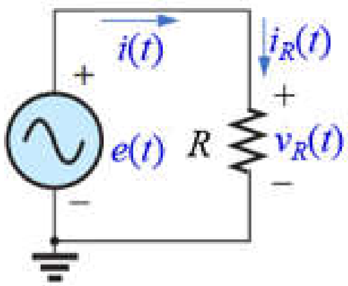
$$W_L = \frac{V_{Lm} I_{Lm}}{2\omega} = \frac{V_L I_L}{\omega} \quad [\text{J}]$$

$$W_L = \frac{1}{2} L I_{Lm}^2 = L I_L^2 \quad [\text{J}] \quad \textbf{(19.18)}$$

$$W_C = \frac{V_{Cm} I_{Cm}}{2\omega} = \frac{V_C I_C}{\omega} \quad [\text{J}]$$

$$W_C = \frac{1}{2} C V_{Cm}^2 = C V_C^2 \quad [\text{J}] \quad \textbf{(19.26)}$$

EXAMPLE The voltage $e(t)$ = 100sin(314t+60°) V is applied across in the following circuits.
(***a***) Write the instantaneous equation of current.
(***b***) Find the power factor, the reactive factor, the power, the reactive power, the apparent power.
(***c***) Write the instantaneous power equation.
(***d***) Find the energy dissipated by the resistor over one full cycle of the input voltage and the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve.



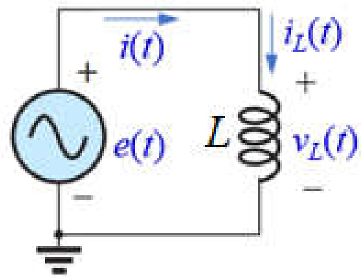
R=10 Ω

$$I_m = \frac{V_m}{R} = \frac{100\,\text{V}}{10\,\Omega} = 10\ \text{A}$$

Here, $i(t)$ and $e(t)$ are in phase

$$\theta_i = \theta_e = 60°$$

***i*(*t*) = 10sin(314*t* + 60°) A**



L = 31.85 mH

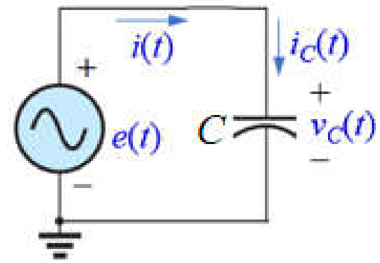
$$X_L = (314\,\text{rad/s})(31.85\,\text{H}) = 10\ \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100\,\text{V}}{10\,\Omega} = 10\ \text{A}$$

Here, $i(t)$ lags $e(t)$ by 90°

$$\theta_i = \theta_e - 90° = 60° - 90° = -30°$$

***i*(*t*) = 10sin(314*t* - 30°) A**



C = 318.47 μF

$$X_C = \frac{1}{(314\,\text{rad/s})(318.47 \times 10^{-6}\,\text{F})} = 10\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{100\,\text{V}}{10\,\Omega} = 10\ \text{A}$$

Here, $i(t)$ leads $e(t)$ by 90°

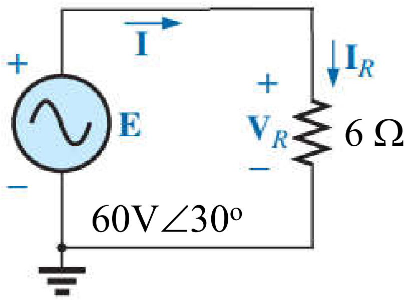
$$\theta_i = \theta_e + 90° = 60° + 90° = 150°$$

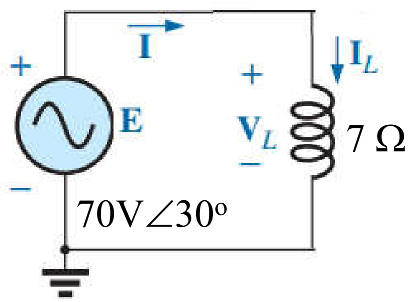
***i*(*t*) = 10sin(314*t* + 150°) A**

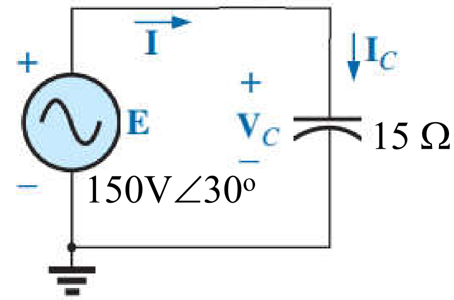
(***b***) Find the power factor, the reactive factor, the power, the reactive power, the apparent power.
(***c***) Write the instantaneous power equation.
(***d***) Find the energy dissipated by the resistor over one full cycle of the input voltage and the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve.

$R=10\,\Omega \quad I_m = 10\text{ A} \quad \theta_i = 60°$	$L = 31.85\text{ mH} \quad X_L = 10\,\Omega$ $I_m = 10\text{ A} \quad \theta_i = 150°$	$C = 318.47\ \mu\text{F} \quad X_C = 10\,\Omega$ $I_m = 10\text{ A} \quad \theta_i = -30°$
$\theta = \theta_z = \theta_v - \theta_i = 0°$ $pf = \cos\theta = \cos\theta_z = \cos(0°) = 1$ $rf = \sin\theta = \sin\theta_z = \sin(0°) = 0$	$\theta = \theta_z = \theta_v - \theta_i = 90°$ $pf = \cos\theta = \cos\theta_z = \cos(90°) = 0$ $rf = \sin\theta = \sin\theta_z = \sin(90°) = 1$	$\theta = \theta_z = \theta_v - \theta_i = -90°$ $pf = \cos\theta = \cos\theta_z = \cos(-90°) = 0$ $rf = \sin\theta = \sin\theta_z = \sin(-90°) = -1$
$S = \frac{V_m I_m}{2} = \frac{(100\text{V})(10\text{A})}{2} = 500\text{ VA}$ $P = S\cos\theta = (500\text{VA})\cos(0°) = 500\text{ W}$ $Q = S\sin\theta = (500\text{VA})\sin(0°) = 0\text{ Var}$ $p(t) = 500(1 - \cos 628t)\text{ W}$	$S = \frac{V_m I_m}{2} = \frac{(100\text{V})(10\text{A})}{2} = 500\text{ VA}$ $P = S\cos\theta = (500\text{VA})\cos(90°) = 0\text{ W}$ $Q = S\sin\theta = (500\text{VA})\sin(90°) = 500\text{ Var}$ $p(t) = 500\sin 628t\text{ W}$	$S = \frac{V_m I_m}{2} = \frac{(100\text{V})(10\text{A})}{2} = 500\text{ VA}$ $P = S\cos\theta = (500\text{VA})\cos(-90°) = 0\text{ W}$ $Q = S\sin\theta = (500\text{VA})\sin(-90°) = -500\text{ Var}$ $p(t) = -500\sin 628t\text{ W}$
$W_R = \frac{V_R I_R}{f} = \frac{V_{Rm} I_{Rm}}{2f}$ $= \frac{(100\text{V})(10\text{V})}{2 \times 50\text{Hz}} = 10\text{ J}$	$W_L = \frac{V_L I_L}{\omega} = \frac{1}{2} L I_m^2$ $= \frac{1}{2}(31.85 \times 10^{-3}\text{H})(10\text{A})^2 = 1.59\text{ J}$	$W_C = \frac{V_C I_C}{\omega} = \frac{1}{2} C V_m^2$ $= \frac{1}{2}(318.47 \times 10^{-6}\text{F})(100\text{V})^2 = 1.59\text{ J}$

EXAMPLE The voltage $\boldsymbol{E}$ with 50 Hz frequency applied across in the following circuits.
(***a***) Find the power factor, the reactive factor, the power, the reactive power, the apparent power.
(***b***) Write the instantaneous power equation.
(***c***) Find the energy dissipated by the resistor over one full cycle of the input voltage and the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve.







Solution: $\boldsymbol{Z} = \boldsymbol{Z}_R = 6\Omega\angle 0° = 6\Omega$

$$\boldsymbol{I} = \boldsymbol{I}_R = \frac{\boldsymbol{V}_R}{\boldsymbol{Z}_R} = \frac{\boldsymbol{E}}{\boldsymbol{Z}}$$

$$= \frac{60\text{V}\angle 30°}{6\Omega} = 10\text{A}\angle 30°$$

$$pf = \cos\theta = \cos\theta_Z = \cos(0°) = 1$$

$$rf = \sin\theta = \sin\theta_Z = \sin(0°) = 0$$

$$\boldsymbol{Z} = \boldsymbol{Z}_L = 7\Omega\angle 90° = j7\ \Omega$$

$$\boldsymbol{I} = \boldsymbol{I}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{Z}_L} = \frac{\boldsymbol{E}}{\boldsymbol{Z}}$$

$$= \frac{70\text{V}\angle 30°}{7\Omega\angle 90°} = 10\text{A}\angle -60°$$

$$pf = \cos\theta = \cos\theta_Z = \cos(90°) = 0$$

$$rf = \sin\theta = \sin\theta_Z = \sin(90°) = 1$$

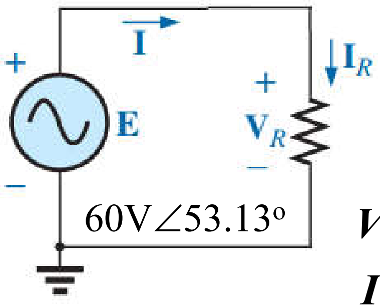
$$\boldsymbol{Z} = \boldsymbol{Z}_C = 15\Omega\angle -90° = -j15\ \Omega$$

$$\boldsymbol{I} = \boldsymbol{I}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{Z}_C} = \frac{\boldsymbol{E}}{\boldsymbol{Z}}$$

$$= \frac{150\text{V}\angle 30°}{15\Omega\angle -90°} = 10\text{A}\angle 120°$$

$$pf = \cos\theta = \cos\theta_Z = \cos(-90°) = 0$$

$$rf = \sin\theta = \sin\theta_Z = \sin(-90°) = -1$$



$$V_R = 60\text{V}\angle 30^\circ$$
$$I_R = 10\text{A}\angle 30^\circ$$

$$S = VI = (60\text{V})(10\text{A}) = 600 \text{ VA}$$

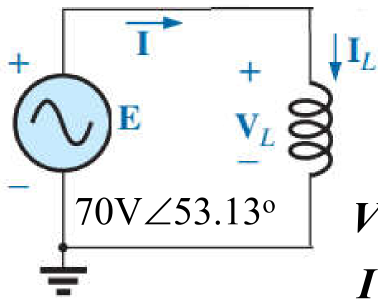
$$P = VI\cos\theta_Z = (60\text{V})(10\text{A})\cos(0^\circ) = 600 \text{ W}$$

$$Q = VI\sin\theta_Z = (60\text{V})(10\text{A})\sin(0^\circ) = 0 \text{ Var}$$

$$\omega = 2\pi f = 2\pi\times 50 = 314 \text{ rad/s}$$

$$p(t) = 600(1-\cos 628t)\text{ W}$$

$$W_R = \frac{V_R I_R}{f} = \frac{(60\text{V})(10\text{V})}{50\,\text{Hz}} = 12 \text{ J}$$



$$V_L = 70\text{V}\angle 30^\circ$$
$$I_L = 10\text{A}\angle -60^\circ$$

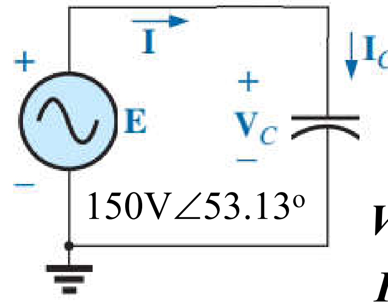
$$S = VI = (70\text{V})(10\text{A}) = 700 \text{ VA}$$

$$P = VI\cos\theta_Z = (70\text{V})(10\text{A})\cos(90^\circ) = 0 \text{ W}$$

$$Q = VI\sin\theta_Z = (70\text{V})(10\text{A})\sin(90^\circ) = 700 \text{ Var}$$

$$p(t) = 700\sin 628t\text{ W}$$

$$W_L = \frac{V_L I_L}{\omega} = \frac{(70\text{V})(10\text{V})}{314 \text{ rad/s}} = 2.23 \text{ J}$$



$$V_C = 150\text{V}\angle 30^\circ$$
$$I_C = 10\text{A}\angle 120^\circ$$

$$S = VI = (150\text{V})(10\text{A}) = 1500 \text{ VA}$$

$$P = VI\cos\theta_Z = (150\text{V})(10\text{A})\cos(-90^\circ) = 0 \text{ W}$$

$$Q = VI\sin\theta_Z = (150\text{V})(10\text{A})\sin(-90^\circ) = -1500 \text{ Var}$$

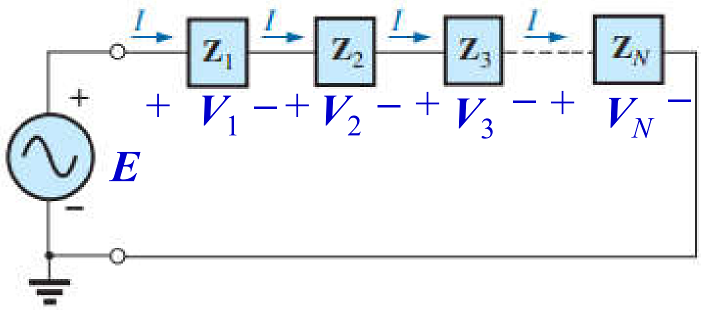
$$p(t) = -1500\sin 628t\text{ W}$$

$$W_C = \frac{V_C I_C}{\omega} = \frac{(150\text{V})(10\text{V})}{314\,\text{rad/s}} = 4.78 \text{ J}$$

Chapter 15

Series Circuits [AC]

15.3 SERIES CONFIGURATION



The total impedance of a series configuration is the sum of the individual impedances:

$$Z_T = Z_1 + Z_2 + Z_3 + ... + Z_N \qquad (15.4)$$

$$I = \frac{E}{Z_T} = EY_T \qquad E = IZ_T = \frac{I}{Y_T}$$

$$V_1 = I_1 Z_1 = IZ_1 \qquad V_2 = I_2 Z_2 = IZ_2$$

$$V_3 = I_3 Z_3 = IZ_3 \qquad V_N = I_N Z_N = IZ_N$$

If $Z_1 = Z_2 = Z_3 = ... = Z_n = Z_s$

$$Z_T = N \times Z_N \quad V_1 = V_2 = V_3 = ... = V_N = IZ_s = \frac{E}{N}$$

Voltage Divider Rule (VDR)

The voltage across an impedance in a series circuit is equal to the value of that impedance (Z_x) times the total applied voltage (E) divided by the total impedance (Z_T) of the series configuration.

$$V_x = \frac{Z_x}{Z_T} E = \frac{Y_T}{Y_x} E$$

Kirchhoff's Voltage Law (KVL)

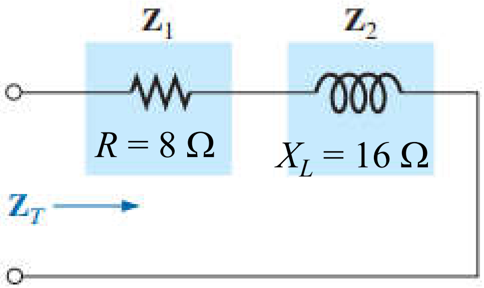
(1) The algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

$$\sum_{\text{closed path}} V = 0$$

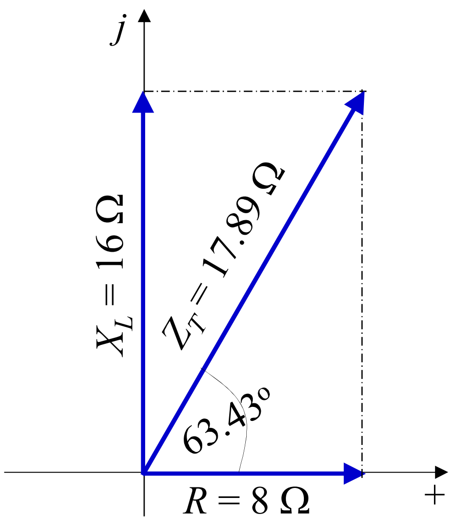
(2) The sum of the applied or supplied or rise voltage of a series circuit will equal the sum of the voltage drops of the circuit.

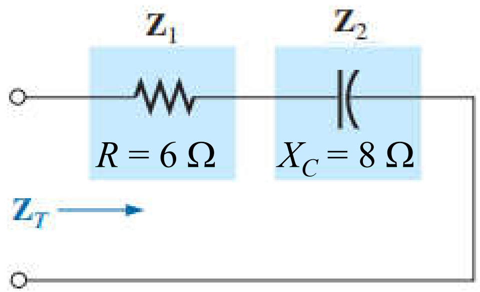
$$\sum_{\text{closed path}} V_{rise} = \sum_{\text{closed path}} V_{drop}$$

EXAMPLE For the following circuits, find the total impedance and draw the impedance diagram.

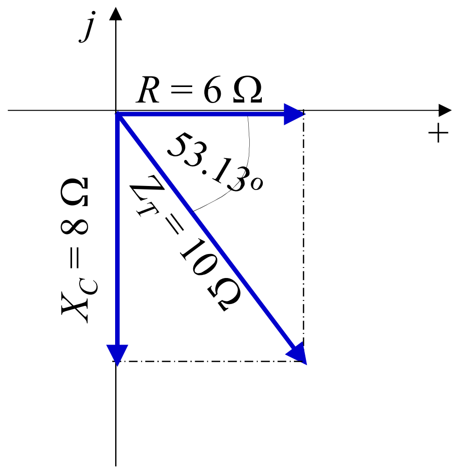


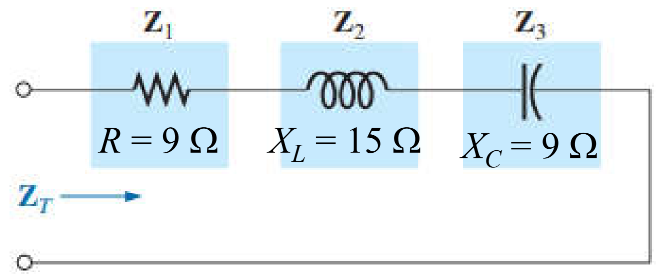
$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 = R + jX_L$$
$$= 8 + j16 = \mathbf{17.89\Omega\angle 63.43^\circ}$$



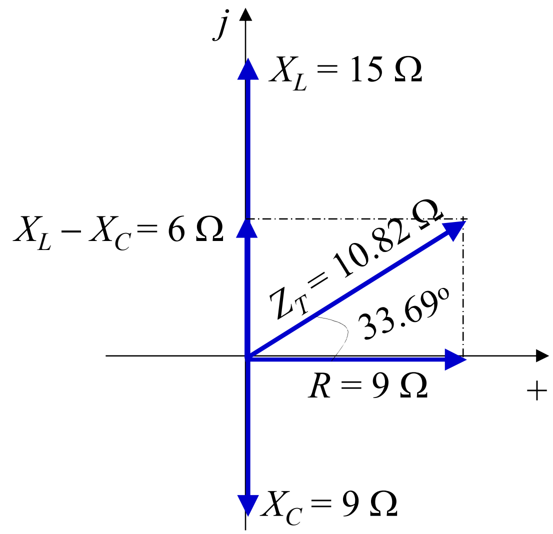


$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 = R - jX_C$$
$$= 6 - j8 = \mathbf{10\Omega\angle -53.13^\circ}$$

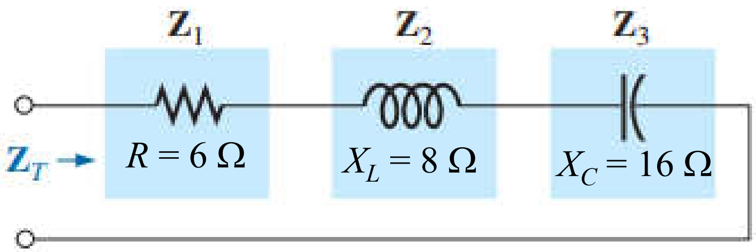


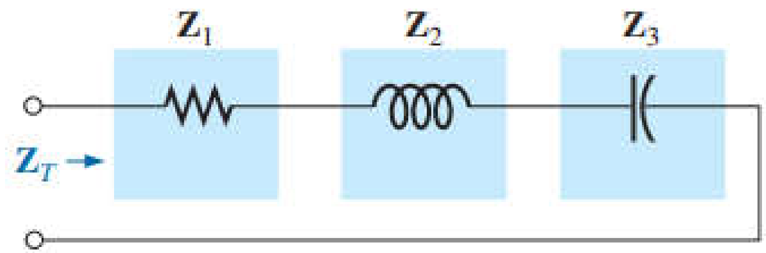


$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 = R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 9\Omega + j(15\Omega - 9\Omega)$$
$$= 9\Omega + j6\Omega = \mathbf{10.82\Omega\angle 33.69^\circ}$$



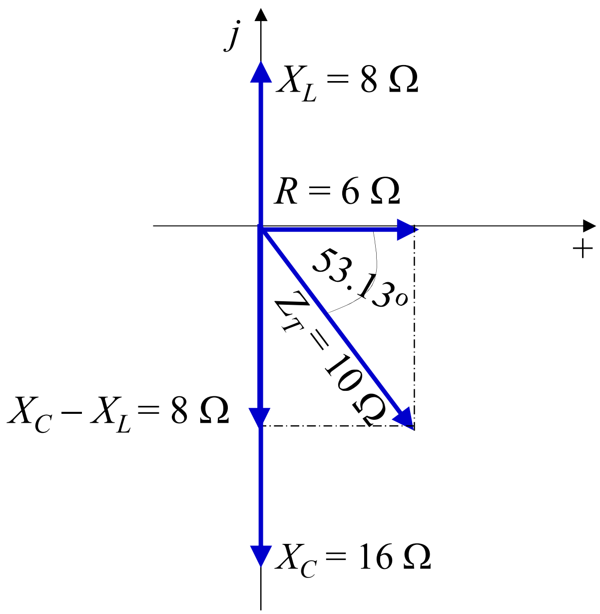
EXAMPLE For the following circuits, fnd the total impedance and draw the impedance diagram.





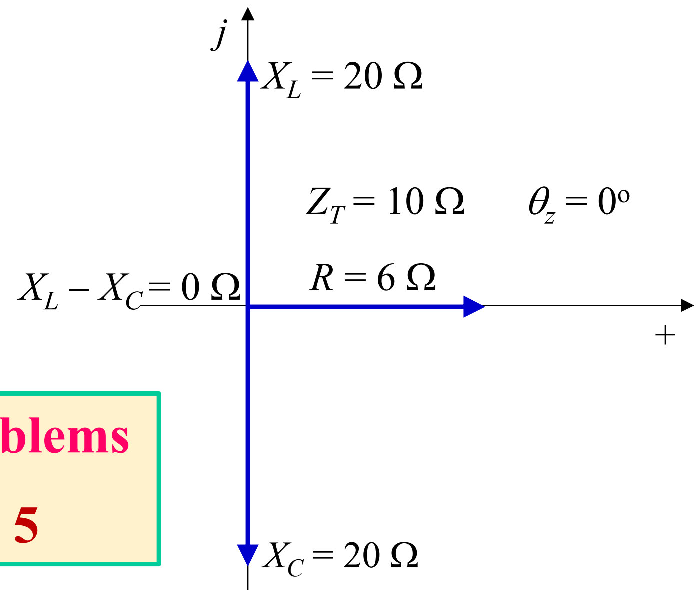
$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 = R + jX_L - jX_C = R + j(X_L - X_C)$$
$$= 6\Omega + j(8\Omega - 16\Omega) = 6\Omega - j8\Omega = \mathbf{10\Omega\angle -53.13^\circ}$$

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 = R + jX_L - jX_C = R + j(X_L - X_C)$$
$$= 10\Omega + j(20\Omega - 20\Omega) = 10\Omega - j0\Omega = \mathbf{10\Omega\angle 0^\circ}$$



Practice Book Problems

[Ch. 15] 4 and 5



R-L Series Circuit

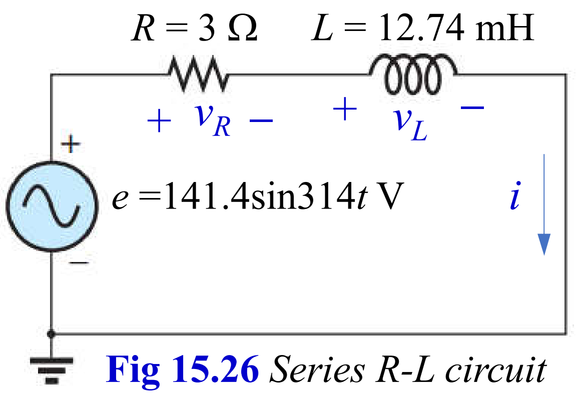


Fig 15.26 *Series R-L circuit*

$$X_L = \omega L = 314 \times (12.47 \times 10^{-3}) = 4\ \Omega$$

$$E = (0.707 \times 141.4)\angle 0^{o} = 100\text{V}\angle 0^{o}$$

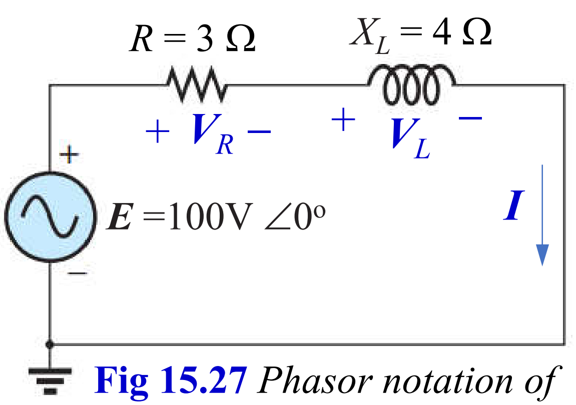


Fig 15.27 *Phasor notation of Fig. 15.26*

R-L Series Circuit

Impedance

$$Z_T = Z_1 + Z_2 = R + jX_L$$
$$= 3 + j4 = \mathbf{5\Omega\angle 53.13^\circ}$$

Impedance Diagram

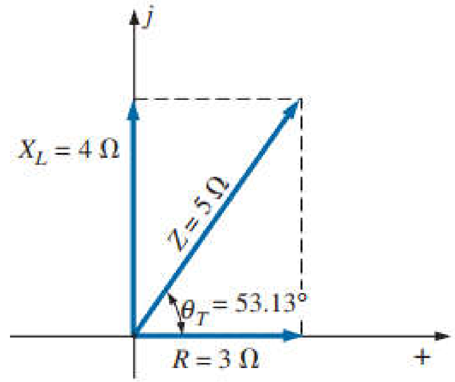


FIG. 15.28 *Impedance diagram*

Current

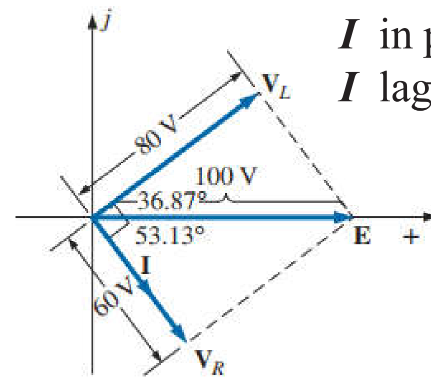
$$I = \frac{E}{Z_T} = \frac{100\text{V}\angle 0^\circ}{5\Omega\angle 53.13^\circ}$$
$$= 20\text{A}\angle -53.13^\circ$$

V_R and V_L

$$\mathbf{V}_R = \mathbf{IZ}_R = (20\text{ A}\ \angle -53.13°)(3\ \Omega\ \angle 0°)$$
$$= \mathbf{60\ V\ \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{IZ}_L = (20\text{ A}\ \angle -53.13°)(4\ \Omega\ \angle 90°)$$
$$= \mathbf{80\ V\ \angle 36.87°}$$

Phasor or Vector Diagram



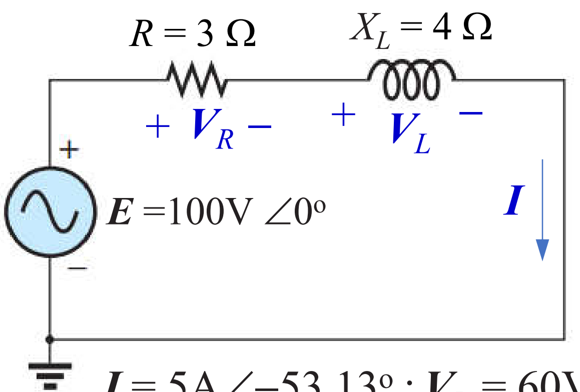
I in phase with V_R

I lags with V_L by 90^{o}

FIG. 15.29 *Phasor diagram*

KVL:

$$E = V_R + V_L$$
$$= 60\text{V}\angle -53.13^{o} + 80\text{V}\angle 36.87^{o}$$
$$= 100\text{V}\ \angle 0^{o}$$



$I = 5\text{A}\angle{-53.13^{\circ}}$; $V_R = 60\text{V}\angle{-53.13^{\circ}}$;
$V_L = 80\text{V}\angle 36.87^{\circ}$

Power Factor and Reactive Factor

$pf = (R/Z_T) = \cos\theta_z = \cos(53.13^{\circ}) = \mathbf{0.6\ Lagging}$
$rf = (X_L/Z_T) = \sin\theta_z = \sin(53.13^{\circ}) = \mathbf{0.8}$

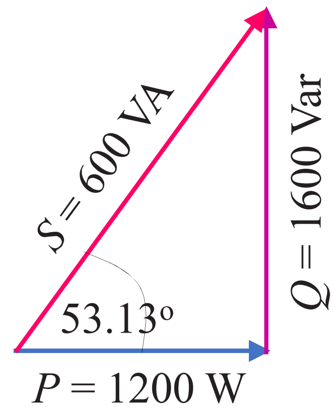
Power [Total watts]

$P_E = EI\cos\theta_z = 100\times20\cos(53.13^{\circ}) = \mathbf{1200\ W}$
$P_R = I^2R = (20\text{A})^2 \times 3\Omega = \mathbf{1200\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 100\times20\sin(53.13^{\circ}) = \mathbf{1600\ Var}$
$Q_L = I^2X_L = (20\text{A})^2 \times 4\Omega = \mathbf{1600\ Var}$

Power Triangle



Apparent Power [volt-ampere]

$S_E = EI = 100\times20 = \mathbf{2000\ VA}$
$S_Z = I^2Z = (20\text{A})^2 \times 5\Omega = \mathbf{2000\ VA}$

Instantaneous Power Equation

$p(t) = P(1 - \cos2\omega t) + Q\sin2\omega t$ W
$= 1200(1-\cos628t) + 1600\sin628t$ W

Instantaneous Current and Voltages Equation

$i(t) = (\sqrt{2}\times20)\sin(314t - 53.13^{\circ})$ A
$= 28.28\sin(314t - 53.13^{\circ})$ A

$v_R(t) = (\sqrt{2}\times60)\sin(314t - 53.13^{\circ})$ V
$= 84.85\sin(314t - 53.13^{\circ})$ V

$v_L(t) = (\sqrt{2}\times80)\sin(314t + 36.87^{\circ})$ V
$= 113.14\sin(314t + 36.87^{\circ})$ V

Practice Book Problems [Ch. 15] 7

R-*C* Series Circuit

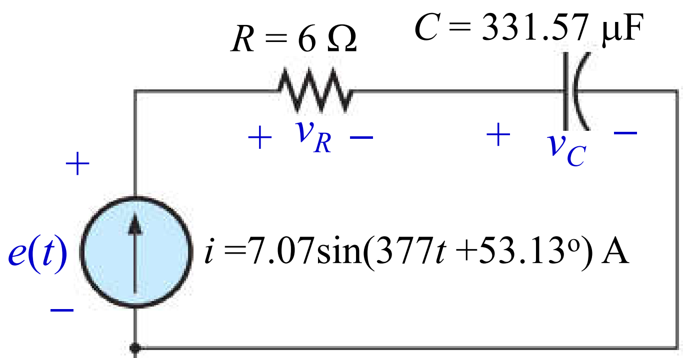


Fig 15.30 *Series R-C circuit*

$$X_C = \frac{1}{377\times\left(331.57\times10^{-6}\right)} = 8\ \Omega$$

$$I = (0.707\times7.07)\angle53.13^{\circ} = 5\text{A}\angle53.13^{\circ}$$

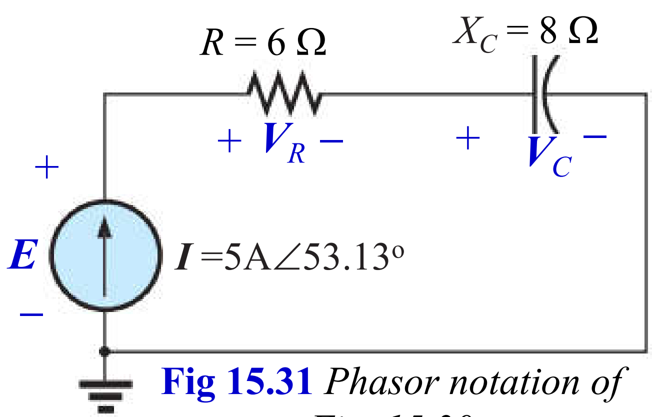


Fig 15.31 *Phasor notation of Fig. 15.30*

R-C Series Circuit

Impedance

$$Z_T = Z_1 + Z_2 = R - jX_C$$
$$= 6 - j8 = \mathbf{10\Omega\angle-53.13^{\circ}}$$

Impedance Diagram

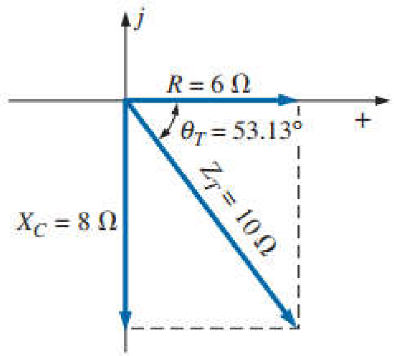


FIG. 15.32 *Impedance diagram*

Voltage

$$E = IZ_T$$
$$= (5\text{V}\angle53.13^{\circ})(10\Omega\angle-53.13^{\circ})$$
$$= 50\text{V}\angle0^{\circ}$$

V_R and V_C

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle0^{\circ})$$
$$= (5\text{ A}\angle53.13^{\circ})(6\ \Omega\angle0^{\circ}) = \mathbf{30\ V\angle53.13^{\circ}}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle-90^{\circ})$$
$$= (5\text{ A}\angle53.13^{\circ})(8\ \Omega\angle-90^{\circ}) = \mathbf{40\ V\angle-36.87^{\circ}}$$

Phasor or Vector Diagram

I in phase with V_R

I leads with V_L by 90°

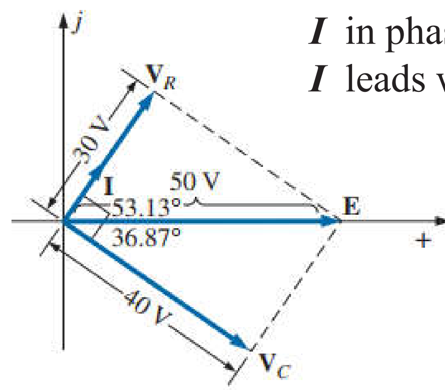
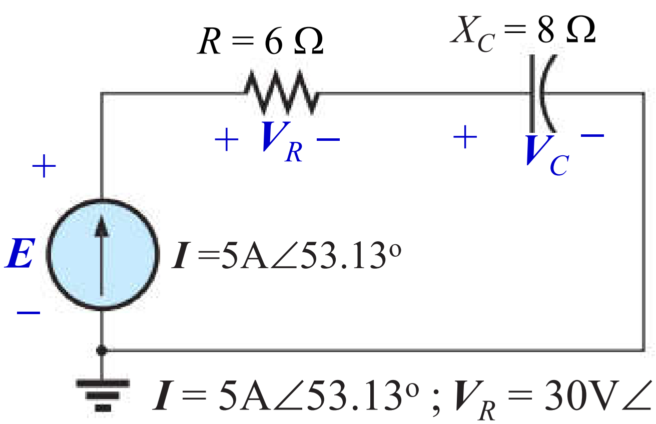


FIG. 15.33 *Phasor diagram*

KVL:

$$E = V_R + V_C$$
$$= 30\text{V}\angle53.13^{\circ} + 40\text{V}\angle-36.87^{\circ}$$
$$= 50\text{V}\angle0^{\circ}$$



$I = 5A\angle 53.13^o$; $V_R = 30V\angle 53.13^o$;
$V_C = 40V\angle -36.87^o$

Power Factor and Reactive Factor

$pf = (R/Z_T) = \cos\theta_z = \cos(-53.13^o) = \mathbf{0.6\ Leading}$

$rf = (X_L/Z_T) = \sin\theta_z = \sin(-53.13^o) = \mathbf{-0.8}$

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times5\cos(-53.13^o) = \mathbf{150\ W}$

$P_R = I^2R = (5A)^2 \times 6\Omega = = \mathbf{150\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times5\sin(-53.13^o) = \mathbf{-200\ Var}$

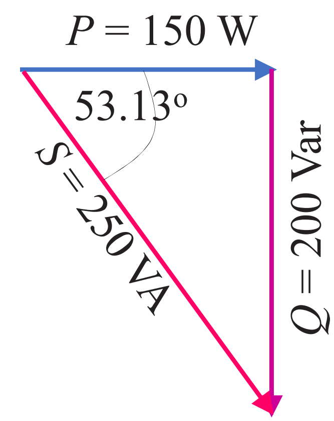
$Q_C = -I^2X_C = -(5A)^2 \times 8\Omega = \mathbf{-200\ Var}$

Apparent Power [volt-ampere]

$S_E = EI = 50\times5 = \mathbf{250\ VA}$

$S_Z = I^2Z = (5A)^2 \times 10\Omega = \mathbf{250\ VA}$

Power Triangle



Instantaneous Power Equation

$$p(t) = P(1 - \cos 2\omega t) + Q\sin 2\omega t \text{ W}$$
$$= 150(1 - \cos 754t) + 200\sin 754t \text{ W}$$

Instantaneous or Time Domain Current and Voltages Equation

$e(t) = (\sqrt{2}\times 50)\sin(377t)$ V = $70.7\sin(314t - 53.13^o)$ V

$v_R(t) = (\sqrt{2}\times 30)\sin(377t + 53.13^o)$ V = $42.43\sin(314t - 53.13^o)$ V

$v_C(t) = (\sqrt{2}\times 40)\sin(377t - 36.87^o)$ V = $56.57\sin(377t - 36.87^o)$ V

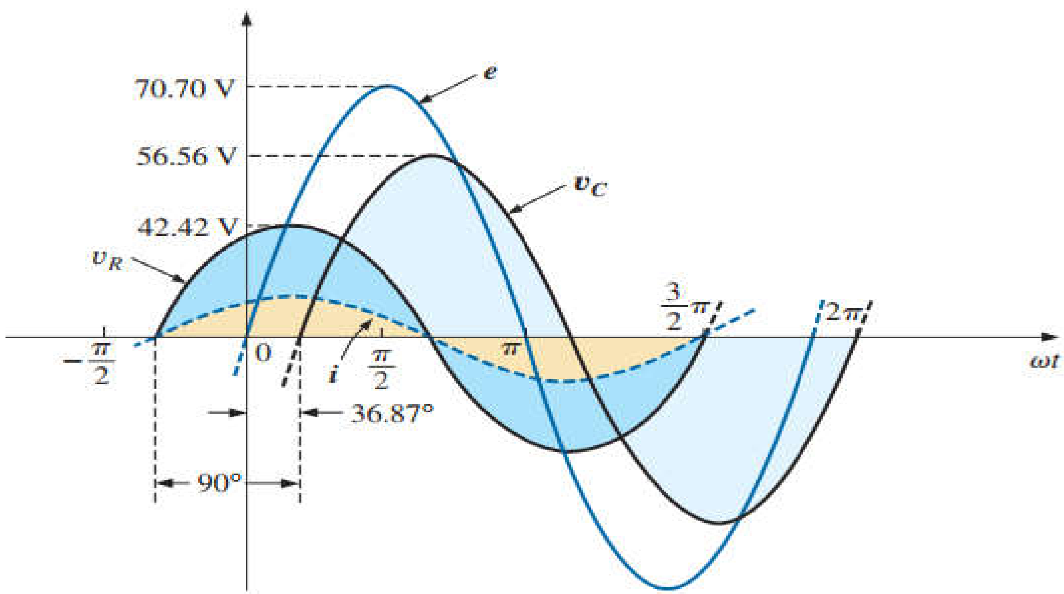


FIG. 15.34
Waveforms for the series R-C circuit in Fig. 15.30.

Practice Book Problems [Ch. 15] 8 and 9

R-L-C Series Circuit

R-L-C Series Circuit

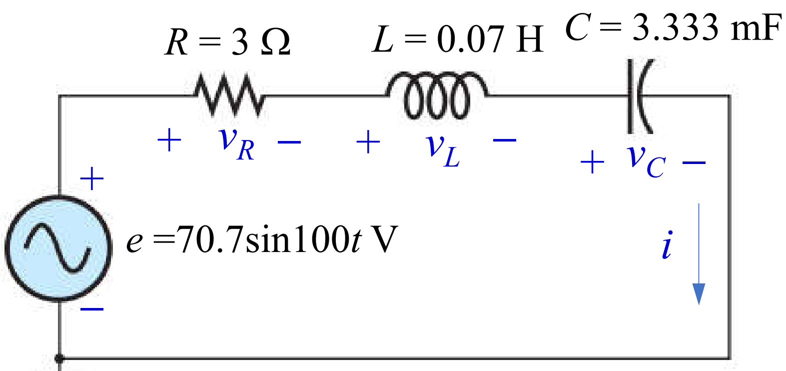


Fig 15.35 *Series R-L-C circuit*

$$X_L = \omega L = 100 \times 0.07 = 7\ \Omega$$

$$X_C = \frac{1}{100 \times \left(3.333 \times 10^{-3}\right)} = 3\ \Omega$$

$$\boldsymbol{E} = (0.707 \times 70.7)\angle 0^{\circ} = 50\text{V}\angle 0^{\circ}$$

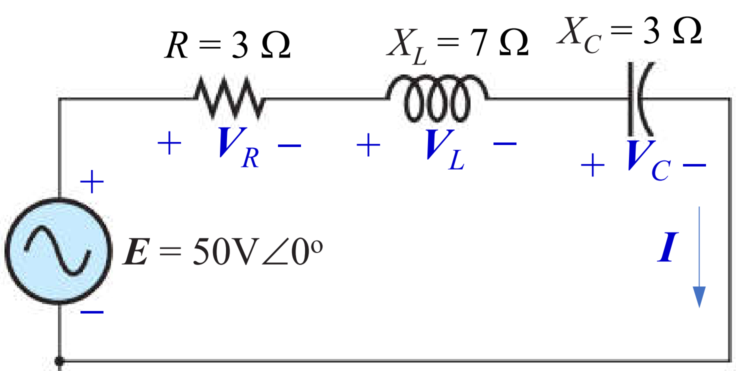


Fig 15.36 *Phasor notation of Fig. 15.35*

Impedance

$$\begin{aligned} \boldsymbol{Z}_T &= \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 = R + jX_L - jX_C \\ &= R + j(X_L - jX_C) = 3 + j(7 - j3) \\ &= 3 + j4 = \mathbf{5\Omega\angle 53.13^{\circ}} \end{aligned}$$

Current

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V}\ \angle 0^{\circ}}{5\ \Omega\ \angle 53.13^{\circ}} = \mathbf{10\ A\ \angle\ -53.13^{\circ}}$$

Impedance Diagram

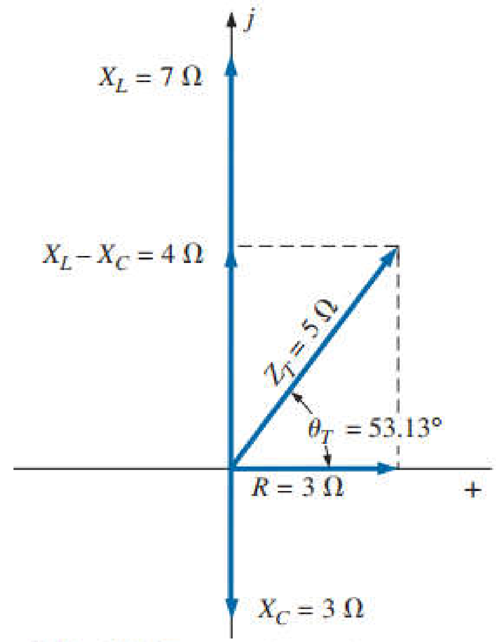


FIG. 15.37 *Impedance diagram*

V_R, V_L and, V_C

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (10\text{ A} \angle -53.13°)(3\ \Omega \angle 0°)$$
$$= \mathbf{30\ V \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90°) = (10\text{ A} \angle -53.13°)(7\ \Omega \angle 90°)$$
$$= \mathbf{70\ V \angle 36.87°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (10\text{ A} \angle -53.13°)(3\ \Omega \angle -90°)$$
$$= \mathbf{30\ V \angle -143.13°}$$

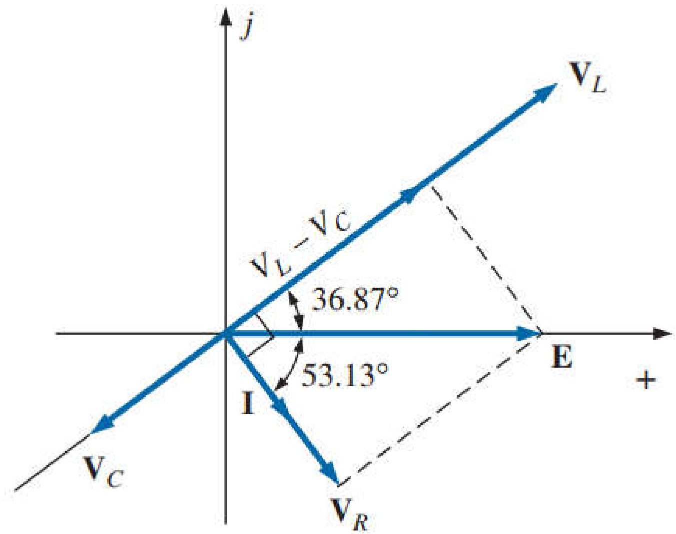


FIG. 15.38
Phasor diagram for the series R-L-C circuit in Fig. 15.35.

KVL:

$$E = V_R + V_C + V_L$$
$$= 30V\angle{-53.13^o} + 70V\angle{-36.87^o} + 30V\angle{-143.13^o}$$
$$= 50V\angle 0^o$$

Power Factor and Reactive Factor

$$pf = (R/Z_T) = \cos\theta_z = \cos(53.13^o) = \mathbf{0.6\ Lagging}$$
$$rf = (X_L - X_C)/Z_T = \sin\theta_z = \sin(53.13^o) = \mathbf{0.8}$$

Power [Total watts]

$$P_E = EI\cos\theta_z = 50\times 10\cos(53.13^o) = \mathbf{300\ W}$$
$$P_R = I^2R = (10A)^2 \times 3\Omega = \mathbf{300\ W}$$

Reactive Power [volt-ampere reactive]

$$Q_E = EI\sin\theta_z = 50\times 10\sin(53.13^o) = \mathbf{400\ Var}$$
$$Q_L = I^2X_L = (10A)^2 \times 7\Omega = \mathbf{700\ Var}$$
$$Q_C = -I^2X_C = -(10A)^2 \times 3\Omega = \mathbf{-300\ Var}$$
$$Q = Q_L + Q_C = 700 - 300 = \mathbf{400\ Var}$$

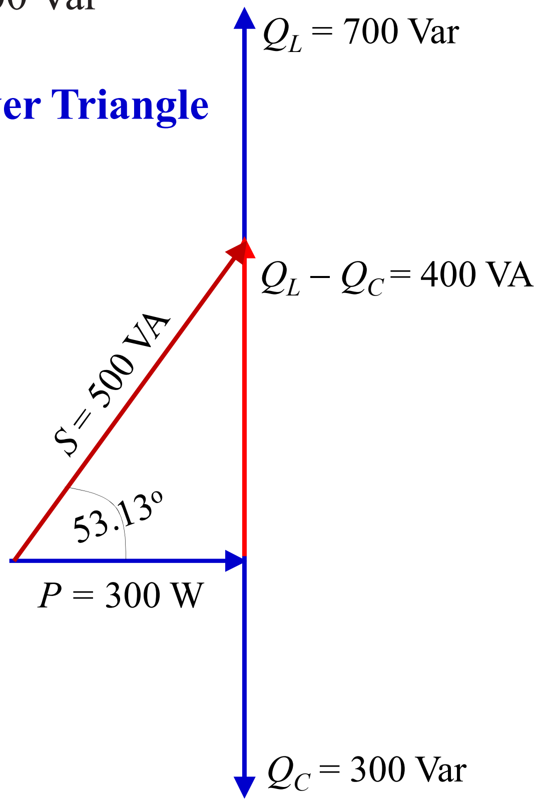
$P = 300$ W;

$Q_L = 700$ Var;

$Q_C = -300$ Var;

$Q = 400$ Var

Power Triangle



Apparent Power [volt-ampere]

$S_E = EI = 50 \times 10 = \mathbf{500\ VA}$
$S_L = I^2 Z = (10\text{A})^2 \times 5\Omega = \mathbf{500\ VA}$

Energy dissipated by the resistor over one full cycle of the input voltage

$$W_R = \frac{V_R I_R}{f} = 2\pi \frac{V_R I_R}{\omega} = 2\pi \frac{30\text{V} \times 10\text{A}}{100\,\text{rad/s}} = 18.84\ \text{J}$$

Energy stored in, or returned by, the inductor over one half-cycle of the power curve

$$W_L = \frac{V_L I_L}{\omega} = \frac{70\,\text{V} \times 10\text{A}}{100\,\text{rad/s}} = 7\ \text{J}$$

Energy stored in, or returned by, the capacitor over one half-cycle of the power curve

$$W_C = \frac{V_C I_C}{\omega} = \frac{30\,\text{V} \times 10\text{A}}{100\,\text{rad/s}} = 3\ \text{J}$$

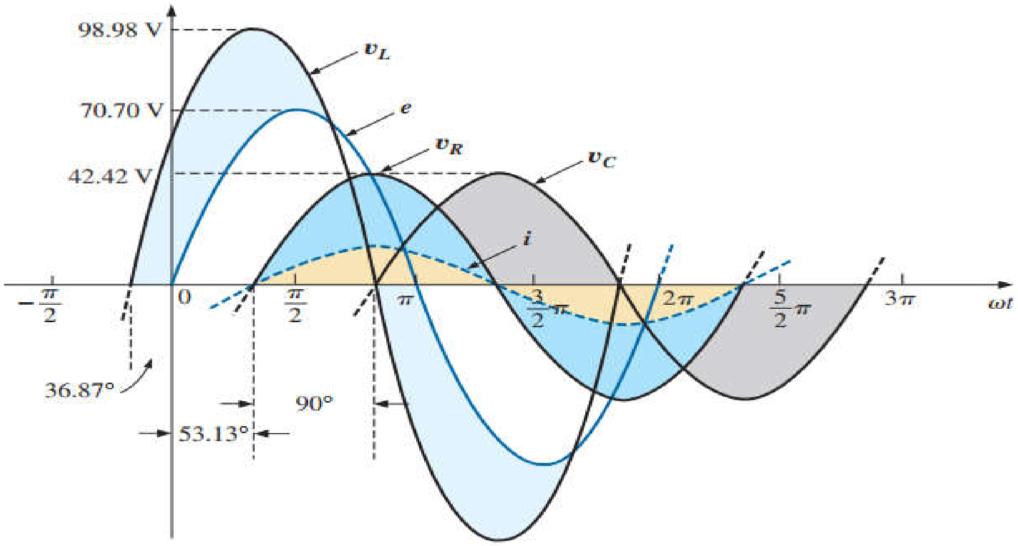
Instantaneous Power Equation

$$p(t) = P(1 - \cos 2\omega t) + Q\sin 2\omega t \text{ W} = 300(1 - \cos 200t) + 400\sin 200t \text{ W}$$

Instantaneous or Time Domain Current and Voltages Equation

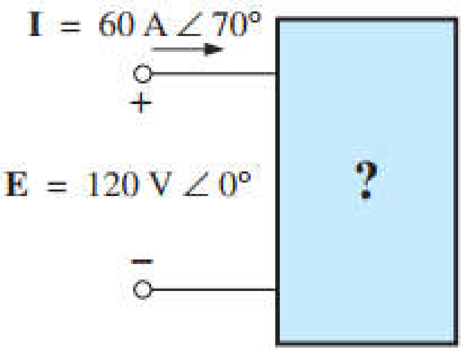
$$i = \sqrt{2}(10)\sin(\omega t - 53.13^\circ) = \mathbf{14.14\sin(\omega t - 53.13^\circ)}$$
$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13^\circ) = \mathbf{42.42\sin(\omega t - 53.13^\circ)}$$
$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87^\circ) = \mathbf{98.98\sin(\omega t + 36.87^\circ)}$$
$$v_C = \sqrt{2}(30)\sin(\omega t - 143.13^\circ) = \mathbf{42.42\sin(\omega t - 143.13^\circ)}$$

Practice Book Problems [Ch. 15] 10 and 11



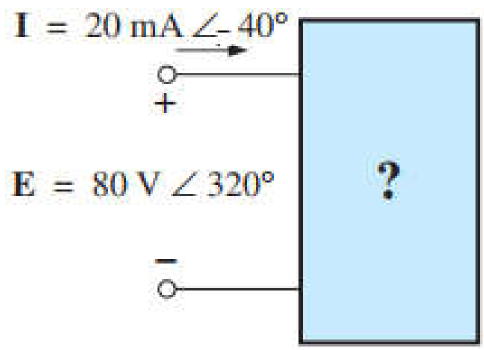
Problem 6 [Ch. 15] Find the type and impedance in ohms of the series circuit elements that must be in the closed container in Fig. 15.125 for the indicated voltages and currents to exist at the input terminals. (Find the simplest series circuit that will satisfy the indicated conditions.)

Practically, **$-90^o \le \theta_z \le 90^o$**



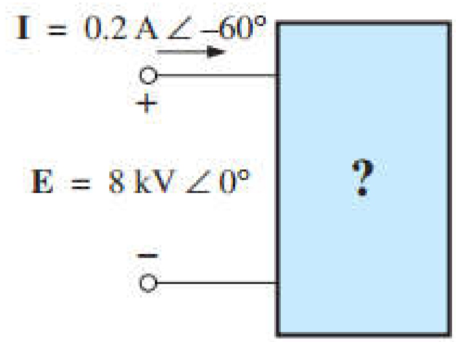
(a)

$\theta_z = \theta_v - \theta_i = 0^o - 70^o = -70^o$
Since $\theta_z < 0^o$ so voltage lags current.
The circuit is R–C series circuit or R–L –C series circuit with $X_C > X_L$.



(b)

$\theta_v = 320^o - 360^o = -40^o$
$\theta_z = \theta_v - \theta_i = -40^o - (-40^o) = 0^o$
Since $\theta_z = 0^o$ so voltage and current are in phase.
The circuit is pure resistive or R–L –C series circuit with $X_L = X_C$.



(c)

$\theta_z = \theta_v - \theta_i = 0^o - (-60^o) = 60^o$
Since $\theta_z > 0^o$ so voltage leads current.
The circuit is R–L series circuit or R–L –C series circuit with $X_L > X_C$.

Voltage Divider Rule (VDR)

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig. 15.40.

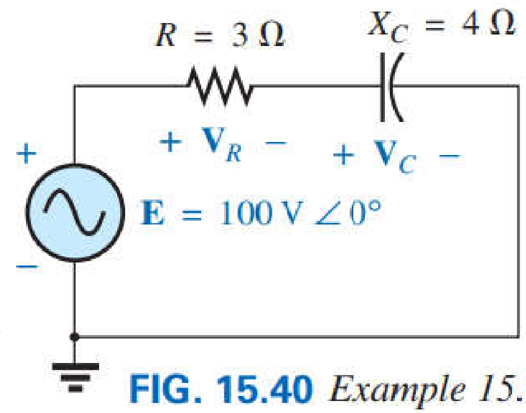


FIG. 15.40 *Example 15.9.*

Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega\ \angle -90°)(100\ \text{V}\ \angle 0°)}{4\ \Omega\ \angle -90° + 3\ \Omega\ \angle 0°} = \frac{400\ \angle -90°}{3 - j4}$$

$$= \frac{400\ \angle -90°}{5\ \angle -53.13°} = \mathbf{80\ V\ \angle\ -36.87°}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega\ \angle 0°)(100\ \text{V}\ \angle 0°)}{5\ \Omega\ \angle -53.13°} = \frac{300\ \angle 0°}{5\ \angle -53.13°}$$

$$= \mathbf{60\ V\ \angle\ +53.13°}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit in Fig. 15.41.

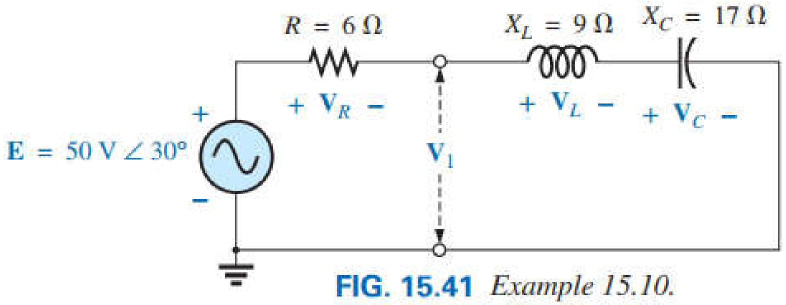


FIG. 15.41 *Example 15.10.*

Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega \angle 0°)(50\ \text{V} \angle 30°)}{6\ \Omega \angle 0° + 9\ \Omega \angle 90° + 17\ \Omega \angle -90°}$$

$$= \frac{300 \angle 30°}{6 + j\,9 - j\,17} = \frac{300 \angle 30°}{6 - j\,8}$$

$$= \frac{300 \angle 30°}{10 \angle -53.13°} = \mathbf{30\ V \angle 83.13°}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90°)(50\ \text{V} \angle 30°)}{10\ \Omega \angle -53.13°} = \frac{450\ \text{V} \angle 120°}{10 \angle -53.13°}$$

$$= \mathbf{45\ V \angle 173.13°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega \angle -90°)(50\ \text{V} \angle 30°)}{10\ \Omega \angle -53.13°} = \frac{850\ \text{V} \angle -60°}{10 \angle -53°}$$

$$= \mathbf{85\ V \angle -6.87°}$$

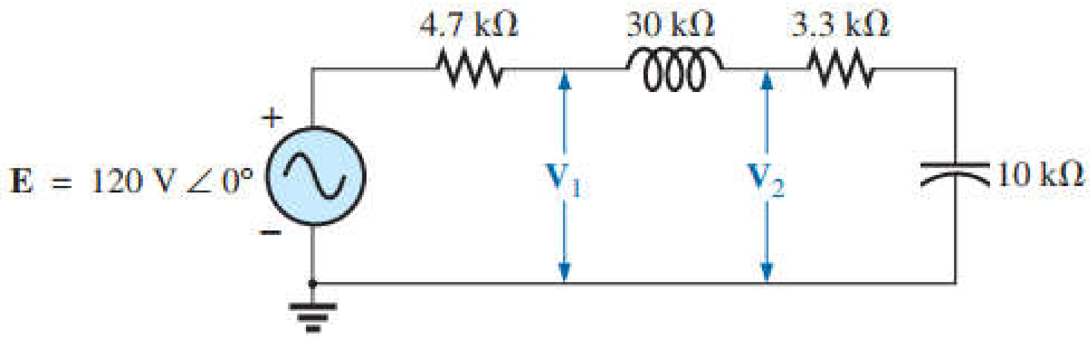
$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90° + 17\ \Omega \angle -90°)(50\ \text{V} \angle 30°)}{10\ \Omega \angle -53.13°}$$

$$= \frac{(8 \angle -90°)(50 \angle 30°)}{10 \angle -53.13°}$$

$$= \frac{400 \angle -60°}{10 \angle -53.13°} = \mathbf{40\ V \angle -6.87°}$$

Practice Book Problems
[Ch. 15] 15 and 16

Problem 16(b) Calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ for the circuits in Fig. 15.135 in phasor form using the voltage divider rule.



$$\boldsymbol{E} = 120\text{V}\angle 0° = 120\,\text{V}$$

$$\boldsymbol{Z}_T = (4.7 + j30 + 3.3 - j10)\,\text{k}\Omega = (8 + j20)\,\text{k}\Omega$$

Let, $\boldsymbol{Z}_1 = (j30 + 3.3 - j10)\,\text{k}\Omega = (3.3 + j20)\,\text{k}\Omega$

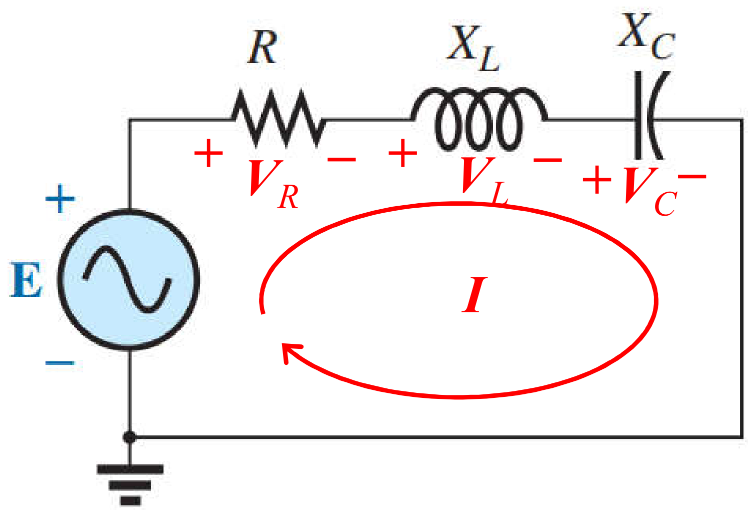
$\boldsymbol{Z}_2 = (3.3 - j10)\,\text{k}\Omega$

$$\boldsymbol{V}_1 = \frac{\boldsymbol{Z}_1}{\boldsymbol{Z}_T}\boldsymbol{E} = \frac{(3.3 + j20)\,\text{k}\Omega}{(8 + j20)\,\text{k}\Omega}(120\text{V})$$
$$= 110.28 + j24.31\ \text{V}$$
$$= 112.93\text{V}\angle 12.43°$$

$$\boldsymbol{V}_2 = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_T}\boldsymbol{E} = \frac{(3.3 - j10)\,\text{k}\Omega}{(8 + j20)\,\text{k}\Omega}(120\text{V})$$
$$= -44.9 - j37.76\ \text{V}$$
$$= 58.67\text{V}\angle -139.937°$$

Kirchhoff's Voltage Law (KVL)

Example: Applying KVL write the loop equation for the following circuit.



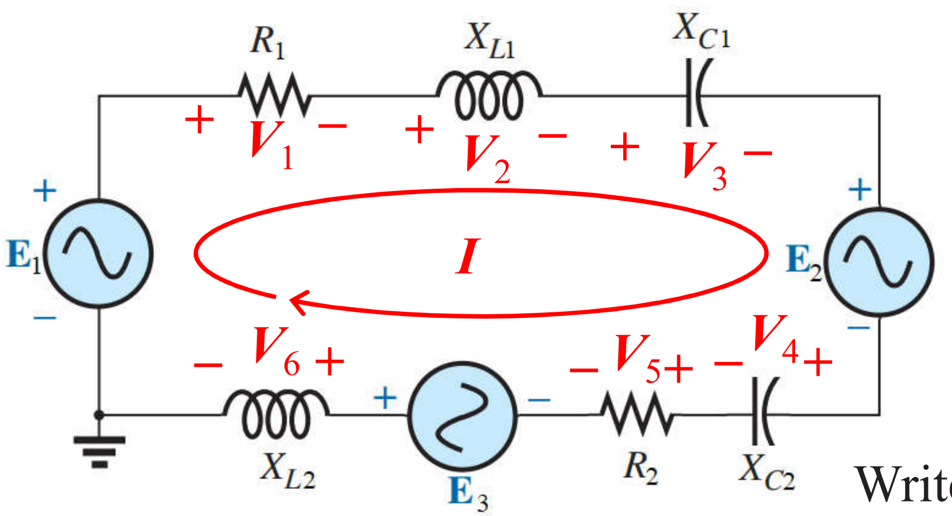
$$\boldsymbol{V}_R = \boldsymbol{Z}_R\boldsymbol{I} = R\boldsymbol{I}$$
$$\boldsymbol{V}_L = \boldsymbol{Z}_L\boldsymbol{I} = jX_L\boldsymbol{I}$$
$$\boldsymbol{V}_C = \boldsymbol{Z}_C\boldsymbol{I} = -jX_C\boldsymbol{I}$$

$$\boldsymbol{Z}_R = R\angle 0° = R\ \ \Omega$$
$$\boldsymbol{Z}_L = X_L\angle 90° = jX_L\ \ \Omega$$
$$\boldsymbol{Z}_C = X_C\angle -90° = -jX_C\ \ \Omega$$

Write loop equation using KVL:

$$\boldsymbol{V}_R + \boldsymbol{V}_L + \boldsymbol{V}_C = \boldsymbol{E}$$
$$\boldsymbol{Z}_R\boldsymbol{I} + \boldsymbol{Z}_L\boldsymbol{I} + \boldsymbol{Z}_C\boldsymbol{I} = \boldsymbol{E}$$
$$(\boldsymbol{Z}_R + \boldsymbol{Z}_L + \boldsymbol{Z}_C)\boldsymbol{I} = \boldsymbol{E}$$
$$(R + jX_L - jX_C)\boldsymbol{I} = \boldsymbol{E}$$
$$[R + j(X_L - X_C)]\boldsymbol{I} = \boldsymbol{E}$$

Example: Applying KVL write the loop equation for the following circuits.



$$\boldsymbol{V}_1 = \boldsymbol{Z}_1\boldsymbol{I} = R_1\boldsymbol{I} \qquad \boldsymbol{V}_2 = \boldsymbol{Z}_2\boldsymbol{I} = jX_{L1}\boldsymbol{I}$$

$$\boldsymbol{V}_3 = \boldsymbol{Z}_3\boldsymbol{I} = -jX_{C1}\boldsymbol{I} \qquad \boldsymbol{V}_4 = \boldsymbol{Z}_4\boldsymbol{I} = -jX_{C2}\boldsymbol{I}$$

$$\boldsymbol{V}_5 = \boldsymbol{Z}_5\boldsymbol{I} = R_2\boldsymbol{I} \qquad \boldsymbol{V}_6 = \boldsymbol{Z}_6\boldsymbol{I} = jX_{L2}\boldsymbol{I}$$

$$\boldsymbol{Z}_1 = R_1\angle 0° = R_1 \ \Omega$$

$$\boldsymbol{Z}_2 = X_{L1}\angle 90° = jX_{L1} \ \Omega$$

$$\boldsymbol{Z}_3 = X_{C1}\angle -90° = -jX_{C1} \ \Omega$$

$$\boldsymbol{Z}_4 = X_{C2}\angle -90° = -jX_{C2} \ \Omega$$

$$\boldsymbol{Z}_5 = R_2\angle 0° = R_2 \ \Omega$$

$$\boldsymbol{Z}_6 = X_{L2}\angle 90° = jX_{L2} \ \Omega$$

Write loop equation using KVL:

$$\boldsymbol{V}_1 + \boldsymbol{V}_2 + \boldsymbol{V}_3 + \boldsymbol{V}_4 + \boldsymbol{V}_5 + \boldsymbol{V}_6 = \boldsymbol{E}_1 - \boldsymbol{E}_2 + \boldsymbol{E}_3$$

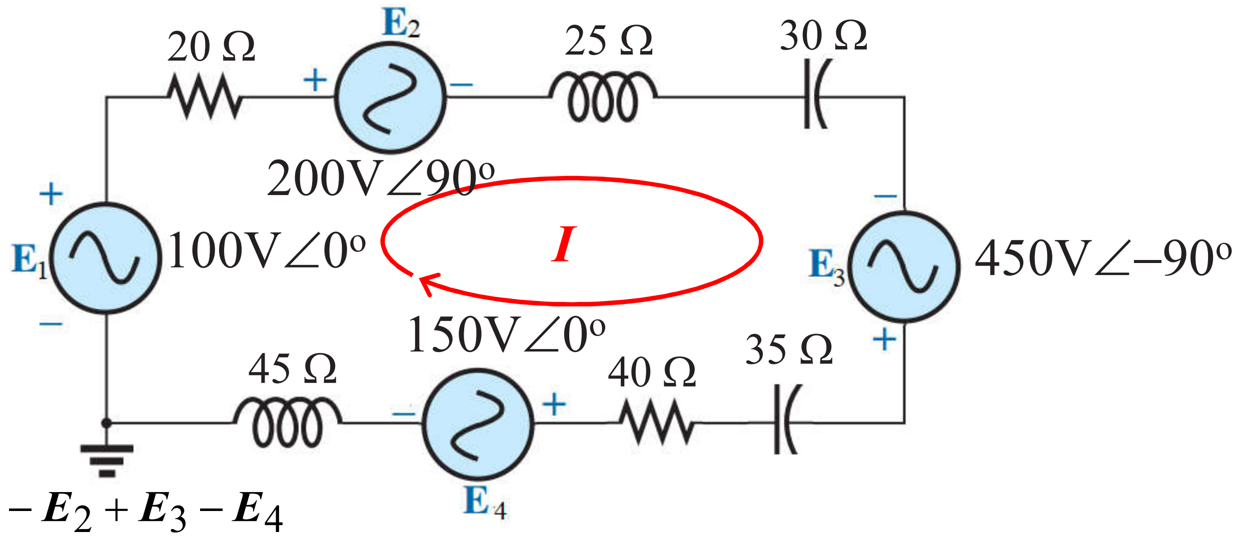
$$\boldsymbol{Z}_1\boldsymbol{I} + \boldsymbol{Z}_2\boldsymbol{I} + \boldsymbol{Z}_3\boldsymbol{I} + \boldsymbol{Z}_4\boldsymbol{I} + \boldsymbol{Z}_5\boldsymbol{I} + \boldsymbol{Z}_6\boldsymbol{I} = \boldsymbol{E}_1 - \boldsymbol{E}_2 + \boldsymbol{E}_3$$

$$(\boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 + \boldsymbol{Z}_4 + \boldsymbol{Z}_5 + \boldsymbol{Z}_6)\boldsymbol{I} = \boldsymbol{E}_1 - \boldsymbol{E}_2 + \boldsymbol{E}_3$$

$$(R_1 + jX_{L1} - jX_{C1} - jX_{C2} + R_2 + jX_{L2})\boldsymbol{I} = \boldsymbol{E}_1 - \boldsymbol{E}_2 + \boldsymbol{E}_3$$

$$[(R_1 + R_2) + j(X_{L1} + X_{L2} - X_{C1} - X_{L2})]\boldsymbol{I} = \boldsymbol{E}_1 - \boldsymbol{E}_2 + \boldsymbol{E}_3$$

Example: Applying KVL write the loop equation for the following circuits.



$$(20+j25-j30-j35+40+j45)\boldsymbol{I}=\boldsymbol{E}_1-\boldsymbol{E}_2+\boldsymbol{E}_3-\boldsymbol{E}_4$$

$$(60+j70-j65)\boldsymbol{I}=\boldsymbol{E}_1-\boldsymbol{E}_2+\boldsymbol{E}_3-\boldsymbol{E}_4$$

$$(60+j5)\boldsymbol{I}=\boldsymbol{E}_1-\boldsymbol{E}_2+\boldsymbol{E}_3-\boldsymbol{E}_4$$

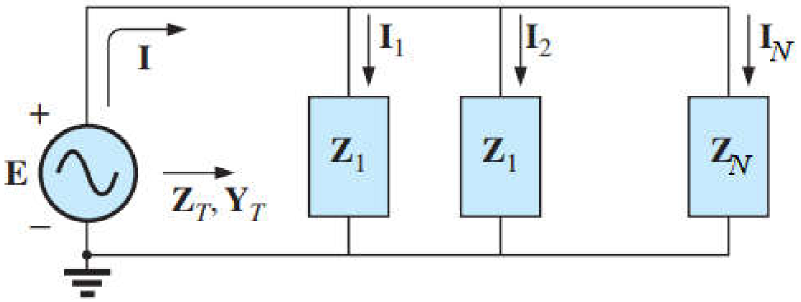
$$\begin{aligned}\boldsymbol{E}_1-\boldsymbol{E}_2+\boldsymbol{E}_3-\boldsymbol{E}_4&=100\text{V}\angle 0^\circ+200\text{V}\angle 90^\circ+450\text{V}\angle-90^\circ+150\text{V}\angle 0^\circ\\&=100\text{V}+j200\text{V}-j450\text{V}+150\text{V}\\&=250-j250\quad\text{V}\end{aligned}$$

$$(60+j5)\boldsymbol{I}=250-j250\ \text{V}$$

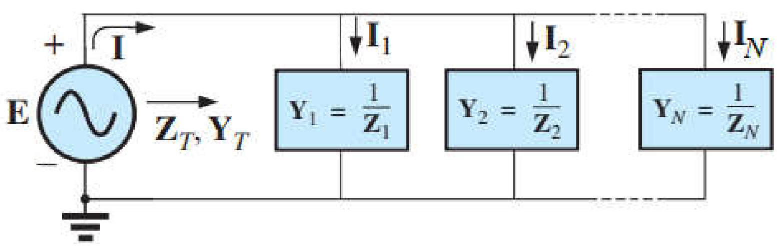
Chapter 15

Parallel Circuits

Parallel Configuration



$$Y_1 = \frac{1}{Z_1}; \quad Y_2 = \frac{1}{Z_2}; \ldots\ldots \quad Y_N = \frac{1}{Z_N}$$



The **total admittance** of a parallel configuration is the sum of the individual admittances:

$$Y_T = Y_1 + Y_2 + \ldots\ldots + Y_N \qquad \textbf{(15.16)}$$

The **total impedance** of a parallel configuration can be calculated as follows:

$$Z_T = \frac{1}{Y_T}$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots + \frac{1}{Z_N} \qquad \textbf{(15.17)}$$

$$Z_T = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots + \frac{1}{Z_N}} \qquad \textbf{(15.18)}$$

For **two impedance in parallel** :

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad \textbf{(15.19)}$$

Current

$$I = \frac{E}{Z_T} = EY_T$$

$$I_1 = \frac{E}{Z_1} = EY_1$$

$$I_2 = \frac{E}{Z_2} = EY_2$$

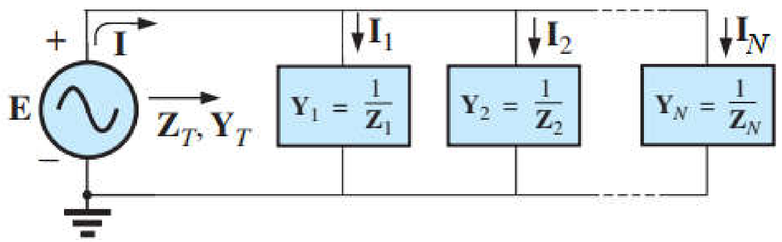
$$I_N = \frac{E}{Z_N} = EY_N$$

If $Z_1 = Z_2 = \ldots . = Z_N = Z_p$

$$Y_1 = Y_2 = \ldots . = Y_N = Y_p$$

$$Y_T = N \times Y_p \qquad Z_T = \frac{Z_p}{N}$$

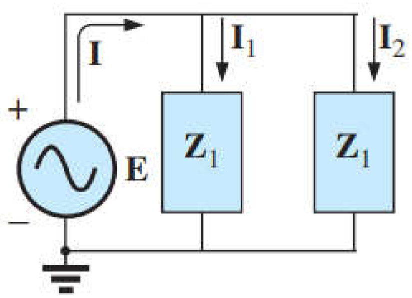
$$I_1 = I_2 = .. = I_N = \frac{I}{N}$$



Current Divider Rule (CDR)

The current flows through an admittance in a parallel circuit is equal to the value of that admittance (Y_x) times the total current (I) divided by the total admittance (Y_T) of the parallel configuration.

$$I_x = \frac{Y_x}{Y_T} I = \frac{Z_T}{Z_x} I$$



$$I_1 = \frac{Z_2}{Z_1 + Z_2} I$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

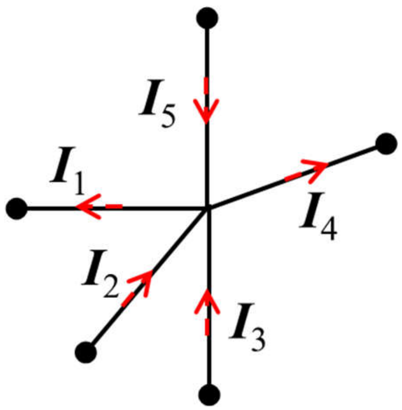
Kirchhoff's Current Law (KCL)

(1) The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

$$\sum I_{entering} - \sum I_{leaving} = 0$$

(2) The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

$$\sum I_{entering} = \sum I_{leaving}$$

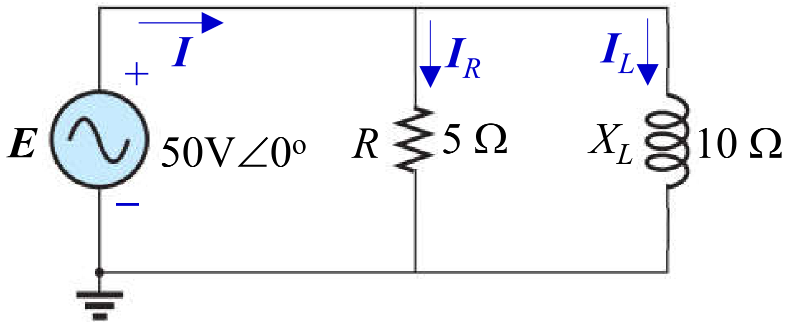


(1) $(I_2 + I_3 + I_5) - (I_1 + I_4) = 0$

(2) $I_2 + I_3 + I_5 = I_1 + I_4$

R-*L* Parallel Circuit

R-L Parallel Circuit



Admittance

$$\boldsymbol{Z}_R = 5\Omega\angle 0° = 5\ \Omega \qquad \boldsymbol{Z}_L = 10\Omega\angle 90° = j10\ \Omega$$

$$\boldsymbol{Y}_R = \frac{1}{\boldsymbol{Z}_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{10\Omega\angle 90°} = 0.1\text{S}\angle -90° = -j0.1\ \text{S}$$

$$\boldsymbol{Y}_T = \boldsymbol{Y}_R + \boldsymbol{Y}_L = 0.2\ \text{S} - j0.1\ \text{S} = 0.224\text{S}\angle -26.57°$$

Admittance Diagram

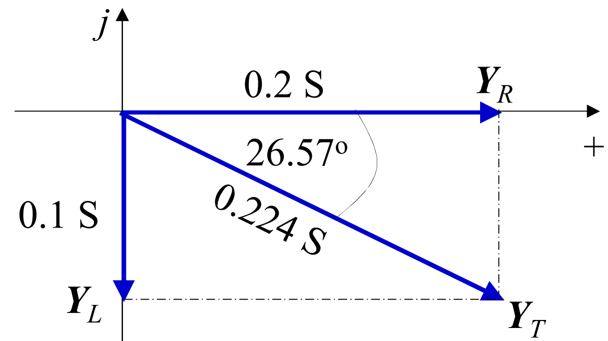


Fig. Admittance diagram

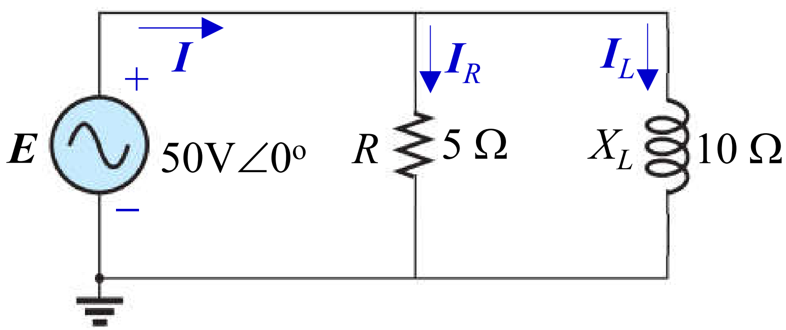
Impedance

$$\boldsymbol{Z}_T = \frac{1}{\boldsymbol{Y}_T} = \frac{1}{0.224\text{S}\angle -26.57°} = 4.46\Omega\angle 26.57° \cong 4 + j2\Omega$$

$$\boldsymbol{Z}_T = \frac{1}{\frac{1}{\boldsymbol{Z}_R} + \frac{1}{\boldsymbol{Z}_L}} = \frac{1}{\frac{1}{5\Omega} + \frac{1}{j10\Omega}} = 4 + j2\Omega = 4.47\Omega\angle 26.57$$

$$\boldsymbol{Z}_T = \frac{\boldsymbol{Z}_R\boldsymbol{Z}_L}{\boldsymbol{Z}_R + \boldsymbol{Z}_L} = \frac{(5\Omega)(j10\Omega)}{5\Omega + j10\Omega} = 4 + j2\ \Omega = 4.47\Omega\angle 26.57°$$

R-L Parallel Circuit



Current

$$\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \boldsymbol{E}\boldsymbol{Y}_T = \frac{50\text{V}\angle 0^\circ}{4.47\Omega\angle 26.57^\circ}$$

$$= 11.18\text{A}\angle -26.57^\circ$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{E}}{\boldsymbol{Z}_R} = \boldsymbol{E}\boldsymbol{Y}_R = \frac{50\text{V}\angle 0^\circ}{5\Omega\angle 0^\circ} = 10\text{A}\angle 0^\circ$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{E}}{\boldsymbol{Z}_L} = \boldsymbol{E}\boldsymbol{Y}_L = \frac{50\text{V}\angle 0^\circ}{10\Omega\angle 90^\circ} = 5\text{A}\angle -90^\circ$$

KCL:

$$\boldsymbol{I}_R + \boldsymbol{I}_L = 10\text{A}\angle 0^\circ + 5\text{A}\angle -90^\circ = 11.18\text{A}\angle -26.57^\circ = \boldsymbol{I}$$

Phasor Diagram

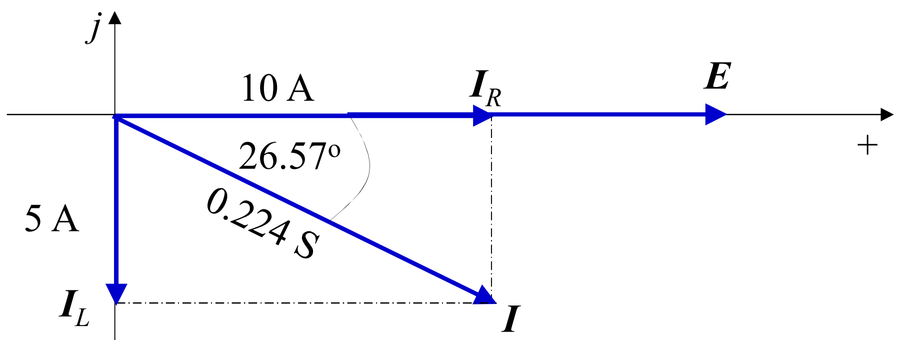
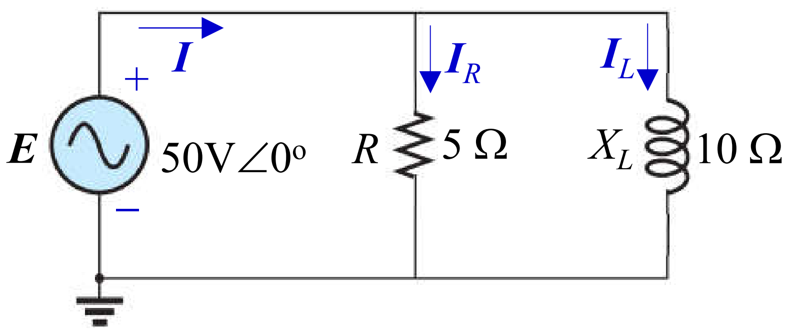


Fig. Phasor diagram

Practice Solution of Fig. 15.68 [Ch. 15], Problems 28 and 30

R-L Parallel Circuit



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(26.57°) = \mathbf{0.894\ lagging}$

$rf = (B_L/Y_T) = \sin\theta_z = \sin(26.57°) = \mathbf{0.447}$

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times11.18\cos(26.57°) = \mathbf{500.19\ W}$

$P_R = I_R^2R = (E^2/R) = (50V)^2/5\Omega = \mathbf{500\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times11.18\sin(26.57°) = \mathbf{250.1\ Var}$

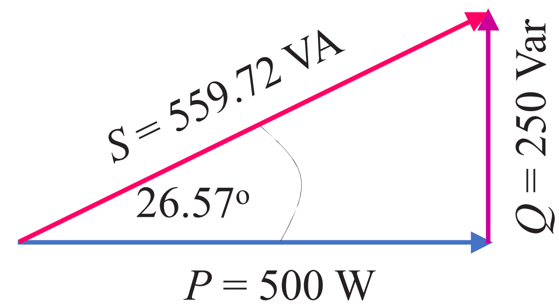
$Q_L = I_L^2X_L = (E^2/X_L) = (50V)^2/10\Omega = \mathbf{250\ Var}$

Apparent Power [volt-ampere]

$S_E = EI = 50\times11.18 = \mathbf{559.5\ VA}$

$S_Z = I^2Z = (E^2/Z) = (50V)^2/4.47\Omega = \mathbf{559.72\ VA}$

Power Triangle



Instantaneous Equation

$p(t) = 500(1 - \cos2\omega t) + 250\sin2\omega t$ W

$e(t) = \left(\sqrt{2}\times50\right)\sin\omega t$ V

$i(t) = \left(\sqrt{2}\times11.18\right)\sin\left(\omega t - 26.57°\right)$ A

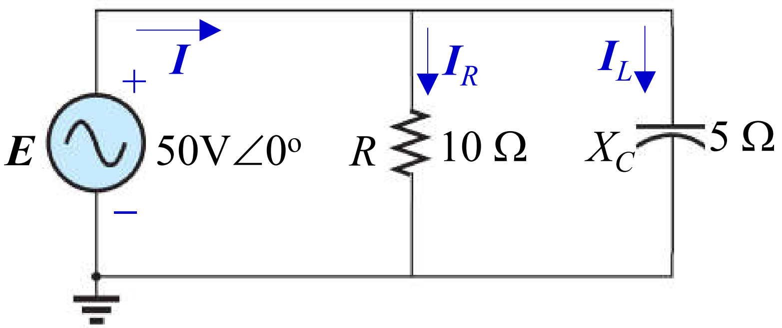
$i_R(t) = \left(\sqrt{2}\times10\right)\sin\omega t$ A

$i_L(t) = \left(\sqrt{2}\times5\right)\sin\left(\omega t - 90°\right)$ A

Practice Solution of Fig. 15.68 [Ch. 15], Problems 28 and 30

R-*C* Parallel Circuit

R-C Parallel Circuit



Admittance

$$Z_R = 10\Omega\angle 0° = 10\ \Omega$$

$$Z_C = 5\Omega\angle -90° = -j5\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{10\Omega\angle 0°} = 0.1\text{S}\angle 0° = 0.1\ \text{S}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{5\Omega\angle -90°} = 0.2\text{S}\angle 90° = j0.2\ \text{S}$$

$$Y_T = Y_R + Y_C = 0.1\ \text{S} + j0.2\ \text{S}$$
$$= 0.224\text{S}\angle 63.43°$$

Admittance Diagram

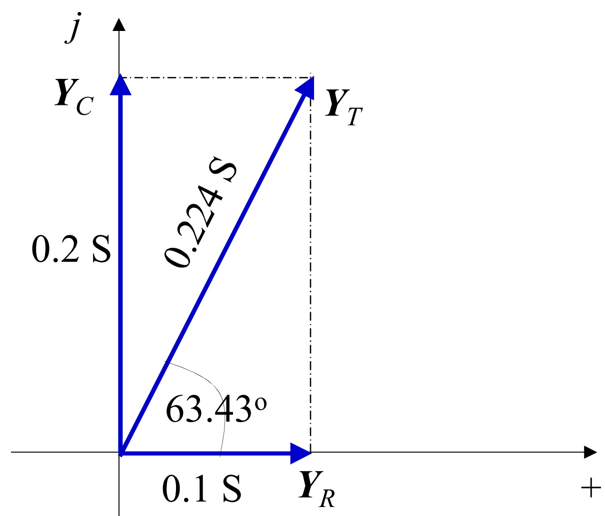


Fig. Admittance diagram

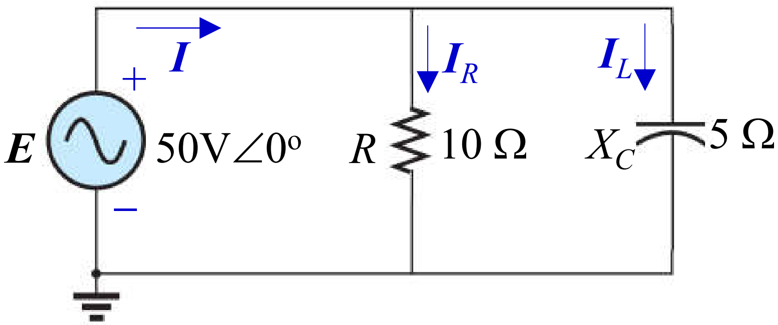
Impedance

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.224\text{S}\angle 63.43°}$$
$$= 4.46\Omega\angle -63.43°$$
$$\cong 2 - j4\Omega$$

$$Z_T = \frac{Z_R Z_C}{Z_R + Z_C}$$
$$= \frac{(10\Omega)(j5\Omega)}{10\Omega -\ j5\Omega}$$
$$= 2 - j4\Omega$$
$$= 4.472\Omega\angle -63.43°$$

$$Z_T = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}}$$
$$= \frac{1}{\frac{1}{10\Omega} + \frac{1}{-j5\Omega}}$$
$$= 2 - j4\Omega$$
$$= 4.472\Omega\angle -63.43°$$

R-C Parallel Circuit



Current

$$I = \frac{E}{Z_T} = EY_T = \frac{50\text{V}\angle 0^\circ}{4.47\Omega\angle -63.43^\circ}$$

$$= 11.18\text{A}\angle 63.43^\circ$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{50\text{V}\angle 0^\circ}{10\Omega\angle 0^\circ} = 5\text{A}\angle 0^\circ$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{50\text{V}\angle 0^\circ}{5\Omega\angle -90^\circ} = 10\text{A}\angle 90^\circ$$

KCL:

$$I_R + I_C = 5\text{A}\angle 0^\circ + 10\text{A}\angle 90^\circ = 11.18\text{A}\angle 63.43^\circ = I$$

Phasor Diagram

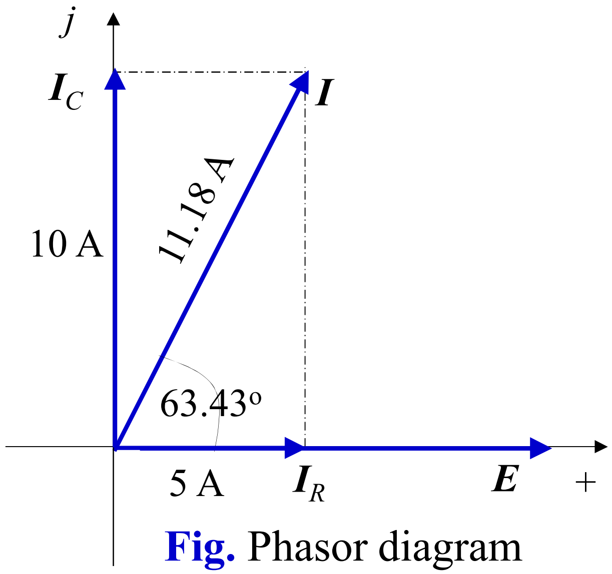
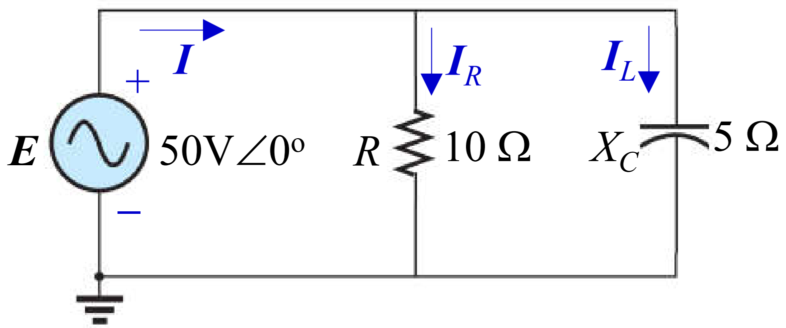


Fig. Phasor diagram

Practice Solution of Fig. 15.72 [Ch. 15], Problem 29

R-C Parallel Circuit



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(-63.43^o)$ = **0.447 leading**

$rf = (B_L/Y_T) = \sin\theta_z = \sin(-63.43^o)$ = **– 0.894**

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times11.18\cos(-63.43^o)$ = **250.1 W**

$P_R = I_R^2R = (E^2/R) = (50V)^2/10\Omega$ = **250 W**

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times11.18\sin(-63.43^o)$ = **– 500.19 Var**

$Q_C = -I_C^2X_C = -(E^2/X_C) = -(50V)^2/5\Omega$ = **– 500 Var**

Apparent Power [volt-ampere]

$S_E = EI$

$= 50\times11.18$ VA

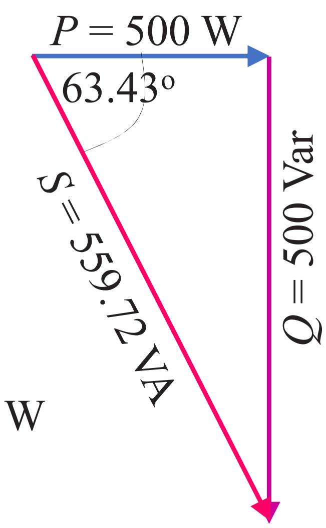
= 559.5 VA

$S_Z = I^2Z = (E^2/Z)$

$=(50V)^2/4.47\Omega$

= **559.72 VA**

Power Triangle



Instantaneous Equation

$p(t) = 250(1-\cos2\omega t) - 500\sin2\omega t$ W

$e(t) = \left(\sqrt{2}\times50\right)\sin\omega t$ V

$i(t) = \left(\sqrt{2}\times11.18\right)\sin(\omega t + 63.43°)$ A

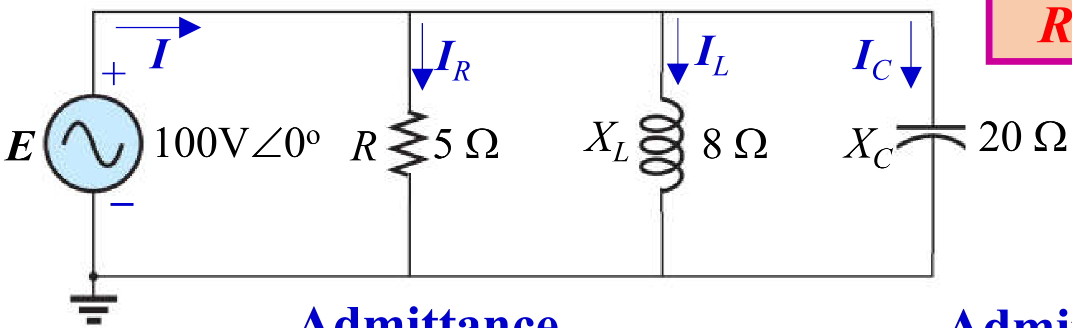
$i_R(t) = \left(\sqrt{2}\times5\right)\sin\omega t$ A

$i_C(t) = \left(\sqrt{2}\times10\right)\sin(\omega t + 90°)$ A

Practice Solution of Fig. 15.72 [Ch. 15], Problem 29

R-L-C Parallel Circuit Example 1

R-L-C Parallel Circuit 1



Admittance

$$\boldsymbol{Z}_R = 5\Omega\angle 0° = 5\ \Omega \qquad \boldsymbol{Z}_L = 8\Omega\angle 90° = j8\ \Omega$$

$$\boldsymbol{Z}_C = 20\Omega\angle -90° = -j20\ \Omega$$

$$\boldsymbol{Y}_R = \frac{1}{\boldsymbol{Z}_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{8\Omega\angle 90°} = 0.125\text{S}\angle -90° = -j0.125\ \text{S}$$

$$\boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{20\Omega\angle -90°} = 0.05\text{S}\angle 90° = j0.05\ \text{S}$$

$$\boldsymbol{Y}_T = \boldsymbol{Y}_R + \boldsymbol{Y}_L + \boldsymbol{Y}_C = 0.2\ \text{S} - j0.125\ \text{S} + j0.05\ \text{S}$$
$$= 0.2\ \text{S} - j0.075\ \text{S} = 0.214\text{S}\angle -20.56°$$

Admittance Diagram

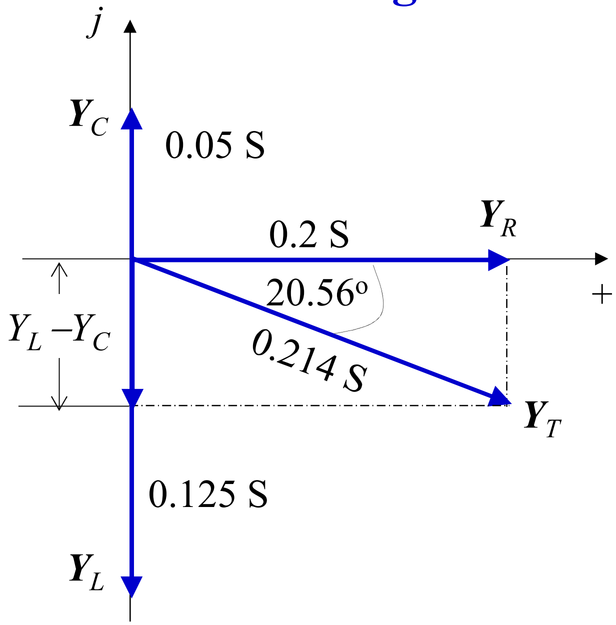
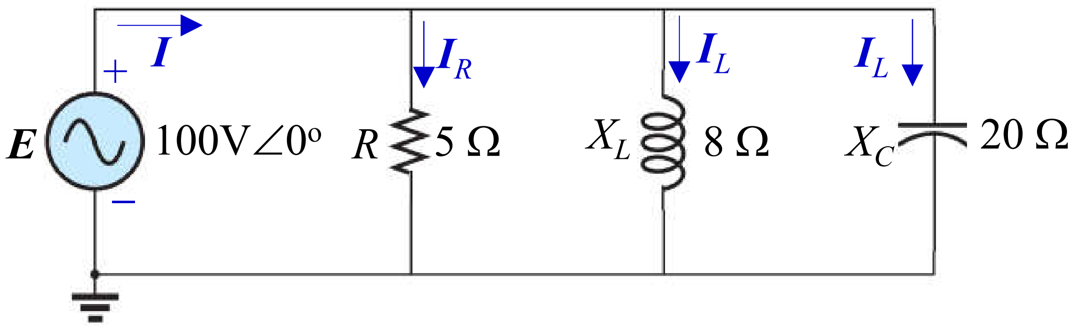


Fig. Admittance diagram

Impedance

$$\boldsymbol{Z}_T = \frac{1}{\dfrac{1}{\boldsymbol{Z}_R} + \dfrac{1}{\boldsymbol{Z}_L} + \dfrac{1}{\boldsymbol{Z}_C}}$$
$$= \frac{1}{\dfrac{1}{5\Omega} + \dfrac{1}{j8\Omega} + \dfrac{1}{-j20\Omega}}$$
$$= 4.38 + j1.64\Omega$$
$$= 4.68\Omega\angle 20.56°$$



Current

$$I = \frac{E}{Z_T} = EY_T = \frac{100\text{V}\angle 0°}{4.68\Omega\angle 20.56°} = 21.37\text{A}\angle -20.56°$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{100\text{V}\angle 0°}{5\Omega\angle 0°} = 20\text{A}\angle 0°$$

$$I_L = \frac{E}{Z_L} = EY_L = \frac{100\text{V}\angle 0°}{8\Omega\angle 90°} = 12.5\text{A}\angle -90°$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{100\text{V}\angle 0°}{20\Omega\angle -90°} = 5\text{A}\angle 90°$$

KCL:

$$I_R + I_L + I_C = 20\text{A}\angle 0° + 12.5\text{A}\angle -90° + 5\text{A}\angle 90°$$
$$= 21.37\text{A}\angle -20.56° = I$$

Phasor Diagram

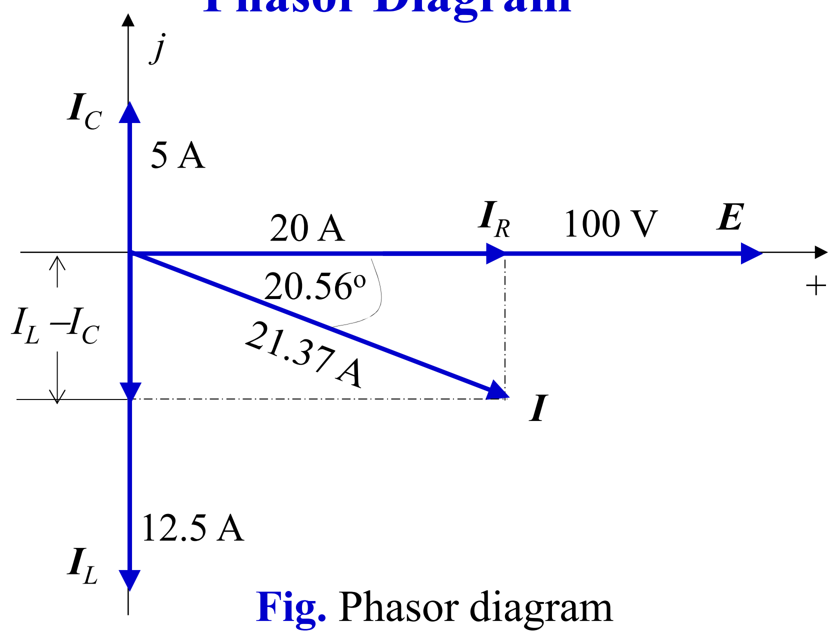
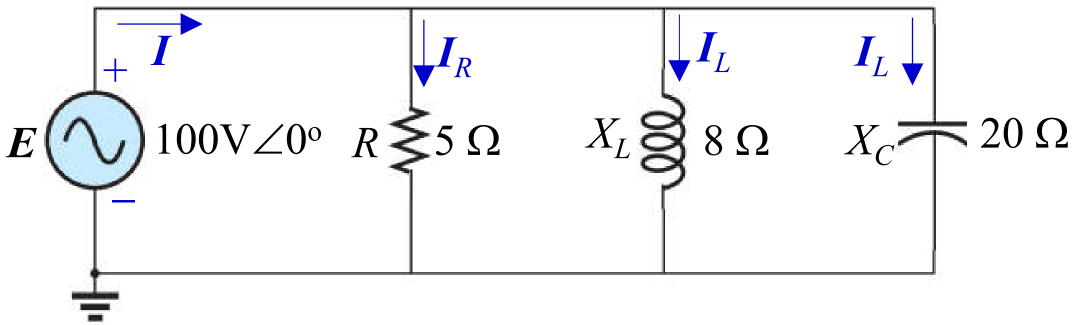


Fig. Phasor diagram

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32



Apparent Power [volt-ampere]

$S_E = EI = 100\times 21.37 = \mathbf{2137\ VA}$
$S_Z = I^2Z = (E^2/Z) = (100V)^2/4.68\Omega = \mathbf{2137.75\ VA}$

Power Triangle

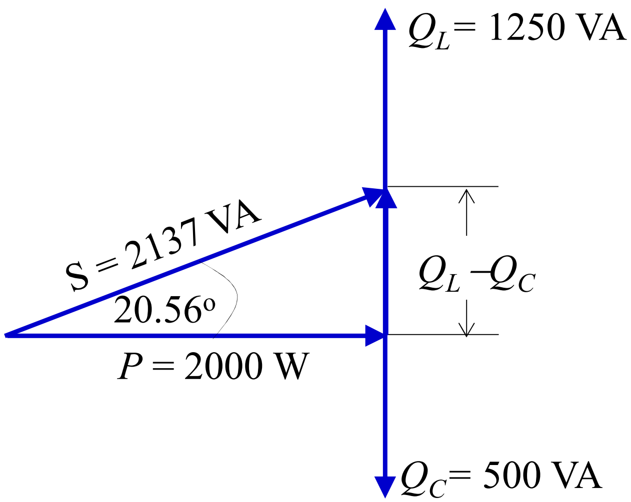


Fig. Admittance diagram

Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(20.56^o) = \mathbf{0.936\ lagging}$
$rf = (B/Y_T) = \sin\theta_z = \sin(20.56^o) = \mathbf{0.351}$

Power [Total watts]

$P_E = EI\cos\theta_z = 100\times 21.37\cos(20.56^o) = \mathbf{2000.23\ W}$
$P_R = I_R^2R = (E^2/R) = (100V)^2/5\Omega = \mathbf{2000\ W}$

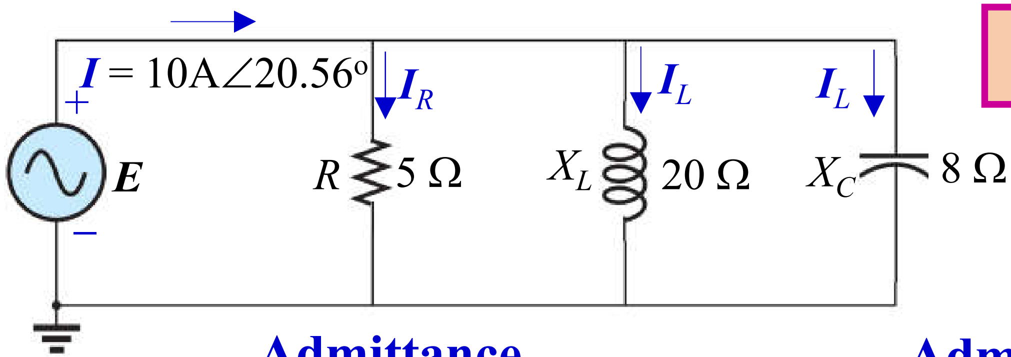
Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 100\times 21.37\sin(20.56^o) = \mathbf{750.09\ Var}$
$Q_L = I_L^2X_L = (E^2/X_L) = (100V)^2/8\Omega = \mathbf{1250\ Var}$
$Q_C = -I_C^2X_C = -(E^2/X_C) = -(100V)^2/20\Omega = \mathbf{-500\ Var}$
$Q = Q_L + Q_C = \mathbf{750\ Var}$

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32

R-L-C Parallel Circuit Example 2

R-L-C Parallel Circuit 2



Admittance

$$Z_R = 5\Omega\angle 0° = 5\ \Omega \qquad Z_L = 20\Omega\angle 90° = j20\ \Omega$$

$$Z_C = 8\Omega\angle -90° = -j8\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{20\Omega\angle 90°} = 0.05\text{S}\angle -90° = -j0.05\ \text{S}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{8\Omega\angle -90°} = 0.125\text{S}\angle 90° = j0.125\ \text{S}$$

$$Y_T = Y_R + Y_L + Y_C = 0.2\ \text{S} - j0.05\ \text{S} + j0.125\ \text{S}$$
$$= 0.2\ \text{S} + j0.075\ \text{S} = 0.214\text{S}\angle 20.56°$$

Admittance Diagram

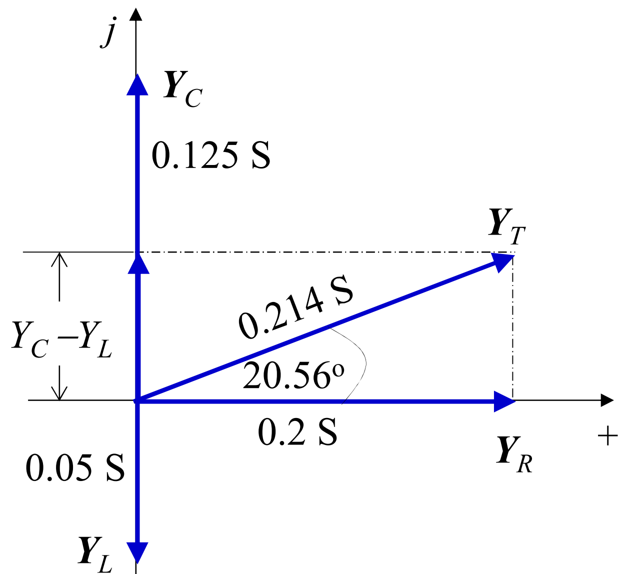


Fig. Admittance diagram

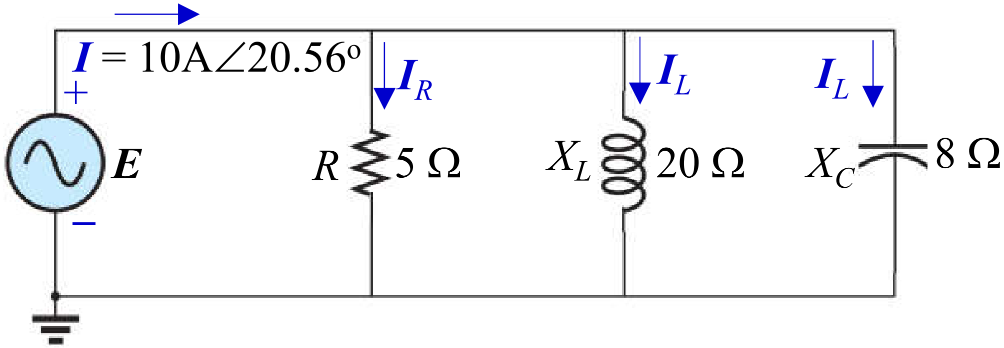
Impedance

$$Z_T = \cfrac{1}{\cfrac{1}{Z_R} + \cfrac{1}{Z_L} + \cfrac{1}{Z_C}}$$

$$= \cfrac{1}{\cfrac{1}{5\Omega} + \cfrac{1}{-j8\Omega} + \cfrac{1}{j20\Omega}}$$

$$= 4.38 - j1.64\Omega$$

$$= 4.68\Omega\angle -20.56°$$



Current

$$E = IZ_T = \frac{I}{Y_T} = \frac{10A\angle 20.56°}{0.214S\angle -20.56°} = 46.73V\angle 0°$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{46.73V\angle 0°}{5\Omega\angle 0°} = 9.35A\angle 0°$$

$$I_L = \frac{E}{Z_L} = EY_L = \frac{46.73V\angle 0°}{20\Omega\angle 90°} = 2.34A\angle -90°$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{46.73V\angle 0°}{8\Omega\angle -90°} = 5.84A\angle 90°$$

KCL:

$$I_R + I_L + I_C = 9.35A\angle 0° + 2.34A\angle -90° + 5.84A\angle 90°$$
$$= 10A\angle 20.56° = I$$

Phasor Diagram

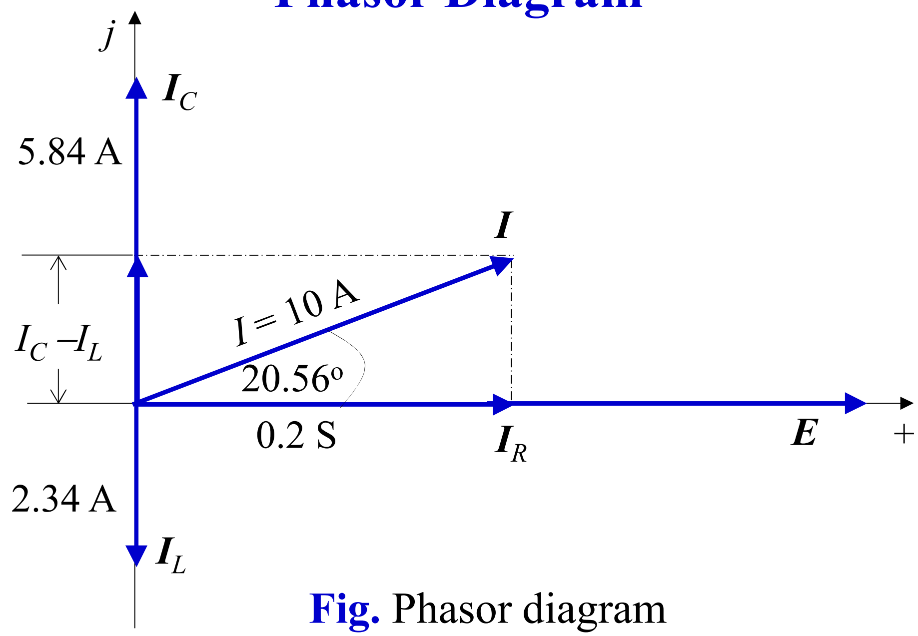
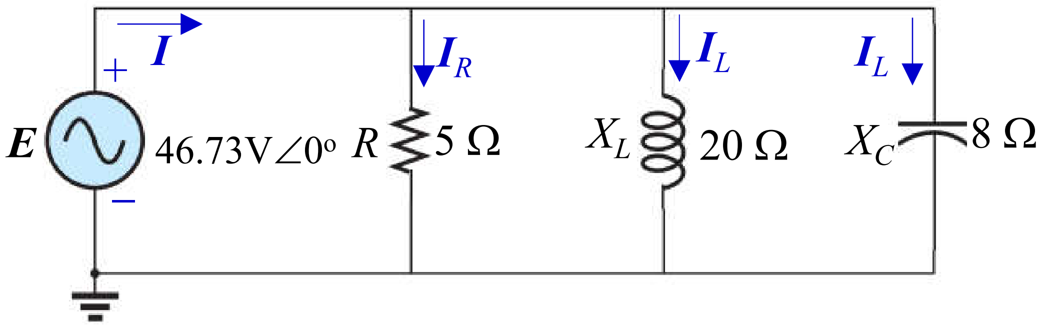


Fig. Phasor diagram



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(-20.56^o)$ **= 0.351 leading**

$rf = (B/Y_T) = \sin\theta_z = \sin(-20.56^o)$ **= – 0.936**

Power [Total watts]

$P_E = EI\cos\theta_z = 46.73\times10\cos(-20.56^o)$ **= 437.39 W**

$P_R = I_R^2R = (E^2/R) = (46.73\text{ V})^2/5\Omega$ **= 437.11 W**

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 46.73\times21.37\sin(-20.56^o)$ **= – 164.02 Var**

$Q_L = I_L^2X_L = (E^2/X_L) = (46.73\text{V})^2/20\Omega$ **= 109.51 Var**

$Q_C = -I_C^2X_C = -(E^2/X_C) = -(46.73\text{V})^2/8\Omega$ **= – 272.84 Var**

$Q = Q_L + Q_C$ **= – 163.33 Var**

Apparent Power [volt-ampere]

$S_E = EI = 46.73\times10$ **= 467.3 VA**

$S_Z = I^2Z = (E^2/Z) = (46.73\text{V})^2/4.68\Omega$ **= 468 VA**

Power Triangle

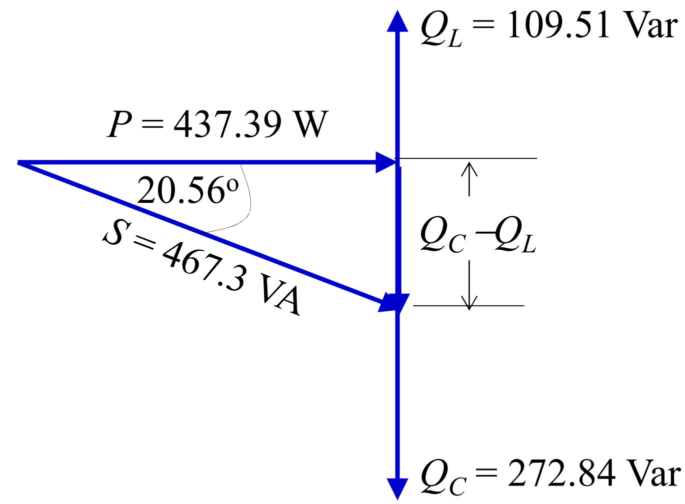


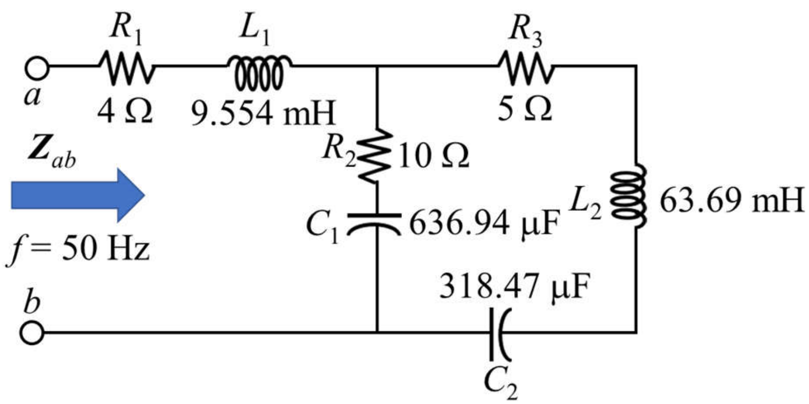
Fig. Phasor diagram

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32

Chapter 16

Series-Parallel Circuits

EXAMPLE: Calculate the impedance at terminals *a* and *b* for the following electrical network.



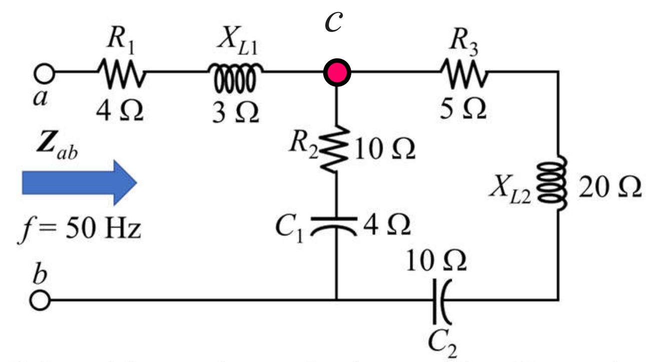
Solution: (1) Calculate all reactance if needed.

$$X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times \left(9.554 \times 10^{-3}\right) = 3\ \Omega$$

$$X_{L2} = 2\pi f L_2 = 2\pi \times 50 \times \left(63.69 \times 10^{-3}\right) = 20\ \Omega$$

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 50 \times \left(636.94 \times 10^{-6}\right)} = 5\ \Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 50 \times \left(318.47 \times 10^{-6}\right)} = 10\ \Omega$$



(2) Identify and mark the nodes/junctions.

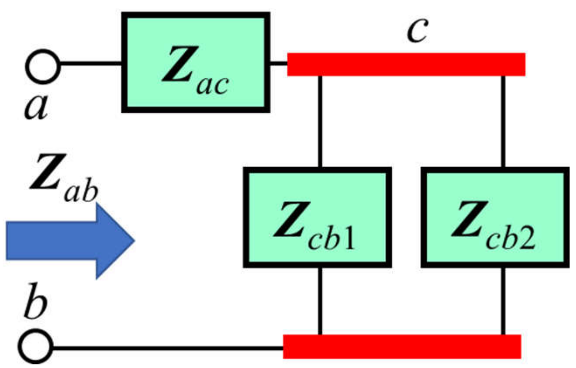
There are two branches are connected between terminals *c* and *b*.

Write the impedances in different branches.

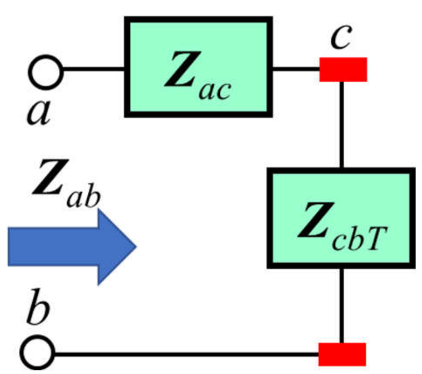
$$\boldsymbol{Z}_{ac} = 4 + j3\ \Omega$$
$$\boldsymbol{Z}_{cb1} = 10 - j5\ \Omega$$
$$\boldsymbol{Z}_{cb2} = 5 + j20 - j10 = 5 + j10\ \Omega$$

Redraw the circuit showing the impedance.

$$Z_{ac} = 4 + j3\ \Omega$$
$$Z_{cb1} = 10 - j5\ \Omega$$
$$Z_{cb2} = 5 + j20 - j10 = 5 + j10\ \Omega$$



$$Z_{cbT} = \frac{Z_{cb1}Z_{cb2}}{Z_{cb1} + Z_{cb2}} = \frac{(10 - j5)(5 + j10)}{(10 - j5) + (5 + j10)} = \frac{100 + j75}{15 + j5} = 7.5 + j2.5\ \Omega$$



$$Z_{ab} = Z_{ac} + Z_{cbT} = 4 + j3 + 7.5 + j2.5 = 11.5 + j5.5\ \Omega$$

EXAMPLE 16.1: For the network in Fig. 16.1:

(*a*) Calculate $\boldsymbol{Z}_T$.
(*b*) Determine $\boldsymbol{I}_s$.
(*c*) Calculate $\boldsymbol{V}_R$, $\boldsymbol{V}_C$ and $\boldsymbol{V}_L$.
(*d*) Find the $\boldsymbol{I}_C$ and $\boldsymbol{I}_L$.
(*e*) Compute the power delivered.
(*e*) Find power factor (F_p) of the network.

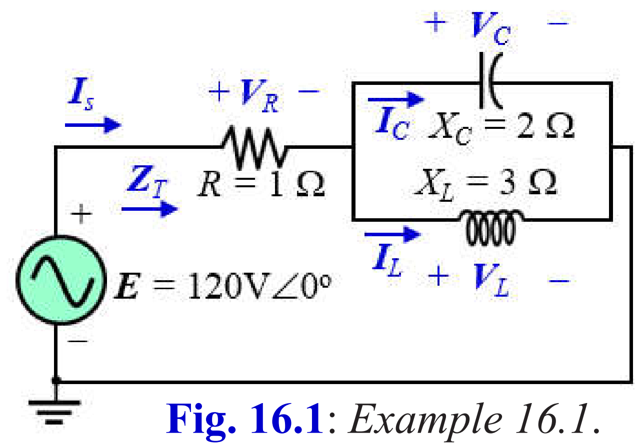


Fig. 16.1: *Example 16.1.*

Solution: (a) Let, $\boldsymbol{Z}_1 = 1\ \Omega = 1\Omega\angle 0^\circ$; $\boldsymbol{Z}_2 = -j2\ \Omega = 2\Omega\angle -90^\circ$; $\boldsymbol{Z}_3 = j3\ \Omega = 3\Omega\angle 90^\circ$;

Fig. 16.1(a) shows the redrawing circuit of Fig. 16.1.

$$\boldsymbol{Z}_4 = \frac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2+\boldsymbol{Z}_3} = \frac{(-j2)(j3)}{(-j2)+(j3)} = -j6\ \Omega = 6\Omega\angle -90^\circ$$

Fig. 16.1(b) shows the redrawing circuit of Fig. 16.1(a).

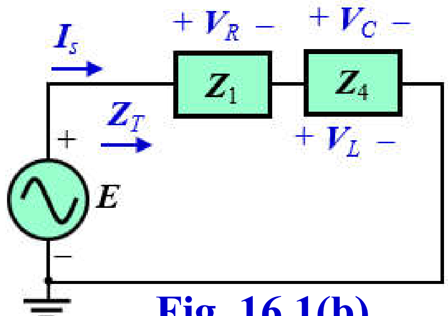


Fig. 16.1(b)

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_4 = 1 - j6\ \Omega$$
$$= \mathbf{6.08\Omega\angle -80.54^\circ}$$

$$(\boldsymbol{b})\ \boldsymbol{I}_s = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{120\text{V}\angle 0^\circ}{6.08\Omega\angle -80.54^\circ}$$
$$= 19.74\text{A}\angle 80.54^\circ$$

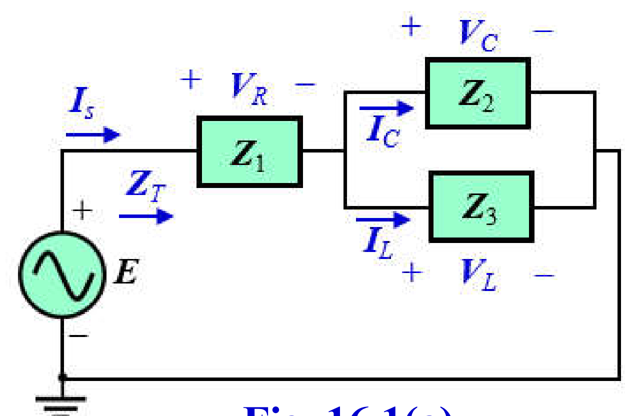


Fig. 16.1(a)

(**c**) Referring to Fig. 16.1(b), we have.

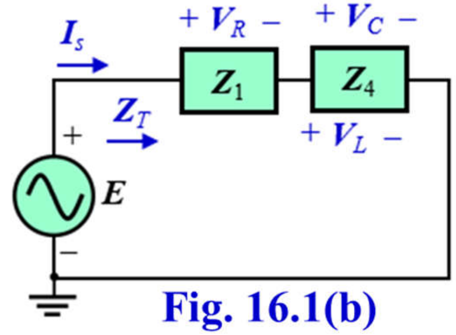


Fig. 16.1(b)

$$\boldsymbol{V}_R = \boldsymbol{I}_s\boldsymbol{Z}_1 = (19.74\text{A}\angle 80.54°)(1\Omega\angle 0°) = \mathbf{19.74}V\angle\mathbf{80.54°}$$

$$\boldsymbol{V}_C = \boldsymbol{V}_L = \boldsymbol{I}_s\boldsymbol{Z}_4 = (19.74\text{A}\angle 80.54°)(6\Omega\angle -90°) = \mathbf{118.44}\text{V}\angle -\mathbf{9.46°}$$

(**d**) Referring to Fig. 16.1(b), we have.

$$\boldsymbol{I}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{Z}_C} = \frac{118.44\text{V}\angle -9.46°}{2\Omega\angle -90°} = \mathbf{59.22A\angle 80.54°}$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{Z}_L} = \frac{118.44\text{V}\angle -9.46°}{3\Omega\angle 90°} = \mathbf{39.48A\angle -99.46°}$$

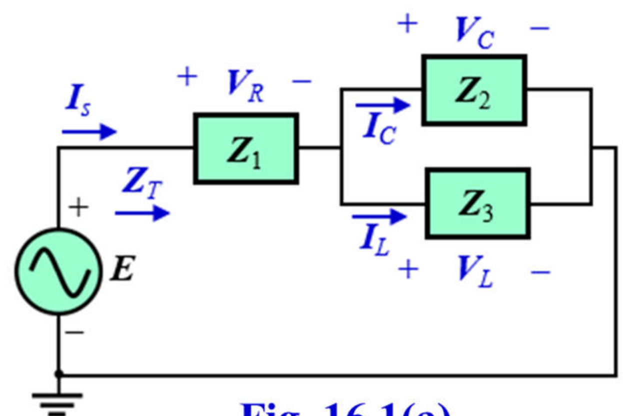


Fig. 16.1(a)

$(\boldsymbol{e})\ P_{del} = I_s^2 R = (19.74)^2\,(1\Omega) = \mathbf{389.67\ W}$

$(\boldsymbol{f})\ pf = F_p = cos\theta = \cos(80.54°) = \mathbf{0.164\ Leading}$

EXAMPLE 16.3: For the network in Fig. 16.5:

(***a***) Calculate the total impedance $\boldsymbol{Z}_T$ and the current $\boldsymbol{I}_s$.

(***b***) Calculate the voltage V_C using the voltage divider rule.

(***c***) Calculate the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the current divider rule.

(***d***) Calculate the power consumption by R, the reactive power consumption by L and the reactive power supplied by C.

(***e***) Calculate the apparent power, the power and the reactive power delivered by source.

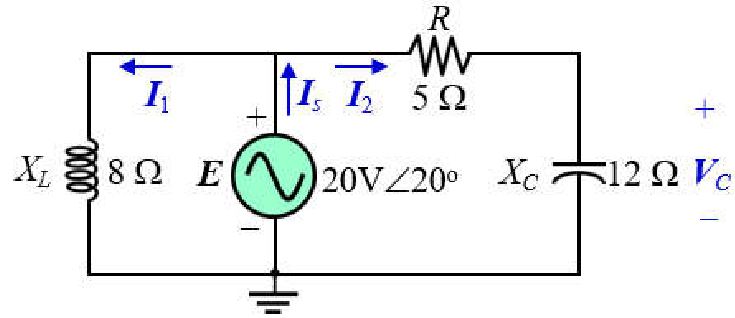


Fig. 16.5: *Example 16.3.*

Solution: (a) Let, $\boldsymbol{Z}_1 = 5\ \Omega = 5\Omega\angle 0^o$;

$\boldsymbol{Z}_2 = -j12\ \Omega = 12\Omega\angle -90^o$;

$\boldsymbol{Z}_3 = j8\ \Omega = 8\Omega\angle 90^o$;

Fig. 16.5(a) shows the redrawing circuit of Fig. 16.5.

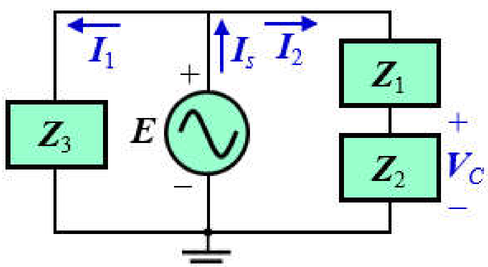


Fig. 16.5(a)

$\boldsymbol{Z}_4 = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 = 5 - j12\ \Omega = 13\Omega\angle -67.38^o$

Fig. 16.5(b) shows the redrawing circuit of Fig. 16.5(a).

$$\boldsymbol{Z}_T = \frac{\boldsymbol{Z}_3\boldsymbol{Z}_4}{\boldsymbol{Z}_3+\boldsymbol{Z}_4} = \frac{(j8)(5-j3)}{(j8)+(5-j3)}$$
$$= 7.8 + j14.24\ \Omega$$
$$= 16.24\Omega\angle 61.29^o$$

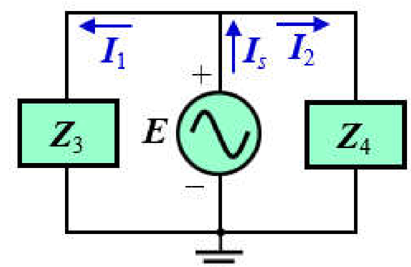


Fig. 16.5(b)

$$\boldsymbol{I}_s = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{20\text{V}\angle 20°}{16.24\Omega\angle 61.29°}$$
$$= 1.23\text{A}\angle 40.29°$$

($\boldsymbol{b}$) Calculate the voltage $\boldsymbol{V}_C$ using the voltage divider rule.

Referring to Fig. 16.5(a), we have.

$$\boldsymbol{V}_C = \frac{\boldsymbol{Z}_2\boldsymbol{E}}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = \frac{(12\Omega\angle -90°)(20\text{V}\angle 20°)}{5 - j12}$$
$$= 18.46\text{V}\angle -2.62°$$

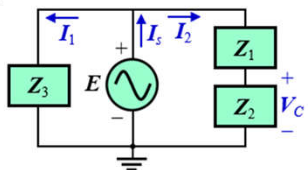


Fig. 16.5(a)

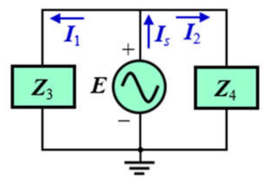


Fig. 16.5(b)

($\boldsymbol{c}$) Calculate the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the current divider rule.

Referring to Fig. 16.5(b), we have.

$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_4\boldsymbol{I}_s}{\boldsymbol{Z}_3 + \boldsymbol{Z}_4} = \frac{(5 - j12\ \Omega)(1.23\text{A}\angle 40.29°)}{(j8) + (5 - j12)}$$
$$= 0.87 - j2.35\ \text{A} = 2.51\text{A}\angle -68.68°$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{Z}_3\boldsymbol{I}_s}{\boldsymbol{Z}_3 + \boldsymbol{Z}_4} = \frac{(j8\ \Omega)(1.23\text{A}\angle 40.29°)}{(j8) + (5 - j12)}$$
$$= 0.06 + j1.54\ \text{A} = 1.54\text{A}\angle 87.77°$$

($\boldsymbol{d}$) Calculate the power consumption by R, the reactive power consumption by L and the reactive power supplied by C.

$$P_R = I_2^2 R = (1.54\text{A})^2 \times (5\Omega) = 11.86\ \text{W}$$

$$Q_L = I_1^2 X_L = (2.51\text{A})^2 \times (8\Omega) = 50.4\ \text{VAR}$$

$$Q_C = -I_2^2 X_C = -(1.54\text{A})^2 \times (12\Omega) = -28.46\ \text{VAR}$$

($\boldsymbol{e}$) Calculate the apparent power, the power and the reactive power delivered by source.

$$S = EI_s = (20\text{V}) \times (1.23\text{A}) = 24.6\ \text{VA}$$

$$P = EI_s cos\theta = EI_s \cos\theta_z$$
$$= (20\text{V}) \times (1.23\text{A}) \cos(61.29^{\text{o}})$$
$$= 11.82\ \text{W}$$

$$Q = EI_s sin\theta = EI_s \sin\theta_z$$
$$= (20\text{V}) \times (1.23\text{A}) \sin(61.29^{\text{o}})$$
$$= 21.58\ \text{VAR}$$

EXAMPLE 16.6 For the network in Fig. 16.12:

a. Determine the current **I**.
b. Find the voltage **V**.

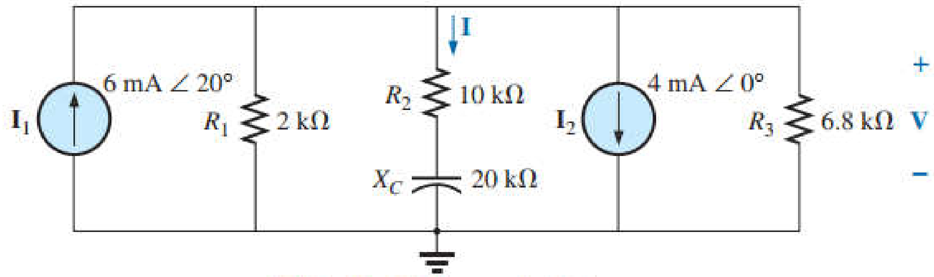


FIG. 16.12 *Example 16.6.*

Solutions:

a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$\begin{aligned}\mathbf{I}_T &= 6\text{ mA}\angle 20^\circ - 4\text{ mA}\angle 0^\circ\\ &= 5.638\text{ mA} + j\,2.052\text{ mA} - 4\text{ mA}\\ &= 1.638\text{ mA} + j\,2.052\text{ mA}\\ &= 2.626\text{ mA}\angle 51.402^\circ\end{aligned}$$

Redrawing the network using block impedances results in the network in Fig. 16.13 where

$$\mathbf{Z}_1 = 2\text{ k}\Omega\angle 0^\circ \parallel 6.8\text{ k}\Omega\angle 0^\circ = 1.545\text{ k}\Omega\angle 0^\circ$$

and $$\mathbf{Z}_2 = 10\text{ k}\Omega - j\,20\text{ k}\Omega = 22.361\text{ k}\Omega\angle -63.435^\circ$$

Note that **I** and **V** are still defined in Fig. 16.13.

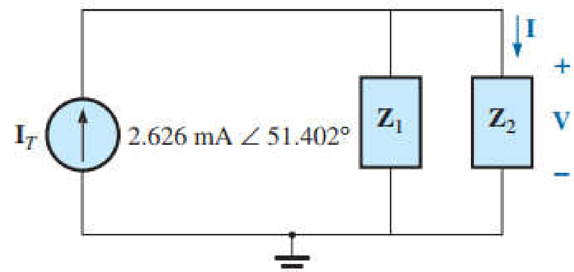


FIG. 16.13
Network in Fig. 16.12 following the assignment of the subscripted impedances.

Calculate the currents ***I***.

Current divider rule:

$$\mathbf{I} = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(1.545\ \text{k}\Omega\ \angle 0°)(2.626\ \text{mA}\ \angle 51.402°)}{1.545\ \text{k}\Omega + 10\ \text{k}\Omega - j\,20\ \text{k}\Omega}$$

$$= \frac{4.057\ \text{A}\ \angle 51.402°}{11.545 \times 10^3 - j\,20 \times 10^3} = \frac{4.057\ \text{A}\ \angle 51.402°}{23.093 \times 10^3\ \angle -60.004°}$$

$$= \mathbf{0.18\ mA\ \angle 111.41°}$$

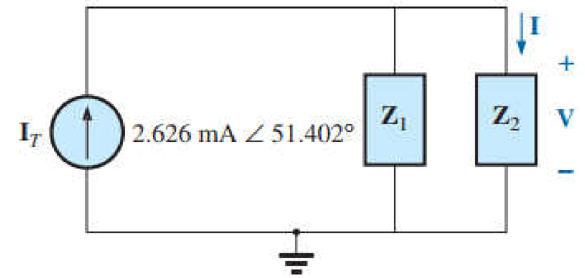


FIG. 16.13
Network in Fig. 16.12 following the assignment of the subscripted impedances.

(***b***) Calculate the voltage ***V***.

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_2$$

$$= (0.176\ \text{mA}\ \angle 111.406°)(22.36\ \text{k}\Omega\ \angle -63.435°)$$

$$= \mathbf{3.94\ V\ \angle 47.97°}$$

EXAMPLE 16.7 For the network in Fig. 16.14:

a. Compute $\mathbf{I}$.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

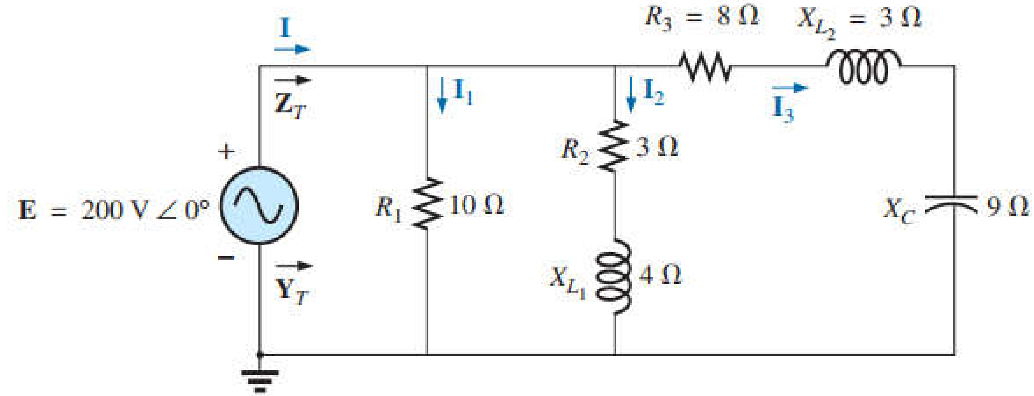


FIG. 16.14
Example 16.7.

Solutions:

a. $\mathbf{Z}_1 = R_1 = 10\ \Omega \angle 0°$

$\mathbf{Z}_2 = R_2 + j\,X_{L_1} = 3\ \Omega + j\,4\ \Omega$

$\mathbf{Z}_3 = R_3 + j\,X_{L_2} - j\,X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$

Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

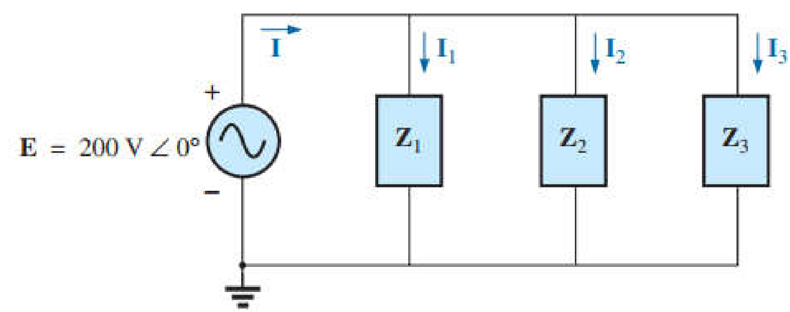


FIG. 16.15 *Network in Fig. 16.14 following the assignment of the subscripted impedances.*

The total admittance is

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3$$

$$= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega}$$

$$= 0.1\ \text{S} + \frac{1}{5\ \Omega \angle 53.13°} + \frac{1}{10\ \Omega \angle -36.87°}$$

$$= 0.3\ \text{S} - j\,0.1\ \text{S} = 0.316\ \text{S} \angle -18.435°$$

The current **I**:

$$\mathbf{I} = \mathbf{E}\mathbf{Y}_T = (200\ \text{V}\ \angle 0^\circ)(0.326\ \text{S}\ \angle -18.435^\circ)$$
$$= \mathbf{63.2\ A\ \angle -18.44^\circ}$$

(***b***) Find the currents I_1, I_2 and I_3.

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200\ \text{V}\ \angle 0^\circ}{10\ \Omega\ \angle 0^\circ} = \mathbf{20\ A\ \angle 0^\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{40\ A\ \angle -53.13^\circ}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200\ \text{V}\ \angle 0^\circ}{10\ \Omega\ \angle -36.87^\circ} = \mathbf{20\ A\ \angle +36.87^\circ}$$

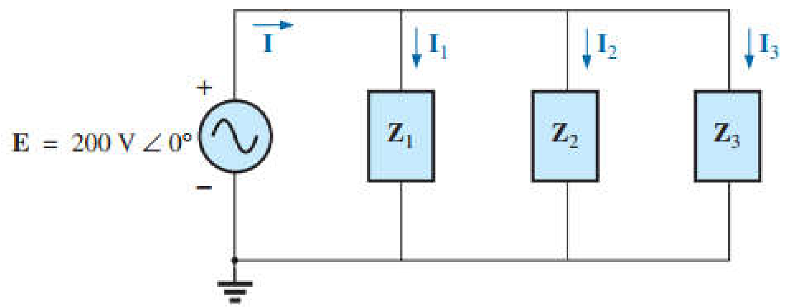


FIG. 16.15 *Network in Fig. 16.14 following the assignment of the subscripted impedances.*

(***c***) Verify KCL:

c.
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$
$$60 - j\,20 = 20\ \angle 0^\circ + 40\ \angle -53.13^\circ + 20\ \angle +36.87^\circ$$
$$= (20 + j\,0) + (24 - j\,32) + (16 + j\,12)$$
$$60 - j\,20 = 60 - j\,20 \qquad \text{(checks)}$$

(***d***) Find the total impedance of the circuit.

d.
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316\ \text{S}\ \angle -18.435^\circ}$$
$$= \mathbf{3.17\ \Omega\ \angle 18.44^\circ}$$

EXAMPLE 16.8 For the network in Fig. 16.18:

a. Calculate the total impedance $\mathbf{Z}_T$.
b. Compute $\mathbf{I}$.
c. Find the total power factor.
d. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$.
e. Find the average power delivered to the circuit.

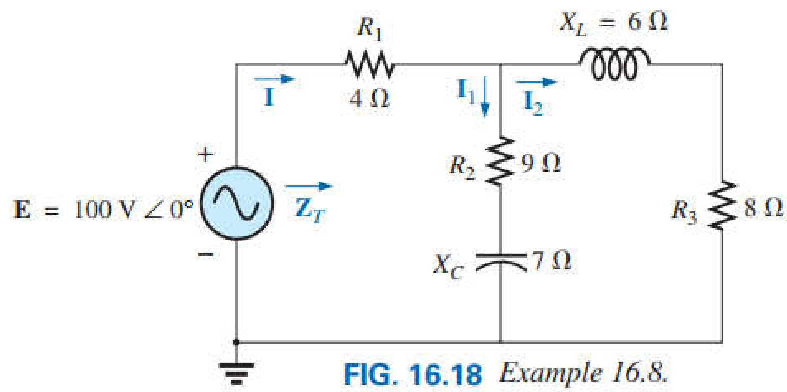


FIG. 16.18 *Example 16.8.*

Solutions:

a. $\mathbf{Z}_1 = R_1 = 4\ \Omega\ \angle 0°$

$\mathbf{Z}_2 = R_2 - j\,X_C = 9\ \Omega - j\,7\ \Omega = 11.40\ \Omega\ \angle -37.87°$

$\mathbf{Z}_3 = R_3 + j\,X_L = 8\ \Omega + j\,6\ \Omega = 10\ \Omega\ \angle +36.87°$

Redrawing the circuit as in Fig. 16.19, we have

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_{T_1} = \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3}$$

$$= 4\ \Omega + \frac{(11.4\ \Omega\ \angle -37.87°)(10\ \Omega\ \angle 36.87°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$

$$= 4\ \Omega + 6.68\ \Omega + j\,0.28\ \Omega = 10.68\ \Omega + j\,0.28\ \Omega$$

$$\mathbf{Z}_T = \mathbf{10.68\ \Omega\ \angle 1.5°}$$

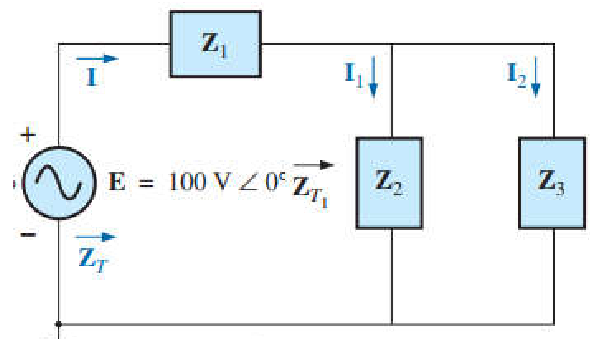


FIG. 16.19

b. $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{100\text{ V} \angle 0°}{10.684\ \Omega \angle 1.5°} = \mathbf{9.36\text{ A} \angle -1.5°}$

c. $F_p = \cos\theta_T = \dfrac{R}{Z_T} = \dfrac{10.68\ \Omega}{10.684\ \Omega} \cong \mathbf{1}$

d. Current divider rule:

$$\mathbf{I}_2 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(11.40\ \Omega \angle -37.87°)(9.36\text{ A} \angle -1.5°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$
$$= \frac{106.7\text{ A} \angle -39.37°}{17 - j\,1} = \frac{106.7\text{ A} \angle -39.37°}{17.03 \angle -3.37°}$$
$$\mathbf{I}_2 = \mathbf{6.27\text{ A} \angle -36°}$$

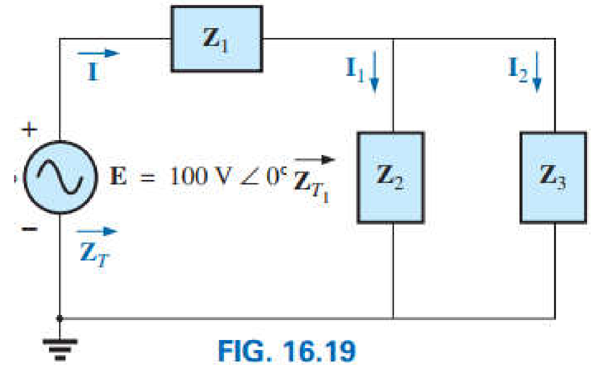


FIG. 16.19

Applying Kirchhoff's current law (rather than another application of the current divider rule) yields

$$\mathbf{I}_1 = \mathbf{I} - \mathbf{I}_2$$

or

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$$
$$= (9.36\text{ A} \angle -1.5°) - (6.27\text{ A} \angle -36°)$$
$$= (9.36\text{ A} - j\,0.25\text{ A}) - (5.07\text{ A} - j\,3.69\text{ A})$$
$$\mathbf{I}_1 = 4.29\text{ A} + j\,3.44\text{ A} = \mathbf{5.5\text{ A} \angle 38.72°}$$

e. $P_T = EI\cos\theta_T$
$$= (100\text{ V})(9.36\text{ A})\cos 1.5°$$
$$= (936)(0.99966)$$
$$P_T = \mathbf{935.68\text{ W}}$$

*10. For the network in Fig. 16.48:

a. Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.
b. Find the source current $\mathbf{I}_s$ in phasor form.
c. Find the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in phasor form.
d. Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_{ab}$ in phasor form.
e. Find the average power delivered to the network.
f. Find the power factor of the network, and indicate whether it is leading or lagging.

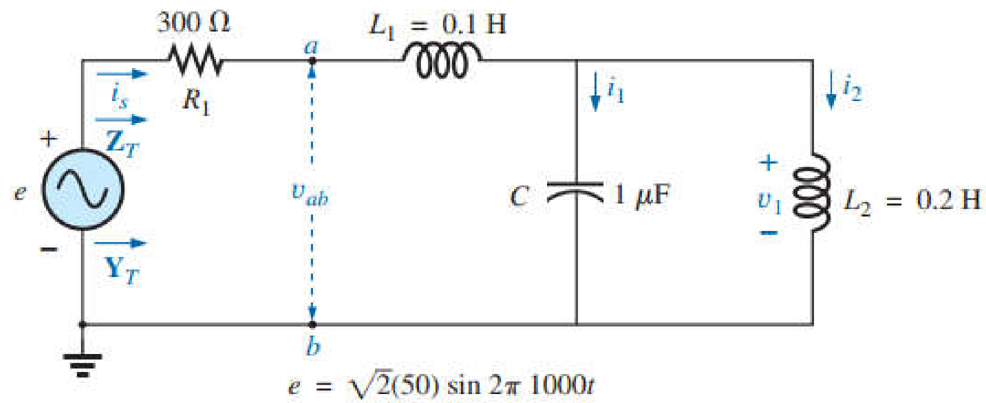


FIG. 16.48 *Problem 10.*

Solution: Here, $E = 50$ V, $f = 1000$ Hz and $\omega = 2\pi\times1000 = 6280$ rad/s $\quad \boldsymbol{E} = 50\text{V}\angle 0°$

$$X_{L1} = \omega L_1 = (6280\text{ rad/s})(0.1\text{ H}) = 628\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(6280\text{ rad/s})(1\times10^{-6}\text{ F})} = 159.24\,\Omega$$

$$X_{L2} = \omega L_2 = (6280\text{ rad/s})(0.2\text{ H}) = 1256\,\Omega$$

Let, $\boldsymbol{Z}_1 = 300\,\Omega = 300\Omega\angle 0°$

$$\boldsymbol{Z}_2 = j628\,\Omega = 628\Omega\angle 90°$$

$$\boldsymbol{Z}_3 = -j159.24\,\Omega = 159.24\Omega\angle -90°$$

$$\boldsymbol{Z}_4 = j1256\,\Omega = 1256\Omega\angle 90°$$

Fig. 16.48(a) shows the redrawing circuit of Fig. 16.48.

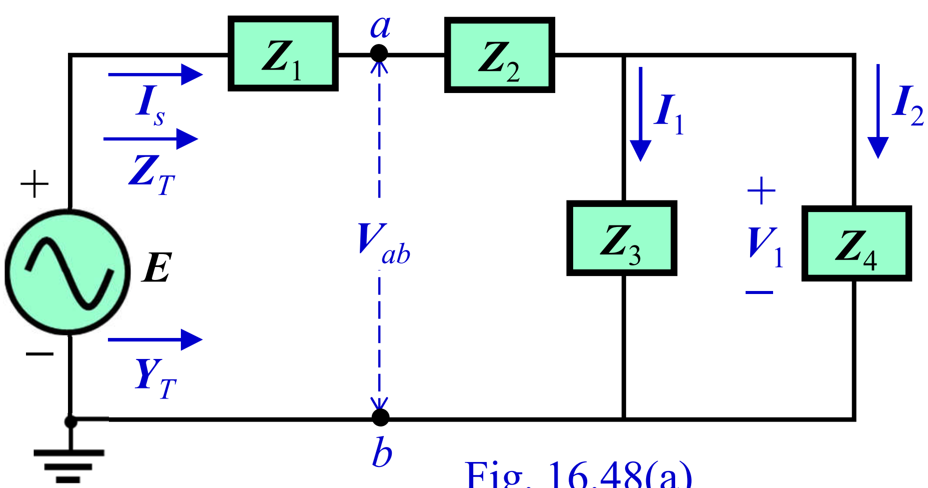


Fig. 16.48(a)

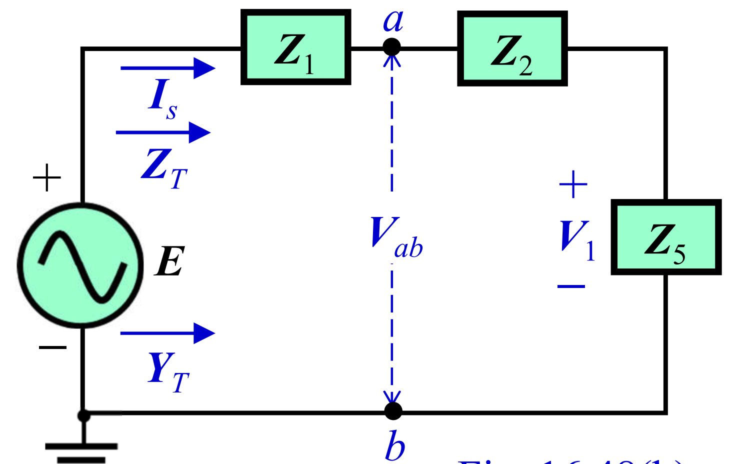


Fig. 16.48(b)

$$Z_5 = \frac{Z_3 Z_4}{Z_3 + Z_4} = \frac{(-j159.24\ \Omega)(j1256\ \Omega)}{(-j159.24\ \Omega) + (j1256\ \Omega)}$$
$$= -j182.36\ \Omega = 182.36\Omega\angle - 90°$$

Fig. 16.48(b) shows the redrawing circuit of Fig. 16.48(a).

$$Z_6 = Z_2 + Z_5 = j445.64\ \Omega = 445.64\Omega\angle 90°$$

Fig. 16.48(c) shows the redrawing circuit of Fig. 16.48(b).

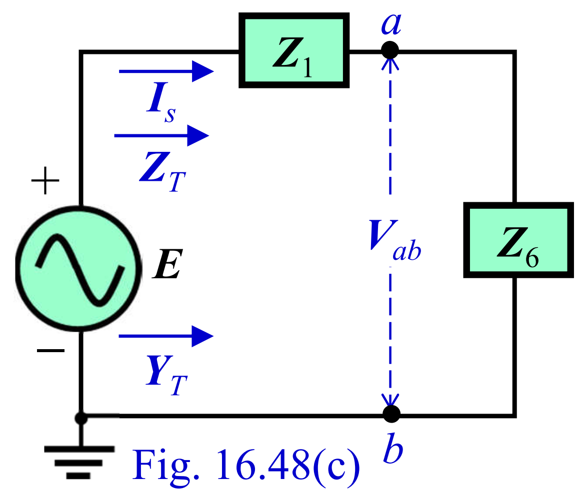


Fig. 16.48(c)

$$Z_T = Z_1 + Z_6 = 300 + j445.64\ \Omega = 537.21\Omega\angle 56.05°$$

Fig. 16.48(d) shows the redrawing circuit of Fig. 16.48(c).

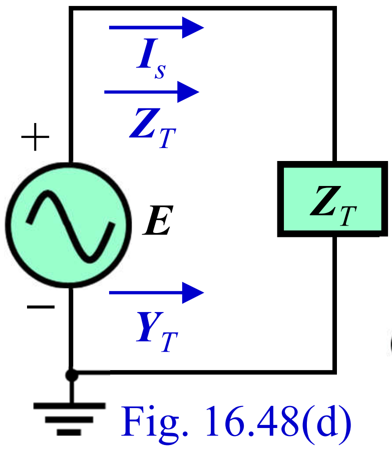


Fig. 16.48(d)

$$Y_T = \frac{1}{Z_T} = \frac{1}{537.21\Omega\angle 56.0°}$$
$$= \mathbf{1.861mS\angle -56.05°}$$

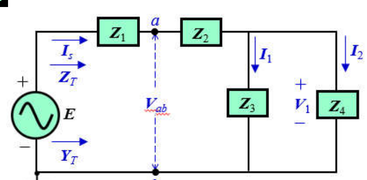


Fig. 16.48(a)

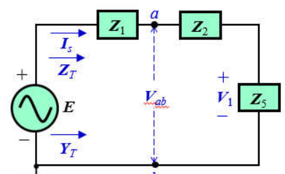


Fig. 16.48(b)

(b) $$\boldsymbol{I_s} = \frac{\boldsymbol{E}}{\boldsymbol{Z_T}} = \frac{50\text{V}\angle 0°}{537.21\Omega\angle 56.05°} = \mathbf{93.07mA\angle -56.05°}$$

(c) Referring to Fig. 16.48(a) and Fig. 16.48(b), $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ are:

$$\boldsymbol{I_1} = \frac{\boldsymbol{Z_5}}{\boldsymbol{Z_3}}\boldsymbol{I_s} = \mathbf{106.58mA\angle -56.05°}$$

$$\boldsymbol{I_2} = \frac{\boldsymbol{Z_5}}{\boldsymbol{Z_4}}\boldsymbol{I_s} = \mathbf{13.52mA\angle 123.96°}$$

(d) Referring to Fig. 16.48(b) and Fig. 16.48(c), $\boldsymbol{V}_{ab}$ and $\boldsymbol{V}_1$ are:

$$\boldsymbol{V_{ab}} = \boldsymbol{Z_6 I_s} = \mathbf{41.48V\angle 33.95°}$$

$$\boldsymbol{V_1} = \boldsymbol{Z_5 I_s} = \mathbf{16.98V\angle 213.95°}$$

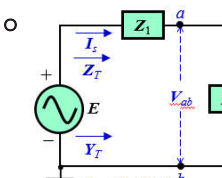


Fig. 16.48(c)

(e) Average power:

$$P = EI_s\cos\theta_T$$
$$= 50 \times (93.07\text{mA})\cos(56.05°) = \mathbf{2.6\ W}$$

(f) Power Factor:

$$F_p = \cos\theta_T = \mathbf{0.558\ Lagging}$$

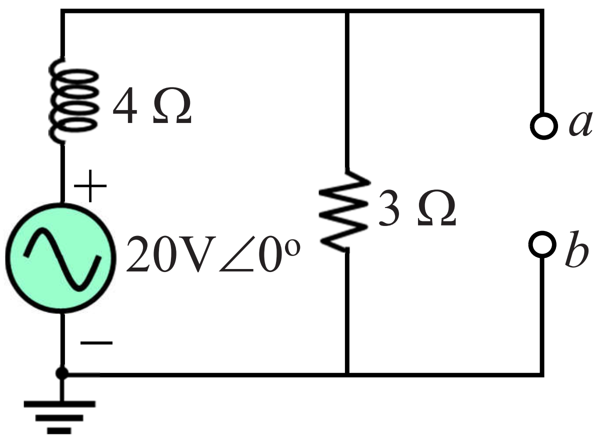
Practice Book Remaining Examples

And

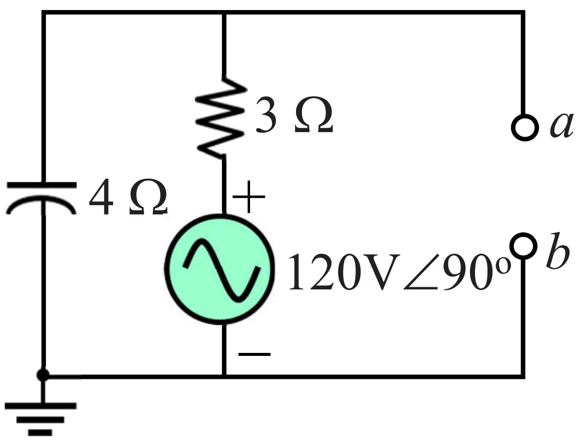
All Problems of Chapter 16

Example Related to Open Circuit and Short Circuit

Example: For the following circuits, find voltage drop across the terminals *a* and *b*.

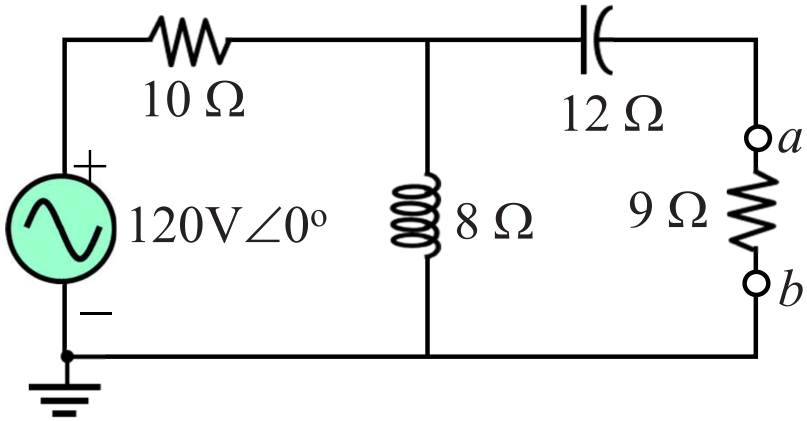


$$V_{ab} = \frac{(3\Omega)}{(3\Omega)+(j4\,\Omega)}(20\text{V}\angle 0°)$$
$$= 7.2 - j9.6 \text{ V}$$
$$= 12\text{V}\angle -53.13°$$

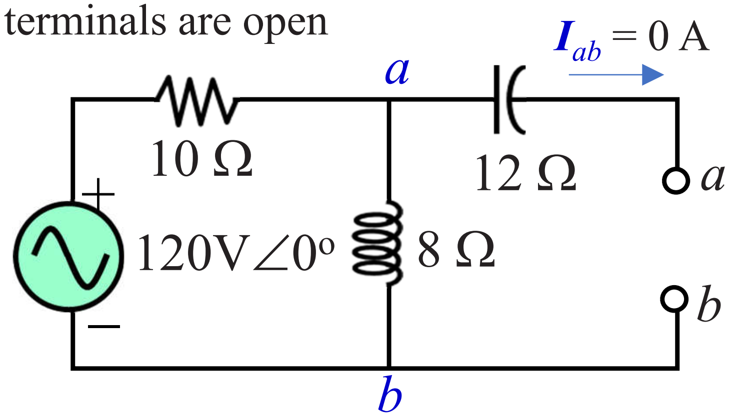


$$V_{ab} = \frac{(-j4\Omega)}{(3\Omega)+(j4\,\Omega)}(120\text{V}\angle 90°)$$
$$= 57.6 + j76.8 \text{ V}$$
$$= 96\text{V}\angle 53.13°$$

Example: Find the voltage drop across the terminals *a* and *b*. when the terminals are open.



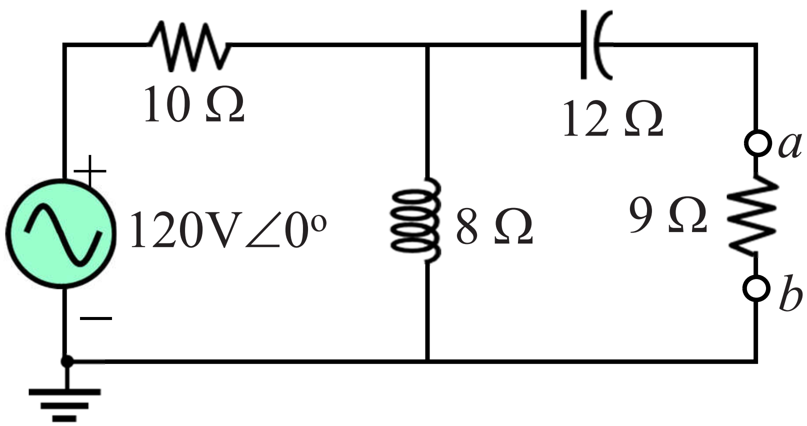
Solution: Redraw the circuit by considering *a* and *b* terminals are open



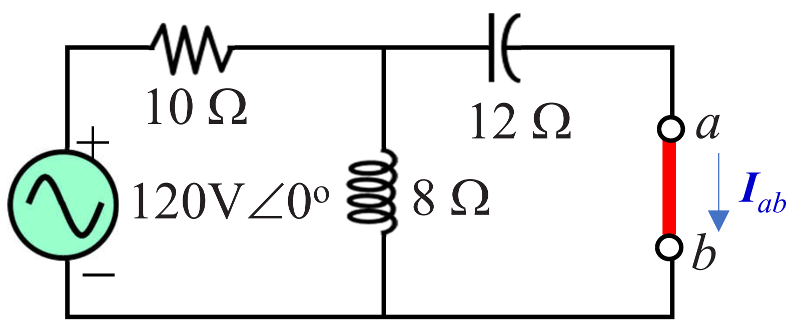
Due to the open circuit no current flows through the 12 Ω capacitive reactance, so the voltage drop across the *a* and *b* terminals is equal to the voltage drop across the 8 Ω inductive reactance.

$$
\begin{aligned}
V_{ab} &= \frac{(j8\Omega)}{(10\Omega)+(j8\ \Omega)}(120\text{V}\angle 0^\circ) \\
&= 46.83 + j58.54\ \text{V} \\
&= 74.97\text{V}\angle 51.34^\circ
\end{aligned}
$$

Example: Find the current passing through the terminals *a* and *b*. when the terminals are shorted.



Solution: (***a***) Redraw the circuit by considering *a* and *b* terminals are open

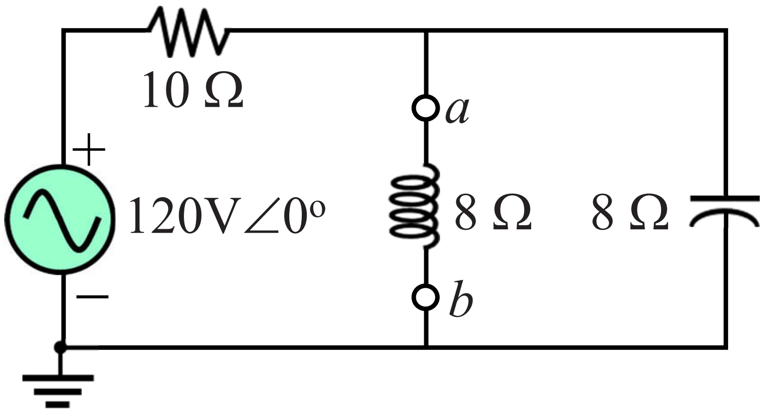


$$\boldsymbol{Z}_p = \frac{(j8\Omega)(-j12\Omega)}{(j8\Omega)+(-j12\Omega)}$$
$$= 46.83 + j58.54\ \Omega$$
$$= 74.97\Omega\angle 51.34°$$

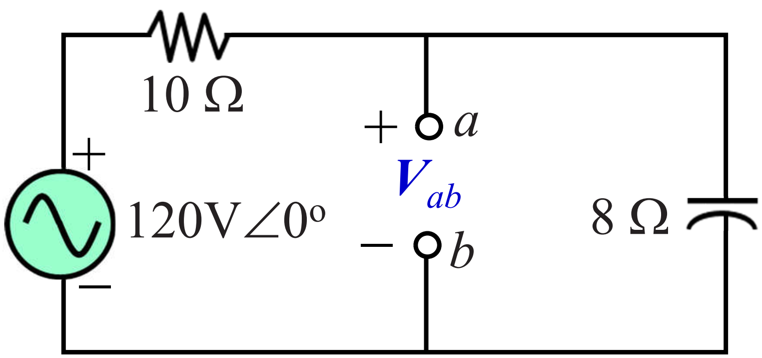
$$\boldsymbol{V}_p = \frac{(10)(\boldsymbol{Z}_p)}{(10)+(\boldsymbol{Z}_p)}\boldsymbol{E}$$
$$= \frac{(10)(46.83+j58.54)}{(10)+(46.83+j58.54)}(120\text{V}\angle 0°)$$
$$= 102.25 + j42.6\ \text{V}$$
$$= 110.77\text{V}\angle 22.62°$$

$$\boldsymbol{I}_{ab} = \frac{\boldsymbol{V}_p}{(-j12\ \Omega)} = \frac{(102.25+j42.6\ \text{V})}{(-j12\ \Omega)}$$
$$= -3.55 + j8.52\ \text{A}$$
$$= 9.23\text{A}\angle 112.62°$$

Example: Find the voltage drop across the terminals *a* and *b*. when the terminals are open.



Solution: Redraw the circuit by considering *a* and *b* terminals are open

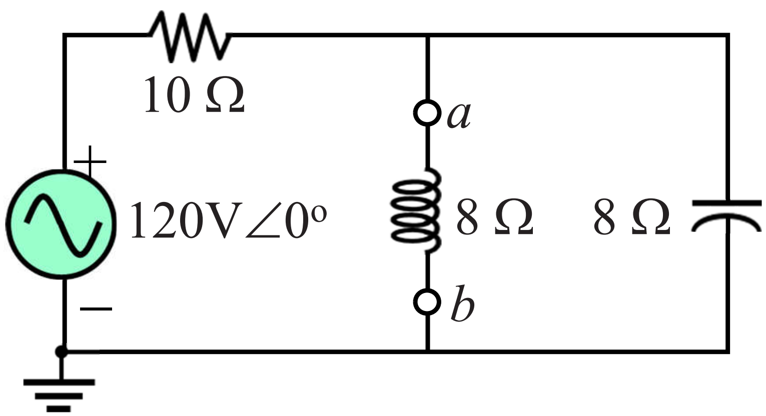


$$V_{ab} = \frac{(10\Omega)(-j8\Omega)}{(10\Omega)(-j8\Omega)}(120\text{V}\angle 0^\circ)$$

$$= 46.83 - j58.54 \text{ V}$$

$$= 74.97\text{V}\angle -51.34^\circ$$

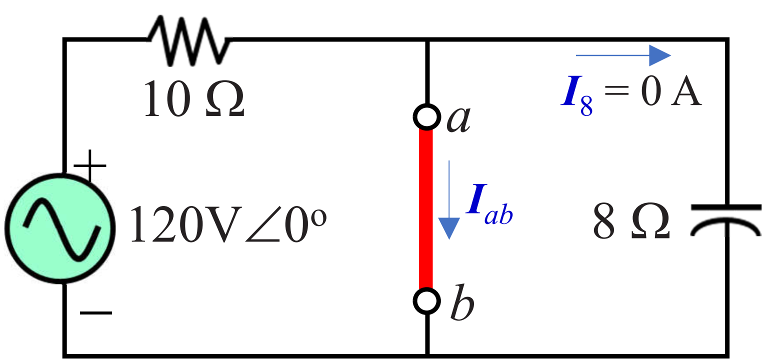
Example: Find the current passing through the terminals *a* and *b*. when the terminals are shorted.



Due to the short circuit no current flows through the 8 Ω capacitive reactance, so the current passes through the *a* and *b* terminals can be calculated as:

$$\boldsymbol{I}_{ab} = \frac{\boldsymbol{E}}{10\ \Omega} = \frac{120\text{V}\angle 0°}{10\ \Omega} = 12\text{A}\angle 0°$$

Solution: (***a***) Redraw the circuit by considering *a* and *b* terminals are open



MAXIMUM POWER TRANSFER THEOREM [AC]

Maximum Power Transfer or Impedance Matching Theorem

Statement: Maximum power will be delivered to a load when the load impedance is the complex conjugate of the Thévenin impedance across its terminals.

If, $\boldsymbol{Z}_L = R \pm jX$ and $\boldsymbol{Z}_{Th} = R_{Th} \pm jX_{Th}$

Then, according to maximum power transfer theorem:

$$\boldsymbol{Z}_L = R \pm jX = (\boldsymbol{Z}_{Th})^* = R_{Th} \mp jX_{Th}$$

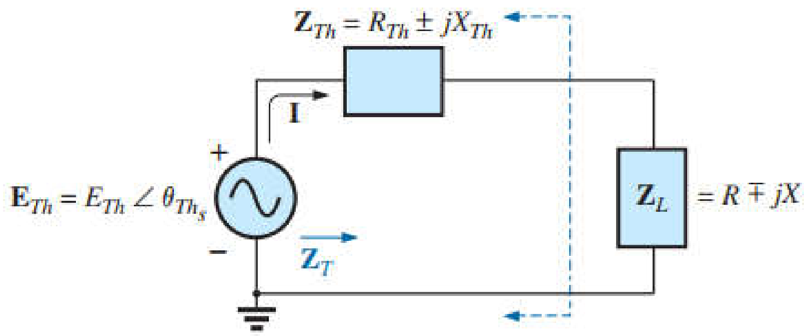


FIG. 18.82
Conditions for maximum power transfer to $\mathbf{Z}_L$.

$$\boxed{Z_L = Z_{Th} \text{ and } \theta_L = -\theta_{Th_Z}} \qquad \textbf{(18.16)}$$

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm j X_{\text{load}} = \mp j X_{Th}} \qquad \textbf{(18.17)}$$

$$\boxed{\mathbf{Z}_T = 2R} \qquad \textbf{(18.18)}$$

$$\boxed{F_p = 1} \qquad \text{(maximum power transfer)} \qquad \textbf{(18.19)}$$

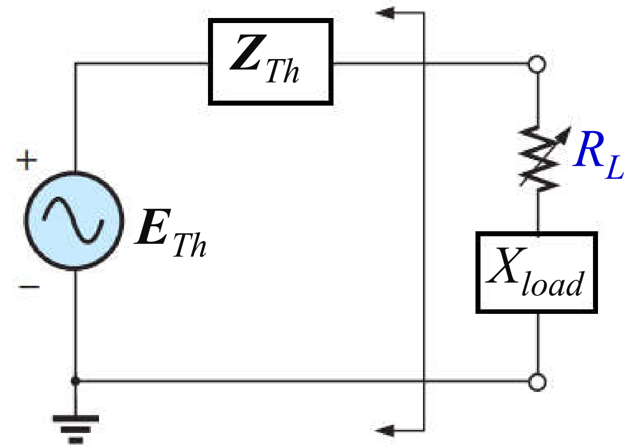
$$\boxed{P_{\max} = \frac{E_{Th}^2}{4R}} \qquad \textbf{(18.20)}$$

Special Situation: If the load resistance is adjustable but the magnitude of the load reactance cannot be set equal to the magnitude of the Thévenin reactance, then the maximum power that can be delivered to the load will occur when the load reactance is made as close to the Thévenin reactance as possible, and the load resistance is set to the following value:

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}} \tag{18.21}$$

$$\boxed{P = E_{Th}^2/4R_{\text{av}}} \tag{18.22}$$

$$\boxed{R_{\text{av}} = \frac{R_{Th} + R_L}{2}} \tag{18.23}$$



EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

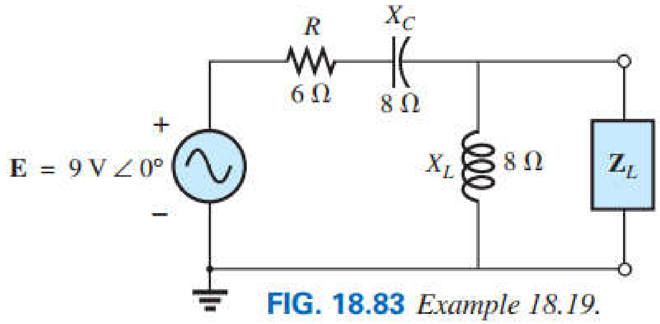


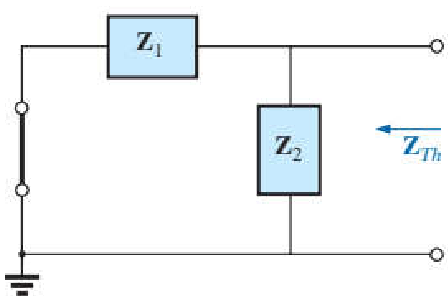
FIG. 18.83 *Example 18.19.*

Solution:

$$\mathbf{Z}_1 = R - j\,X_C = 6\ \Omega - j\,8\ \Omega = 10\ \Omega\ \angle -53.13^\circ$$

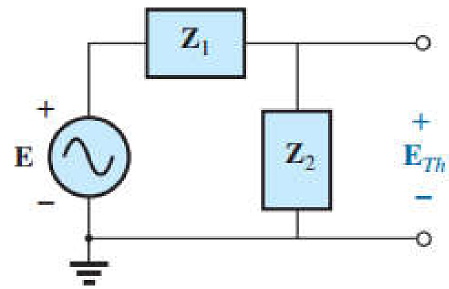
$$\mathbf{Z}_2 = +j\,X_L = j\,8\ \Omega$$

Determine $\mathbf{Z}_{Th}$ [Fig. 18.84(a)]:



$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13^\circ)(8\ \Omega\ \angle 90^\circ)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = 13.33\ \Omega\ \angle 36.87^\circ = 10.66\ \Omega + j\,8\ \Omega$$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.84(b)], as follows:



$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega\ \angle 90^\circ)(9\ \text{V}\ \angle 0^\circ)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90^\circ}{6\ \angle 0^\circ} = 12\ \text{V}\ \angle 90^\circ$$

According to maximum power transfer theorem:

$$\mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87^\circ = \mathbf{10.66\ \Omega - j\,8\ \Omega}$$

Maximum power received by load:

$$P_{\max} = \frac{E_{Th}^2}{4R} = \frac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \frac{144}{42.64} = \mathbf{3.38\ W}$$

EXAMPLE 18.21 For the network in Fig. 18.90:

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.

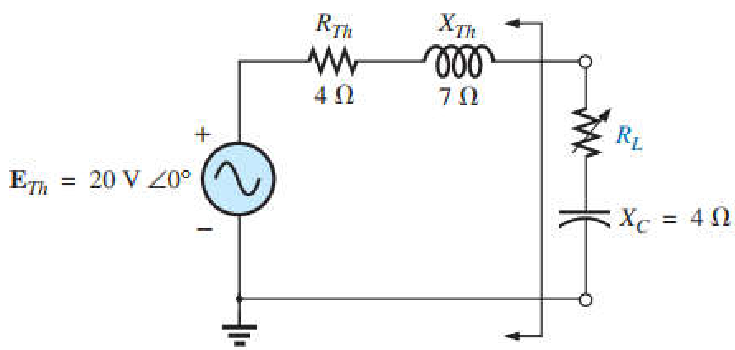


FIG. 18.90 *Example 18.21.*

Solutions:

a. Eq. (18.21):

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}$$
$$= \sqrt{(4\ \Omega)^2 + (7\ \Omega - 4\ \Omega)^2}$$
$$= \sqrt{16 + 9} = \sqrt{25}$$
$$R_L = \mathbf{5\ \Omega}$$

b. Eq. (18.23):

$$R_{\text{av}} = \frac{R_{Th} + R_L}{2} = \frac{4\ \Omega + 5\ \Omega}{2}$$
$$= \mathbf{4.5\ \Omega}$$

Eq. (18.22):

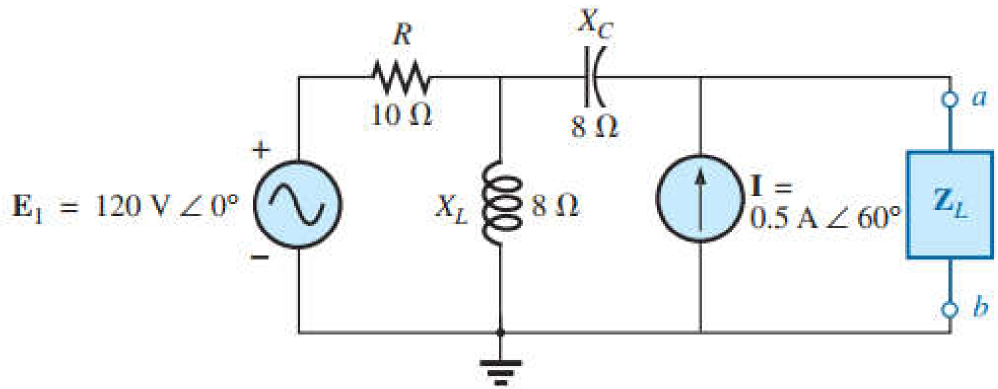
$$P = \frac{E_{Th}^2}{4R_{\text{av}}}$$
$$= \frac{(20\ \text{V})^2}{4(4.5\ \Omega)} = \frac{400}{18}\ \text{W}$$
$$\cong \mathbf{22.22\ W}$$

c. For $\mathbf{Z}_L = 4\ \Omega - j\,7\ \Omega$,

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\ \text{V})^2}{4(4\ \Omega)}$$
$$= \mathbf{25\ W}$$

exceeding the result of part (b) by 2.78 W.

PROBLEM: Find the load impedance $\boldsymbol{Z}_L$ for the networks in following Figure for maximum power to the load, and find the maximum power to the load.



Solution: $\boldsymbol{Z}_1 = R\ \Omega = 10\ \Omega$; $\boldsymbol{Z}_2 = jX_L\ \Omega = j8\ \Omega$;
$\boldsymbol{Z}_3 = -jX_C\ \Omega = -j8\ \Omega$;

Step 1 and **Step 2**:

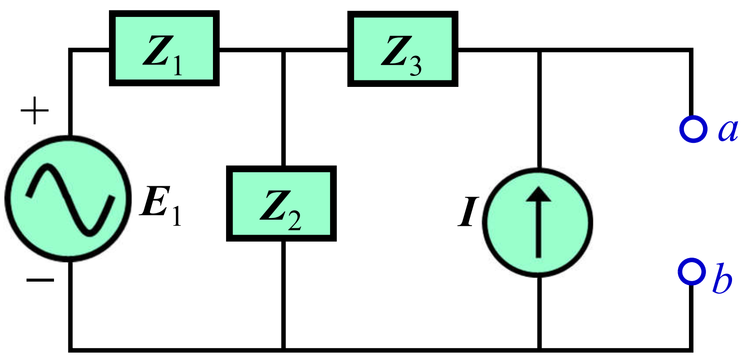


Fig. (a)

Step 3: $\boldsymbol{Z}_{Th}$ calculation

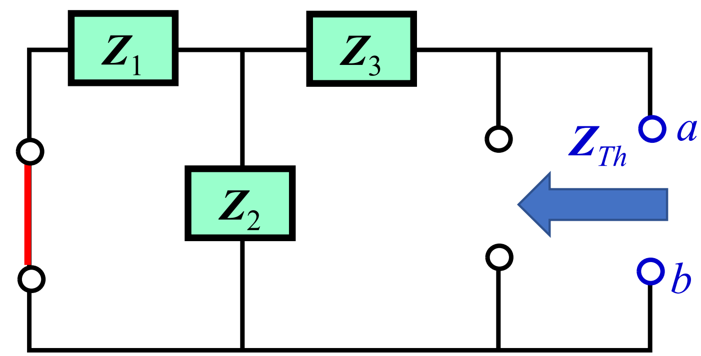


Fig. (b)

$$\boldsymbol{Z}_{Th} = \boldsymbol{Z}_3 + \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = 3.9 - j3.12\ \ \Omega = 5\Omega\angle -38.66^\circ$$

Step 4: $\boldsymbol{E}_{Th}$ calculation

Since there are two sources, Thevenin's voltage can be calculated by using Superposition Theorem.

Considering $\boldsymbol{E}_1$:

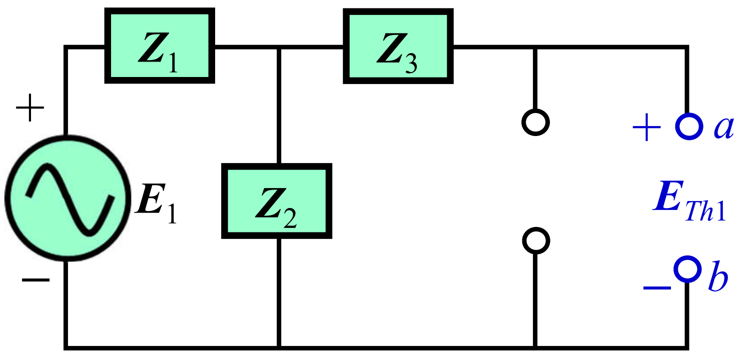


Fig. (c)

$$\boldsymbol{E}_{Th1} = \frac{\boldsymbol{Z}_2\boldsymbol{E}_1}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = 46.83 + j58.54\ \Omega$$

$$= 74.96\Omega\angle 51.34°$$

Considering $\boldsymbol{I}$:

$$\boldsymbol{Z}_{T2} = \boldsymbol{Z}_3 + \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2}$$

$$= 3.9 - j3.12\ \Omega$$

$$= 5\Omega\angle -38.66°$$

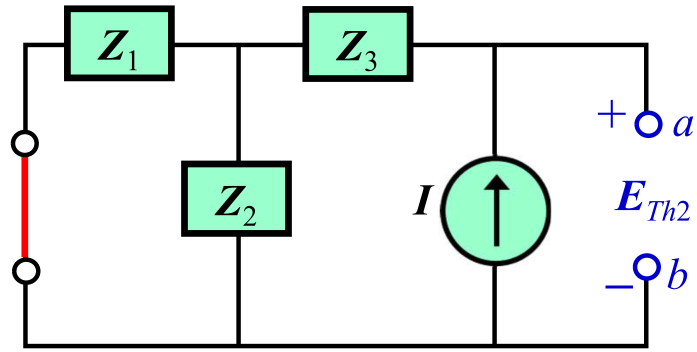
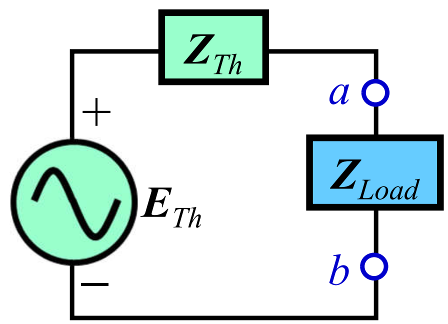


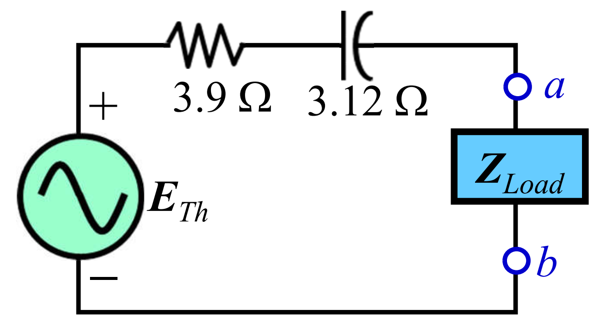
Fig. (d)

$$\boldsymbol{E}_{Th2} = \boldsymbol{Z}_{T2}\boldsymbol{I} = 2.32 + j0.897\ \text{V} = 2.49\text{V}\angle 21.16°$$

According Superposition Theorem:

$$\boldsymbol{E}_{Th} = \boldsymbol{E}_{Th1} + \boldsymbol{E}_{Th2} = 49.15 + j59.43\ \text{V} = 77.12\text{V}\angle 50.41°$$





According to maximum power transfer theorem:

$$\boldsymbol{Z}_L = (\boldsymbol{Z}_{Th})^* = (3.9 - j3.12)^* = 3.9 + j3.12\ \Omega$$

Maximum power received by load:

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}}$$

$$= \frac{(77.12\text{V})^2}{4 \times 3.9\Omega}$$

$$= \mathbf{381.25\ W}$$

Practice Book Remaining Examples
And
Problem 39, 40, 45 and 46 [Ch. 18]

Chapter 24

Poly-phase System

Poly-Phase Generator

An ac generator designed to **develop a single sinusoidal voltage** for each rotation of the shaft (rotor) is referred to as a **single-phase ac generator**.
If the number of coils on the rotor is increased in a specified manner, the result is a **polyphase ac generator**, which **develops more than one ac phase voltage** per rotation of the rotor.

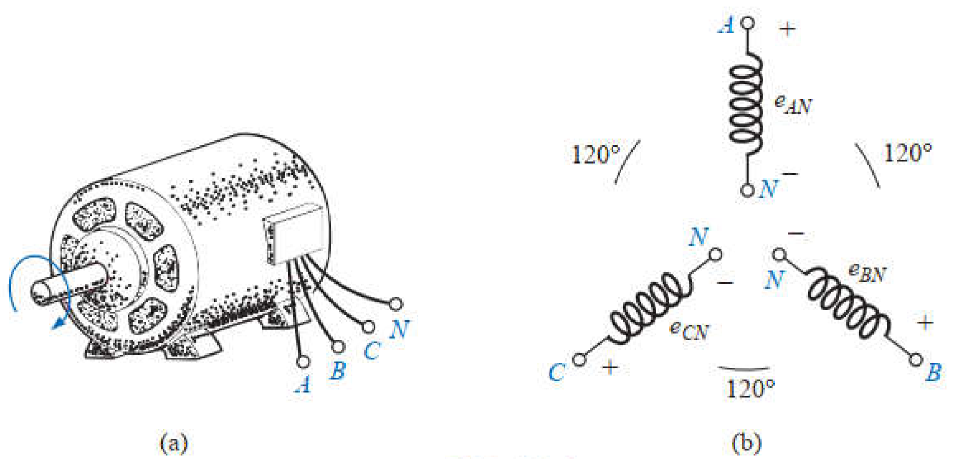


FIG. 22.1
(a) Three-phase generator; (b) induced voltages of a three-phase generator.

For a balanced three phase source, the peak value of voltage $e_{AN}(t)$, $e_{BN}(t)$, and $e_{CN}(t)$ are equal and the phase displacement from each other is 120°.

$$e_{AN}(t) = E_m \sin \omega t$$

$$e_{BN}(t) = E_m \sin(\omega t - 120°)$$

$$e_{CN}(t) = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

Let, the rms value of voltages $e_{AN}(t)$, $e_{BN}(t)$, and $e_{CN}(t)$ is E_p then these voltage are in phasor form as folows:

$$\boldsymbol{E}_{AN} = E_p \angle 0°$$

$$\boldsymbol{E}_{BN} = E_p \angle -120°$$

$$\boldsymbol{E}_{CN} = E_p \angle -240° = E_p \angle 120°$$

where, $E_p = \frac{1}{\sqrt{2}} E_m = 0.707 E_m$

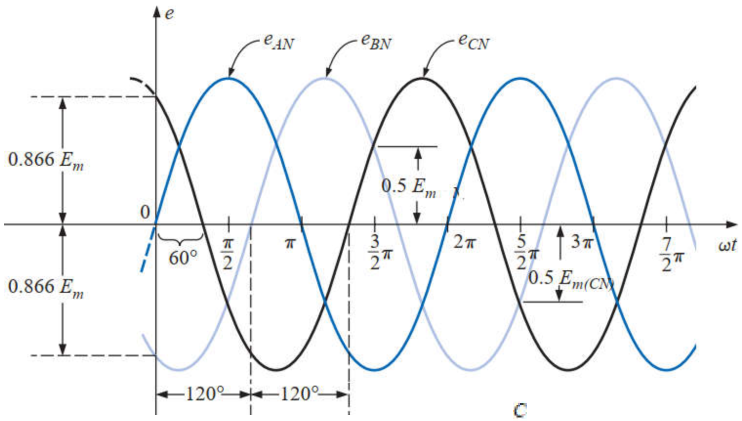
In a balanced system, **at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is *zero***. That means:

$$e_{AN}(t) + e_{BN}(t) + e_{CN}(t) = 0 \qquad \boldsymbol{E}_{AN} + \boldsymbol{E}_{CN} + \boldsymbol{E}_{CN} = 0$$

Phase Sequence or Phase Order

There are two phase sequence or phase orders:

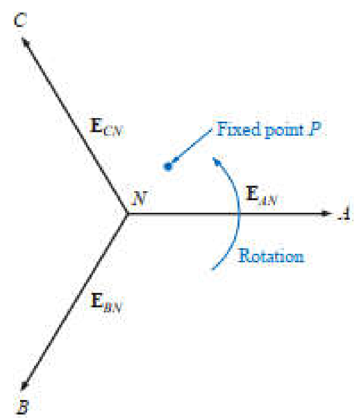
(***a***) ***ABC*–sequence** [B lags A by 120° and C lags B by 120° that means C lags A by 240°]



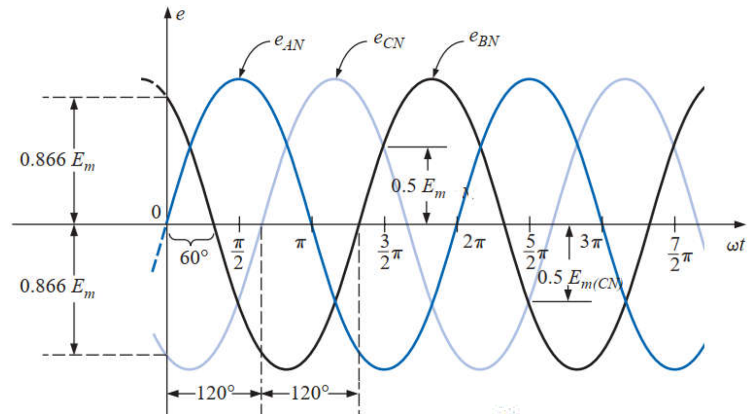
$$E_{AN} = E_p\angle 0°$$

$$E_{BN} = E_p\angle -120°$$

$$E_{CN} = E_p\angle -240° = E_p\angle 120°$$



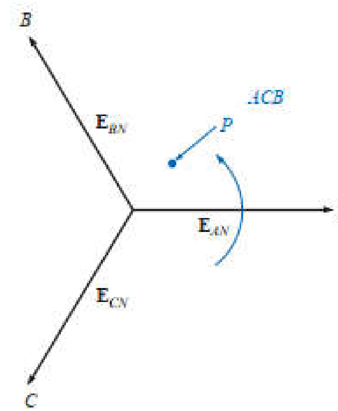
(***b***) ***ACB*–sequence** [C lags A by 120° and B lags C by 120° that means B lags A by 240°]



$$E_{AN} = E_p\angle 0°$$

$$E_{CN} = E_p\angle -120°$$

$$E_{BN} = E_p\angle -240° = E_p\angle 120°$$

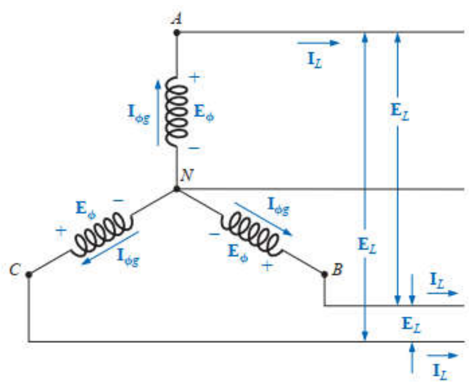


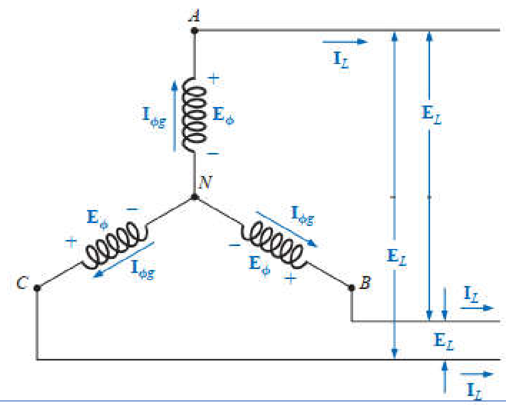
Connection of Three-Phase System

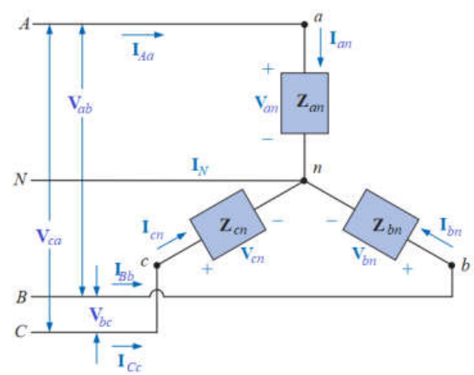
Three phase system can be connected two different ways:

(*a*) **Star** or **Y** (Wye) or **T** (Tee) connection

Balanced source voltages are equal in magnitude and are out of phase with each other by 120°.



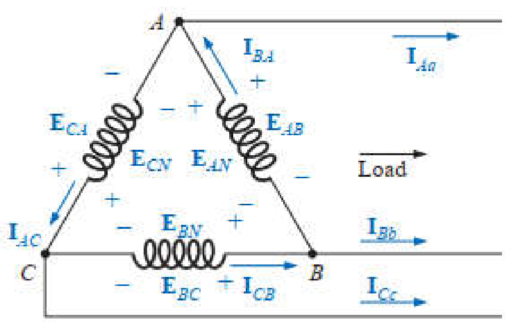


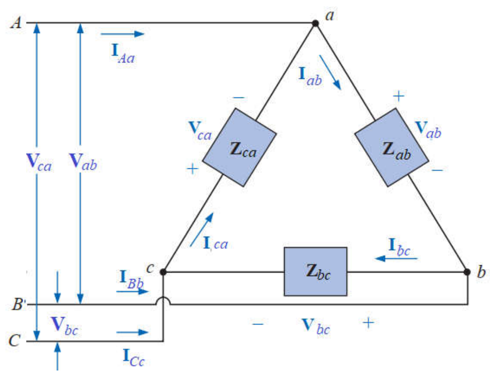


$$\boldsymbol{Z}_{an} = \boldsymbol{Z}_{bn} = \boldsymbol{Z}_{cn} = \boldsymbol{Z}_{Y}$$

$$\boldsymbol{Z}_{Y} = Z\angle\theta_z = R \pm jX$$

(*b*) **Mesh** or Δ (delta) or Π (pai) connection



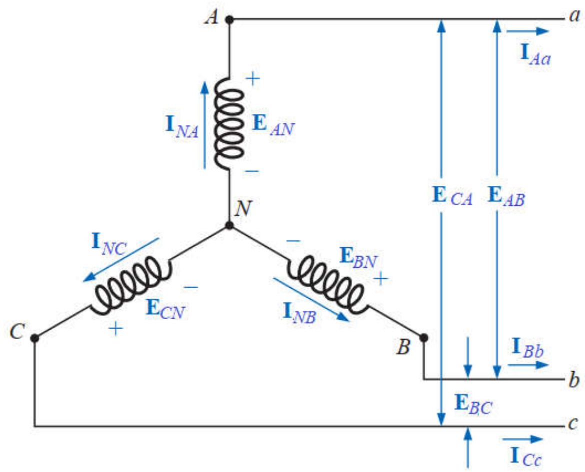


$$\boldsymbol{Z}_{ab} = \boldsymbol{Z}_{bc} = \boldsymbol{Z}_{ca} = \boldsymbol{Z}_{\Delta}$$

$$\boldsymbol{Z}_{\Delta} = Z\angle\theta_z = R \pm jX$$

A balanced load is one which the phase impedance are equal in magnitude and in phase (also, equal in real part and equal in imaginary part).

Star or Y (Wye) or T (Tee) connection



Phase Voltages: $\mathbf{E}_{AN}$, $\mathbf{E}_{BN}$ and $\mathbf{E}_{CN}$

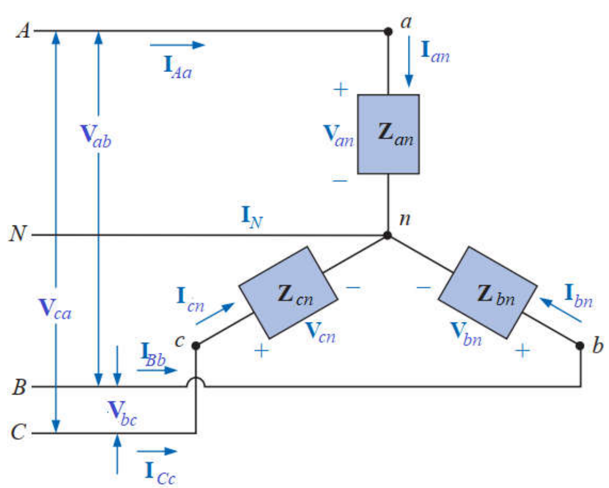
Line Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Phase Currents: $\mathbf{I}_{NA}$, $\mathbf{I}_{NB}$ and $\mathbf{I}_{NC}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Currents = Phase Currents

$\mathbf{I}_{Aa} = \mathbf{I}_{NA}$; $\mathbf{I}_{Bb} = \mathbf{I}_{NB}$; and $\mathbf{I}_{Cc} = \mathbf{I}_{NC}$



Phase Voltages: $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$

Line Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Phase Currents: $\mathbf{I}_{an}$, $\mathbf{I}_{bn}$ and $\mathbf{I}_{cn}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Currents = Phase Currents

$\mathbf{I}_{Aa} = \mathbf{I}_{an}$; $\mathbf{I}_{Bb} = \mathbf{I}_{bn}$; and $\mathbf{I}_{Cc} = \mathbf{I}_{cn}$

Line Voltage ≠ Phase Voltage

$$\boldsymbol{E}_{AB} = \boldsymbol{E}_{AN} - \boldsymbol{E}_{BN}$$
$$\boldsymbol{E}_{BC} = \boldsymbol{E}_{BN} - \boldsymbol{E}_{CN}$$
$$\boldsymbol{E}_{CA} = \boldsymbol{E}_{CN} - \boldsymbol{E}_{AN}$$

$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn}$$
$$\boldsymbol{V}_{bc} = \boldsymbol{V}_{bn} - \boldsymbol{V}_{cn}$$
$$\boldsymbol{V}_{ca} = \boldsymbol{V}_{cn} - \boldsymbol{V}_{an}$$

Let,

V_P and E_P: RMS value of phase voltage

V_L and E_L: RMS value of line voltage

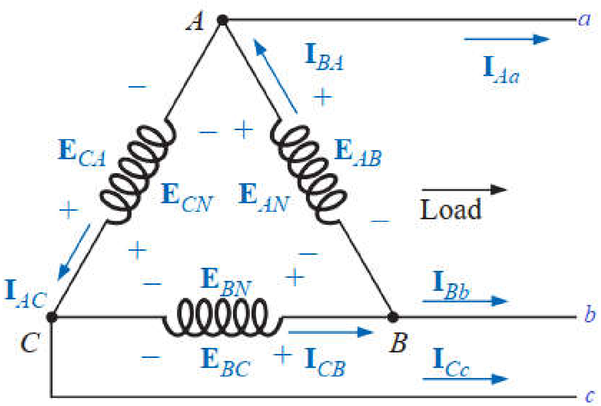
I_P: RMS value of phase current

I_L: RMS value of line current

$$E_L = \sqrt{3}E_P \qquad V_L = \sqrt{3}V_P$$

$$I_L = I_P$$

Mesh or Δ (delta) or Π (pai) connection



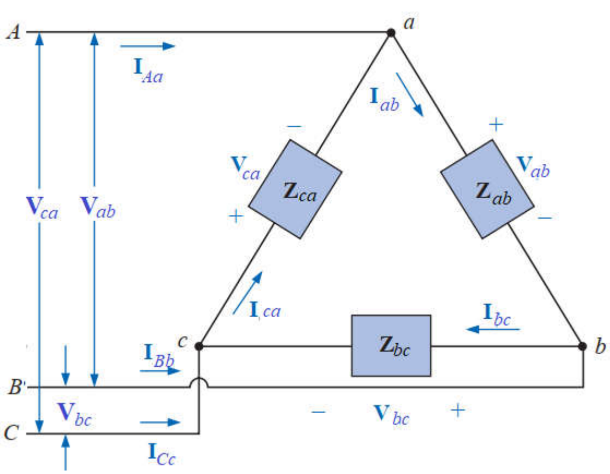
Phase Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Line Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Phase Currents: $\mathbf{I}_{BA}$, $\mathbf{I}_{AC}$ and $\mathbf{I}_{CB}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Voltage = Phase Voltage



Phase Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Line Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Phase Currents: $\mathbf{I}_{ab}$, $\mathbf{I}_{bc}$ and $\mathbf{I}_{ca}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Voltage = Phase Voltage

Line Current ≠ Phase Current

$$\boldsymbol{I}_{Aa} = \boldsymbol{I}_{BA} - \boldsymbol{I}_{AC}$$
$$\boldsymbol{I}_{Bb} = \boldsymbol{I}_{CB} - \boldsymbol{I}_{BA}$$
$$\boldsymbol{I}_{Cc} = \boldsymbol{I}_{AC} - \boldsymbol{I}_{CB}$$
$$\boldsymbol{I}_{Aa} = \boldsymbol{I}_{ab} - \boldsymbol{I}_{ca}$$
$$\boldsymbol{I}_{Bb} = \boldsymbol{I}_{bc} - \boldsymbol{I}_{ab}$$
$$\boldsymbol{I}_{Cc} = \boldsymbol{I}_{ca} - \boldsymbol{I}_{bc}$$

Let,

V_P and E_P: RMS value of phase voltage

V_L and E_L: RMS value of line voltage

I_P: RMS value of phase current

I_L: RMS value of line current

$$I_L = \sqrt{3} I_P$$

$$E_L = E_P \qquad V_L = V_P$$

Power Calculation

Instantaneous Power Equation: $p(t) = \frac{3}{2} E_m I_m \cos\theta = 3E_p I_p \cos\theta$ [W]

V_m and E_m: Peak value of phase voltage
I_m : Peak value of phase current
V_P and E_P: RMS value of phase voltage
V_L and E_L: RMS value of line voltage
I_P: RMS value of phase current
I_L: RMS value of line current

For Y – Connection :

$$\theta = \theta_z = \theta_{e(an)} - \theta_{i(an)} = \theta_{e(bn)} - \theta_{i(bn)} = \theta_{e(cn)} - \theta_{i(cn)}$$

For Δ – Connection :

$$\theta = \theta_z = \theta_{e(ab)} - \theta_{i(ab)} = \theta_{e(bc)} - \theta_{i(bc)} = \theta_{e(ca)} - \theta_{i(ca)}$$

Source Side

$pf = \cos\theta$ $\qquad$ $rf = \sin\theta$

$$S = 3E_P I_P = \sqrt{3} E_L I_L$$

$$P = 3E_P I_P \cos\theta = \sqrt{3} E_L I_L \cos\theta = S\cos\theta$$

$$Q = 3E_P I_P \sin\theta = \sqrt{3} E_L I_L \sin\theta = S\sin\theta$$

Load Side

$S = 3I_P^2 Z$ $\qquad$ $P = 3I_P^2 R$ $\qquad$ $Q_L = 3I_P^2 X_L$

$Q_C = -3I_P^2 X_C$ $\qquad$ $Q = Q_L + Q_C$

$pf = \frac{P}{S}$ $\qquad$ $rf = \frac{Q}{S}$

Example: The line voltage and line current of a three-phase system are 440 V and 40 A. Calculate the phase voltage and phase current for (***i***) star-connection, and (***ii***) mesh-connection.

Solution: Given: V_L =440 V, I_L = 40 A

For Star connection $I_P = I_L = 40 \text{ A}$

$$V_P = \frac{V_L}{\sqrt{3}} = 254 \text{ V}$$

For Mesh connection $V_P = V_L = 440 \text{ V}$

$$I_P = \frac{I_L}{\sqrt{3}} = 23.1 \text{ A}$$

Example 4.1.6: The phase voltage and phase current of a three-phase system are 400 V and 20 A. Calculate the line voltage and line current for (***i***) star or Wye-connection, and (***ii***) mesh or Delta-connection.

Solution: Given: V_P =400 V, I_P = 20 A, n=3

Star or Wye connection

$$I_L = I_P = 20 \text{ A}$$

$$V_L = \sqrt{3}V_P = \sqrt{3} \times 400 = 692.82\ V$$

Mesh or Delta connection

$$V_L = V_P = 400 \text{ V}$$

$$I_L = \sqrt{3}I_P = \sqrt{3} \times 20 = 34.64 \text{ A}$$

Example: A 500 volts three-phase supply is connected with a Δ-connected load having R = 18 Ω and X_C = 24 Ω in series in per phase. Calculate (***i***) the real power, (***ii***) the reactive power, (***iii***) the apparent, (***iv***) the power factor and (***v***) the reactive factor.

Solution: Given, $V_L = 500 \;\; \text{V} \qquad \boldsymbol{Z}_\Delta = \boldsymbol{Z}_{ab} = \boldsymbol{Z}_{bc} = \boldsymbol{Z}_{ca} = 18 - j24 = 30\angle -53.13° \quad \Omega$

$$Z_p = 30 \;\Omega \qquad \theta = \theta_z = -53.13°$$

$$V_p = V_L = 500 \;\; \text{V} \qquad I_p = \frac{V_p}{Z_p} = \frac{500}{30} = 16.67 \;\; \text{A} \qquad I_L = \sqrt{3} I_p = \sqrt{3} \times 16.67 = 28.87 \;\; \text{A}$$

$$P = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 500 \times 28.87 \times \cos(-53.13°) = 15001.33 \;\text{W}$$

$$Q = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta = \sqrt{3} \times 500 \times 28.87 \times \sin(-53.13°) = -20001.7 \;\text{Var}$$

$$S = 3V_p I_p = \sqrt{3} V_L I_L = \sqrt{3} \times 500 \times 28.87 = 25002.15 \;\text{VA}$$

$$pf = \cos\theta = \cos(-53.13°) = 0.6 \qquad rf = \sin\theta = \sin(-53.13°) = -0.8$$

Example: A three-phase Y-connected motor draws 5.6 kW at a power factor of 0.8 lagging when the line voltage is 220 V. Determine (***i***) the line current and (***ii***) the impedance of the motor.

Solution: Given, P = 5.6 kW = 5600 W; $pf = \cos\theta_z = \cos\theta = 0.8$ lagging, $V_L = 220$ V

Since $P = \sqrt{3}V_L I_L \cos\theta = 5600\ \text{W}$

$$I_L = \frac{P}{\sqrt{3}V_L \cos\theta} = \frac{5600}{\sqrt{3}\times 220\times 0.8} = 18.37\ \text{A}$$

For Y-connection:

$$I_p = I_L = 18.37\ \text{A} \qquad V_L = \sqrt{3}V_p \qquad V_p = \frac{1}{\sqrt{3}}V_L = \frac{1}{\sqrt{3}}\times 220 = 127.02\ \text{V}$$

The magnitude of impedance is: $Z_p = \dfrac{V_p}{I_p} = \dfrac{127.02}{18.37} = 6.91\ \Omega$

Since power factor is lagging, we have $\theta_z = \theta = \cos(pf) = \cos(0.8) = 36.87°$

The impedance is: $\boldsymbol{Z} = Z_p\angle\theta_z = 6.91\Omega\angle 36.87° = 5.53 + j4.15\ \Omega$