LESSON 2

BOOK CHAPTER 4

Projectile Motion

Projectile Motion:

A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a **projectile** (meaning that it is projected or launched), and its motion is called **projectile motion**.

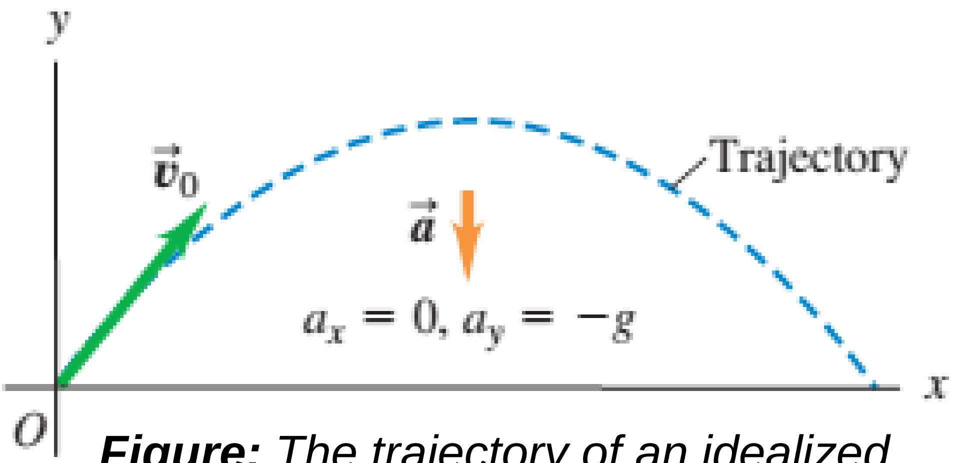
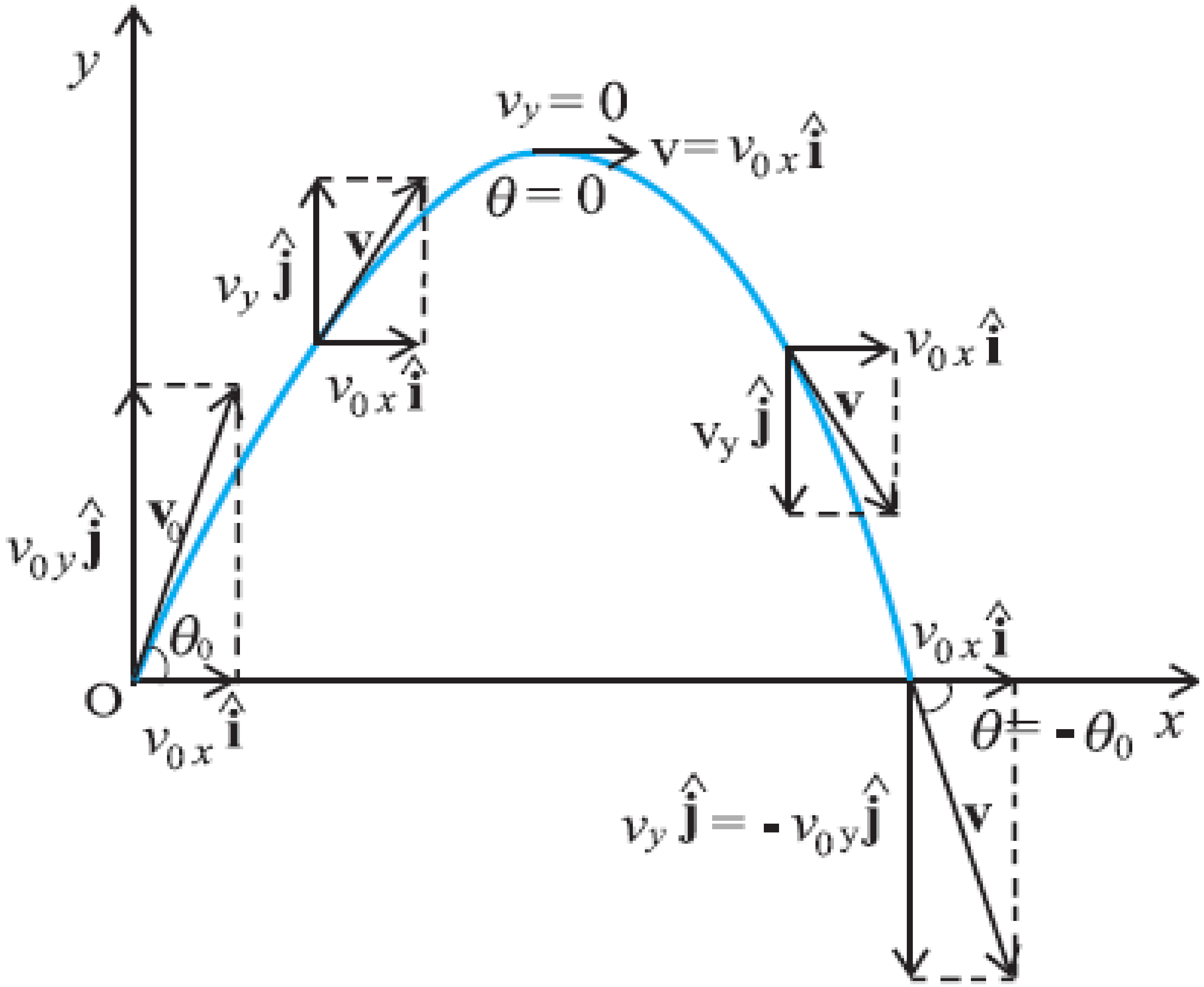


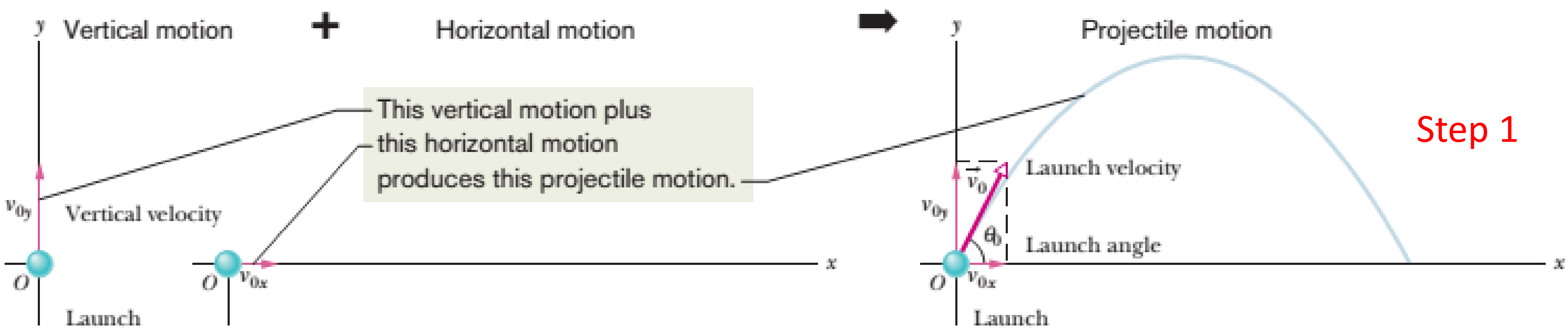
Figure: *The trajectory of an idealized projectile.*

Examples: A batted baseball, a thrown football, a package dropped from an airplane, and a bullet shot from a rifle are all projectiles.

Sketch of the path taken in projectile motion:



Sketch of the path taken in projectile motion (Step-by-Step):



Step 1

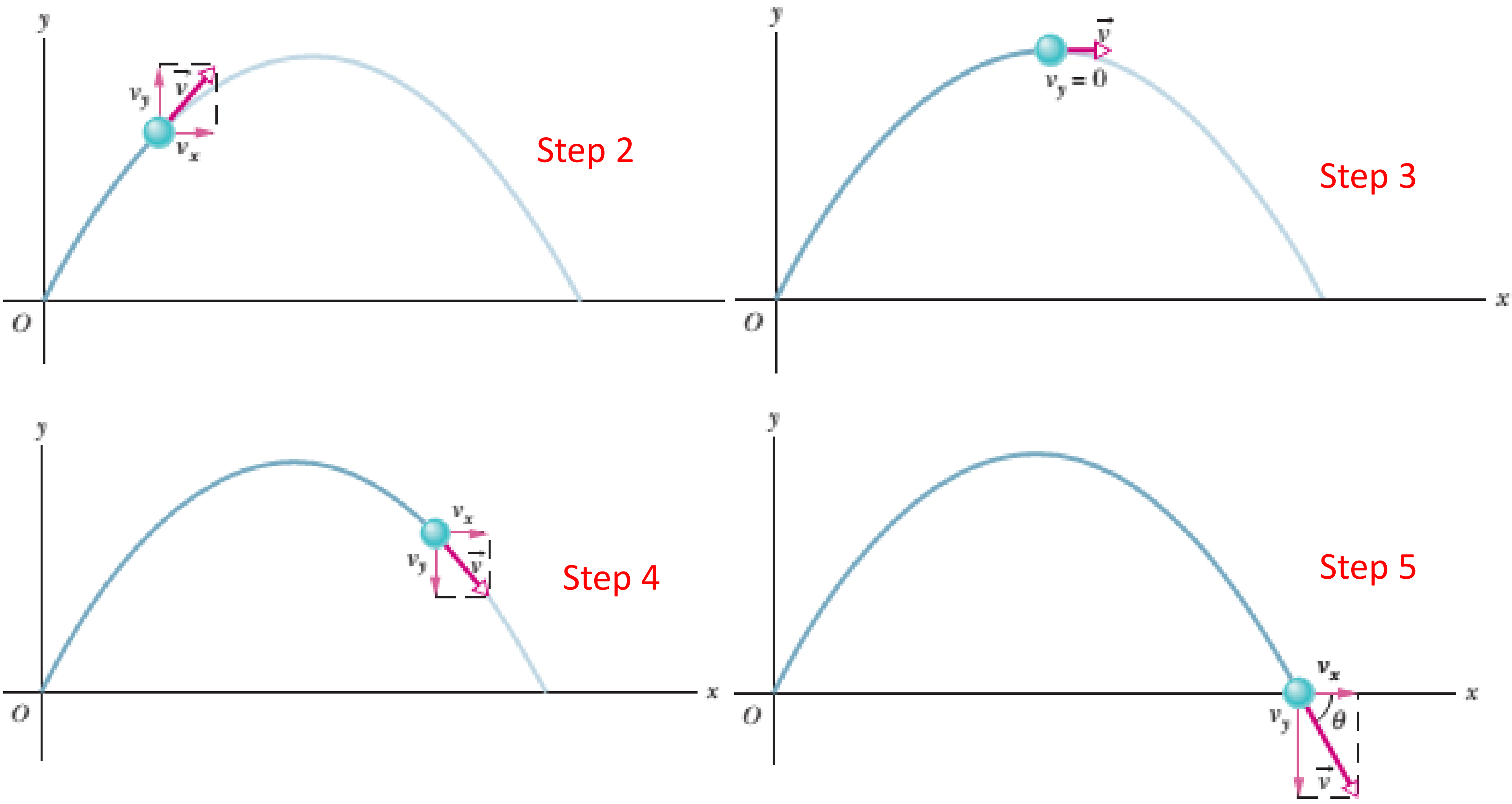
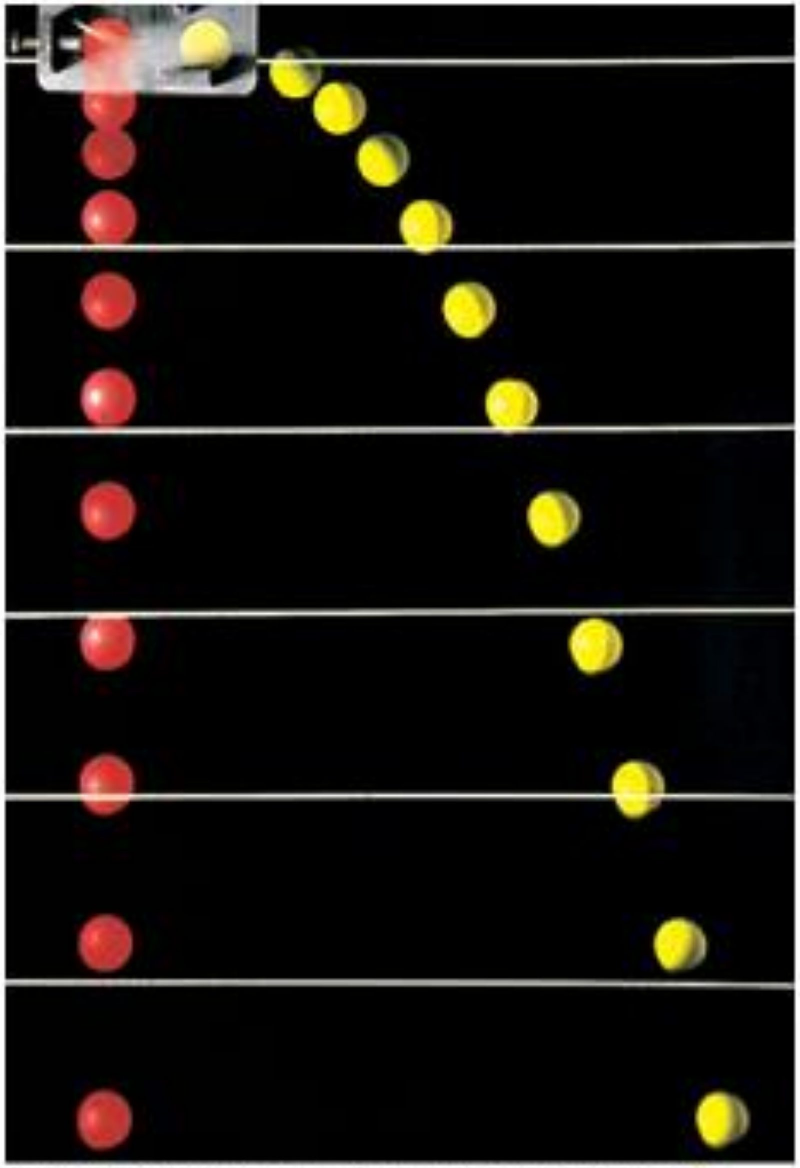


Figure: The *projectile motion* of an object launched into the air at the origin of a coordinate system and with launch velocity $\vec{v}_0$ at angle θ_0. The motion is a combination of vertical motion (constant acceleration) and horizontal motion (constant velocity), as shown by the velocity components.

The adjacent Figure shows two balls with different *x*-motion but identical *y*-motion; one is dropped from rest and the other is projected horizontally, but both balls fall the same distance in the same time.

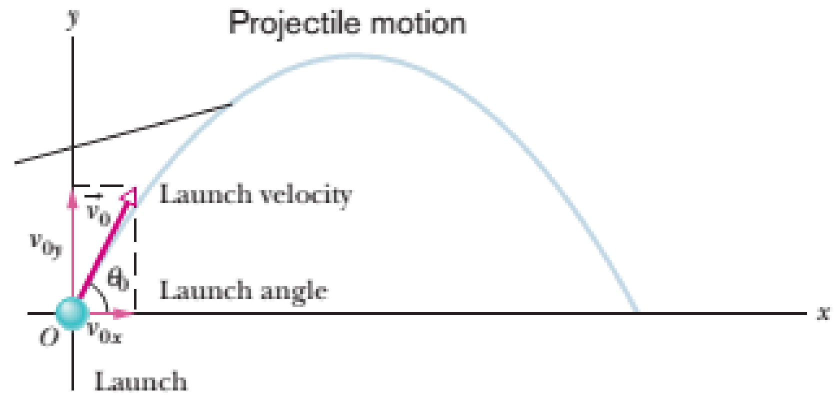


Richard Megna/Fundamental Photographs

The Horizontal Motion:

At any time t, the projectile's horizontal displacement $x - x_0$ from an initial position x_0 is given by

$$x - x_0 = v_{0x}\, t + \frac{1}{2} a_x t^2$$



Where $acceleration\ along\ x - axis, a_x = 0$

Using $v_{0x} = v_0 \cos\theta_0$ we can write

$$x - x_0 = (v_0 \cos\theta_0)\ t \qquad \text{.......... (1)}$$

At any time t, the projectile's horizontal velocity $v_{0x} = v_x$

The Vertical Motion:

At any time t, the projectile's vertical displacement $y - y_0$ from an initial position y_0 is given by

$$y - y_0 = v_{0y}\, t - \frac{1}{2} g t^2 \qquad [\text{ where, } a_y = -g]$$

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2} g t^2 \qquad [\text{ where, } v_{0y} = v_0 \sin\theta_0]$$

.... (2)

At any time *t*, the projectile's vertical velocity

$$v_y = v_0 \sin\theta_0 - gt$$

And also we can express $v_y \ as$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2\,g(y - y_0)$$

❑ Show that the path of a projectile is a parabola.

From equation (1) we can write

$$t = \frac{x - x_0}{v_0 \cos\theta_0}$$

Using the value of t in equation (2), we get

$$y - y_0 = v_0 \sin\theta_0 \frac{x-x_0}{v_0 \cos\theta_0} - \frac{1}{2} g \left(\frac{x-x_0}{v_0 \cos\theta_0}\right)^2$$

For simplicity, we let $x_0 = 0 \; and \; y_0 = 0$.

Therefore, the equation becomes

$$y = (\tan\theta_0)x - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta_0}\right)^2 \qquad \text{.....................} \; (3)$$

Where $\theta_0, g \; and \; v_0 \; are \; constants$.

Equation (3) is of the form $y = ax \mp bx^2 \;, where \; a \; and \; b \; are \; constants$.

This is the equation of a parabola, so the path is *parabolic*.

❑ Equations for the horizontal range and the maximum horizontal range of a projectile:

The ***horizontal range R*** of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). That is $x - x_0 = R \; when \; y - y_0 = 0$.

Using $x - x_0 = R \; in \; equation \; (1) and \; y - y_0 = 0 \; in \; equation \; (2)$, we get

$$R = (v_0 \cos\theta_0)\, t$$ [From equation (1)]

And $0 = (v_0 \sin\theta_0)\; t - \frac{1}{2} g t^2$ [From equation (2)]

$$or \quad (v_0 \sin\theta_0)\; t = \frac{1}{2} g t^2 \qquad or \quad t = \frac{2 v_0 \sin\theta_0}{g}$$

Therefore, $$R = (v_0 \cos\theta_0)\, \frac{2 v_0 \sin\theta_0}{g} = \frac{v_0^2\, (2 \sin\theta_0 \cos\theta_0)}{g}$$

$$\boxed{R = \frac{v_0^2 \sin 2\theta_0}{g}} \quad ……(3)$$

***Caution:* This equation does not give the horizontal distance traveled by a projectile when the final height is not the launch height.**

The value of R is maximum in equation (3) when $\sin 2\theta_0 = 1$

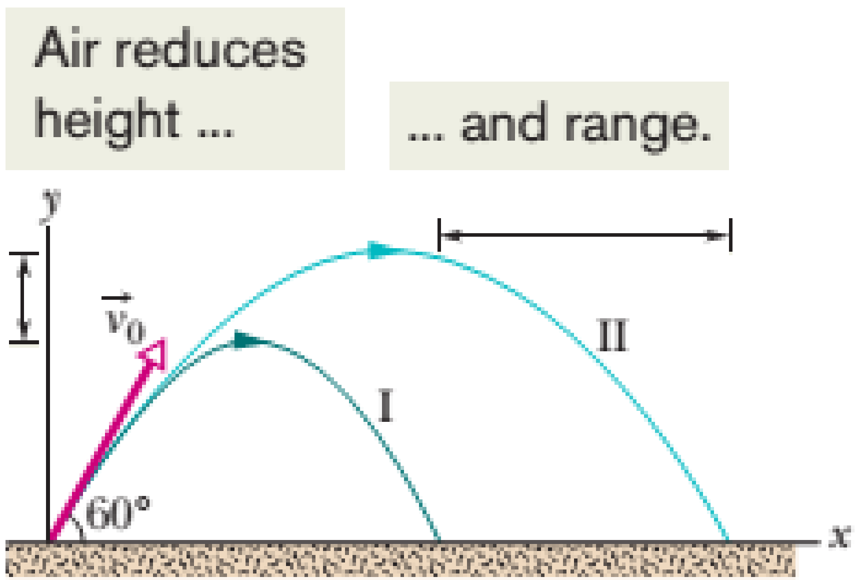
$$or\ \ 2\theta_0 = \sin^{-1} 1$$

$$or\ 2\theta_0 = 90^0 \qquad [since\ \ \sin^{-1} 1 = 90^0]$$

$$\boxed{\theta_0 = 45^0}$$

The Effects of the Air (in the projectile motion):

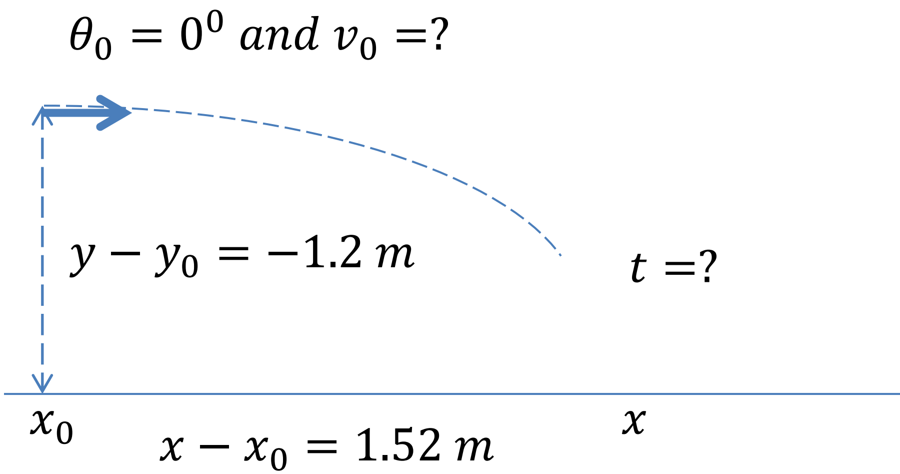
	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s



The launch angle is 60° and the launch speed is 44.7 m/s.

Problem 22 (Book chapter 4):

A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?



Answer:

(a) We know

$$y - y_0 = (v_0 \sin\theta_0)\; t - \frac{1}{2} g t^2$$

$$-1.20 = (v_0 \; sin \; 0^0)\; t - 4.9t^2$$

$$-1.20 = 0 - 4.9t^2$$

$$t = \sqrt{\frac{1.2}{4.9}} = 0.495 \; s$$

(b) We know

$$x - x_0 = (v_0 \cos\theta_0)\; t$$

$$1.52 = (v_0 \cos 0^0)(\;0.495)$$

$$1.52 = (v_0 \cos 0^0)(\;0.495)$$

$$1.52 = (v_0)(1)(0.495)$$

$$\boxed{v_0 = \frac{1.52}{0.495} = 3.07 \; m/s}$$

Thank You