EQUATION OF A CIRCLE

The equation of a circle comes in two forms:

1) The standard form: $(x - h)^2 + (y-k)^2 = r^2$
2) The general form : $x^2 + y^2 + Dx + Ey + F = 0$, where D, E, F are constants.

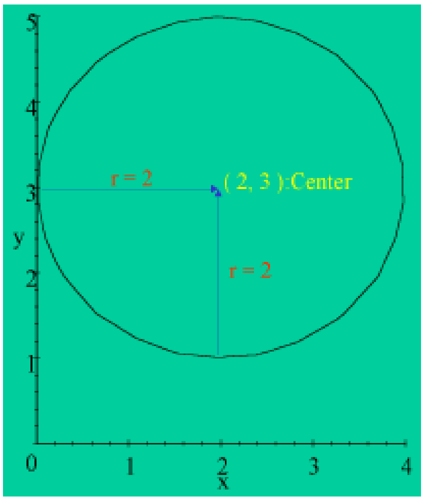
If the equation of a circle is in the standard form, we can easily identify the centre of the circle, (h, k), and the radius, r . Note: The radius, r, is always positive.

Example 1: $(x-2)^2 + (y-3)^2 = 4$. (a) Find the centre and radius of the circle. (b) Graph the circle.

Note: A common mistake is to take h= -2 and K= -3. In an equation, if the sign preceding h and k , (h, k) are negative, then h and k are positive. That is, h= 2 and k= 3.

(a) Centre: (h= 2, k= 3) = (2, 3) and radius r=2 since $r^2 = 4$ => r = Ö4 = 2

(b) The graph is



Example 2: $(x+1)^2 + (y-2)^2 = 9$. (a) Find the centre and radius of the circle. (b) Graph the circle.

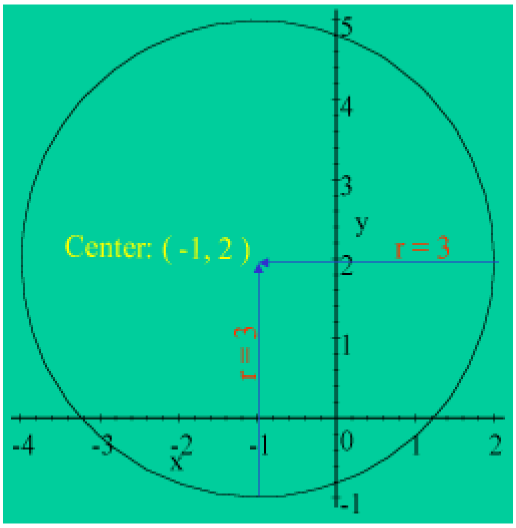
Note: To correctly identify the centre of the circle we have to place the equation in the standard form:

The standard form is: $(x - h)^2 + (y-k)^2 = r^2$

$(x - (-1))^2 + (y-2)^2 = (3)^2$. Now, you can identify the centre correctly.

(a) Centre: (h= -1, k= 2) = (-1, 2) and radius r=3 since $r^2 = 9$ > r=Ö9=3

(b) The graph is



Example 3: $2x^2 + 2y^2 = 8$. (a) Find the centre and radius of the circle. (b) Graph the circle.

Note: To correctly identify the centre of the circle we have to place the equation in the standard form.

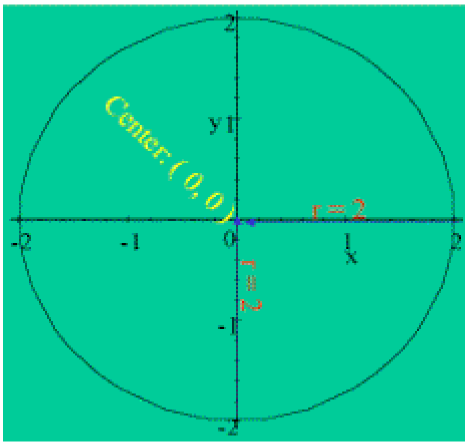
First divide the equation by 2. The new equation is : $x^2 + y^2 = 4$.

The standard form: $(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 0)^2 = (2)^2$. Now, you can identify the centre correctly.

(a) Centre: (h= 0, k= 0) = (0, 0) and radius r = 2 since $r^2 = 4$ => r = Ö4 = 2

(b) The graph is



If the equation is in the general form, we have to complete the square and bring the equation in the standard form. Then, we can identify the centre and radius correctly. We learned how to complete the square when working with quadratic equations (E III). We will review it through an example.

Example 4: $x^2 + y^2 - 6x + 4y + 9 = 0$. (a) Find the centre and radius of the circle. (b) Graph the circle.

Completing the square:

- Write the equation in this form: $(x^2 - 6x + \ ?_1\ ) + (y^2 + 4y + \ ?_2\ ) = -9 + ?_1 + ?_2$. In the first parenthesis, we group the x-terms and in the second the y-terms. The constant is moved on the right hand side. The question mark, ?, is the number needed in each parenthesis to complete the square. Note that we have to add this number to both sides of the equation. That is why you see $?_1$ and $?_2$, added to both sides.

- How to find the number to replace the question mark, $?_1$. Take the coefficient of x and divide it by 2, (-6/2), and then square it, $(-3)^2 = 9$. $?_1$ is going to be replaced by the number 9.

- How to find the number to replace the question mark, $?_2$. Take the coefficient of y and divide it by 2, (4/2), and then square it, $(2)^2 = 4$. $?_2$ is going to be replaced by the number 4.

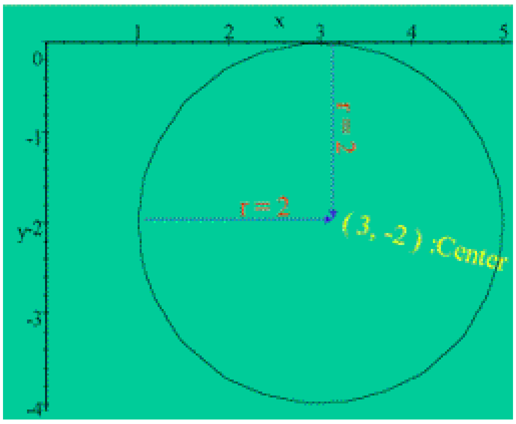
Putting steps 1-3 together we have the following:

$$(x^2 - 6x + \ ?_1\ ) + (y^2 + 4y + \ ?_2\ ) = -9 + ?_1 + ?_2$$
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$
$$(x - 3)^2 + (y + 2)^2 = 4$$
$$(x - 3)^2 + (y - (-2))^2 = 4$$

This equation is in the standard form.

(a) Centre: (h= 3, k= -2) = (3, -2) and radius r = 2 since $r^2 = 4 \Rightarrow r = Ö4 = 2$

(b) The graph is



Example 5: $x^2 + y^2 - 6x + 2y + 4 = 0$. (a) Find the center and radius of the circle. (b) Graph the circle.

Completing the square:

- Write the equation in this form: $(x^2 - 6x + \ ?_1\ ) + (y^2 + 2y + \ ?_2\ ) = -4 + ?_1 + ?_2$. In the first parenthesis, we group the x-terms and in the second the y-terms. The constant is moved on the right hand side. The question mark, ?, is the number needed in each parenthesis to complete the square. Note that we have to add this number to both sides of the equation. That is why you see $?_1$ and $?_2$, added to both sides.

- How to find the number to replace the question mark, $?_1$. Take the coefficient of x and divide it by 2, (-6/2), and then square it, $(-3)^2 = 9$. $?_1$ is going to be replaced by the number 9.

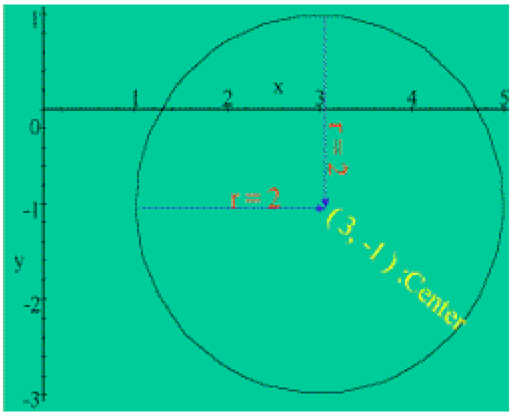
- How to find the number to replace the question mark, $?_2$. Take the coefficient of y and divide it by 2, (2/2), and then square it, $(1)^2 = 1$. $?_2$ is going to be replaced by the number 1.

Putting steps 1-3 together we have the following:

$$(x^2 - 6x + ?_1) + (y^2 + 2y + ?_2) = -4 + ?_1 + ?_2$$
$$(x^2 - 6x + 9) + (y^2 + 2y + 1) = -4 + 9 + 1$$
$$(x - 3)^2 + (y + 1)^2 = 4$$
$$(x - 3)^2 + (y - (-1))^2 = 4$$ This equation is in the standard form.

(a) Center: (h= 3, k= -1) = (3, -1) and radius r = 2 since $r^2 = 4$ => r = Ö4 = 2

(b) The graph is



HOMEWORK-For each problem, (a) find the center and radius of the circle and (b) Graph the circle.

1. $(x-2)^2 + (y+1)^2 = 4$.
2. $(x-3)^2 + (y-2)^2 = 9$
3. $x^2 + y^2 - 6x - 10y + 30 = 0$.
4. $x^2 + y^2 - 6x + 4y + 9 = 0$.
5. $x^2 + y^2 - 10x = 0$.
6. $x^2 + y^2 = 8$.
7. $x^2 + y^2 = 1$.
8. $4x^2 + 4y^2 = 9$.