Bresenham
Mid point Circle

Derivation

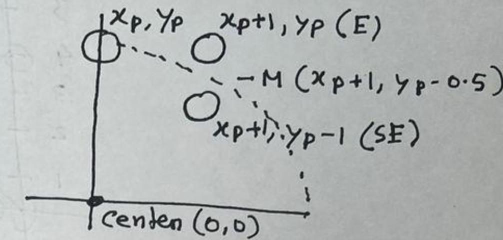
Function of a circle is,

$f(x,y) = (x^2-h)^2 + (y^2-K)^2 = R^2$

For center $(0,0)$

$f(x,y) = x^2+y^2 = R^2$

$= x^2+y^2-R^2 = 0$



So, The first pixel (x_p, y_p) is $(0, R)$

$d_{start} = f(x_p+1, y_p-0.5)$

$= (x_p+1)^2 + (y_p-0.5)^2 - R^2$

$= x_p^2 + 2\cdot x_p \cdot 1 + 1^2 + y_p^2 - 2\cdot y_p \cdot 0.5 + (0.5)^2 - R^2$

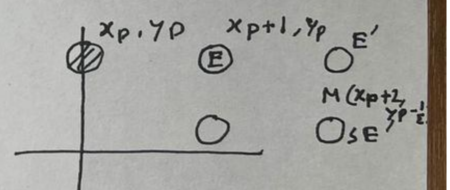
$= 0^2 + 2\cdot 0 \cdot 1 + 1^2 + R^2 - 2\cdot R \cdot 0.5 + (0.5)^2 - R^2$

$= 1 + \cancel{R^2} - R + 0.25 - \cancel{R^2}$

$= 1.25 - R$

$\approx 1 - R$

$\boxed{\begin{array}{ll} \text{if } d >= 0 & SE \\ d < 0 & E \end{array}}$



For pixel E,

$d_{new} = f(x_p+2, y_p-0.5)$

So, $d_E = d_{new} - d_{old}$ (d_{old} is M for d_{start})

$= f(x_p+2, y_p-0.5) - f(x_p+1, y_p-0.5)$

$= (x_p+2)^2 + (y_p-0.5)^2 - R^2 - (x_p+1)^2 - (y_p-0.5)^2 + R^2$

$= x_p^2 + 2\cdot x_p \cdot 2 + 2^2 + \cancel{y_p^2} - 2\cdot y_p \cdot \cancel{0.5} + \cancel{(0.5)^2} - \cancel{R^2} - x_p^2 - 2\cdot x_p \cdot 1 - 1 - \cancel{y_p^2} + \cancel{2\cdot y_p \cdot 0.5} - \cancel{(0.5)^2} - R^2$

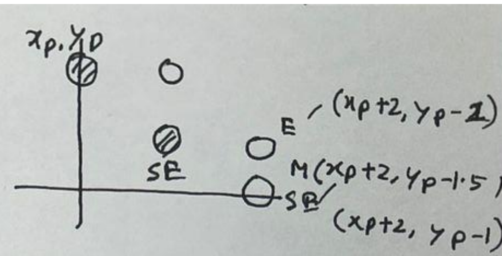
$= \cancel{x_p^2} + 4x_p + 4 - \cancel{x_p^2} - 2x_p - 1$

$= 2x_p + 3$

$\therefore d_E = 2x_p + 3$

For pixel SE,

$d_{new} = f(x_p+2, y_p-1.5)$



$d_{SE} = d_{new} - d_{old}$ [d_{old} is M for d_{start}]

$= f(x_p+2, y_p-1.5) - f(x_p+1, y_p-0.5)$

$= (x_p+2)^2 + (y_p-1.5)^2 - R^2 - (x_p+1)^2 - (y_p-0.5)^2 + R^2$

$= 2x_p - 2y_p + 5$

$\therefore d_{SE} = 2x_p - 2y_p + 5$.

Mathematics

Given R of a circle is 10 and center is (0,0)

So,

$d_{start} = 1-R = 1-10 = -9$

$d_E = 2x_p+3$

$d_{SE} = 2x_p - 2y_p + 5$

Initial pixel (0,10)

d	comp	dec	x	y
$d_{start} = -9$	$d<0$	E	1	10
~~d_{start} =~~ $d_E = -9+(2x_p+3)$ $= -9+(2\cdot0+3)$ $= -9+3 = -6$	$d<0$	E	2	10
$d_E = -6+(2\cdot x_p+3)$ $= -6+(2\cdot1+3)$ $= -1$	$d<0$	E	3	10
$d_E = -1+(2\cdot x_p+3)$ $= -1+(2\cdot2+3)$ $= 6$	$d>0$	SE	4	9
$d_{SE} = 6+2(3-10)+5$ $= -3$	$d<0$	E	5	9

⋮

We continue until $x > y$

Mathematics

For center $(2,2)$, Radius $= 10$

$d_{start} = 1 - R = 1 - 10 = -9$

$(0, R)$

d	comp	dec	x	y
		E	$2+1$	$2+10$
$d_{start} = 9$	$d<0$	E	$2+2$	$2+10$
$d_E = -9 + (2x+3)$	$d<0$			
$= -9 + (2\cdot 0 + 3)$				
$= -6$		E	$2+3$	$2+10$
$d_E = -6 + (2x_p + 3)$	$d<0$			
$= -6 * (2\cdot 1 + 3)$				
$= -1$		⋮		

Until we reach $x > y$