



## American International University- Bangladesh

Department of Electrical and Electronic Engineering

COE 2102: Introduction to Electrical Circuits Laboratory

**Title:** Analysis of RC, RL, RLC series circuits and verification of KVL in RLC series Circuit related to AC circuit

### **Abstract:**

The purpose of this experiment is to develop an understanding of circuits containing  $R, L$  and  $C$  components, to be able to analyze the outputs of RC & RL series circuit obtained practically with simulated or theoretical results along with the determination of phase relationship between  $V$  and  $I$  in an RLC series circuit, and finally to draw the complete vector diagram of and verify KVL for an RLC series circuit.

### **Introduction:**

The RC & RL circuit is used to determine the input and output relationship of voltage and current for different frequencies. In RC series circuit the voltage lags the current by  $90^\circ$  and in RL series circuit the voltage leads the current by  $90^\circ$ .

An RLC circuit is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance respectively. Series RLC circuits are classed as second-order circuits because they contain two energy storage elements, an inductance and a capacitance.

The primary objectives of the lab experiment are-

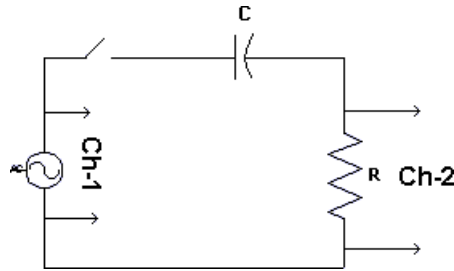
- To determine the reactance of the RL and RC circuits and the impedance equation both practically and theoretically.
- To determine phase relationship between voltage and current in an RLC circuit.
- To draw the complete vector diagram.
- Design an RLC series circuit and verify KVL.

### **Theory and Methodology:**

#### ***RC Series Circuit:***

A resistor-capacitor circuit (RC circuit), or RC network, is an electric circuit composed of resistors and capacitor in series driven by a voltage or current source (See the Figure-1). A first order RC circuit is composed of one resistor and one capacitor and is the simplest type of RC circuit.

In Figure 1, Channel 1 of the oscilloscope indicates the input voltage  $E$ , and Channel 2 of the oscilloscope indicates the voltage  $V_{RL}$  across the resistor which is proportional to the current  $I$  flowing in the circuit. The phase difference  $\theta$  between voltage  $E$  and current  $I$  in the RC series circuit can thus be found from the phase difference between Channel 1 and 2 waveforms.



**Figure 1: RC Series circuit.**

### ***Analysis of a Series RC Circuit:***

For doing a complete analysis of a series RC circuit, given the values of R, C, f, and  $V_T$ .

Step 1. Calculate the value of  $X_C$ :

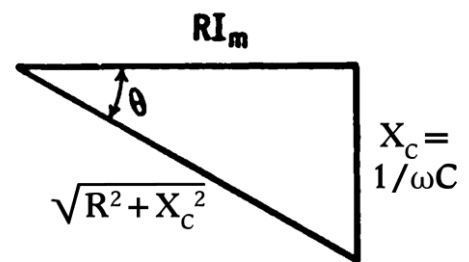
$$X_C = 1 / (2\pi fC)$$

Step 2. Calculate the total impedance Z:

$$Z = \sqrt{X_C^2 + R^2}$$

Step 3. Use Ohm's Law to calculate the total current  $I_T$ :

$$I_T = V_T / Z$$



### ***Difference between Rectangular & Polar representation of Impedance:***

- In Rectangular form:

$$Z_T = R - j X_C$$

- In Polar form:

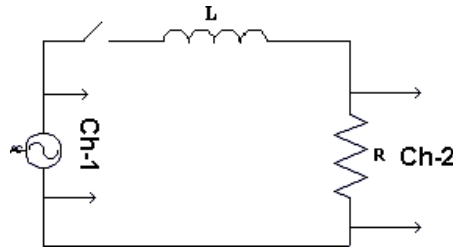
$$Z_T = \sqrt{R^2 + (X_C)^2}$$

$$\theta = \tan^{-1}(-X_C/R) = \tan^{-1}(-1/\omega RC)$$

### ***RL Series Circuit:***

A resistor–inductor circuit (RL circuit), or RL network, is an electric circuit composed of resistors and inductor is in series driven by a voltage or current source (See the Figure-2). A first order RL circuit is composed of one resistor and one inductor and is the simplest type of RL circuit.

In Figure 2, Channel 1 of the oscilloscope indicates the input voltage E, and Channel 2 of the oscilloscope indicates the voltage  $V_{RL}$  across the resistor which is proportional to the current I flowing in the circuit. The phase difference  $\theta$  between voltage E and current I in the RL series circuit can thus be found from the phase difference between Channel 1 and 2 waveforms.



**Figure 2: RL Series circuit.**

***Analysis of a Series RL Circuit:***

For doing a complete analysis of a series RL circuit, given the values of R, L, f, and  $V_T$ .

Step 1. Calculate the value of  $X_L$ :

$$X_L = 2\pi fL$$

Step 2. Calculate the total impedance Z:

$$Z = \sqrt{X_L^2 + R^2}$$

Step 3. Use Ohm's Law to calculate the total current  $I_T$ :

$$I_T = V_T / Z$$

***Difference between Rectangular & Polar representation of Impedance:***

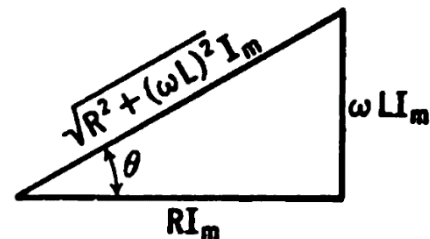
- In Rectangular form:

$$Z_T = R + jX_L$$

- In Polar form:

$$Z_T = \sqrt{R^2 + (X_L)^2}$$

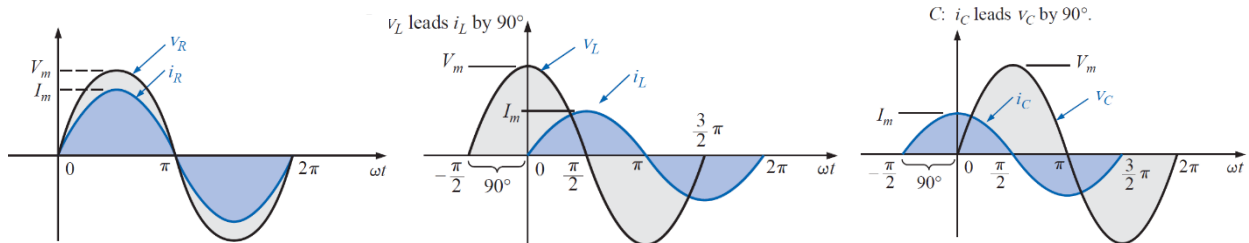
$$\theta = \tan^{-1}(X_L / R) = \tan^{-1}(\omega L / R)$$



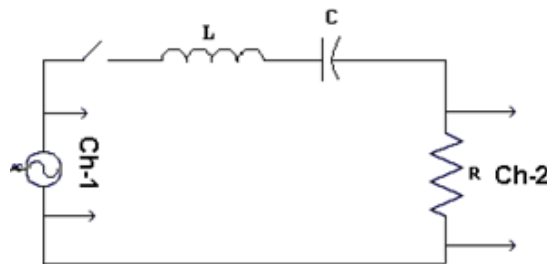
***RLC Series Circuit:***

Three basic passive components- R, L and C have very different phase relationships to each other when connected to a sinusoidal AC supply. In case of a resistor, the voltage waveforms are "in-phase" with the current. In case of pure inductor, the voltage waveform "leads" the current by  $90^\circ$  whereas in case of pure capacitor, the voltage waveform "lags" the current by  $90^\circ$ . This phase difference depends upon the reactive value of the components being used. Reactance is zero if the element is resistive, positive if the element is inductive and negative if the element is capacitive.

Simple Circuit	$R$	$L$	$C$	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
purely resistive	$R$	0	$\infty$	0	0	0	$R$
purely inductive	0	$L$	$\infty$	$X_L$	0	$\pi / 2$	$X_L$
purely capacitive	0	0	$C$	0	$X_C$	$-\pi / 2$	$X_C$



Instead of analyzing each passive element separately, we can combine all three together into a series RLC circuit. The analysis of the series RLC circuit of Figure 3 is the same as that for the dual series RL and RC circuits we studied previously, except this time we need to take account the magnitudes of both inductive reactance and capacitive reactance to find the overall circuit reactance.



**Figure 3: RLC series circuit**

**Relevant Equations:**

Inductive reactance,  $X_L = 2\pi fL$

Capacitive reactance,  $X_C = 1/2\pi fC$

Net reactance,  $X = X_L - X_C$

Total impedance,  $Z = \sqrt{R^2 + X^2}$

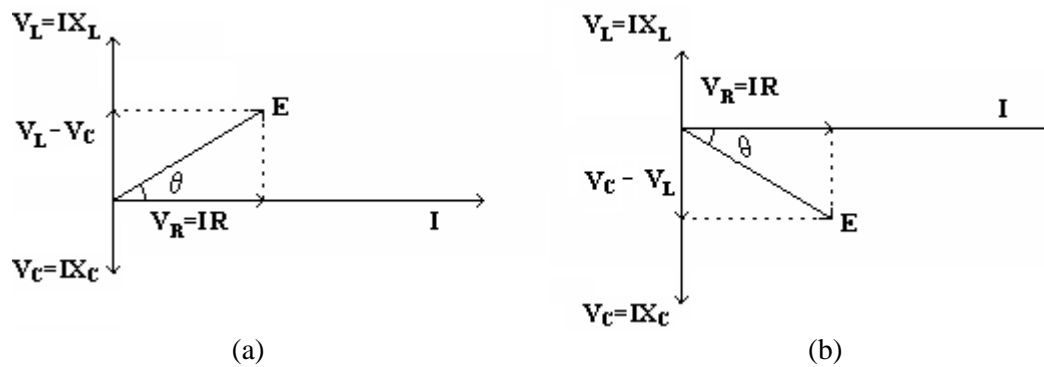
Current,  $I = V/Z$

Resistive voltage drop,  $V_R = I \cdot R$

Reactive voltage drops =  $V_L - V_C$ , where  $V_L = I \cdot X_L$  and  $V_C = I \cdot X_C$

Total voltage drop =  $\sqrt{V_R^2 + (V_L - V_C)^2}$

**Vector Diagram:**



**Figure 4: RLC series circuit Phasor Diagram when (a)  $X_L > X_C$  , and (b)  $X_C > X_L$**

It can be observed from the phasor diagrams that when  $X_L > X_C$  ,  $\theta$  is positive and overall circuit is inductive. When  $X_C > X_L$ ,  $\theta$  is negative and overall circuit is capacitive.

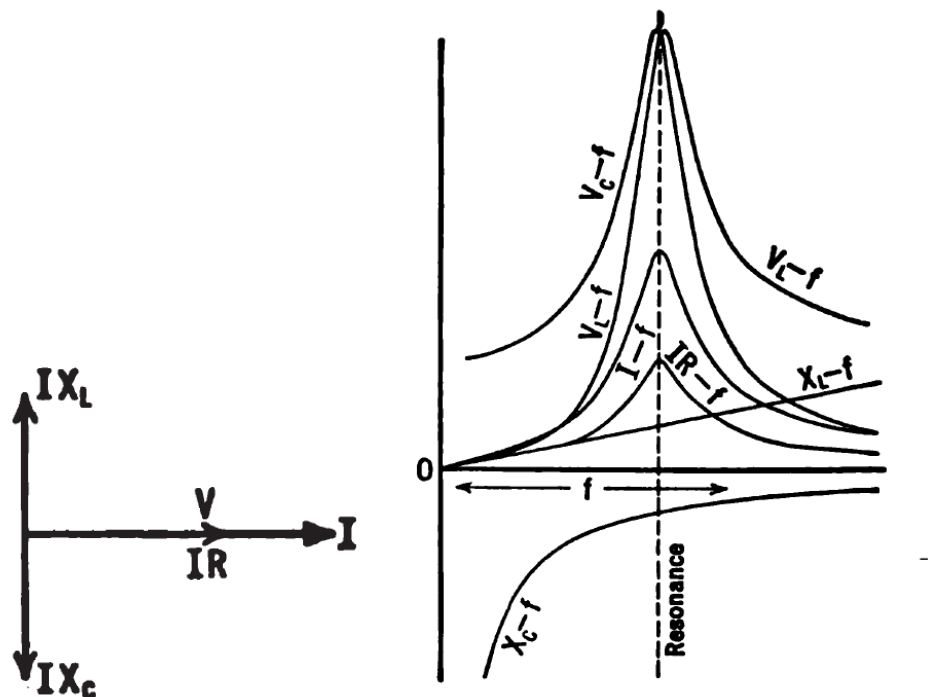
Figure 3 represents a RLC series circuit where we can apply KVL to check if applied voltage  $E$  equals to total voltage drop found from the phasor diagram  $\sqrt{V_R^2 + (V_L - V_C)^2}$

#### ***Impact of frequency on resonance:***

A series circuit containing  $R$ ,  $L$ , and  $C$  is in resonance when the resultant reactance is zero. Since the drop across the inductance leads the current by  $90^\circ$  whereas that across the capacitor lags the current by  $90^\circ$ , the two drops are opposite. If they are made equal as in Fig. 5 (a), the reactive voltage drops neutralize and the impressed voltage is equal only to the resistance drop. This condition is called series resonance, and it occurs when  $X_C = X_L$  and  $\theta = 0$ . Inspection of the vector diagram of Fig. 5 (a) also shows that the applied voltage is in phase with the current at resonance.

Figure 5 (b) shows the impact of varying the value of frequency on current ( $I$ ), resistor voltage ( $V_R = IR$ ), inductor voltage ( $V_L$ ), capacitor voltage ( $V_C$ ) during series resonance.

Since  $2\pi fL = 1/2\pi fC$  at the point of series resonance, the series resonant frequency is  $f_m = 1/2\pi\sqrt{LC}$ .



**Figure 5: Series Resonance.**

**Pre-Lab Homework:**

Read about the characteristics of RC, RL and RLC series circuit from “Alternating Current Circuit” by George F Corcoran and use simulation software to generate the output of the circuits provided in this lab sheet. Compare the wave shapes given in the text book with your results. Save the simulation results and bring it to the lab.

**Apparatus:**

- Oscilloscope
- Function generator
- Resistor: 100  $\Omega$  (For RC and RL)
- Inductor: 2.4mH (For RC and RL)
- Capacitor: 1  $\mu$ F/ 10  $\mu$ F (For RC and RL)
- Resistor (200  $\Omega$ ) For RLC
- Inductor (6.3 mH) For RLC
- Capacitor (1  $\mu$ F) For RLC
- Connecting wire
- Bread board

**Precautions:**

- Oscilloscopes should be properly calibrated using the information provided at the calibration port before obtaining the wave shapes using the experimental set up.
- Do not short any connections. Short connection can produce heat (due to high current flow) which is harmful for the components.

**Experimental Procedure:*****For RC and RL Series Circuit:***

1. Construct the circuit as shown in the Fig. 1. Connect channel 1 of the oscilloscope across function generator to measure E and channel 2 of the oscilloscope across R to measure  $V_R$ .
2. Set the amplitude of the input signal 5V peak and the frequency at 1 kHz. Select sinusoidal wave shape.
3. Measure peak value of the both wave shapes.
4. Determine phase relationship  $\theta$  between the waves.
5. Write down the wave equations for I and E.
6. Calculate resistance and reactance from the relevant data.
7. Do the same experiment setting input frequency 5kHz and 10kHz.
8. Complete the following table.

**Table~1**

f	E	$I=V_R/R$ (A)	$\theta$	$Z=(E/I)\angle\theta$ (Polar) $\Omega$	Z (Rectangular) $\Omega$	R	$X_C$ $=1/2\pi fC$ $\Omega$	$V_R$	$V_C=IX_C$
1KHz									
5KHz									
10KHz									

1. Now construct the circuit as shown in fig.2. Connect channel 1 of the oscilloscope across function generator and channel 2 of the oscilloscope across R.
2. Do the same procedure stated in 2 to 7. Complete the following table.

**Table~2**

f	E	$I=V_R/R$ (A)	$\theta$	$Z=(E/I)\angle\theta$ (Polar) $\Omega$	Z (Rectangular) $\Omega$	R	$X_L=2\pi fL$ $\Omega$	$V_R$	$V_L=IX_L$
1KHz									
5KHz									
10KHz									

**For RLC Series Circuit:**

1. Construct the circuit as shown in the Figure 3. Connect channel 1 of the oscilloscope across the ac voltage source and channel 2 of the oscilloscope across R.
2. Set the amplitude of the input signal 5V peak.
3. Set the frequency of the signal generator 1 kHz.
4. Determine phase relationship  $\theta$  between the waves.
5. Measure value of I.
6. Measure value of  $V_R$ ,  $V_L$  &  $V_C$ .
7. Verify KVL using the experimental data and draw the complete vector diagram.
8. Set the frequency of the signal generator 2 kHz. Repeat step 4-7.
9. Set the frequency of the signal generator 4 kHz. Repeat step 4-7.
10. Complete the following table.

**Table~3**

f	E (V)	$\theta = \tan^{-1}$ X/R	$V_R$ (V)	$I=V_R/R$ (A)	$X_L=2\pi fL$ ( $\Omega$ )	$V_L=IX_L$ (V)	$X_C=1/2\pi fC$ ( $\Omega$ )	$V_C$ (V)	X = $X_L - X_C$	V* (V)
1kHz										
2kHz										
4kHz										

$$*V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

**Simulation and Results:**

Compare the simulation results with your experimental data/ wave shapes and comment on the differences (if any). Example of how the simulated wave shape should look is provided below.

***Result and Calculation:***

1. Complete Table~1 and Table~2 and Table~3.
2. Calculate the value of  $X_C$ ,  $X_L$ , Z, I,  $\theta$ .
3. Draw the complete vector diagram for the RL and RC and RLC circuit.
4. Comment on the role of frequency to inductive reactance and capacitive reactance.

**Questions for report writing:**

1. Draw the complete vector diagrams.
2. Verify KVL.

**Discussion and Conclusion:**

Interpret the data/findings and determine the extent to which the experiment was successful in complying with the goal that was initially set. Discuss any mistake you might have made while conducting the investigation and describe ways the study could have been improved.

**Reference:**

- [1] "Fundamental of Electric Circuit" by AlekzendreSadiku
- [2] "Alternating Current Circuit" by George F Corcoran
- [3] <http://physics.bu.edu/~duffy/py106/ACcircuits.html>