LESSON 6

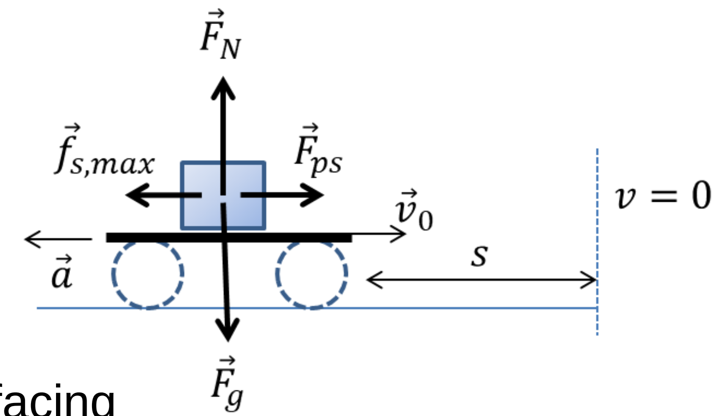
BOOK CHAPTER 6
(Force and Motion-II)
And
BOOK CHAPTER 7
(Kinetic energy and Work)

Problem 1 (Book chapter 6)

The floor of a railroad flatcar is loaded with loose crates having a coefficient of static friction of 0.25 with the floor. If the train is initially moving at a speed of 48 km/h, in how short a distance can the train be stopped at constant acceleration without causing the crates to slide over the floor?

Answer:

Since the crates are not sliding, the net force on the crates along horizontal axis (x-axis) is zero. That is



$$F_{ps} + (-f_{s,max}) = 0$$

[Assuming the crates facing maximum static friction because they are not sliding]

$$F_{ps} = f_{s,max}$$

$$ma = \mu_s F_N = \mu_s mg$$

[Since, along vertical axis (y-axis), $F_N - mg = 0$

$$a = \mu_s g = (0.25)(9.8) = 2.45\ m/s^2$$

To find the value of s for the train, we use the formula

$$v^2 = v_0^2 + 2(-a)s$$

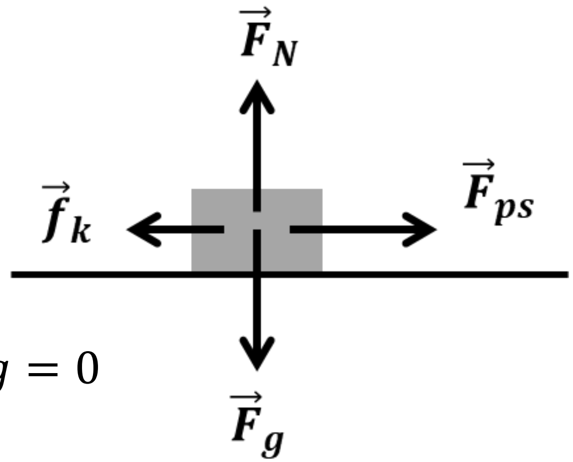
[a is negative for the train]

$$0 = (13.33)^2 + 2(-2.45)s$$

$$s = \frac{177.69}{4.9} = 36.26\ m$$

Problem 7 (Book chapter 6)

A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction between the crate and the floor is 0.35. What is the magnitude of (a) the frictional force and (b) the acceleration of the crate?



Answer:

(a) For the kinetic frictional force, we have

$$f_k = \mu_k F_N = \mu_k(mg)$$

[Along y-axis, $F_N - mg = 0$
Therefore, $F_N = mg$]

$$f_k = (0.35)(55)(9.8) = 188.65\ N$$

(b) The net force along x-axis, [where a is the acceleration of the crate along x-axis]

$$F_{ps} - f_k = ma$$

$$220 - 188.65 = (55)a$$

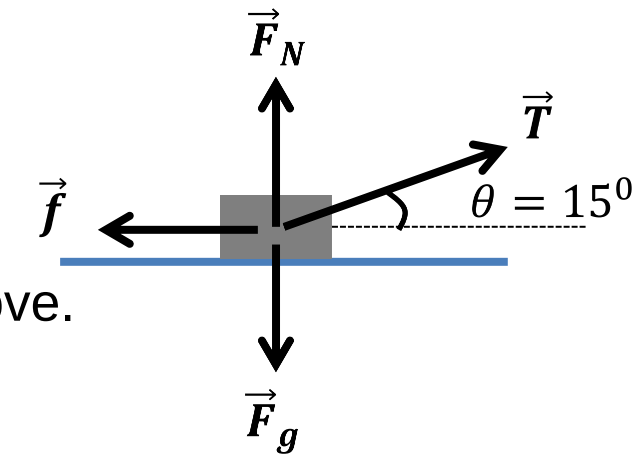
$$31.35 = (55)a$$

Therefore,

$$a = \frac{31.35}{55} = 0.57\ m/s^2$$

Problem 11 (Book chapter 6)

68 kg crate is dragged across a floor by pulling on a rope attached to the crate and inclined 15^0 above the horizontal. (a) If the coefficient of static friction is 0.50, what minimum force magnitude is required from the rope to start the crate moving? (b) If $\mu_k = 0.35$, what is the magnitude of the initial acceleration of the crate?



Answer:

The cerate is facing maximum static frictional force ($f_{s,max}$), because it is just start to move.

Hence, the net force along x-axis is

$$T\cos\theta - f_{s,max} = 0$$

$$T\cos\theta - \mu_s F_N = 0$$

$$T\cos\theta = \mu_s F_N$$

$$T = \frac{\mu_s F_N}{\cos\theta} = \frac{0.50 F_N}{\cos 15^0} = \frac{0.50 F_N}{0.9659}$$

$$T = 0.5176\, F_N \quad \text{------------------------} \quad (1)$$

The net force along y-axis is

$$F_N + T\sin\theta - mg = 0$$

$$F_N = mg - T\sin\theta$$

$$F_N = (68)(9.8) - T\sin 15^0$$

$$F_N = 666.4 - 0.2588\,T \quad \text{------------------------} \quad (2)$$

By substituting the value of F_N *from equation* (2) *in equation* (1), *we get*

$$T = 0.5176(666.4 - 0.2588\,T)$$

$$T = 344.93 - 0.134\,T$$

$$T + 0.134T = 344.93$$

$$1.134\,T = 344.93$$

$$T = \frac{344.93}{1.134} = 304.17\,N$$

(b) Now, we assume that the crate is moving with an acceleration a.

Hence, the net force along x-axis is

$$T\cos\theta - f_k = ma$$

$$T\cos\theta - \mu_k F_N = ma$$

$$a = \frac{T\cos\theta - \mu_k(666.4 - 0.2588\,T)}{m}$$

$$a = \frac{304.17\cos 15^0 - 0.35[666.4 - (0.2588)(304.17)]}{68}$$

$$a = \frac{(304.17)(0.9615) - 233.24 + 27.55)}{68} = \frac{86.769}{68} = 1.276\ m/s^2$$

BOOK CHAPTER 7

(Kinetic energy and Work)

Kinetic Energy:

Kinetic energy *K is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.*

For an object of mass *m* whose speed *v* is well below the speed of light,

$$K = \frac{1}{2}mv^2$$

The SI unit of kinetic energy (and all types of energy) is the ***joule*** *(J), named for James Prescott Joule, an English scientist of the 1800s and defined as*

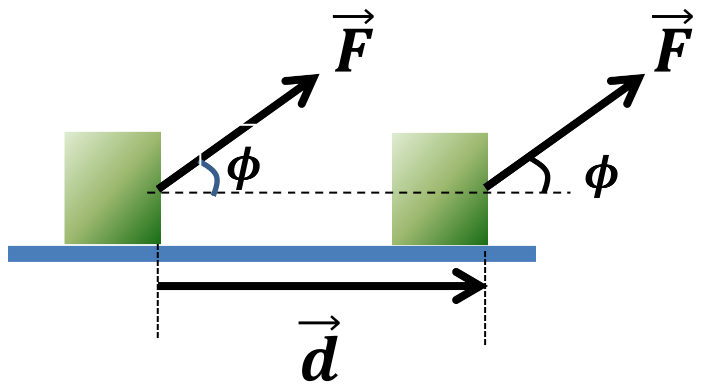
$$1\text{joule} = 1\text{ J} = 1\text{kg}.\, m^2/s^2$$

Work:

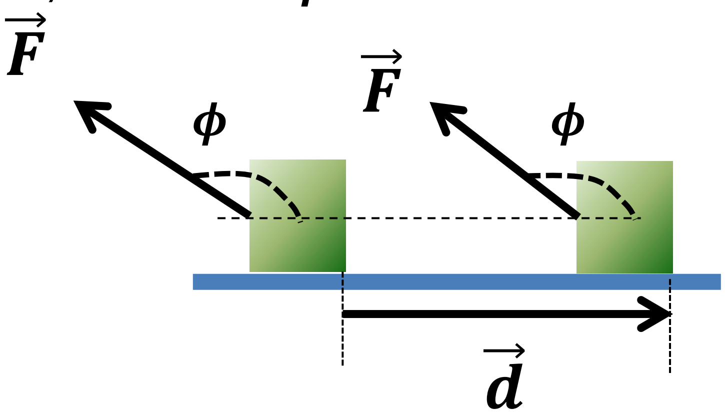
Work W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

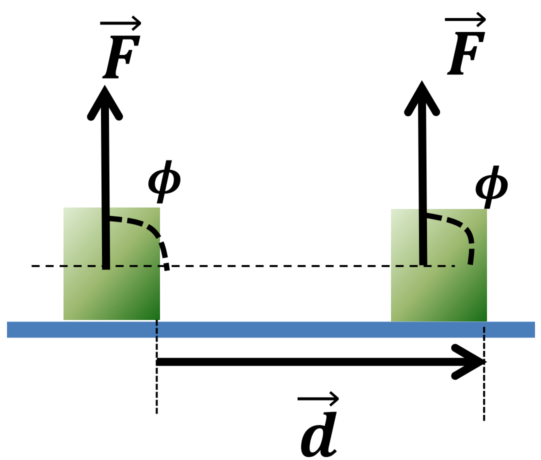
$$\boldsymbol{W = \vec{F}.\vec{d} = Fd\cos\phi}$$



Where ϕ is constant angle between the directions of $\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{d}}$. This is positive work, because $\boldsymbol{\phi < 90^0}$



Work is Negative,
because $\boldsymbol{90^0 < \phi}$



The force does *no* work on the object, because $\boldsymbol{\phi = 90^0}$

The principle of work-kinetic energy theorem:

For a particle, a change ΔK in the kinetic energy equals the net work *W* done on the particle:

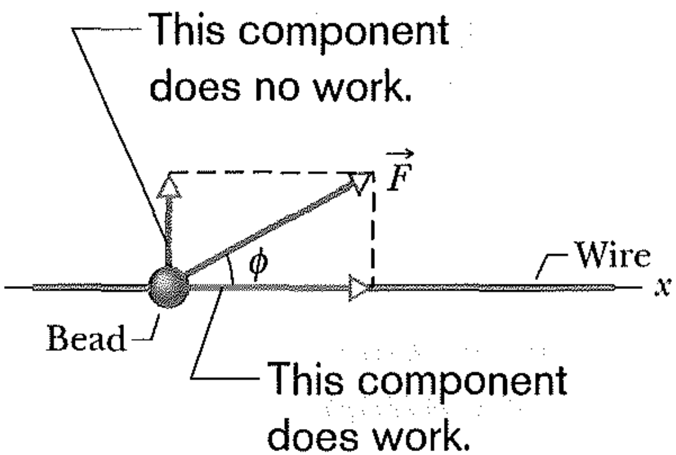
$$\Delta \boldsymbol{K} = \boldsymbol{K_f} - \boldsymbol{K_i} = \boldsymbol{W}$$

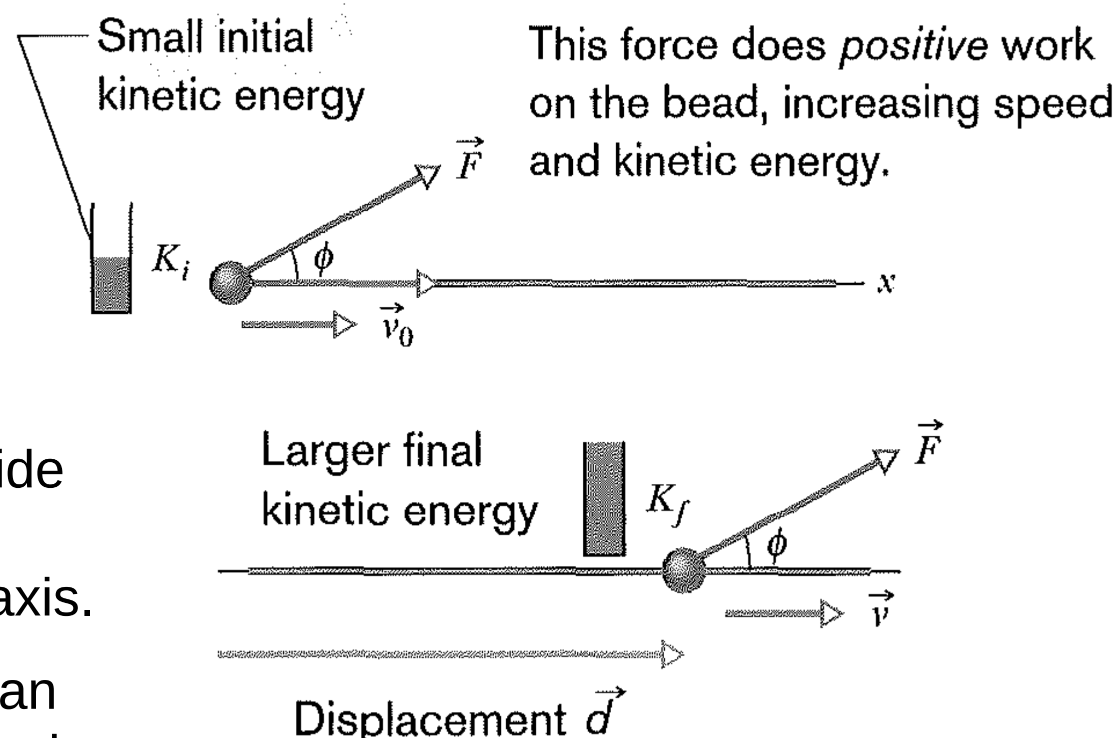
This is known as work-kinetic energy theorem, in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done.

The SI unit of work is joule, the same as kinetic energy. The corresponding unit in the British system is the foot-pound (ft.lb). 1joule is equivalent to

$$1\, joule = (\mathrm{newton})(1\ \mathrm{meter}) = 1\mathrm{N.m} = 0.738\ \mathrm{ft.lb}$$

Finding an Expression for Work-Kinetic energy:





Considering a bead that can slide along a frictionless wire that is stretched along a horizontal *x* axis.

A constant force $\vec{F}$, directed at an angle ϕ to the wire, accelerates the bead along the wire.

Using Newton's law we can write,

$$F_x = ma_x \qquad \text{..............} \ (1)$$

$where\ m\ is\ mass\ of\ the\ bead$ and $a_x\ is\ the\ acceleration\ along\ x - axis.$

Using the equation for motion with constant acceleration, we can write

$$v^2 = v_0^2 + 2a_x d$$

$$v^2 - v_0^2 = 2a_x d$$

$$a_x = \frac{(v^2 - v_0^2)}{2d} \quad \text{..............} (2)$$

Substituting the value of a_x in equation (1) we can write,

$$F_x = m\left(\frac{v^2 - v_0^2}{2d}\right)$$

$$F_x d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\boxed{W = K_f - K_i}$$

in which $K_i = \frac{1}{2}mv_0^2$ is the initial kinetic energy of the particle and $K_f = \frac{1}{2}mv^2$ is the kinetic energy after the work, $W = F_x d$ is done.

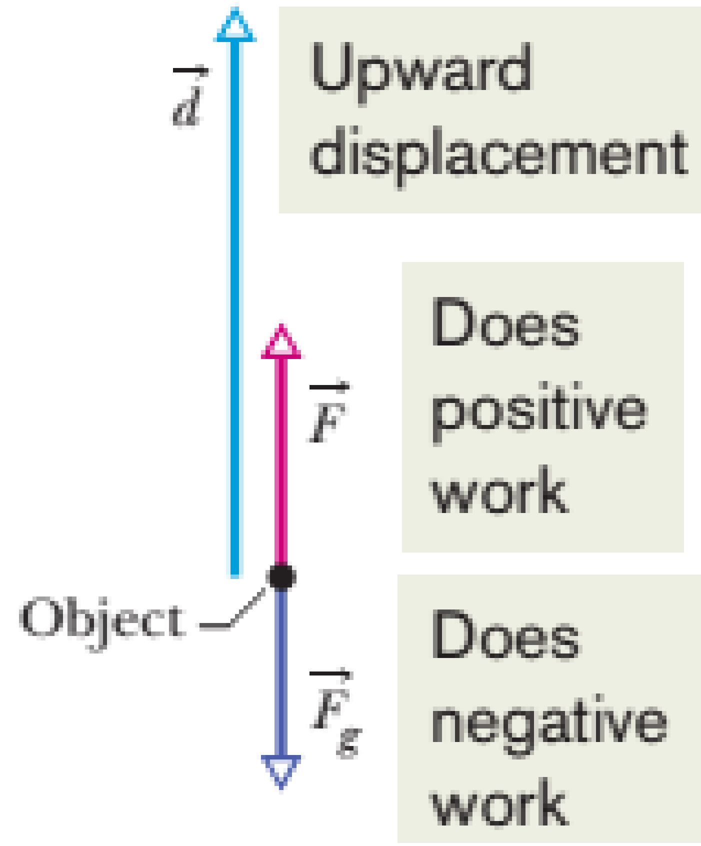
Work Done by the Gravitational Force:

We know that the work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$\boldsymbol{W = \vec{F}.\vec{d} = Fd\cos\phi}$$

Accordingly, the work W_g done by the gravitational force $\vec{F}_g$ on a object (particle/body) of mass *m* as the object moves through a displacement $\vec{d}$ is given by

$$W_g = F_g d\cos\phi$$



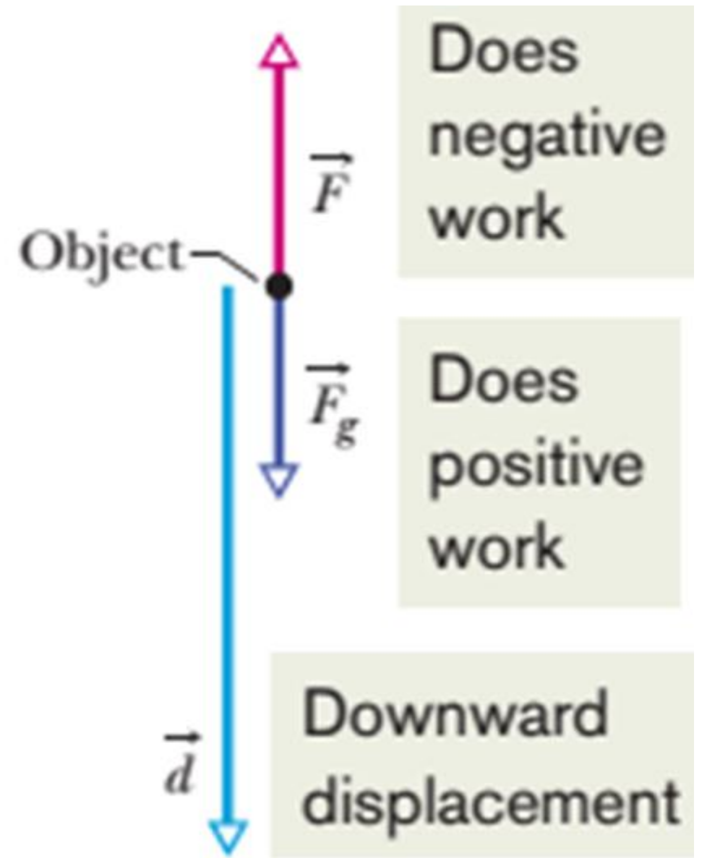
Where ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

- **For rising an object:** Force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$ (as shown in the adjacent figure). Hence, W_g becomes

$$W_g = F_g d\cos 180^0 = mgd(-1) = -mgd$$

- **For lowering an object:** Force $\vec{F}_g$ is directed along the displacement $\vec{d}$ (as shown in the adjacent figure). Hence, W_g becomes

$$W_g = F_g d \cos 0^0 = mgd(+1) = mgd$$



Thank You