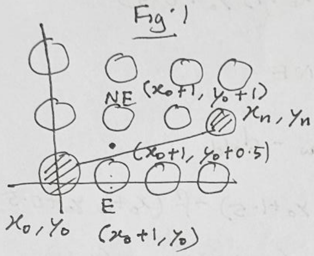
Bresenham's Midpoint Line Algorithm Derivation and Example

Bresenham Mid-point Line

$4>0$

$2\times 4>0$

Fig' 1



$m = \frac{dy}{dx} = \frac{y_n - y_0}{x_n - x_0}$

Eq. st Line : $y = mx + B$ — (i)

Eq. st Line : $f(x,y) = ax + by + c = 0$ — (ii)

From eq (i)

$y = mx + c$

$\Rightarrow y = \frac{dy}{dx}x + B$

$\Rightarrow y = \frac{dy \cdot x + dx \cdot B}{dx}$

$\Rightarrow y \cdot dx = dy \cdot x + dx \cdot B$

$\Rightarrow dy \cdot x - y \cdot dx + dx \cdot B = 0$

From eq (i) & eq (ii)

$a = dy$, $b = -dx$, $c = dx \cdot B$

From fig 1,

$d_{start} = f(x_0+1, y_0+0.5)$

$= a(x_0+1) + b(y_0+0.5) + c$

$= ax_0 + a + by_0 + 0.5b + c$

$= \underline{ax_0 + by_0 + c} + a + 0.5b$

$= f(x_0, y_0) + a + 0.5b$

$= 0 + a + 0.5b \quad [\because f(x_0, y_0) = 0]$

$= dy - \frac{1}{2}dx$

$2 \times d_{start} = 2dy - dx$ (decision parameter)

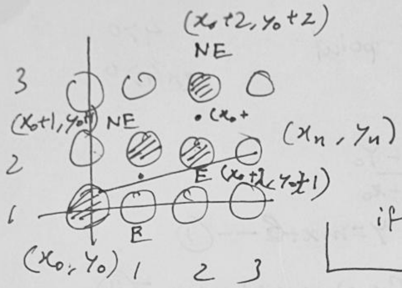
if $d_{start} > 0$ NE

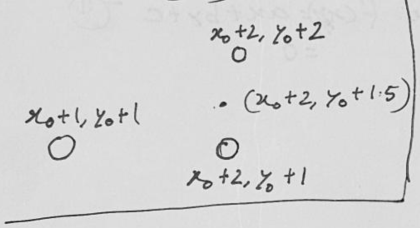
$d_{start} < 0$ E

$2\times d > 0$

$d < 0$

$\approx d_{start} = 2dy - dx$





From last page,

$d_{start} = 2dy - dx$

Midpoint $(x_0+1, y_0+0.5)$

if $d > 0$, we choose NE

$d_{NE} = d_{new} - d_{old}$

$= f(x_0+2, y_0+1.5) - f(x_0+1, y_0+0.5)$

$= a(x_0+2) + b(y_0+1.5) + c - a(x_0+1)$

$- b(y_0+0.5) - c$

$= \cancel{ax_0} + 2a + \cancel{by_0} + 1.5b + \cancel{c} - \cancel{ax_0} - a - \cancel{by_0} - 0.5b$
$- \cancel{c}$

$= a + b$

$= dy - dx$

If we choose new NE, $d_{new} = d_{NE} + d_{old}$

$= (dy - dx) + (dy - dx)$

$\boxed{\therefore d_{new} = 2 \times (dy - dx)}$

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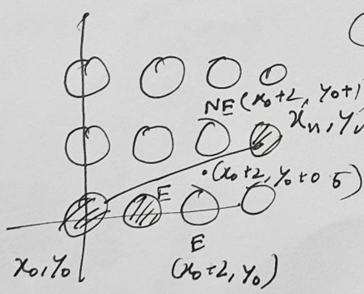
if $d <= 0$ we choose E,



(1) — $d_E = d_{new} - d_{old}$

$= f(x_0+2, y_0+0.5) - f(x_0+1, y_0+0.5)$

$= a(x_0+2) + b(\cancel{y_0}+0.5) + \cancel{c} - a(x_0+1) - b(\cancel{y_0}+0.5)$
$- \cancel{c}$

$= \cancel{ax_0} + 2a - \cancel{ax_0} - a$

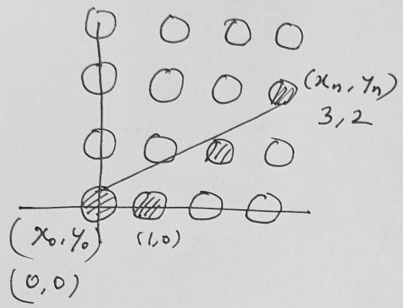
$= a$

$= dy$

(1)

$d_{new} = \boxed{d_E + d_{old} = dy + dy = 2\,dy}$

Midpoint Line Algo



$dy = y_n - y_0$
$= 2 - 0$
$= 2$

$dx = x_n - x_0$
$= 3 - 0$
$= 3$

$d_{start} = 2dy - dx = (2 \times 2) - 3 = 1$

$d_E = 2dy = 2 \times 2 = 4$

$d_{NE} = 2(dy - dx) = 2 \times (2 - 3) = -2$

d-value	compare	decision	x	y
$d_{start} = 1$	$d > 0$	NE	0	0
$d_{new} = d_{NE} + d_{old} = -2 + 1 = -1$	$d < 0$	E	1	0
$d_{new} = d_E + d_{old} = 4 - 1 = 3$	$d > 0$	NE	2	1
$d_{new} = d_{NE} + d_{old} = -2 + 3 = 1$	$d > 0$	NE	3	2

we continue until $x = x_n$
$x = 3$