Linear Regression on a Transformed Time Variable

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To illustrate linear regression on a transformed time variable we fit a quadratic polynomial.

To reproduce the results, it is necessary to prepare the data set, plot base, and training and test data sets, as outlined in the "Data Preparation" section.

Preparation

Loading Required Packages and Data

Load the necessary packages, data sets, and other supporting files. Each element serves a specific purpose:

- `tidyverse`: For data manipulation and visualisation.

- **`lme4` and `lmerTest`**: To fit and analyse mixed-effects models.
- **`caret`**: To compute model performance indices.
- **`plot_base`**: A pre-configured ggplot object for visualisation.
- **Training and Test Data sets**: Required for cross-validation.

```r
# Load necessary packages
library(tidyverse)
library(lme4)
library(lmerTest)
library(caret)

# Load the data set
load("data/wido.rdata")

# Load the pre-configured plot base
plot_base <- readRDS("objects/plot_base.rds")

# Load training and test datasets for cross-validation
training_datasets <- readRDS("objects/training_datasets.rds")
test_datasets <- readRDS("objects/test_datasets.rds")
```

Applying an Orthogonal Polynomial

To avoid multicollinearity arising from using two terms of time (a linear and a quadratic term), we use an orthogonal polynomial. This ensures that the linear and quadratic time terms are uncorrelated. The `poly()` function generates two orthogonal terms: the linear and the quadratic components of time, stored in `poly_time`.

```r
# Apply orthogonal polynomial transformation to the time variable
wido$poly_time <- poly(wido$mnths, 2)
```

Analysis

Fitting the Model

Fit the linear mixed-effects model using the transformed time variable (`poly_time`). This model includes both fixed effects for the linear and quadratic time terms and random effects for these terms to account for person-specific trajectories.

```
# Fit the linear mixed-effects model
lin <- lmer(
  lifesatisfaction ~ poly_time[,1] + poly_time[,2] +
                     (poly_time[,1] + poly_time[,2] | id),
  data = wido
)

# Display the summary of the model
summary(lin)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: lifesatisfaction ~ poly_time[, 1] + poly_time[, 2] + (poly_time[,
    1] + poly_time[, 2] | id)
   Data: wido

REML criterion at convergence: 5512.7

Scaled residuals:
    Min      1Q  Median      3Q     Max
-5.4176 -0.4947  0.0778  0.5705  3.3371

Random effects:
 Groups   Name           Variance Std.Dev. Corr
 id       (Intercept)      0.6131  0.7830
          poly_time[, 1] 442.7048 21.0406   0.15
          poly_time[, 2]  66.0257  8.1256  -0.23  0.11
 Residual                  0.4358  0.6601
Number of obs: 2322, groups:  id, 208

Fixed effects:
                Estimate Std. Error        df t value Pr(>|t|)
(Intercept)      4.95087    0.05902 207.34316  83.887  < 2e-16 ***
poly_time[, 1]  -8.94865    1.89670 102.63469  -4.718 7.54e-06 ***
```

```
poly_time[, 2]   5.93670    1.46277  69.70255   4.059 0.000127 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
             (Intr) p_[,1]
poly_tm[,1] 0.109
poly_tm[,2] 0.148  0.086
```

```r
# Compute confidence intervals for the model parameters
round(confint(lin), 2)
```

```
Computing profile confidence intervals ...

                2.5 % 97.5 %
.sig01           0.70   0.87
.sig02          -0.04   0.34
.sig03          -0.89   0.15
.sig04          17.36  24.92
.sig05          -0.84   0.70
.sig06           1.05  12.74
.sigma           0.64   0.68
(Intercept)      4.83   5.07
poly_time[, 1] -13.08  -4.76
poly_time[, 2]   2.49   9.37
```

Visualisation

Bootstrapping Confidence Intervals

Use bootstrapping to estimate the confidence intervals for the predicted values of the model. This provides a robust measure of uncertainty.

```r
# For reproducibility
set.seed(123)

# Bootstrapping for confidence intervals of the predictions
boot_results <- bootMer(
```

```
  lin,
  FUN = function(x) predict(x, newdata = wido, re.form = NA),
  nsim = 1000
)

# Extract the 95% confidence intervals from the bootstrapped results
ci <- apply(boot_results$t, 2, quantile, probs = c(0.025, 0.975))

# Assign the lower and upper bounds to the data
wido$lower_bound <- ci[1, ]
wido$upper_bound <- ci[2, ]
```

Predicting Average and Individual Trajectories

Predict both the population-level (fixed effects) and individual-level (random effects) trajectories of life satisfaction.

```
# Predict population-level trajectories based on fixed effects
wido$lifesatisfaction_lin_f <- predict(lin, newdata = wido, re.form = NA)

# Predict individual-level trajectories based on random effects
wido$lifesatisfaction_lin_r <- predict(lin, newdata = wido, re.form = NULL,
↪   allow.new.levels = TRUE)
```

Selecting a Random Sample for Plotting

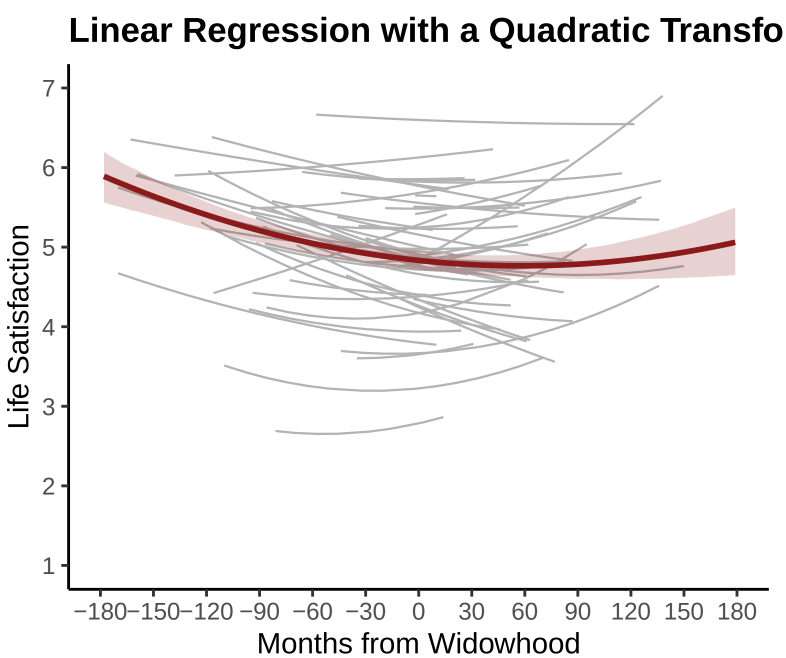
For better visualisation, select a random sample of individuals to display their individual trajectories.

```
# For reproducibility
set.seed(123)

# Randomly sample 50 participants
rsample_ids <- sample(unique(wido$id), 50)

# Filter the data to include only the randomly selected participants
wido_rsample <- wido %>%
  filter(id %in% rsample_ids)
```

Creating the Plot

Combine all elements to create the plot, which includes individual trajectories, the population trajectory, and the confidence interval of the population trajectory.

```
# Create the plot using the pre-configured plot base
plot_base +
  geom_line(
    data = wido_rsample,
    aes(x = mnths, y = lifesatisfaction_lin_r, group = id),
    color = "grey70", linewidth = 0.4
  ) +
  geom_ribbon(
    data = wido,
    aes(x = mnths, ymin = lower_bound, ymax = upper_bound),
    fill = "firebrick4", alpha = 0.2
  ) +
  geom_line(
    data = wido,
    aes(x = mnths, y = lifesatisfaction_lin_f),
    color = "firebrick4", linewidth = 1
  ) +
  ggtitle("Linear Regression with a Quadratic Transformation") +
  theme(plot.title = element_text(size = 13, face = "bold"))
```



Model Performance

Evaluating the Model

Assess the model's performance using the Bayesian Information Criterion (BIC), R-squared (R^2), Mean Absolute Error (MAE), and Root Mean Squared Error (RMSE).

```
# Compute BIC for the fitted model
round(BIC(lin), 2)
```

```
[1] 5590.19
```

```
# Calculate R², MAE, and RMSE for the fixed effects predictions
data.frame(
  R2_FE = round(R2(wido$lifesatisfaction_lin_f, wido$m_lifesat_per_mnth), 2),
  MAE_FE = round(MAE(wido$lifesatisfaction_lin_f, wido$m_lifesat_per_mnth),
  ↪  2),
  RMSE_FE = round(RMSE(wido$lifesatisfaction_lin_f, wido$m_lifesat_per_mnth),
  ↪  2)
)
```

```
  R2_FE MAE_FE RMSE_FE
1  0.14   0.33    0.43
```

```
# Calculate R², MAE, and RMSE for the random effects predictions
data.frame(
  R2_RE = round(R2(wido$lifesatisfaction_lin_r, wido$lifesatisfaction), 2),
  MAE_RE = round(MAE(wido$lifesatisfaction_lin_r, wido$lifesatisfaction), 2),
  RSME_RE = round(RMSE(wido$lifesatisfaction_lin_r, wido$lifesatisfaction),
  ↪  2)
)
```

```
  R2_RE MAE_RE RSME_RE
1   0.7   0.46    0.61
```

Cross-Validation

To assess the replicability of the model, perform cross-validation using the training and test data sets. For each training data set, fit the model and compute performance metrics for the associated test data set R^2, MAE, and RMSE.

```
# Initialise vectors to store performance metrics
R2_values <- c()
MAE_values <- c()
RMSE_values <- c()

# Perform cross-validation
for (i in seq_along(training_datasets)) {
  train_data <- training_datasets[[i]]
  test_data <- test_datasets[[i]]

  # Apply polynomial transformation to time variable
  train_data$poly_time <- poly(train_data$mnths, 2)
  test_data$poly_time <- poly(test_data$mnths, 2)

  # Fit the linear mixed-effects model on training data
  lin <- lmer(
    lifesatisfaction ~ poly_time[,1] + poly_time[,2] +
                       (poly_time[,1] + poly_time[,2] | id),
    data = train_data
  )

  # Make predictions on the test data
  test_predictions <- predict(lin, newdata = test_data, re.form = NA)

  # Compute average trajectory in the test data
  test_data <- test_data %>%
    group_by(mnths) %>%
    mutate(m_lifesat_per_mnth = mean(lifesatisfaction, na.rm = TRUE))

  # Calculate performance metrics
  R2_values <- c(R2_values, R2(test_predictions,
↪   test_data$m_lifesat_per_mnth))
  MAE_values <- c(MAE_values, MAE(test_predictions,
↪   test_data$m_lifesat_per_mnth))
  RMSE_values <- c(RMSE_values, RMSE(test_predictions,
↪   test_data$m_lifesat_per_mnth))
```

```
}

# Compute average performance metrics (mean)
  average_R2 <- mean(R2_values)
  average_MAE <- mean(MAE_values)
  average_RMSE <- mean(RMSE_values)

# Compute average performance metrics (SD)
  sd_R2 <- sd(R2_values)
  sd_MAE <- sd(MAE_values)
  sd_RMSE <- sd(RMSE_values)

# Combine the mean and standard deviation into one data.frame
combined_metrics <- data.frame(
  Metric = c("R²", "MAE", "RMSE"),
  Mean = round(c(average_R2, average_MAE, average_RMSE), 2),
  SD = round(c(sd_R2, sd_MAE, sd_RMSE), 2)
)

# Print the combined metrics
print(combined_metrics)
```

```
  Metric Mean   SD
1     R² 0.06 0.04
2    MAE 0.63 0.07
3   RMSE 0.83 0.10
```