Fourier Neural Operator for learning the dynamic of Ionic models

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Todo list

1 FitzHugh-Nagumo

$$\begin{cases} \dfrac{dV}{dt} = bV(V-\beta)(\delta - V) - cw + I_{app}, & t \in [0,T] \\ \dfrac{dw}{dt} = e(V - \gamma w), & t \in [0,T] \\ V(0) = V_0, w(0) = w_0 \end{cases}$$

Where

- b = 5
- β = 0.1
- c = 1
- δ = 1
- γ = 0.25
- e =1

Our objective is to learn the operator

LP: Regolarita' di HH,FHN

$$\begin{aligned} \mathcal{G}^\dagger : &\mathbb{R} \to H^1([0,T];\mathbb{R}) \times H^1([0,T];\mathbb{R}) \\ &I_{app}(t) \mapsto (V(t), w(t)) \end{aligned}$$

1.1 Dataset

The data set is created using Matlab ode15s with $T = 1$. We randomly choose the intensity of the current and the duration of the stimulus T_{stim}. In order to get all the important dynamics of the system, the generation of the dataset for training is divided into three parts formed in according to the following table

MG: Finire il traning con meno esempi per nap 20

Name	Range of values for the current	Range of values for T_{stim}	$n_{examples}$
t_0	$(0.1, 2)$	$(0, 0)$	200
General	$(0.1, 2)$	$(0.01, 1)$	2300
nap	$(1e-7, 0.01)$	$(0.1, 1)$	500

For testing, we generate 375 examples where the dataset is split into four.

Name	Range of values for the current	Range of values for T_{stim}	$n_{examples}$
t_0	$(0.1, 2)$	$(0, 0)$	20
General	$(0.1, 2)$	$(0.01, 1)$	285
nap	$(1e-7, 0.01)$	$(0.1, 1)$	30
t_{fin}	$(0.3, 2)$	$(1, 1)$	50

In figure 1 we present the results obtained by taking five randomly selected examples for each part of the dataset (General, nap, and t_{fin}). (non messi i t_0 perchè è semplicemente la soluzione nulla, qui abbiamo riportato gli esempi con gli errori ma si possono avere li stessi esempi con invece che l'errore lo spazio delle fasi)

LP: Cambiare il titolo con index of the examples

LP: Grafici usando python

1.2 Architecture

MG: Finire ray

2 Hodgkin-Huxley

$$
\begin{cases}
C_m \dfrac{dV}{dt} = -\Big(\bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_K n^4 (V - V_K) + \bar{g}_L (V - V_L)\Big) + I_{app}, & t \in [0, T], \\
\dfrac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V) m, & t \in [0, T], \\
\dfrac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V) h, & t \in [0, T], \\
\dfrac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V) n, & t \in [0, T], \\
V(0) = 2.7570e-02, \\
m(0) = 5.2934e-02, \\
h(0) = 5.9611e-01, \\
n(0) = 3.1768e-01.
\end{cases}
$$

Where

- $C_m = 1$
- $\bar{g}_{Na} = 120$
- $\bar{g}_K = 36$
- $\bar{g}_L = 0.3$
- $V_{Na} = 115$
- $V_K = -12$
- $V_L = 10.6$

$$
\begin{aligned}
\alpha_m(V) &= 0.1(25 - V)\left[\exp\left(\frac{25 - V}{10}\right)\right]^{-1} & \beta_m(V) &= 4\exp\left(-\frac{V}{18}\right) \\
\alpha_h(V) &= 0.07\exp\left(-\frac{V}{20}\right) & \beta_h(V) &= \left[\exp\left(\frac{30 - V}{10}\right)\right]^{-1} \\
\alpha_n(V) &= 0.01(10 - V)\left[\exp\left(\frac{10 - V}{10}\right) - 1\right]^{-1} & \beta_n(V) &= 0.125\exp\left(-\frac{V}{80}\right)
\end{aligned}
$$

Our objective is to learn the operator

$$\begin{aligned} \mathcal{G}^\dagger :& \mathbb{R} \to H^1([0,T];\mathbb{R}) \times H^1([0,T];\mathbb{R}) \times H^1([0,T];\mathbb{R}) \times H^1([0,T];\mathbb{R}) \\ & I_{app}(t) \mapsto (V(t), m(t), h(t), n(t)) \end{aligned}$$

2.1 Dataset

The dataset is formed using odes15s of matlab with $T = 1$. Where we radomly pick the intenstiy of the current and the duration of the stimolus T_{stim}. In order to get all the imporant dynamics of the system the generation of the dataset for the training is splitted in three formed in the following way (i nomi sono quelli usati su matlab)

MG: training con il nuovo dataset con nap = 20

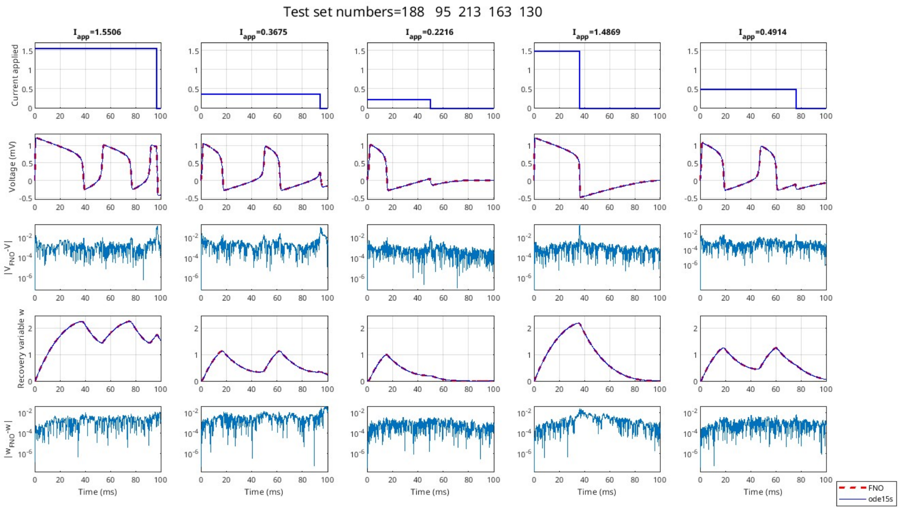
LP: Ridurre il numero di esempi per il caso del t_0 e metterre aggiungere i casi del t_{fin}

Name	Range of sampling for the current	Range of sampling for T_{stim}	$n_{examples}$
t_0	$(2, 10)$	$(0, 0)$	200
General	$(2, 10)$	$(0.01, 1)$	2200
nap	$(1e-7, 2)$	$(0.1, 1)$	300
i_{high}	$(50, 200)$	$(0.1, 1)$	300

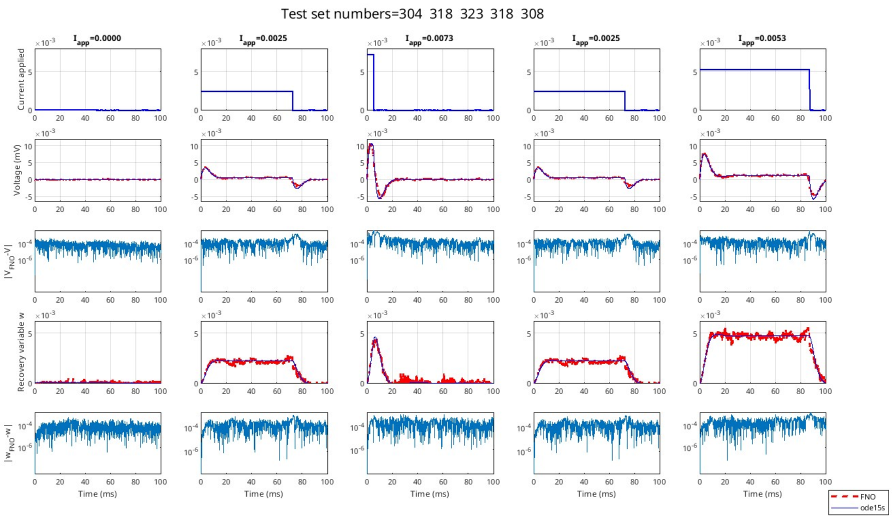
For the testing we genera 375 example where the dataset is splitted in four.

Name	Range of sampling for the current	Range of sampling for T_{stim}	$n_{examples}$
t_0	$(0.1, 2)$	$(0, 0)$	10
General	$(0.1, 2)$	$(0.01, 1)$	275
nap	$(1e-7, 0.01)$	$(0.1, 1)$	30
i_{high}	$(50, 200)$	$(0.1, 1)$	30
t_{fin}	$(10, 30)$	$(1, 1)$	30

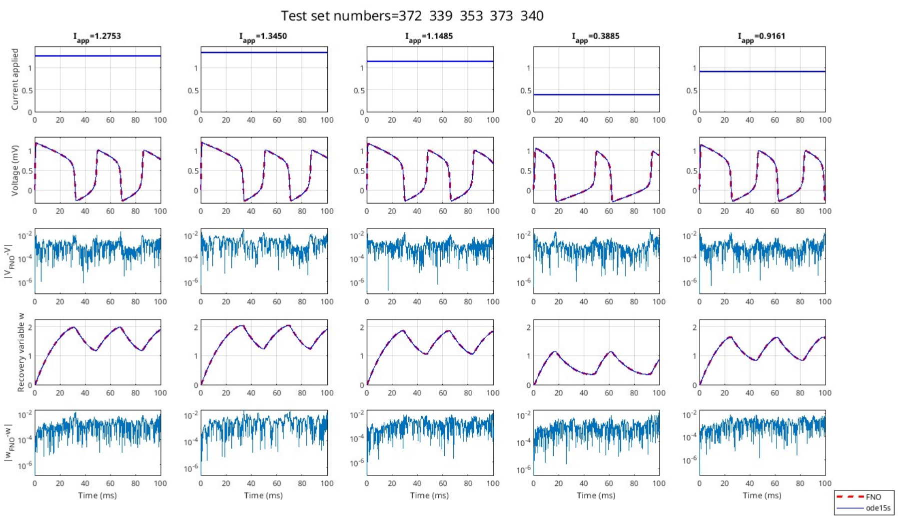
As before we take five examples for each part of the test dataset (General,i_{high},nap,t_fin) and visualize the results in figure 3.



(a) Five examples for the general part of the dataset

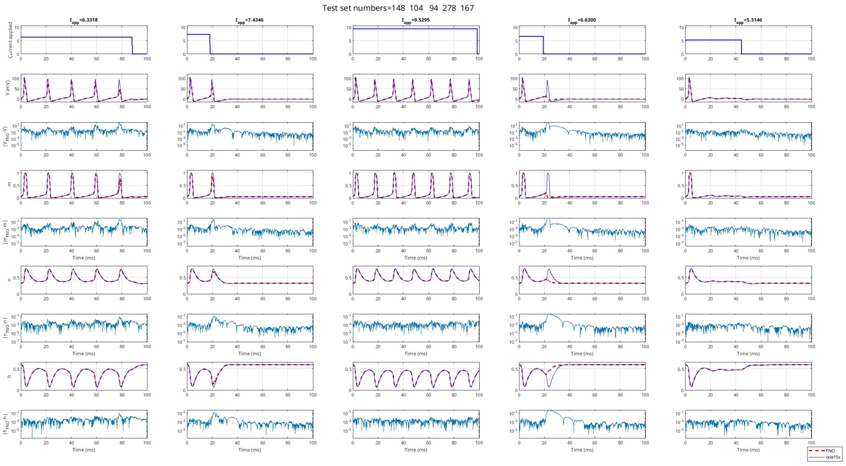


(b) Five examples for the nap part of the dataset

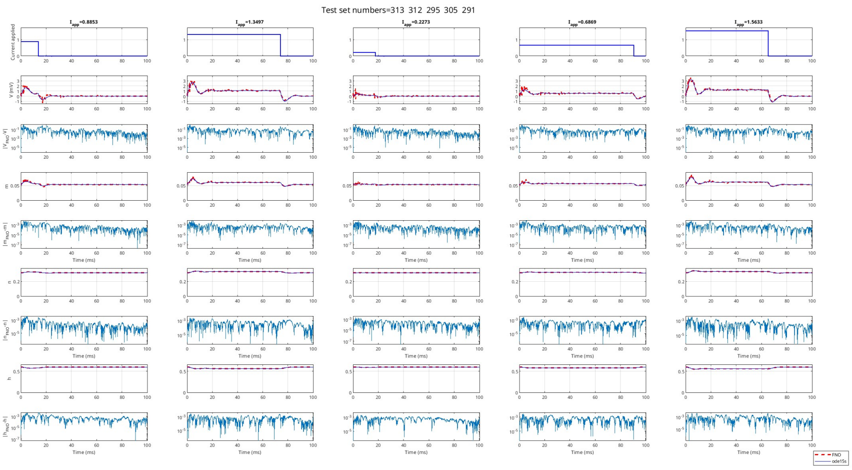


(c) Five examples for the t_{fin} part of the dataset

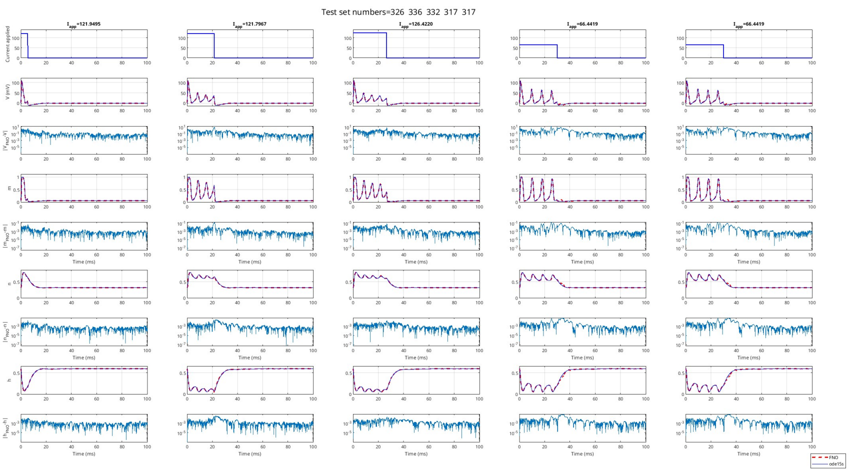
Figure 1: Each figure contains five plots, arranged in order from the top row as follows: current applied, voltage, point-wise error for the voltage, recovery variable w, and point-wise error for the recovery variable.



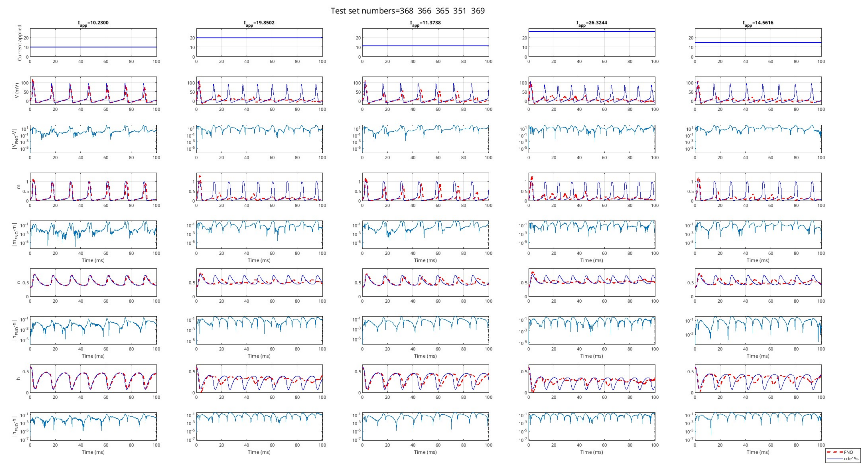
(a) Five examples for the general dataset



(b) Five examples for the nap dataset



(c) Five examples for the i_{high} dataset



(a) Five examples for the t_{fin} dataset

Figure 3: Results obtained each of the figures has the following graph in order from the top row: current applied, Voltage, point-wise error for the Voltage,gating variable m, point-wise error for m,gating variable n,point-wise error for n,gating variable h, error for the gating variable h