Reference for Homework 5

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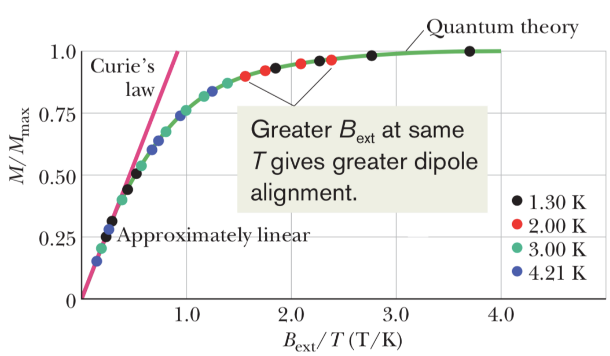
Please read the preface before reading this document!!!

1

Consider a solid containing N atoms per unit volume, each atom having a magnetic dipole moment $m\vec{u}$. Suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field $\vec{B}$ (this will be the case if $\vec{\mu}$ is due to the spin of a single electron). According to statistical mechanics, the probability of an atom being in a state with energy U is proportional to $e^{-U/kT}$, where T is the temperature and k is Boltzmann's constant. Thus, because energy U is $-\vec{\mu}\cdot\vec{B}$, the fraction of atoms whose dipole moment is parallel to $\vec{B}$ is proportional to $e^{\mu B/kT}$ and the fraction of atoms whose dipole moment is antiparallel to $\vec{B}$ is proportional to $e^{-\mu B/kT}$.
(a) Show that the magnitude of the magnetization of this solid is

$$M = N\mu\tanh(iN/kT).$$

Here tanh is the hyperbolic tangent function: $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$.
(b) Show that the result given in (a) reduces to $M = N\mu^2 B/kT$ for $\mu B \gg kT$.
(c) Show that the result of (a) reduces to $M = N\mu$ for $\mu B \ll kT$.
(d) SHow that both (b) and (c) agree qualitatively with the following figure.



Reference:
(a) the quantity of net magnetic dipole moment per unit volume is $n = N\frac{e^{\frac{\mu N}{kT}} - e^{-\frac{\mu N}{kT}}}{e^{\frac{\mu B}{kT}} + e^{-\frac{\mu B}{kT}}} = N \tanh\frac{\mu B}{kT} \Rightarrow$ the magnitude of the magnetization of the solid is $M = n\mu = N\mu \tanh\frac{\mu B}{kT}$
(b) apply Taylor's Law and we can get that $e^x = 1 + x + O(x)$ and $e^{-x} = 1 - x + O(x)$
$\Rightarrow$ when $x \to 0, \tanh x \approx \frac{2x}{2} = x$
$\Rightarrow$ when $\mu B \ll kT, \frac{\mu B}{kT} \to 0, M = N\mu \tanh\frac{\mu B}{kT} = \frac{N\mu^2 B}{kT}$
(c) when $x \to +\infty, e^{-x} \to 0, \Rightarrow \tanh x \to 1$
$\Rightarrow$ when $\mu B \gg kT, \frac{\mu B}{kT} \to +\infty, M = N\mu \tanh\frac{\mu B}{kT} = N\mu$
(d) when $\frac{\mu B}{kT}$ is small, M and $\frac{\mu B}{kT}$ is approximately linearly related; when $\frac{\mu B}{kT}$ is large, M is approximately a constant, so it is consistent with the figure

2

You place a magnetic compass on a horizontal surface, allow the needle to settle, and then give the compass a gentle wiggle to cause the needle to oscillate about its equilibrium position. The oscillation frequency is 0.312 Hz. Eartg;s magnetic field at the location of the compass has a horizontal component of 18.0μ T. The needle has a magnetic moment of 0.680 mJ / T, What is the needle's rotational inertia about its (vertical) axis of rotation?
Reference:
$\Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B} \Rightarrow \tau = -\mu B_{\|} \sin\theta \approx -\mu B_{\|}\theta = I\alpha = -\omega^2 I\theta$, in which $\omega = 2\pi f$
$\Rightarrow I = \frac{\mu B_{\|}}{\omega^2} = \frac{\mu B_{\|}}{4\pi^2 f^2} = 3.19 \times 10^{-9}\mathrm{kg{\cdot}m^2}$

3

Prove that the displacement current in a parallel-plate capacitor of capacitance C can be written as $i_d = C(dV/dt)$, where V is the potential difference between the plates.
Reference:
$\Rightarrow \nabla \times \vec{B} = \vec{0} \Rightarrow \mu_0 J = \mu_0\epsilon_0\frac{\partial E}{\partial t}$
$\Rightarrow \mu_0 J = \mu_0\frac{id}{A}$ and $\mu_0\epsilon_0\frac{\partial E}{\partial t} = \mu_0\epsilon_0\frac{1}{d}\cdot\frac{\partial V}{\partial t} \Rightarrow \mu_0\frac{id}{A} = \mu_0\epsilon_0\frac{1}{d}\cdot\frac{\partial V}{\partial t}$
$\Rightarrow id = \frac{\epsilon_0 A}{d}\cdot\frac{\partial V}{\partial t} = C\frac{\partial V}{\partial t}$

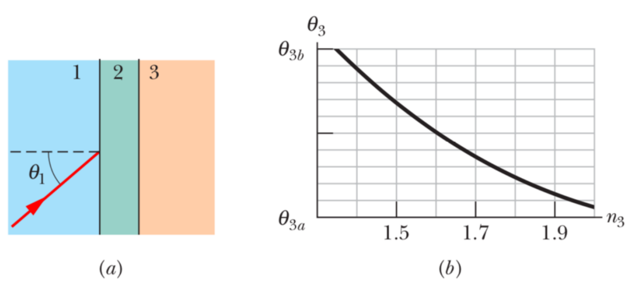
4

A plane electronmagnetic wave traveling in the positive direction of an x axis in vacuum has components $E_x = E_y = 0$ amd $E_z = (2.0V/m)\cos[(\pi \times 10^{15}s^{-1})(t-$

$x/c)]$.
(a) What is the amplitude of the magnetic field component?
(b) Parallel to which axis does the magnetic field oscillate?
(c) When the electric field component is in the positive direction of the z axis at a cretain point P, what is the direction of the magnetic field component there?
Reference:
(a) $B_m = \frac{E_m}{C} = 6.67 \times 10^{-9}$ T
(b) $\Rightarrow$ the magnetic field component is parallel to the y axis
(c) $\vec{B} = \frac{1}{c}(\vec{k} \times \vec{E}) \Rightarrow$ the direction of $\vec{B}$ is along the positive direction of the y axis

5

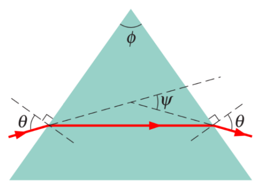
A beam of light in material 1 is incident on a boundary at an angle $\theta_1 = 40°$. Some of the light travels through material 2, and then some of it emerges into material 3. The two boundaries between the three materials are parallel. The final direction of the beam depends, in part, on the index of refraction n_3 of the third material.
Figure b gives the angle of refraction θ_3 in that material versus n_3 for a range of possible n_3 values. The vertical axis scale is set by $\theta_{3a} = 30.0°$ and $\theta_{3b} = 50.0°$.
(a) What is the index of refraction of material 1, or is the index impossible to calculate without more information?
(b) What is the index of refraction of material 2, or is the index impossible to calculate without more information?
(c) If θ_1 is changed to 70° and the index of refraction of material 3 is 2.4, what is θ_3?



Reference:
(a) $\Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$
select $(1.6, 40°)$ in the figure at which point $\theta_1 = \theta_3 \Rightarrow n_1 = 1.6$
(b) insufficent information
(c) $\Rightarrow n_1 \sin\theta_1 = n_3 \sin\theta_3 \Rightarrow \sin\theta_3 = \frac{n_1}{n_3} \sin\theta_1 = 0.626 \Rightarrow \theta_3 = 38.8°$

6

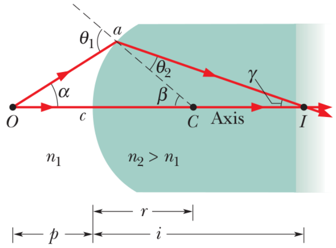
A ray is incedent on one face of a triangular glass prism in air. The angle of incident θ is chosen so that the emerging ray also makes the same angle θ with the normal to the other face. Show that the index of refraction n of the glass prism is given by $n = \frac{\sin\frac{1}{2}(\psi+\phi)}{\sin\frac{1}{2}\phi}$ where ϕ is the vertex angle of the prism and ψ is the deviation angle, the total angle through which the beam is turned in passing through the prism. (Under these conditions the deviation angle ψ has the smallest possible value, which is called the angle of minimum deviation.)



Reference:
$\Rightarrow \theta = \frac{\pi}{2} - (\frac{1}{2}(\pi - \phi) - \frac{\psi}{2}) = \frac{1}{2}(\phi + \psi)$
suppose the refraction angle is $\varphi \Rightarrow \varphi = \theta - \frac{\psi}{2} = \frac{1}{2}\phi$
from Snell's Law $\sin\theta = n\sin\varphi \Rightarrow n = \frac{\sin\theta}{\sin\varphi} = \frac{\sin\frac{1}{2}(\phi+\psi)}{\sin\frac{1}{2}\phi}$

7

Consider images formed by refraction through spherical surfaces of transparent materials, such as glass. As shown in the figure, light emits from a point object O in a medium with index of refraction n_1. It will refract through a spherical surface of radius r and center of curvature C into a medium of index of refraction n_2. Show that, for light rays making only small angles with the central axis, the object distance p and the image distance i are related by $\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$.



Reference:
$\Rightarrow \tan\gamma \approx \gamma = \beta - \theta_2 = \frac{\beta r}{i} \Rightarrow \theta_2 = \beta(1 - \frac{r}{i})$
$\Rightarrow \tan\alpha \approx \alpha = \theta_1 - \beta = \frac{\beta r}{p} \Rightarrow \theta_1 = \beta(1 + \frac{r}{p})$
from Snell's Law and small angle approximation $\Rightarrow n_1\theta_1 = n_2\theta_2 \Rightarrow n_1(1 + \frac{r}{p}) = n_2(1 - \frac{r}{i})$, simplify it we can get $\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$

8

When a thin lens with index of refraction n is surrounded by some meduymwith index of refraction n_{medium}, show that the focal length f is given by $\frac{1}{f} = (\frac{n}{n_{medium}} - 1)(\frac{1}{r_1} - \frac{1}{r_2})$, where r_1 is the radius of curvature of the lens surface nearer the object and r_2 is that of the other surface.
No reference yet.