Reference for Homework 3

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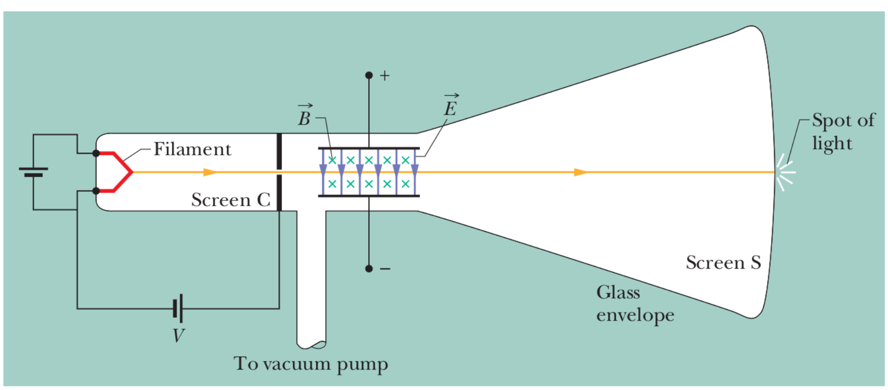
Please read the preface before reading this document!!!

1

In Thomson's experimental apparatus, charged particles are emitted by a hot filament and form a narrow beam after they pass through a slit. They then pass through a region of crossed $\vec{E}$ and $\vec{B}$ fields, headed toward a fluorescent screen S, where they produce a spot of light.
(a) Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the un deflected beam. Turn on $\vec{E}$ and measure the resulting beam deflection. Show that the deflection of the particle at the far end of the plates is $y = \frac{|q|EL^2}{2mv^2}$, where v is the particle's speed, m its mass, q its charge, and L is the length of the plates.
(b) Maintaining $\vec{E}$, now turn on $\vec{B}$ and adjust its value until the beam retuerns to the undeflected position. SHow that the crossed fields allow us to measure the speed of the charged particles passing through them, and therefore, $\frac{|q|}{m} = \frac{2yE}{B^2L^2}$, in which all quantities on the right can be measured.

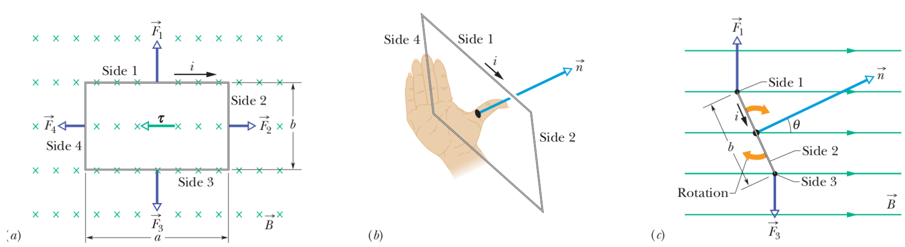


Reference:
(a) $\Rightarrow t = \frac{L}{v}, a = \frac{|q|E}{m} \Rightarrow y = \frac{1}{2}at^2 = \frac{|q|EL^2}{2mv^2}$

(b) $\Rightarrow qvB = qE \Rightarrow v = \frac{E}{B}$
$\Rightarrow \frac{|q|}{m} = \frac{2v^2 y}{EL^2} = \frac{2y}{EL^2}(\frac{E}{B})^2 = \frac{2yE}{B^2L^2}$

2

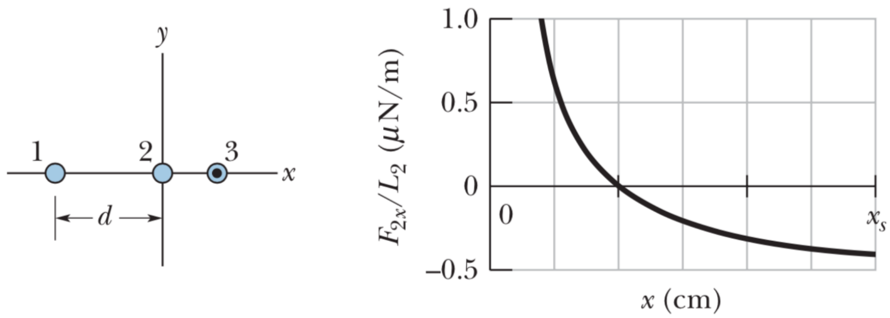
For a rectangular loop, of length a and width b and carrying a current i, a uniform perpendicular magnetic field $\vec{B}$ produces a torque, tending to rotate the loop about its central axis. The torque can be expressed to be $\vec{\tau} = i\vec{L_2}\times(\vec{L_3})\times\vec{B}$, where $\vec{L_2}$ and $\vec{L_3}$ are two perpendicular length vectors.
(a) Using the vector triple product identities
$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$,
$(\vec{A}\times\vec{B})\times\vec{C} = -\vec{A}(\vec{B}\cdot\vec{C}) + \vec{B}(\vec{A}\cdot\vec{C})$,
show that $\vec{\tau} - i\vec{A}\times\vec{B}$, where $\vec{A} = ab\hat{n}$ is the area vector for the loop.
(b) Discuss why the expression $t\vec{au} = i\vec{A}\times\vec{B}$ holds for all flat coils, no matter what their shapes are, provided $\vec{B}$ is uniform.



Reference:
(a) $\vec{\tau} = i\vec{L_2}\times(\vec{L_3}\times\vec{B})$
$= i(\vec{L_3}\cdot(\vec{L_2}\cdot\vec{B}) - \vec{B}\cdot(\vec{L_2}\cdot\vec{L_3}))$
$= i(\vec{L_3}\cdot(\vec{L_2}\cdot\vec{B}))$
$= i(\vec{L_3}\cdot(\vec{L_2}\cdot\vec{B}) - \vec{L_2}\cdot(\vec{L_3}\cdot\vec{B}))$
$= i(\vec{L_2}\times\vec{L_3})\times\vec{B}$
$= i\vec{A}\times\vec{B}$
(b) divide the circuit into n small loops $A_1, ..., A_n \Rightarrow \vec{\tau_k} = i\vec{A_k}\times\vec{B}$
when $n\to\infty$, the small loops can be seen as rectangular, so $\lim_{n\to\infty}\sum_{k=1}^{n}\vec{A_k} = \vec{A} \Rightarrow \vec{\tau} = \lim_{n\to\infty}\sum_{k=1}^{n}\vec{\tau_k} = i\lim_{n\to\infty}\sum_{k=1}^{n}\vec{A_k}\times\vec{B} = i\vec{A}\times\vec{B}$

3

In cross section, three current-carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an x axis, with separation d. Wire 1 has a current of 0.750 A, but the direction of the current is not given. Wire 3, with a current of 0.250 A out of the page, can be moved along the x axis to the right of wire 2. As wire 3 is moved, the magnitude of the net magnetic force $\vec{F_2}$ on wire 2 due to the currents in wires 1 and 3 changes. The x component of that force is F_{2x} and the value per unit length of wire 2 is F_{2x}/L_2. Figure b gives F_{2x}/L_2 versus the position x of wire 3. The plot has an asymptote $F_{2x}/L_2 = -0.627\mu$N / m as $x \to \infty$. The horizontal scale is set by $x_s = 12.0$cm. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2?



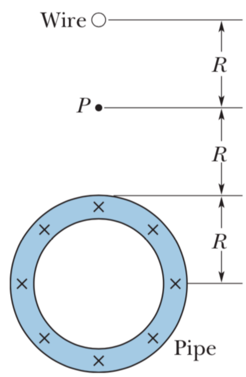
(a) (b)

Reference:
(a)(b) $\Rightarrow$ the direction of current in wire 2 is the same as that in wire 1, and the magnitude of the current in wire 2 is the same as that in wire 3
$\Rightarrow$ the current in wire 2 is out of the page, and the current in wire 1 is out of the page
$F_{2x} = \frac{\mu_0 I_3}{2\pi x} I_2 L_2 - \frac{\mu_0 I_1}{2\pi d} I_2 L_2 \Rightarrow -\frac{\mu_0 I_1 I_2}{2\pi d} = -0.627\mu\text{N/m}$
and when $x_0 = 4.00$cm, $F_2 x = 0 \Rightarrow \frac{\mu_0 I_3}{2\pi x_0} I_2 L_2 = \frac{\mu_0 I_1}{2\pi d} I_2 L_2$
$\Rightarrow d = \frac{I_1}{I_3} x_0 = 12.0$cm, $\Rightarrow I_2 = 0.502$ A

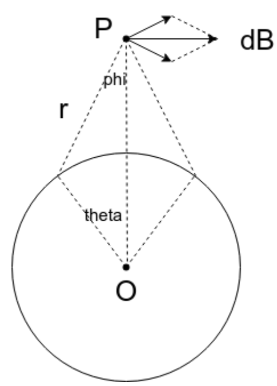
4

A long circular pipe with outside radius $R = 2.6$cm carries a (uniformly distributed) current $i = 8.00$mA into the page. A wire runs parallel to the pipe at a distance of 3.00 R from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point P has the same magnitude at the center of the pipe but is in the opposite direction.

Reference:
(a)(b) suppose that the positive direction of the current in the pipe is into the page, the circular pipe can be seen as the many paralleled small wires, and suppose the positive direction of the magnetic field is to the right



for the center of the pipe
the magnetic field generated by the two wires on the two ends of the diameter has the same magnitude and the opposite direction, so the net magnetic field generated by the circular pipe is zero
$\Rightarrow B_O = -\frac{\mu_0 I}{2\pi \cdot 3R}$
for point P
consider the magnetic field generated by the two small wires, whose angular position is θ to $\theta + d\theta$, as the figure below shows



$\Rightarrow di = \frac{id\theta}{2\pi}, r = \sqrt{(2R)^2 + R^2 - 4R^2\cos\theta} = R\sqrt{5 - 4\cos\theta}$
$\Rightarrow \cos\phi = \frac{(2R)^2+r^2-R^2}{4Rr} = \frac{2-\cos\theta}{\sqrt{5-4\cos\theta}}$
$\Rightarrow dB = \frac{\mu_0 di}{2\pi r} \cdot 2\cos\phi = \frac{\mu_0 i}{2\pi^2 R} \cdot \frac{2-\cos\theta}{5-4\cos\theta} d\theta$
$\Rightarrow B = \int dB = \frac{\mu_0 i}{2\pi^2 R} \int_0^{\pi} \frac{2-\cos\theta}{5-4\cos\theta} d\theta = \frac{\mu_0 i}{4\pi R}$
$\Rightarrow B_P = \frac{\mu_0 i}{4\pi R} - \frac{\mu_0 I}{2\pi R} = -B_O$
$\Rightarrow I = \frac{3}{8} i = 3.00$ mA, and the direction of the current is into the page

5

An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 m/s and its velocity vector makes an angle of 30° with the central axis of the solenoid. The solenoid carrries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)
Reference:
$\Rightarrow B = \frac{\mu_0 in}{L}, v_{\|} = v\cos\theta, v_{\perp} = v\sin\theta, t = \frac{L}{v_{\|}} = \frac{L}{v\cos\theta}$
$v_{\perp}B = mv_{\perp}\frac{2\pi}{T} \Rightarrow T = \frac{2\pi m}{eB}$
$\Rightarrow N_{revolution} = \frac{t}{T} = \frac{e\mu_0 in}{2\pi mv\cos\theta} = 1.62 \times 10^6$ rad

6

A student makes a short electronmagnet by winding 300 turns of wire around a wooden cylinder of diameter $d = 5.0$cm. The coil is connected to a battery producing a current of 4.0A in the wire.
(a) What is the magnitude of the magnetic dipole moment of this device¿
(b) At what axial distance $z \gg d$ will the magnetic field have the magtinude 5.0μT (approximattely one-tenth that of Earth's magnetic field)?
Reference:
(a) $\Rightarrow \mu = Ni\frac{\pi d^2}{4} = 2.36$ A·m^2
(b) $\Rightarrow B = \frac{\mu_0}{2\pi} \cdot \frac{\mu}{z^3} = 5.0\mu\text{T} \Rightarrow z = 45.5\text{cm}$

7

The free electron density in copper is $n = 8.5 \times 10^{28}\text{m}^{-3}$. Copper has a resistivity $\rho = 1.7 \times 10^{-8}\Omega$·m at room temperature. In the classical theory of metals, estimate the mean free path l of electrons in coppper, in units of meters?
Reference:
$\Rightarrow l = \tau v = \frac{m}{\rho ne^2}\sqrt{\frac{8k_BT}{\pi m}} = 2.64 \times 10^{-9}$ m

8

In a Hall effect measurement a copper sample of size $L_x \times L_y \times L_z = 10.0 \times 1.0 \times 0.1$ cm^3 is used. An external magnetic field $B_z = 0.1$, as well as longitudinal voltage $V_x = 10$ V, is applied. Estimate the ratio of the number of carriers that set up the Hall (electric) field to the total number of carriers in the metal. The dielectric constant for copper is assumed to be $\kappa = 1$. What is the ratio if the sample is a semiconductor with $\kappa = 12$? Assume the carrier density and the resistivity of the semiconductor are $n = 10^{22}$ m^{-3} and $\rho = 10^{-2}\Omega{\cdot}$m, respectively. No reference yet.