Reference for Homework 4

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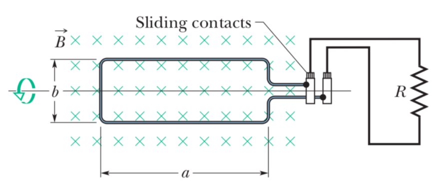
Please read the preface before reading this document!!!

1

A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$. The coil is connected to co-rotating cylinders, against which metal brushed slide to make contact.
(a) Show that the emf induced in the coil is given (as a function of time t) by $\varepsilon = 2\pi f NabB \sin(2\pi ft) = \varepsilon_0 \sin(2\pi ft)$. This is the principle of the commercial alternating-current generator.
(b) What value of Nab gives an emf with $\varepsilon_0 = 150$ V when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T?

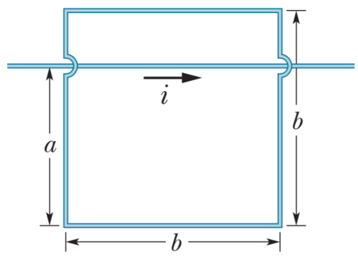


Reference:
(a) $\Rightarrow \Phi = BNab\cos(2\pi ft) \Rightarrow \varepsilon = -\frac{\partial \Phi}{\partial t} = 2\pi ftNabB\sin(2\pi ft)$
(b) $\Rightarrow Nab = \frac{\varepsilon_0}{2\pi fB} = 0.796\text{m}^2$

2

For a wire arrangement, $a = 12.0$cm and $b = 16.0$cm. The current in the long straight wire is $i = 4.50t^2 - 10.0t$, where i is in amperes and t is in seconds.
(a) Find the emf in the square loop at $t = 3.00$ s.
(b) What is the direction of the induced current in the loop?

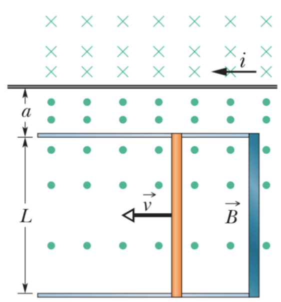
Reference:



(a) suppose the positive direction of the magnetic flux is into the page
$\Rightarrow \Phi = \int_{b-a}^{a} \frac{\mu_0 ib}{2\pi r} dr = \frac{\mu_0 ib}{2\pi} \ln \frac{a}{b-a} = \frac{\mu_0 ib}{2\pi} \ln 3$
$\Rightarrow \varepsilon = \frac{d\Phi}{dt} = \frac{\mu_0 b}{2\pi} \ln 3(9t - 10)$
$\Rightarrow$ when $t = 3.00$s, $\varepsilon = 5.98 \times 10^{-7}$
(b) when $0 \le t < \frac{10}{9}$s, the magnetic flux is decreasing, the induced current is clockwire; when $t = \frac{10}{9}$s, there's no induced current; when $t = \frac{10}{9}$s, the magnetic flux is increasing, the induced current is counterclockwire

3

A rod of length $L = 10.0$ cm that is forced to move at constant speed $v = 5.00$ m/s along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance 0.400Ω; the rest of the loop has negligibel resistance. A current $i = 100$ A through the long straight wire at distance $a = 10.0$ mm from the loop sets up a (nonuniform) magnetic field through the loop. Find the (a) emf and (b) current induced in the loop. (c) At what rate is thermal energy generated in the rod? (d) What is the magnitude of the force that must be applied to the rod to make it move at constant speed? (e) At what rate does this force do work on the rod?



Reference:
(a) $\Rightarrow \varepsilon = \int_{a}^{a+b} \frac{\mu_0 I_0 v}{2\pi r} dr = \frac{\mu_0 I_0 v}{2\pi} \ln \frac{a+b}{a} = 2.40 \times 10^{-4}$ V

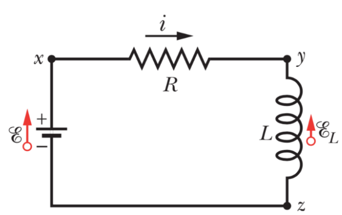
(b) $\Rightarrow I = \frac{\varepsilon}{R} = 5.99 \times 10^{-4}$A, clockwise
(c) $\Rightarrow P_1 = \frac{\varepsilon_0^2}{R} = 1.44 \times 10^{-7}$ W
(d) $\Rightarrow F = \int_a^{a+b} \frac{\mu_0 I_0}{2\pi r} I dr = \frac{\mu_0 I_0 I}{2\pi} \ln \frac{a+b}{b} = 2.87 \times 10^{-8}$N
(e) $\Rightarrow P_2 = Fv = 1.44 \times 10^{-7}$ W

4

The magnetic field of a cylindrical magnet that has a pole-face diameter of 3.3 cm can be varied sinusoidally betweem 29.6 T and 30.0 T at a frequency of 15 Hz. (The current in a wire wrapped around a permanent magnet is varied to give this variation in the net field.) At a radial distance of 1.6 cm, what is the amplitude of the electric field induced by the variation?
No reference yet.

5

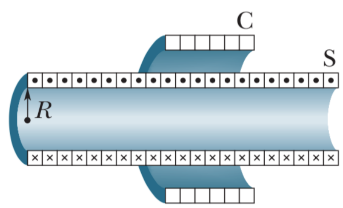
For the circuit, assume that $\varepsilon = 10.0$ V, $R = 6.70\Omega$ and $L = 5.50$ H. The ideal battery is connected at time $t = 0$.
(a) How much energy is delivered by the battery during the first 2.00 s?
(b) How much of this energy is stored in the magnetic field of the inductor?
(c) How much of this energy is dissipated in the resistor?



Reference:
(a) $\Rightarrow i(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) \Rightarrow W_1 = \int_0^t \varepsilon i dt = \frac{\varepsilon^2}{R}t + \frac{\varepsilon^2 L}{R^2}(e^{-\frac{R}{L}t} - 1) = 18.7$ J
(b)$\Rightarrow W_2 = \int_0^t i^2 R dt = \frac{\varepsilon^2}{R}t + \frac{2\varepsilon^2 L}{R^2}(e^{-\frac{R}{L}t} - 1) + \frac{\varepsilon^2 L}{2R^2}(1 - e^{-\frac{2R}{L}t}) = 13.7$J
(c) $\Rightarrow Q = \frac{1}{2}Li^2 = \frac{\varepsilon^2 L}{2R^2}(1 - e^{-\frac{R}{L}t})^2 = 5.10$ J

6

A coil C of N turns is placed around a long solenoid S of radius R and n turns per unit length.
(a) Show that the mutual inductance for the coil-solenoid combination is given by $M = \mu_0 \pi R^2 nN$.
(b) Explain why M does not depend on the shape, size of possible lack of close packing of the coil.



Reference:
(a) $\Rightarrow B = \mu_0 ni \Rightarrow \Phi - \pi R^2 N \mu_0 ni$
$\Rightarrow \varepsilon = -M\frac{di}{dt} = -\frac{d\Phi}{dt} = -\pi R^2 N \mu_0 n \frac{di}{dt} \Rightarrow M = \mu_0 \pi R^2 nN$
(b) the magnetic field in the coil C is generated by the long solenoid S, so the magnetic flux is only related to the turns of the coil C

7

In an oscillating series RLC circuit, show that $\delta U/U$, the fraction of the energy lost per cycle of oscillation, is given to a close approximation by $2\pi R/\omega L$. The quantity $\omega L/R$ is often called the Q of the circuit (for quality). A high-Q circuit has low resistance and a low fractional energy loss $(= 2\pi/Q)$ per cycle.
Reference:
in the RLC circuit, $q = Qe^{-\frac{t}{\tau}}\cos(\omega t + \phi)$
$\Rightarrow i = \frac{dq}{dt} = Qe^{-\frac{t}{\tau}}(-\frac{1}{\tau}\cos(\omega t + \phi) - \omega\sin(\omega t + \phi))$
in the k-th cycle, $U_k = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{1}{2C}Q^2 e^{-\frac{2}{\tau}kT}\cos^2\phi + \frac{1}{2}LQ^2 e^{-\frac{2}{\tau}kT}(\frac{1}{\tau}\cos\phi + \omega\sin\phi)^2$
$\Rightarrow \frac{\Delta U}{U} = \frac{U_k - U_{k+1}}{U_k} = 1 - e^{=\frac{2}{\tau}T} =\approx \frac{2}{\tau}T$
we have $\frac{1}{\tau} = \frac{R}{2L}$ and $T = \frac{2\pi}{\omega} \Rightarrow \frac{\Delta U}{U} = \frac{2\pi R}{\omega L}$

8

Assume that an electron of mass m and charge magnitude e moves in a circular orbit of radius r about a nucleus. A uniform magnetic field $\vec{B}$ is then estab-

lished perpendicular to the plane of the orbit. Assuming also that the radius of orbit does not change and that the change in the speed of the electron due to dield $\vec{B}$ is small, find an expression for the change in the orbital magnetic dipole moment of the electron due to the field.
Reference:
we suppose that before the magnetic field is established the electron's velocity is v, and after the magnetic field is established, the electron's velocity is $v^{'} \Rightarrow m\frac{v^2}{r} + ev^{'}B = m\frac{v^{'2}}{r}$
$ev^{'}B = \frac{m}{r}(v^{'^2} - v^2) = \frac{m}{r}(v^{'} + v)(v^{'} - v) \approx \frac{m}{r} \cdot 2v^{'} \cdot (v^{'} - v) \Rightarrow v^{'} - v = \frac{eBr}{2m}$
and the magnetic dipole moment $\mu = NiA = \pi r^2 \frac{e}{T} = \pi r^2 \cdot \frac{e}{2\pi r/v} = \frac{1}{2}erv$
$\Rightarrow \Delta\mu = \frac{er}{2}(v^{'} - v) = \frac{er}{2} \cdot \frac{eBr}{2m} = \frac{e^2r^2B}{2m}$