Reference for Homework 2

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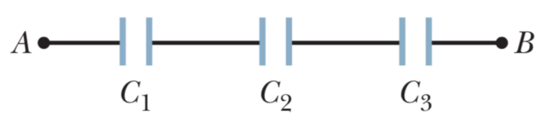
Please read the preface before reading this document!!!

1

In a three-capacitor, $C_1 = 10.0\mu\text{F}$, $C_2 = 20.0\mu\text{F}$, and $C_3 = 25.0\mu\text{F}$. If no capacitor can withstand a potential difference of more than 100 V without failure, what are
(a) the magnitude of the maximum potential difference that can exist between points A and B and
(b) the maximum energy that can be stored in the three-capacitor arrangement?

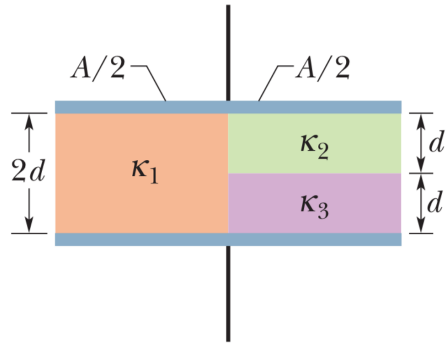


Reference:
(a) $\Rightarrow Q_{\text{max}} = C_1 U_{\text{max}} = 1^{-3}$ C
$U_1 = U_{\text{max}} = 100$ V, $U_2 = Q_{\text{max}}/C_2 = 50$ V, $U_3 = Q_{\text{max}}/C_3 = 40$ V
$\Rightarrow U = U_1 + U_2 + U_3 = 190$ V
(b) $\Rightarrow E = \frac{1}{2}C_1U_1^2 + \frac{1}{2}C_2U_2^2 + \frac{1}{2}C_3U_3^2 = 9.5 \times 10^{-2}$ J

2

A parallel-plate capacitor of plate area $A = 10.5\text{cm}^2$ and plate separation $2d = 7.12$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 21.0$; the top of the right half is filled with material of dielectric constant $\kappa_2 = 42.0$; the bottom of the right half is filled with material of dielectric constant $\kappa_3 = 58.0$. What is the capacitance?

Reference:



$\Rightarrow C_1 = \frac{\epsilon_0 \kappa_1 A/2}{2d}, C_2 = \frac{\epsilon_0 \kappa_2 A/2}{d}, \frac{\epsilon_0 \kappa_3 A/2}{d}$
$\Rightarrow C = C_1 + \frac{1}{\frac{1}{C_2}+\frac{1}{C_3}} = \frac{\epsilon_0 A}{2d}(\frac{\kappa_1}{2} + \frac{1}{\frac{1}{\kappa_2}+\frac{1}{\kappa_3}}) = 4.55 \times 10^{-11}$ F

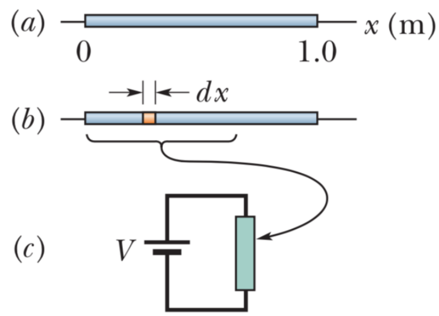
3

The rain-soaked shoes of a person may explode if ground current from nearby vaporizes the water. THe sudden conversion of water to water vapor causes a dramatic expansion that can rip apart shoes. Water has density 1000 kg / m^3 and requires 2256 kJ / kg to be vaporized. If horizontal current lasts 2.00 ms and encounters water with resistivity 150Ω·m, length 12.0cm, and vertical cross-sectional area 15×10^{-5} m^2, what average current is required to vaporize the water?
Reference:
$\Rightarrow R = \rho_R \frac{l}{A}$ and $E = Lm\rho Al$
and from $\bar{I}^2 Rt = E$ we can get that $barI = \sqrt{\frac{E}{tR}} = 13.0$

4

There is a rod of resistive material (Figure a). The resistance per unit length of the rod increases in the positive direction of the x axis. At any position x along the rod, the resistance dR of a narrow (differential) section of width dx is given by $dR = 5.00dx$ where dR is in ohms and x is in meters. FIgure b shows such a narrow section. You are to slice off a length of the rod between $x = 0$ and some position $x = L$ and then connect that length to a battery with potential difference $V = 5.0$ V (Figure c). You want the current in the length to transfer enerfy to thermal energy at the rate of 200 W. At what position $x = L$ should you cut the rod?

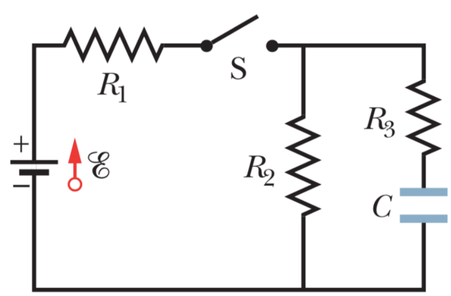
Reference:



$\Rightarrow R = \int_0^L 5.00x dx = 2.50L^2 = \frac{U^2}{P} \Rightarrow L = 0.224$ m

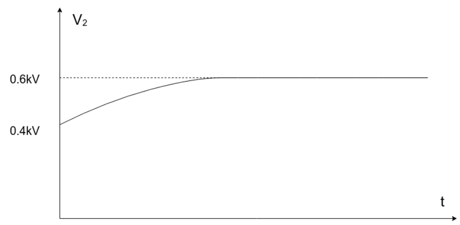
5

In a circuit, $\varepsilon = 1.2$ kV, $C = 6.5\mu$F, $R_1 = R_2 = R_3 = 0.73$ MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$).
At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3?
At $t = \infty$ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3 ? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$ (i) Sketch V_2 versus t between these two extreme times.



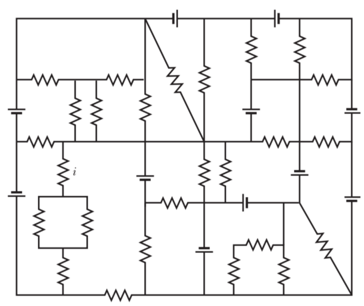
Reference:
(a) $R = R_1 + \frac{R_2 R_3}{R_2+R_3} \Rightarrow i_1 = \frac{\varepsilon}{R} = 1.10$ mA
(b)(c) $\Rightarrow i_2 = i_3 = \frac{1}{2} i_1 = 0.548 mA$
(d)(e) $\Rightarrow i_1 = i_2 = \frac{\varepsilon}{R_1+R_2} 0.822$ mA
(f) $\Rightarrow i_3 = 0$
(g) $\Rightarrow V_2 = \frac{R-R_2}{R}\varepsilon = 0.4$ kV
(h) $\Rightarrow v_2 = \frac{R_2}{R_1+R_2}\varepsilon = 0.6$ kV

(i)

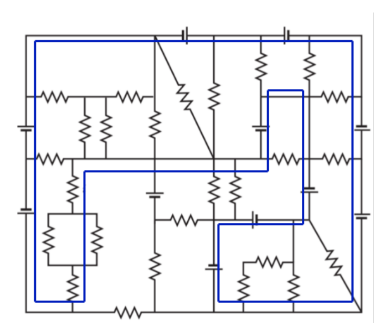


6

What are the (a) size and (b) direction (up or down) of current i, where all resistances are 4.0Ω and all batteries are ideal and have an emf of 10 V? (Hint: Find a special loop such that you can answer by mental calculation only.)



Reference:
(a)(b) consider the circuit shown below:



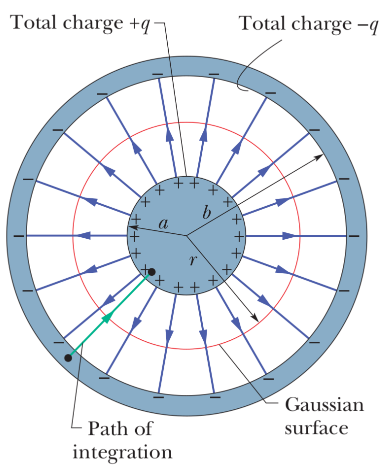
$\Rightarrow i = \frac{(7-3)\varepsilon}{R+R+\frac{R^2}{R+R}} = 4.0$ A, the direction is up

7

A metal sphere of radius 15 cm has a net charge of 3.0×10^{-8} C.
(a) What is the electric field at the sphere's surface?
(b) If $V = 0$ at infinity, what is the electric potential at the sphere's surface?
(c) At what distance from the sphere's surface has the electric potential decreased by 500 V?
Reference:
(a) $\Rightarrow E = \frac{q}{4\pi\epsilon_0 r^2} = 1.20 \times 10^4$ V/m, perpendicular to the surface
(b) $\Rightarrow V = \frac{q}{4\pi\epsilon_0 r} = 1.80 \times 10^3$ V
(c) $\Rightarrow \Delta V = \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 (r+d)} \Rightarrow d = 5.78$ cm

8

Consider two concentric spherical shells, of radii a and b. Show that the capacitance of the shells is $C = 4\pi\epsilon_0 \frac{ab}{b-a}$. What is the capacitance to a single isolated spherical conductor of radius R, then?



Reference:
$\Rightarrow \Delta U = \frac{Q}{4\pi\epsilon_0 a} - \frac{Q}{4\pi\epsilon_0 b} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{b-a}{ab}$
$\Rightarrow C = \frac{Q}{\Delta U} = \frac{4\pi\epsilon_0 (b-a)}{ab}$

9

Show that the curl of a central force $\vec{F}(\vec{r}) = f(r)\hat{r}$ is zero, i.e. $\nabla \times \vec{F}(\vec{r}) = \vec{0}$. Hence, central forces are conservative.

Reference:
$\Rightarrow \int \vec{F}(\vec{r}) \cdot d\vec{s} = \int f(r)\hat{r} \cdot (d\vec{r} + d\vec{r_\theta} + d\vec{r_\phi}) = \int f(r)dr$
that is, the work done by the force is independent of the path, so the force is conservative, $\nabla \times \vec{F}(\vec{r}) = \vec{0}$

10

Consider a two-dimensional electric field $\vec{E}(x,y) = \frac{-y\hat{i}+x\hat{j}}{x^2+y^2}$.
(a) Calculate the curl of the field $\nabla \times \vec{E}$.
(b) Show that the circulation of the field $\Gamma = \oint_C \vec{E} \cdot d\vec{s} = 2\pi$ around a unit circle centered at origin. Therefore, a vanishing curl does not implies, in general, that the force is conservative. They are equivalent only when the space is simply connected.
Reference:
(a) $\Rightarrow \nabla \times \vec{E}(x,y) = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}) \times (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}) = (\frac{y^2-x^2}{x^2+y^2} + \frac{x^2-y^2}{x^2+y^2})\vec{k} = \vec{0}$
(b) let $x = \cos\theta, y = \sin\theta$
$\Rightarrow \vec{E}(x,y) = -\sin\theta\vec{i} + \cos\theta\vec{j}$ and $d\vec{s} = d(\cos\theta\vec{i} + \sin\theta j) = (-\sin\theta\vec{i} + \cos\theta\vec{j})d\theta$
$\Rightarrow \Gamma = \oint_C \vec{E} \cdot d\vec{s} = \int_0^{2\pi} (\sin^2\theta + \cos^2\theta)d\theta = 2\pi$