Innopolis University
Introduction to Optimization
Fall 2024

General information and requirements for
Programming Task 4

Task 1: Bisection Method for Root-Finding

Write a program to implement the Bisection Method to find a root of a given function. The function and the interval $[a, b]$ will be provided as inputs.

Requirements:

1. The function is $f(x) = x^3 - 6x^2 + 11x - 6$.
2. Input: Initial interval $[a, b]$, tolerance (ϵ).
3. Output: Approximate root of the function.
4. Stop when $|f(c)| < \epsilon$, where $c = \frac{a+b}{2}$.

Additional Questions:

1. How does the choice of $[a, b]$ affect convergence?
2. Test your program with $[a, b] = [1, 2]$ and $\epsilon = 10^{-6}$.

Task 2: Golden Section Method for Unimodal Function Optimization

Write a program to implement the **Golden Section Method** to find the minimum of a unimodal function.

Requirements:

1. The function is $f(x) = (x - 2)^2 + 3$.
2. Input: Interval $[a, b]$, tolerance (ϵ).
3. Output: Approximate x_{min} and $f(x_{\text{min}})$.
4. Stop when the interval length is smaller than ϵ.

Additional Questions:

1. Why does the Golden Section Method work only for unimodal functions?
2. Test your program with $[a, b] = [0, 5]$ and $\epsilon = 10^{-4}$.

Task 3: Gradient Ascent Method for Maximizing a Function

Write a program to implement the **Gradient Ascent Method** to find the maximum of a differentiable function.

Requirements:

1. The function is $f(x) = -x^2 + 4x + 1$.
2. Input: Initial guess x_0, learning rate (α), and number of iterations (N).
3. Output: Approximate x_{max} and $f(x_{\text{max}})$.
4. Use the derivative $f'(x) = -2x + 4$.

Additional Questions:

1. How does the choice of α affect convergence?

2. Test your program with $x_0 = 0$, $\alpha = 0.1$, and $N = 100$.

Report

Deliverables:

1. Use Template as previous
2. Code files with proper comments.
3. A short report (1-2 pages) explaining:
   - The logic behind each method.
   - Observations about convergence and results for each task.
   - Challenges faced during implementation.