Mathematics 327
Introduction to eigenvectors and eigenvalues

1. Consider the matrix $A = \begin{bmatrix} 7 & 6 \\ 6 & -2 \end{bmatrix}$. Find the product $A\mathbf{v}$ to verify that $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector of A. What is the associated eigenvalue λ?

2. Go to `http://gvsu.edu/s/0Ja` where you will find an interactive diagram that allows you to choose a matrix A using the sliders along the top. The red vector $\mathbf{v}$ may be moved by clicking in the head of the vector and dragging it to a new location. The gray vector is $A\mathbf{v}$.

   Choose the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Move the red vector $\mathbf{v}$ so that the eigenvector condition holds. What is the eigenvector $\mathbf{v}$ and what is the associated eigenvalue?

   Are you able to find another linearly independent eigenvector $\mathbf{v}$? If so, what is the eigenvector and what is the associated eigenvalue?

   Are you able to find a basis for $\mathbb{R}^2$ consisting of eigenvectors of A?

3. Now consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Use the diagram to describe any eigenvectors and their associated eigenvalues.

   Are you able to find a basis for $\mathbb{R}^2$ consisting of eigenvectors of A?

4. Now consider the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Use the diagram to describe any eigenvectors and their associated eigenvalues. What geometric transformation does this matrix perform on vectors? How does this explain the presence of any eigenvectors?

   Are you able to find a basis for $\mathbb{R}^2$ consisting of eigenvectors of A?

5. The eigenvalues of a matrix A appear as the roots of the characteristic equation $\det(A - \lambda I) = 0$. For the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, write the characteristic equation and use it to find the eigenvalues.

Do the same for the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

and for the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.