

Mathematics 227

Bases of Eigenvectors

Here are some useful Sage commands:

- If A is a square matrix, `A.fcp()` will produce its factored characteristic polynomial.
- If A is a matrix, `A.right_kernel()` will produce a basis for $\text{Nul}(A)$.
- The $n \times n$ identity is `I = identity_matrix(n)`.

1. Consider the matrix $A = \begin{bmatrix} -10 & 12 \\ -6 & 8 \end{bmatrix}$.

Find the eigenvalues of A and their multiplicity (maybe using `A.fcp()`).

For each eigenvalue λ , find a basis for the eigenvectors E_λ .

Are you able to construct a basis for \mathbb{R}^2 consisting of eigenvectors of A ? If so, what is the basis?

2. Consider the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$.

Find the eigenvalues of A and their multiplicity.

For each eigenvalue λ , find a basis for the eigenvectors E_λ .

Are you able to construct a basis for \mathbb{R}^2 consisting of eigenvectors of A ? If so, what is the basis?

3. Consider the matrix $A = \begin{bmatrix} -3 & -5 & -1 \\ 1 & 3 & 1 \\ 1 & 5 & -1 \end{bmatrix}$.

Find the eigenvalues of A and their multiplicity.

Find a basis for the eigenspaces E_λ .

Are you able to construct a basis for \mathbb{R}^3 consisting of eigenvectors of A ? If so, what is the basis?