Mathematics 327 Symmetric matrices

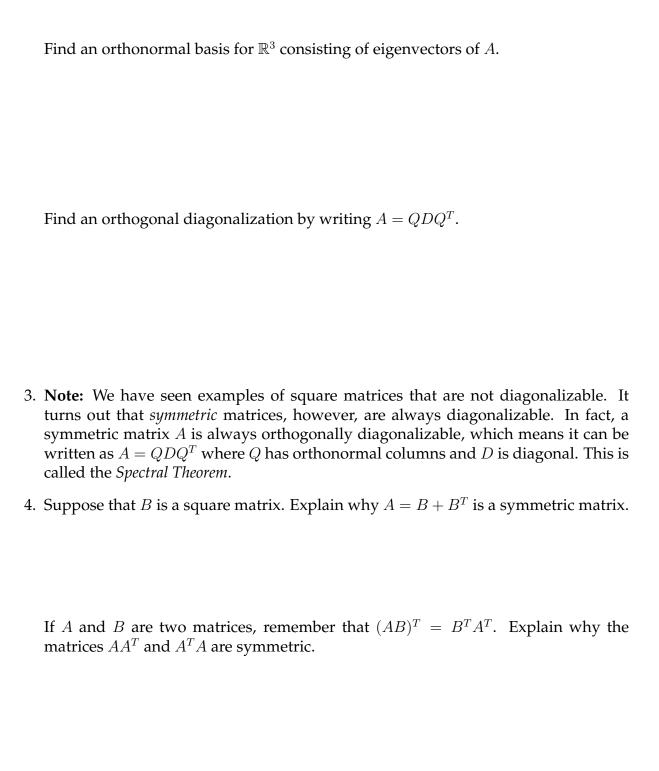
1. Consider the symmetric matrix $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$. Find the eigenvalues and corresponding eigenvectors.

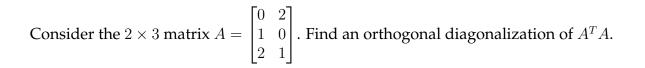
Is there an orthonormal basis of \mathbb{R}^2 consisting of eigenvectors of A?

Find an orthogonal diagonalization by finding matrices Q and D such that $A = QDQ^T$.

2. Consider the symmetric matrix $A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$. Find the eigenvalues of A.

For each eigenvalue, describe a basis for the corresponding eigenspace.





Find an orthogonal diagonalization of AA^{T} .

5. We can see that the eigenvalues of A^TA are nonnegative. For instance, suppose that ${\bf v}$ is an eigenvector of A^TA with corresponding eigenvalue λ . Explain the following steps and why they show that $\lambda \geq 0$.

$$|A\mathbf{v}|^2 = (A\mathbf{v}) \cdot (A\mathbf{v}) = (A\mathbf{v})^T (A\mathbf{v}) = (\mathbf{v}^T A^T)(A\mathbf{v}) = \mathbf{v}^T (A^T A\mathbf{v}) = \mathbf{v} \cdot (\lambda \mathbf{v}) = \lambda |\mathbf{v}|^2.$$

6. Part of the Spectral Theorem says that two eigenvectors of a symmetric matrix corresponding to different eigenvalues are orthogonal. To see this, suppose that \mathbf{v}_1 is an eigenvector with corresponding eigenvalue λ_1 and \mathbf{v}_2 is an eigenvector with corresponding eigenvalue λ_2 and that $\lambda_1 \neq \lambda_2$.

Explain the following steps and why they show that v_1 and v_2 are orthogonal.

$$\lambda_1 \mathbf{v}_1 \cdot \mathbf{v}_2 = (\lambda_1 \mathbf{v}_1) \cdot \mathbf{v}_2 = (A\mathbf{v}_1)^T \mathbf{v}_2 = \mathbf{v}_1^T A^T \mathbf{v}_2 = \mathbf{v}_1^T (A\mathbf{v}_2) = \mathbf{v}_1^T (\lambda_2 \mathbf{v}_2) = \lambda_2 \mathbf{v}_1 \cdot \mathbf{v}_2.$$

Therefore,

$$\lambda_1 \mathbf{v}_1 \cdot \mathbf{v}_2 = \lambda_2 \mathbf{v}_1 \cdot \mathbf{v}_2.$$