

**Mathematics 227**

**Matrix transformations**

1. Suppose that the matrix transformation  $T(\mathbf{x}) = A\mathbf{x}$  where  $A = \begin{bmatrix} 3 & 3 & -2 & 1 \\ 0 & 2 & 1 & -3 \\ -2 & 1 & 4 & -4 \end{bmatrix}$ .

(a) what is the dimension of the vectors  $\mathbf{x}$  that are input into  $T$ ? What is the dimension of the vectors  $T(\mathbf{x})$  that are output from  $T$ ?

(b) Describe all the vectors for which  $T(\mathbf{x}) = \mathbf{0}$ .

2. Suppose that we work for a company that produces baked goods, including cakes, donuts, and eclairs. Our company operates two plants, Plant 1 and Plant 2. In one hour of operation,

- Plant 1 produces 10 cakes, 50 donuts, and 30 eclairs.
- Plant 2 produces 20 cakes, 30 donuts, and 30 eclairs.

(a) If plant 1 operates for  $x_1$  hours and plant 2 for  $x_2$  hours, how many cakes  $C$  do they produce in total? How many donuts  $D$ ? How many eclairs  $E$ ?

(b) We define a matrix transformation  $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} C \\ D \\ E \end{bmatrix}$  where  $\begin{bmatrix} C \\ D \\ E \end{bmatrix}$  represents the quantities of baked goods produced when the plants run for  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  hours. If  $T(\mathbf{x}) = A\mathbf{x}$ , what are the dimensions of  $A$ ?

(c) Find the vector  $T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$  and the vector  $T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$ . Then write the matrix  $A$ .

(d) If we operate plant 1 for 40 hours and plant 2 for 50 hours, how many baked goods have we produced?

(e) The marketing department says we need to produce 1500 cakes, 4700 donuts, and 3300 eclairs. Is it possible to meet this order? If so, how long should the two plants operate?

(f) Suppose that

- Each cake requires 4 units of flour and 2 units of sugar.
- Each donut requires 1 unit of flour and 1 unit of sugar.
- Each eclair requires 1 unit of flour and 2 units of sugar.

Suppose we make  $C$  cakes,  $D$  donuts, and  $E$  eclairs. How many units  $F$  of flour and units  $S$  of sugar are required?

(g) Write a matrix  $B$  that defines the matrix transformation  $R \left( \begin{bmatrix} C \\ D \\ E \end{bmatrix} \right) = \begin{bmatrix} F \\ S \end{bmatrix}$ .

(h) If plant 1 operates for 30 hours and plant 2 for 20 hours, how many units of flour and sugar are required?

(i) Find the matrix that represents the transformation  $P \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} F \\ S \end{bmatrix}$  that represents the ingredients needed when the plants are operated for  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  hours.

3. Many geometric operations can be described as matrix transformations. Suppose, for instance, that we want to describe the geometric transformation  $T$  that rotates vectors in the plane counterclockwise by  $90^\circ$ . Then  $T(\mathbf{x}) = A\mathbf{x}$  where  $A$  is a  $2 \times 2$  matrix.

What is  $T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$  and  $T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$ ? Use these vectors to find the matrix  $A$ .

What vector results when we rotate  $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$  counterclockwise by  $90^\circ$ ?

4. Find the  $2 \times 2$  matrix that reflects vectors across the vertical axis. What is the result of reflecting  $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$  across the vertical axis?