Mathematics 227
Final Review, Part II

1. Consider the matrix
$$A = \begin{bmatrix} -2 & 1 & 3 \\ 2 & 1 & -1 \\ -4 & 0 & 8 \end{bmatrix}.$$
Perform (by hand) a sequence of row operations to find a row equivalent matrix that is upper-triangular. Use this to compute $\det(A)$.

2. Suppose that A is the 2×2 matrix that describes the reflection in the line $y = 2x$. Think geometrically to give a description of the eigenvalues and eigenvectors of A.

3. Suppose that A is a square $n \times n$ matrix. If A is invertible, what is the reduced row echelon form of A?

Explain an algorithm to find A^{-1} and why it works.

Apply this algorithm to find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -3 \\ 1 & -2 & 0 \end{bmatrix}$.

If A is invertible, what can you say about $\mathrm{Col}(A)$?

If A is invertible, what can you say about $\mathrm{Nul}(A)$?

4. What do we mean by a basis of $\mathbb{R}^n$?

Verify that

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}, , \qquad \mathbf{v}_3 = \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix}$$

form a basis $\mathcal{B}$.

If $\mathbf{x} = \begin{bmatrix} -6 \\ 7 \\ 0 \end{bmatrix}$, find $\{\mathbf{x}\}_{\mathcal{B}}$.

If $\{\mathbf{y}\}_{\mathcal{B}} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$, find $\mathbf{y}$.

Suppose that A is a 3×3 matrix having eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ with associated eigenvalues $\lambda_1 = 2$, $\lambda_2 = -3$, and $\lambda_3 = 1$. If $\{\mathbf{x}\}_{\mathcal{B}} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, find $\{A\mathbf{x}\}_{\mathcal{B}}$.

Find a matrix P such that $P\{\mathbf{x}\}_{\mathcal{B}} = \mathbf{x}$.

Find a matrix D such that $\{A\mathbf{x}\}_{\mathcal{B}} = D\{\mathbf{x}\}_{\mathcal{B}}$.

Find $A\begin{bmatrix}1\\0\\0\end{bmatrix}$ by first finding $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}_{\mathcal{B}}$, then $\left\{A\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}_{\mathcal{B}}$, and finally $A\begin{bmatrix}1\\0\\0\end{bmatrix}$.

Explain why $A = PDP^{-1}$.