Mathematics 227
Vectors

Visit the web site `gvsu.edu/s/0Je` where you find an interactive figure you can use to investigate linear combinations. We will consider the vectors

$$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and consider linear combinations $a\mathbf{v} + b\mathbf{w}$.

1. The weight b is initially set to 0. Explain what happens as you vary a with $b = 0$? How is this related to scalar multiplication?

2. What is the linear combination of $\mathbf{v}$ and $\mathbf{w}$ when $a = 1$ and $b = -2$? You may find this result using the diagram, but you should also verify it by computing the linear combination.

3. Describe the vectors that arise when the weight b is set to 1 and a is varied. How is this related to our investigations in the previous activity?

4. Can the vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$? If so, what are weights a and b?

5. Use the diagram to determine whether the vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ can be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$? If so, what are weights a and b?

6. Verify the result from the previous part by algebraically finding the weights a and b that form the linear combination $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

7. Can the vector $\begin{bmatrix} 1.3 \\ 1.7 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$? What about the vector $\begin{bmatrix} 15.2 \\ 7.1 \end{bmatrix}$?

8. Are there any two-dimensional vectors that cannot be expressed as linear combinations of $\mathbf{v}$ and $\mathbf{w}$?