Mathematics 227
Review

1. Consider the matrix
$$A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}.$$

What is the characteristic equation of A?

What are the eigenvalues of A?

Find a basis for the eigenspaces.

2. Suppose that A is a 2×2 matrix having eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and associated eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 4$. Find $A \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $A^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

3. Suppose that $\lambda = 0$ is an eigenvalue of A. What does this say about the invertibility of A?

If A is invertible and λ is an eigenvalue, explain why $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

4. Consider the matrix $A = \begin{bmatrix} -15 & 26 \\ -10 & 17 \end{bmatrix}$ Identify the type of dynamical system this matrix defines and sketch a phase portrait.
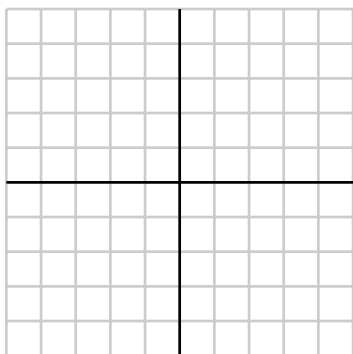
5. Consider the matrix $A = \begin{bmatrix} -1 & 0 & 2 \\ -2 & -2 & -4 \\ 0 & 0 & -2 \end{bmatrix}$. Can you find a basis for $\mathbb{R}^3$ consisting of eigenvectors of A?
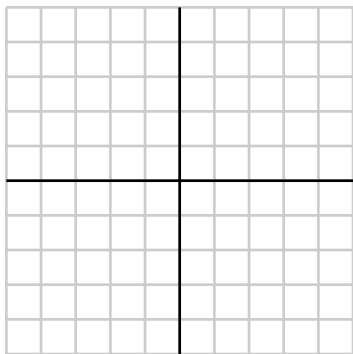
6. The populations of two species R and S in year k are denoted by R_k and S_k. Their populations in the following year are given by

$$R_{k+1} = R_k + S_k$$
$$S_{k+1} = 0.5R_k + 1.5S_k.$$

Denote the state vector $\mathbf{x}_k = \begin{bmatrix} R_k \\ S_k \end{bmatrix}$. Find the matrix A such that $\mathbf{x}_{k+1} = A\mathbf{x}_k$.

Identify the type of this dynamical system and sketch a phase portrait.

After a very long time, what is the ratio of the populations R_k/S_k and what is the growth rate of the populations?

7. Consider the matrix $A = \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.5 & 0.8 & 0.2 \\ 0.3 & 0.0 & 0.7 \end{bmatrix}$.

Find the eigenvalues of A.

Find a steady-state vector **q**.

What happens to a Markov chain that begins with the initial state vector $\mathbf{x}_0 = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix}$?

Does the Perron-Frobenius theorem apply to this Markov chain? Explain why or why not.