Mathematics 227
Matrix multiplication

1. Find the matrix product

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 4 & -3 & -2 \\ -1 & -2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

2. Suppose that A is the matrix

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & -2 & 4 \\ 2 & 1 & 5 \\ 1 & 0 & 3 \end{bmatrix}.$$

   If $A\mathbf{x}$ is defined, what is the dimension of the vector $\mathbf{x}$? What is the dimension of $A\mathbf{x}$?

3. A vector whose entries are all zero is denoted by $\mathbf{0}$. If A is a matrix, what is the product $A\mathbf{0}$?

4. Suppose that $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and that $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Find the product $I\mathbf{x}$ and explain why I is called the *identity matrix*.

5. Suppose we write the matrix A in terms of its columns as

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_n \end{bmatrix}.$$

If the vector $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, what is the product $A\mathbf{e}_1$?

6. Suppose that

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}.$$

Is there a vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$? Explain your process for answering this question clearly.

In Sage, we may define a vector as

```
v = vector([2, 3, -1]).
```

Scalar multiplication, vector addition, and matrix multiiplication work just as you would expect: `3*v`, `v + w`, and `A*v`.

Also, if `A` is a matrix and `b` is a vector, then

```
A.augment(b)
```

gives the new matrix obtained by augmenting `A` by the vector `b`.

In Sage, define the matrix and vectors

$$A = \begin{bmatrix} -2 & 0 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Compare the result of evaluating $A(3\mathbf{v})$ to that of evaluating $3A\mathbf{v}$.

Compare the result of evaluating $A(\mathbf{v}+\mathbf{w})$ to that of evaluating $A\mathbf{v}+A\mathbf{w}$.

7. Suppose that A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 4 & -3 & -2 \\ -1 & -2 & 6 & 1 \end{bmatrix}$$

Describe the solution space to the equation $A\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$.

Describe the solution space to the equation $A\mathbf{x} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$.

Given two matrices A and B, we can form the prodcut AB by multiplying the columns of B by A. That is,

$$\text{if} B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \dots & \mathbf{b}_p \end{bmatrix}, \text{then } AB = \begin{bmatrix} A\mathbf{b}_1 & A\mathbf{b}_2 & A\dots & A\mathbf{b}_p \end{bmatrix}.$$

To do this, the number of columns of A must equal the number of rows of B.

Suppose that

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -3 & 4 & -1 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ -2 & -1 \end{bmatrix}.$$

Find the product AB.

Also, compute the product BA. Are the results the same?

8. Form the matrices matrices

$$A = \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}.$$

What is the product AB in this case?