Mathematics 227
Lab 1, Due: September 10, 2018

Instructions: Complete the following exercises in groups of 2 or 3 students, but you only need to hand in one report for your group. You may write directly on this handout; be sure to include complete explanations of the work you have done.

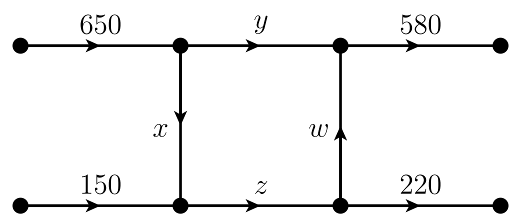
There is a page of Sage cells available at `http://gvsu.edu/s/0Ng`

1. Consider the system of linear equations:

$$\begin{aligned} x + 2y - z &= 1 \\ 3x + 2y + 2z &= 7 \\ -x + 4z &= -3 \end{aligned}$$

   Write this system as an augmented matrix and use Sage to find a description of the solution space.

2. Shown below is the traffic pattern in the downtown area of a large city. The figures give the number of cars per hour traveling along each road. Any car that drives into an intersection must also leave the intersection. This means that the number of cars entering an intersection in an hour is equal to the number of cars leaving the intersection.

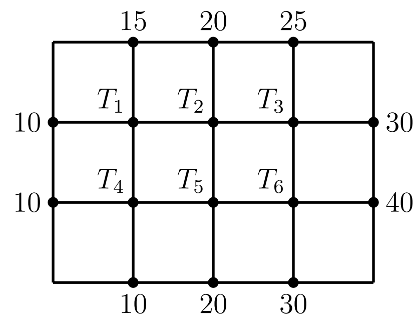
   Write a system of equations for the quantities x, y, z, and w and describe the set of solutions. Is there a single solution, infinitely many solutions, or no solutions?

   Explain why you would expect infinitely many solutions for this particular traffic pattern.

   What is the smallest amount of traffic flowing through x?

3. A typical problem in thermodynamics is to determine the temperature distribution across a thin plate if you know the temperature around the boundary. Assume, for instance, that the plate represents a cross section of a metal beam with negligible heat flow in the direction perpendicular to the plate. Let $T_1, \ldots, T_6$ be the temperatures at the six nodes inside the beam. The temperature at a node is approximately the average of the four nearest nodes: for instance,

$$T_1 = (10 + 15 + T_2 + T_4)/4 \quad \text{or} \quad 4T_1 - T_2 - T_4 = 25.$$



In the real world, the approximation becomes better the closer the points are together or as we add more and more into the grid.

Set up a system of linear equations to find the temperature inside the plate.

Solve your equations to find the temperatures inside the plate.

Helpful Sage hint: If you have a matrix `B` containing rational entries (e.g. fractions), you can obtain a decimal approximation using `B.numerical_approx(digits=4)`. You may, of course, change 4 to any other appropriate value.

4. This exercise is about balancing chemical reactions.

   Chemists denote a molecule of water as H_2O, which means it is composed of two atoms of hydrogen (H) and one atom of oxygen (O). The process by which hydrogen is burned is described by the chemical reaction

   $$x\,\mathrm{H_2} + y\,\mathrm{O_2} \to z\,\mathrm{H_2O}$$

   This means that x molecules of hydrogen H_2 combine with y molecules of oxygen O_2 to produce z water molecules. The number of hydrogen atoms is the same before and after the reaction; the same is true of the oxygen atoms.

   How many hydrogen atoms are there before the reaction? How many hydrogen atoms are there after the reaction? Find a linear equation in x, y, and z by equating these quantities.

   Find a second linear equation in x, y, and z by equating the number of oxygen atoms before and after the reaction.

   Describe the solutions of this linear system. Why would you expect infinitely many solutions when balancing a chemical reaction?

   In this chemical setting, x, y, and z should be positive integers. Find the solution where x, y, and z are the smallest possible positive integers.

Consider the reaction where potassium permanganate and manganese sulfate combine with water to produce manganese dioxide, potassium sulfate, and sulfuric acid:

$$x_1\,\mathrm{KMnO_4} + x_2\,\mathrm{MnSO_4} + x_3\,\mathrm{H_2O} \to x_4\,\mathrm{MnO_2} + x_5\,\mathrm{K_2SO_4} + x_6\,\mathrm{H_2SO_4}.$$

As in the previous exercise, find the appropriate values for $x_1, x_2, \ldots, x_6$ to balance the chemical reaction.