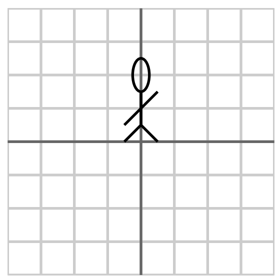
**Mathematics 227**
**Lab 2, Due: October 17, 2018**

**Instructions:** The exercises here should be completed in groups of 2 or 3 students with one write-up submitted from each group.

In this lab, we'll see how computer animators use matrix transformations to tell stories. In fact, if you saw *Incredibles 2* last summer, you were seeing some very elegant applications of linear algebra.

Now, let me introduce you to Wooody, the lovable star of the animated kids' film *Toy Tale* due out this holiday season.



The computer animators who made *Toy Tale* assembled the film frame by frame. To make us think that Wooody is moving, they make changes in Wooody's position from one frame to the next. This is where linear algebra is useful. (Actually, Wooody really lives in a three-dimensional world and each part of his body may be moving in different ways so the situation is a little more complicated but hopefully you will get the idea.)

If you go to `http://gvsu.edu/s/0Jb`, you will find a figure that can make Wooody move. On the left, you see a picture of Wooody, while on the right you will see a picture of him after a matrix transformation has been applied.

Here is the function we will consider; it is not quite as simple as what we have seen in class:

$$\begin{aligned} x_r &= ax_l + by_l + c \\ y_r &= dx_l + ey_l + f \end{aligned}$$

The six sliders along the top of the diagram represent the quantities

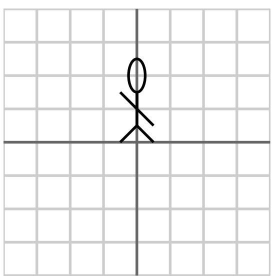
$$\begin{matrix} a & b & c \\ d & e & f. \end{matrix}$$

1. Using homogeneous coordinates, we represent this change as

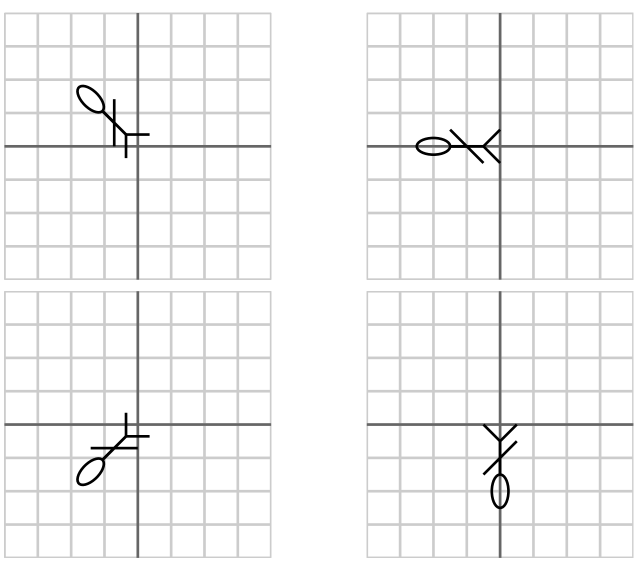
$$\begin{bmatrix} x_r \\ y_r \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ 1 \end{bmatrix}.$$

Compute this matrix product and verify that it produces the appropriate expressions for $x_r$ and $y_r$.

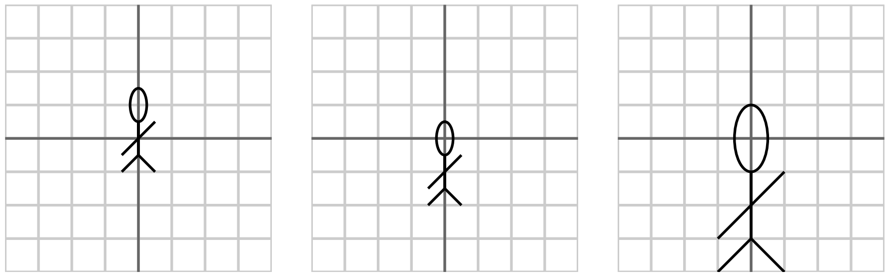
2. Press the **Reset** button if you have modified Wooody. As shown above, Wooody is waving with his left hand. In the first scene of the movie, Wooody comes on screen and waves with his right hand. What is the matrix transformation that has been applied?



3. Next, Wooody begins his morning callesthenics and performs a cartwheel. Write the transformations which produce the following sequence of pictures.



4. At this point in the story, Wooody suddenly remembers a long-ago love and the camera moves in for a close up.

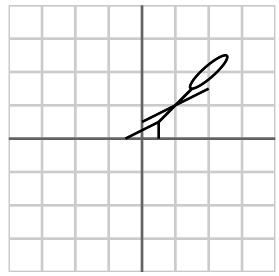


There are two ways to do this, but one is easy after a little thought. The first two frames are rather straightforward. The third is a little tricky but the diagram gives you another tool to help. Once you have the diagram configured in some way you like, you may press the button **Apply**, the image on the left is set to that on the right and the transformation reset to the identity. You may then operate on the new figure from scratch. (Mathematically, you are composing two functions.)

Explain a simple way to make the sequence of frames above.

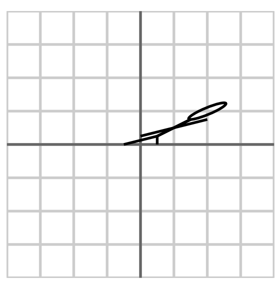
Determine the function that takes our original picture of Wooody into the final close up?

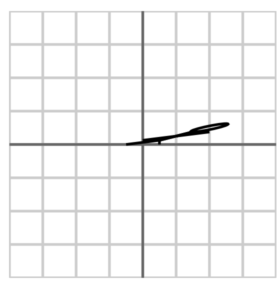
5. Wooody decides to go out for a walk. In the morning sun, he casts a shadow that looks like this:



Find a matrix transformation which creates the shadow. You might remember how we have found the matrix representing a matrix transformation by looking at the image of the standard basis vectors.

As the sun comes up, his shadow get shorter.





Explain what matrix transformations achieve this and how you found them. (Think about composing transformations again.)

6. Write an ending to our heart-felt story and describe how to illustrate it.