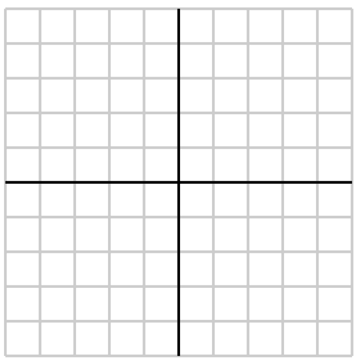
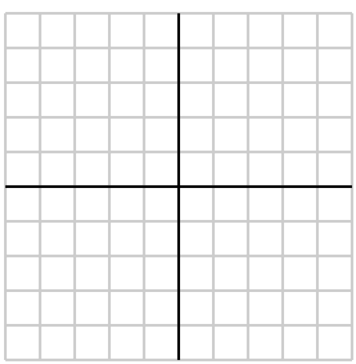
Mathematics 227
Dynamical systems

For each of the following matrices, determine whether the associated dynamical system is a repellor, saddle, or attractor. Sketch the eigenspaces and some trajectories to indicate the behavior of the system below.

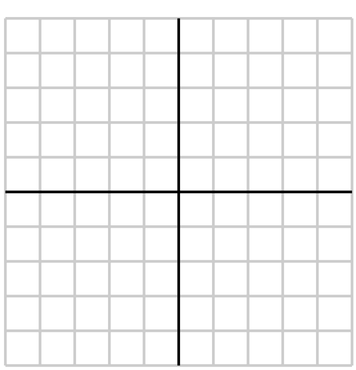
1. $A = \begin{bmatrix} 1.3 & 0.2 \\ 0.2 & 1.3 \end{bmatrix}$.



2. $A = \begin{bmatrix} 0.3 & 0.6 \\ -0.6 & 1.8 \end{bmatrix}$.



3. $A = \begin{bmatrix} 0.7 & -0.4 \\ -0.1 & 0.7 \end{bmatrix}$.



4. Suppose we have two species R and S that interact with one another and that we record the change in their populations from year to year. When we begin our study, the populations, measured in thousands, are R_0 and S_0; after k years, the populations are R_k and S_k.

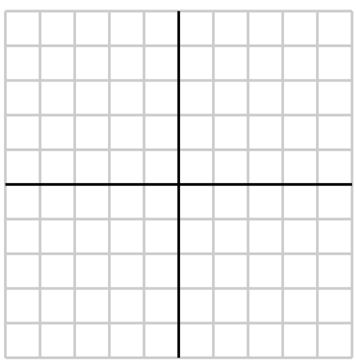
Knowing the populations in one year, we find their populations the next year:

$$\begin{aligned} R_{k+1} &= 0.9R_k + 0.8S_k \\ S_{k+1} &= 0.2R_k + 0.9S_k. \end{aligned}$$

We will combine the populations into a vector $\mathbf{x}_k = \begin{bmatrix} R_k \\ S_k \end{bmatrix}$ and write $\mathbf{x}_{k+1} = A\mathbf{x}_k$ where

$$A = \begin{bmatrix} 0.9 & 0.8 \\ 0.2 & 0.9 \end{bmatrix}.$$

Classify this dynamical system as an attractor, repellor, or saddle and sketch some trajectories below.



Suppose that $\mathbf{x}_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Write $\mathbf{x}_0$ as a linear combination of eigenvectors of A.

Write the vectors $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ as a linear combination of eigenvectors of A.

When k becomes very large, what happens to the ratio of the population R_k/S_k?

After a long time, by approximately what factor does the population of R grow every year? By approximately what factor does the population S grow every year?

5. Consider the dynamical system defined by the matrix

$$A = \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}.$$

Find the eigenvalues and eigenvectors of A.

Using the basis $\mathcal{B}$ of eigenvectors, sketch the trajectories in the $\mathcal{B}$-coordinate system on the left and the trajectories of the dynamical system on the right.