Mathematics 227
Span

1. Suppose that

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

Is $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$ in the span of $\mathbf{v}_1$ and $\mathbf{v}_2$?

Is $\mathbf{b} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$ in the span of $\mathbf{v}_1$ and $\mathbf{v}_2$?

2. Suppose you find a matrix A such that $A\mathbf{x} = \mathbf{b}$ is consistent for every vector $\mathbf{b}$. What does this imply about the pivots of A?

   Suppose that every vector $\mathbf{b}$ in $\mathbb{R}^7$ is in the span of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$. What does this imply about the pivots of the matrix

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_n \end{bmatrix}?$$

3. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}.$$

Find the reduced row echelon form of the matrix

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}.$$

For what vectors $\mathbf{b}$ is the equation

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

consistent?

What does this imply about the span of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$?

4. What is the smallest number of 8-dimensional vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ whose span is $\mathbb{R}^8$?