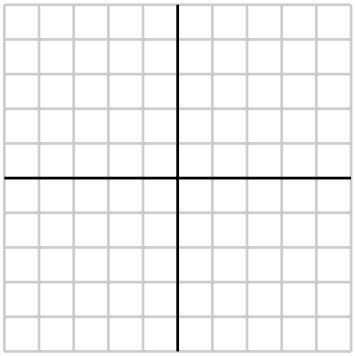
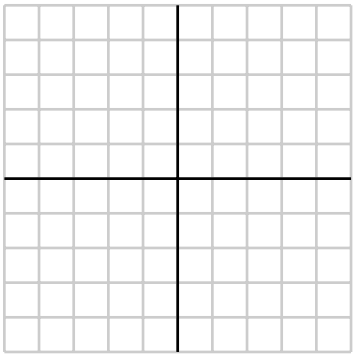
Mathematics 227
Dynamical systems

For each of the following matrices, determine whether the associated dynamical system is one of the six basic types we have seen. Sketch some trajectories to indicate the behavior of the system below.

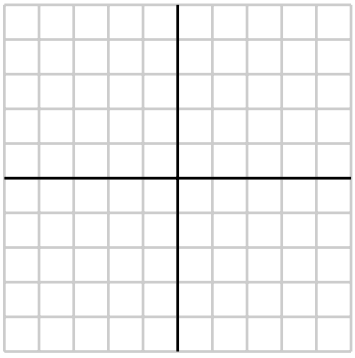
1. $A = \begin{bmatrix} 1.5 & -1 \\ 0.5 & 1 \end{bmatrix}$.



2. $A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$.

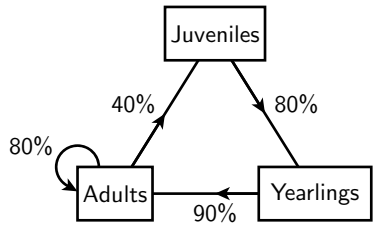


3. $A = \begin{bmatrix} 0.8 & -0.4 \\ -0.2 & 1.0 \end{bmatrix}$.



4. We can divide a population of female bison into three groups: juveniles who are less than one year old; yearlings between one and two years old; and adults who are older than two years. Each year,

   - 80% of the juveniles survive to become yearlings.
   - 90% of the yearlings survive to become adults.
   - 80% of the adults survive.
   - 40% of the adults give birth to a juvenile.



   We will represent the number of juveniles, yearlings, and adults in year k by J_k, Y_k, and A_k. The state of the system in year k will be represented by the vector $\mathbf{x}_k = \begin{bmatrix} J_k \\ Y_k \\ A_k \end{bmatrix}$.

   (a) Find a matrix B such that $\mathbf{x}_{k+1} = B\mathbf{x}_k$.

   (b) Find the eigenvalues of B.

(c) What does the size of the complex eigenvalue tell you about its effect on the long-term behavior of this system?

(d) An eigenvector corresponding to the real eigevalue is approximately

$$\mathbf{v} = \begin{bmatrix} 1.000 \\ 0.756 \\ 2.644 \end{bmatrix}.$$

Make a prediction about the long-term behavior of the herd. For instance, at what rate does it grow? For every 100 adults, how many juveniles and how many yearlings are there?

(e) As stated, the birth rate is 40%; that is, 40% of adults give birth to a juvenile. Suppose this birth rate drops to 20%. How does this affect the growth rate of the herd? What does this mean about the long-term behavior?