Mathematics 227
Pivot positions

1. Shown below are some matrices in reduced row echelon form. Identify the pivot positions and state whether the linear system is consistent or inconsistent. If the linear system is consistent, imagine the variables are $x_1, x_2, \ldots, x_n$ and identify the free and basic variables.

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -9 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

2. A linear system is called *homogeneous* if the right hand side of every equation is zero. The augmented matrix would look like this:

$$\left[\begin{array}{cccc|c} * & * & * & * & 0 \\ * & * & * & * & 0 \\ * & * & * & * & 0 \end{array}\right]$$

   Use pivot positions to explain why a homogeneous linear system is always consistent.

   What values for the variables are guaranteed to give a solution?

3. Give an example of an augmented matrix representing a linear system of 4 equations in 3 unknowns that has a unique solution.

   Give an example of an augmented matrix representing a linear system of 4 equations in 3 unknowns that has infinitely many solutions.

4. Suppose a coefficient matrix represents a consistent linear system no matter what is on the right-hand side of the equations. What can you say about the pivot positions?

   For the coefficient matrix of the last problem, suppose that the solution to the corresponding homogeneous equation is unique. What can you say about the pivot positions?

   What does this say about the dimensions of the coefficient matrix?

5. If a linear system with 26 variables has a unique solution, what can you say about the number of equations? Use pivot positions to explain your response.

6. Suppose that you have a homogeneous linear system with 10 equations and 10 unknowns and that there is a unique solution. What can you say about the pivot positions?

   If you change the right-hand side of the equations, can you guarantee that the system is consistent? If so, can you guarantee the solution is unique.

7. Suppose you have a linear system whose 3×4 coefficient matrix has three pivot columns. What, if anything, can you say about the questions of existence and uniqueness?

8. For what values of the parameters k and l is the following system consistent? For which value of k and l is there a unique solution? Justify your responses by identifying the pivot positions.

$$\left[\begin{array}{cc|c} -1 & k & l \\ 2 & 4 & 3 \end{array}\right]$$