

Mathematics 227**Reduced row echelon form**

1. Consider each of the following augmented matrices. Determine if the matrix is in reduced row echelon form. If it is not, perform a sequence of scaling, interchange, and replacement operations to obtain a row equivalent matrix that is in reduced row echelon form. Then use the reduced row echelon matrix to determine whether there is a single solution, infinitely many solutions, or no solutions. If there are infinitely many, identify free and basic variables.

$$\left[\begin{array}{ccc|c} 2 & 0 & 4 & -8 \\ 0 & 1 & 3 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 4 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

2. Suppose that the reduced row echelon form of a matrix looks like $\left[\begin{array}{cccc|c} 1 & * & 0 & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$.
- What, if anything, does the last row tell you about the solution space?

Suppose that the reduced row echelon form of a matrix looks like $\left[\begin{array}{cccc|c} 1 & * & 0 & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$.

What, if anything, does the last row tell you about the solution space?

If a linear system has no solutions, how can you tell by looking at the reduced row echelon form of its augmented matrix.

3. Consider the system of linear equations:

$$x + 2y - z = 2$$

$$-2x - 2y + 4z = -4$$

$$x - 2y - 6z = 3$$

Form the associated augmented matrix and perform Gaussian elimination to find its reduced row echelon form. What does this tell you about the solution space of the system?