

**Mathematics 227**  
**Vectors**

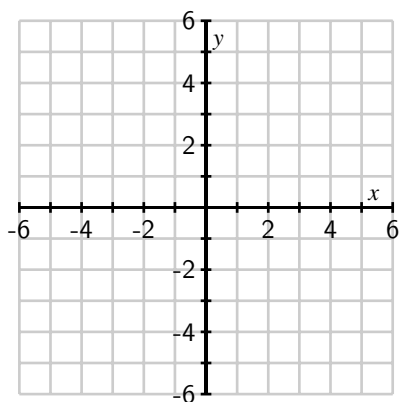
Suppose that

$$\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

1. Find expressions for the vectors

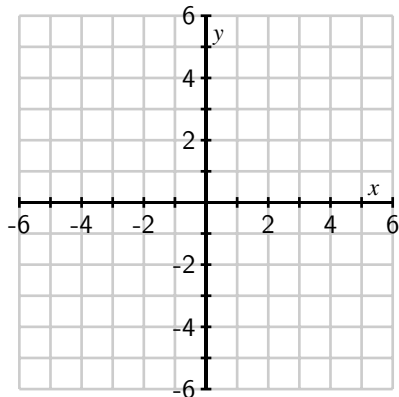
$$\begin{array}{cccc} \mathbf{v}, & 2\mathbf{v}, & -\mathbf{v}, & -2\mathbf{v}, \\ \mathbf{w}, & 2\mathbf{w}, & -\mathbf{w}, & -2\mathbf{w}. \end{array}$$

and sketch them below.

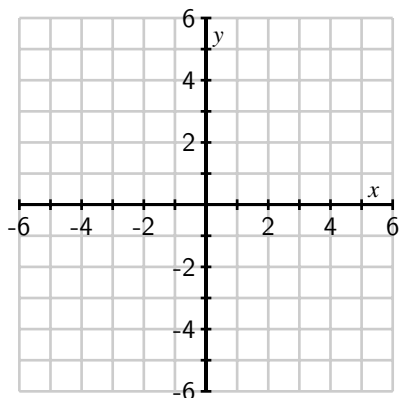


2. What geometric effect does scalar multiplication have on a vector? Also, describe the effect that multiplying by a negative scalar has.

3. Sketch the vectors  $\mathbf{v}$ ,  $\mathbf{w}$ , and  $\mathbf{v} + \mathbf{w}$ .



4. Consider vectors that have the form  $\mathbf{v} + a\mathbf{w}$  where  $a$  is any scalar. Sketch a few of these vectors when, say,  $a = 2, 1, 0, 1, \text{ and } 2$ . Give a geometric description of this set of vectors.



5. If  $a$  and  $b$  are two scalars, then the vector

$$a\mathbf{v} + b\mathbf{w}$$

is called a *linear combination* of the vectors  $\mathbf{v}$  and  $\mathbf{w}$ . Find the vector that is the linear combination when  $a = -2$  and  $b = 1$ .

6. Can the vector  $\begin{bmatrix} -31 \\ 37 \end{bmatrix}$  be represented as a linear combination of  $\mathbf{v}$  and  $\mathbf{w}$ ? In other words, can you find scalars  $a$  and  $b$  such that  $a\mathbf{v} + b\mathbf{w} = \begin{bmatrix} -31 \\ 37 \end{bmatrix}$ .