Mathematics 227
Exam 1 Review

1. Describe the solution space to the linear system:

$$\begin{aligned} 2x_1 - x_2 \quad\quad\quad &+ 3x_4 = 6 \\ x_1 \quad\quad + 3x_3 &- x_4 = 6 \\ 2x_1 \quad\quad + x_3 &+ 3x_4 = 7 \end{aligned}$$

Describe the solution space to the vector equation

$$\begin{bmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & 3 & -1 \\ 2 & 0 & 1 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 6 \\ 6 \\ 7 \end{bmatrix}.$$

Suppose that

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 6 \\ 7 \end{bmatrix}.$$

Can $\mathbf{b}$ be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$? If so, find one set of weights.

Is $\mathbf{b}$ is Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$?

Describe Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$. Explain your thinking.

2. Give a condition on the pivots
   (a) of an augmented matrix such that the associated linear system is not consistent.

   (b) of an augmented matrix such that the associated linear system has a unique solution.

   (c) of a matrix A such that $A\mathbf{x} = \mathbf{b}$ is consistent for any vector $\mathbf{b}$.

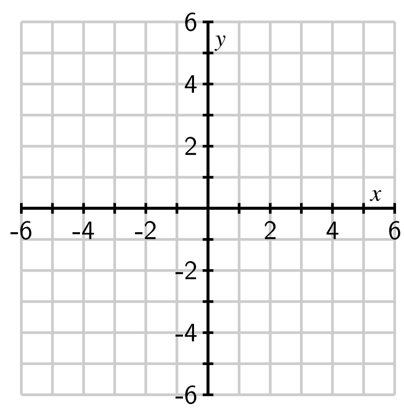
   (d) of a 10×10 matrix A such that the homogeneous equation has a unique solution.

3. What is the smallest number of vectors that span $\mathbb{R}^{1234}$? Explain your thinking.

4. Determine if the following matrix is in reduced row echelon form. If not, perform a sequence of row operations to put it in reduced row echelon form (without using any computational device). Then give a description of the solution space of the associated linear system.

$$\left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 & -2 \end{array}\right]$$

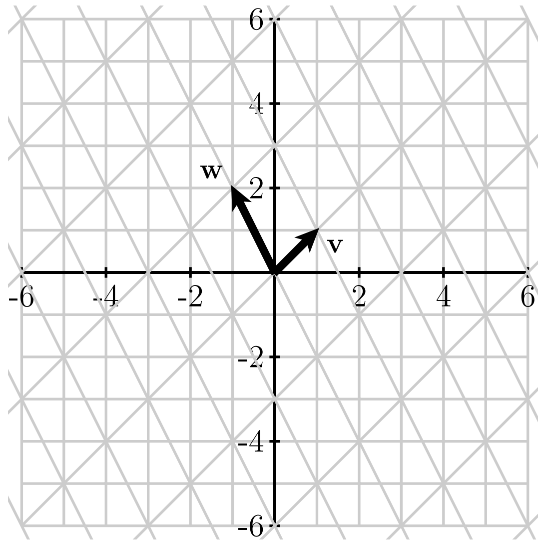
5. Suppose that $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Sketch the vectors $\mathbf{v}$, $\mathbf{w}$, $2\mathbf{v}$, and $\mathbf{v} + \mathbf{w}$.



Sketch all vectors of the form $t\mathbf{v}$ where t is any scalar.

Sketch all vectors of the form $\mathbf{w} + t\mathbf{v}$ where t is any scalar.

6. Consider the 2-dimensional vectors $\mathbf{v}$ and $\mathbf{w}$ shown below.



Sketch the linear combination $a\mathbf{v} + b\mathbf{w}$ with weights $a = -3$ and $b = 2$.

Can the vector $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$ be written as a linear combination of $\mathbf{v}$ and $\mathbf{w}$? If so, how?

Describe the solution space to the equation

$$\begin{bmatrix} \mathbf{v} & \mathbf{w} \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

7. Determine whether the following statements are true or false including a justification for your response.

   (a) If $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ are vectors in $\mathbb{R}^3$, then their span is $\mathbb{R}^3$.

   (b) Suppose that the span of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{27}$ is $\mathbb{R}^{27}$. Then every vector in $\mathbb{R}^{27}$ can be written as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{27}$ is $\mathbb{R}^{27}$ in exactly one way.

   (c) Is $\mathbf{b}$ is a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, then the equation

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

   is consistent.