**Mathematics 227**
**Matrix transformations**

1. Students in a school are sometimes absent due to illness. We will record the fractions of healthy and ill students on one day in a vector $\mathbf{x} = \begin{bmatrix} H \\ I \end{bmatrix}$ where $H$ is the fraction of healthy students and $I$ is the fraction of ill students. For instance, if $\mathbf{x} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$, 80% of the students are healthy and 20% of the students are ill.

   Suppose that

   - 95% of the students who are healthy one day are healthy the next.
   - 50% of the students who are ill one day are ill the next.

   If we know the percentages of healthy and ill students one day, a matrix transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ tells us the percentages the next day.

   (a) Find $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$. That is, if 100% of the students are healthy and none are ill one day, what are the percentages the next day?

   (b) Find $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$. That is, if 100% of the students are healthy and none are ill one day, what are the percentages the next day?

   (c) Use these results to find the matrix $A$ such that $T(\mathbf{x}) = A\mathbf{x}$.

(d) Suppose that 80% of the students are healthy and 20% are ill on Tuesday. What are the percentages on Wednesday?

(e) What were the percentages on Monday?

(f) What will be the percentages on Thursday and Friday?

(g) You can study how the percentages evolve over a long time using Sage. First, define the matrix $A$ and Tuesday's vector $\mathbf{x}$. Then use the following piece of code to show how $\mathbf{x}$ evolves over the next 20 days.

```
for i in range(20):
    x = A*x
    print x
```

What happens to the fraction of healthy and ill students after a very long time?

2. Matrix transformation perform geometric operations. Go to `gvsu.edu/s/0Jf` to study the effect that various matrix transformations have on the plane. On the left is the plane before the transformation; on the right is the plane after the transformation.

   Describe the geometric effect of the matrix transformations defined by

   (a) $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(e) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(f) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(g) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(h) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

What matrix produces a $180^\circ$ rotation?

What matrix produces a reflection over the line $y = -x$?