Mathematics 227
Markov chains

1. Consider the stochastic matrix $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$. Is this a positive matrix?

   Find the unique steady-state vector $\mathbf{q}$.

   What does the Perron-Froebenius theorem say about the convergence of a Markov chain beginning with some initial state vector $\mathbf{x}_0$?

   Consider the initial state vector $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and describe what happens when you generate 20 terms in the Markov chain using the following code after defining `A` and `x`:

```
for i in range(20):
    x = A*x
    print x
```

   What happens to the Markov chain when you begin with the initial state vector $\mathbf{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?

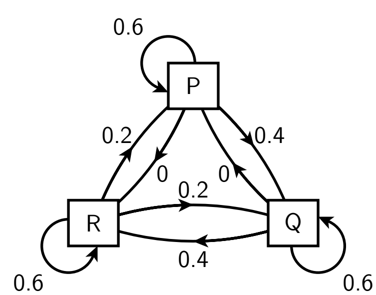
2. Consider the stochastic matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Look at a few powers of A and determine whether A is a positive matrix.

   What happens when you create a Markov chain beginning with the initial state vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$?

   Does the Perron-Frobenius theorem apply in this case?

3. Suppose that you live in a country having three political parties P, Q, and R. We will use P_k, Q_k and R_k to denote the percentage of voters voting for each party in election k. The state vector for an election is $\mathbf{x}_k = \begin{bmatrix} P_k \\ Q_k \\ R_k \end{bmatrix}$.

   Voters will change parties from one election to the next as shown in the figure.



Find the matrix A such that $\mathbf{x}_{k+1} = A\mathbf{x}_k$.

Explain why A is a stochastic matrix.

Suppose that initially 40% of voters vote for party P, 30% vote for party Q, and 30% of voters vote for party R. What is $\mathbf{x}_0$ and why should it be a probability vector?

Does the Perron-Frobenius theorem apply? If so, what does it imply?

Find the unique steady-state vector $\mathbf{q}$.

What happens to the Markov chain after a very long time?

Generate 20 terms in a Markov chain to confirm your prediction.