Mathematics 227
Matrix multiplication

1. Suppose that $A = \begin{bmatrix} -2 & 3 & -8 \\ 1 & -1 & 3 \end{bmatrix}$. Give a description of the solution space to the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

   Give a geometric description of the solution space.

   Give a description of the solution space to the equation $A\mathbf{x} = \begin{bmatrix} -12 \\ 4 \end{bmatrix}$.

   What is the geometric relationship between these two solution spaces.

2. Suppose that A is a 4×4 matrix and that, for some vector $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution.

   What does this say about the pivot positions of A? Explain your thinking.

What does this imply about the solution space to the homogeneous equation $A\mathbf{x} = \mathbf{0}$?

Is it possible to find a 4-dimensional vector $\mathbf{c}$ such that $A\mathbf{x} = \mathbf{c}$ is inconsistent?

3. Visit the web page `gvsu.edu/s/0Jg`. At the top of that page, you will be able to construct linear combinations of the vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$.

   What vector is the linear combination of $\mathbf{v}$ and $\mathbf{w}$ with weights

   - $a = 2$ and $b = 0$?
   - $a = 1$ and $b = 1$?
   - $a = 0$ and $b = -1$?

   Can the vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$?

   Can the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$?

Describe the set of vectors that can be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

For what vectors $\mathbf{b}$ is the equation $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \mathbf{x} = \mathbf{b}$ consistent?

4. At the bottom of that page, there is a diagram allowing you to construct linear combinations of $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

   What vector is the linear combination of $\mathbf{v}$ and $\mathbf{w}$ with weights

   - $a = 2$ and $b = 0$?
   - $a = 1$ and $b = 1$?
   - $a = 0$ and $b = -1$?

   Can the vector $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$?

   Can the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$?

Describe the set of vectors that can be expressed as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

For what vectors $\mathbf{b}$ is the equation $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x} = \mathbf{b}$ consistent?