Mathematics 227
Span

1. Suppose that $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$. Give a geometric description of Span$\{\mathbf{v}\}$.

2. Consider the two vectors in $\mathbb{R}^3$:

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

   What can you say about the components of a vector in Span$\{\mathbf{e}_1, \mathbf{e}_2\}$?

   Give a geometric description of the vectors in Span$\{\mathbf{e}_1, \mathbf{e}_2\}$

3. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}.$$

   Can every vector in $\mathbb{R}^3$ be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$?

What is Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

4. If the span of a set of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ is $\mathbb{R}^3$, what can you guarantee about the number of vectors in this set?

5. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}.$$

Can every vector in $\mathbb{R}^3$ be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$?

Explain why $\mathbf{v}_3$ can be written as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Explain why any linear combination $a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$ can be written as a linear combination of just $\mathbf{v}_1$ and $\mathbf{v}_2$.

Explain why Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ = Span$\{\mathbf{v}_1, \mathbf{v}_2\}$.