Mathematics 227
Pivot positions

1. Shown below are three augmented matrices in reduced row echelon form. For each matrix, identify the pivot positions and describe the solution space.

$$\left[\begin{array}{ccc|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccr|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccr|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

   Each of these augmented matrices above has a row in which each entry is zero. What, if anything, does the presence of such a row tell us about the solution space to the linear system?

   Give an example of a 3×5 augmented matrix in reduced row echelon form that represents a consistent system. Indicate the pivot positions in your matrix and explain why these pivot positions guarantee a consistent system.

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How do the pivot positions determine whether a linear system is consistent or not?

Suppose we have a linear system for which the *coefficient* matrix has the following reduced row echelon form.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

Even though we don't have any information about the right-hand side of the equations, what can you say about the consistency of the linear system? Explain your thinking.

The following augmented matrix is in reduced row echelon form. Describe the solution space; in particular, identify any free and basic variables.

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

How can we identify the free and basic variables by looking at the pivot positions?

If possible, give an example of a 3×5 augmented matrix that corresponds to a system of linear equations having a unique solution. If it is not possible, explain why.

If possible, give an example of a 5×3 augmented matrix that corresponds to a system of linear equations having a unique solution. If it is not possible, explain why.

What condition on the pivot positions guarantees that a system of linear equations has a unique solution?

If a system of linear equations has a unique solution, what can we say about the relationship between the number of equations and the number of unknowns? Use the concept of pivot positions to explain your thinking.