

Mathematics 327
Final review, part I

1. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 4 & 2 \\ -1 & 1 & -2 & -4 \\ 1 & -1 & 4 & 5 \end{bmatrix}$. Find an LU factorization by performing Gaussian elimination.

Why is an LU factorization useful?

2. Find the dot product $\mathbf{u} \cdot \mathbf{v}$ if $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

What is the angle between these two vectors?

3. Suppose that A is a 4×2 matrix $A = [\mathbf{a}_1 \ \mathbf{a}_2]$ and that \mathbf{x} is a 4-dimensional vector. What is $A^T \mathbf{x}$ in terms of dot products?

Suppose that $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 2 \\ -2 & -4 \end{bmatrix}$. Find a basis for $\text{Col}(A)$.

What do we mean by the orthogonal complement of $\text{Col}(A)$?

Why is $\text{Col}(A)^\perp = \text{Nul}(A^T)$?

Find a basis for $\text{Col}(A)^\perp$.

4. Find the orthogonal projection of the vector $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ onto the line spanned by the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

Find the orthogonal projection of the vector $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ onto the plane spanned by the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.