# Übungsserie 9

1) Geg: $I(a) = 2\int_1^a x \cdot \ln(x^2)\,dx$ mit stützpunkte

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $a$ | $e-\frac{1}{2}$ | $e-\frac{1}{4}$ | $e+\frac{1}{4}$ | $e+\frac{1}{2}$ |
| $I(a)$ | 3.9203 | 5.9169 | 11.3611 | 14.8550 |

$\rightarrow n = 3$

a) Ges: $I(a)$ für $a = e$ mittels Lagrange-Interpolation interpolieren

$$P_n(x) = \sum_{i=0}^{n} l_i(x) \cdot y_i \qquad l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j} \qquad (i = 0, \dots, n)$$

$(j \neq 0)$

$$l_0(e) = \frac{[e-(e-\frac{1}{4})]}{[(e-\frac{1}{2})-(e-\frac{1}{4})]} \cdot \frac{[e-(e+\frac{1}{4})]}{[(e-\frac{1}{2})-(e+\frac{1}{4})]} \cdot \frac{[e-(e+\frac{1}{2})]}{[(e-\frac{1}{2})-(e+\frac{1}{2})]} = \frac{\frac{1}{4} \cdot -\frac{1}{4} \cdot -\frac{1}{2}}{-\frac{1}{4} \cdot -\frac{3}{4} \cdot -1} = \frac{\frac{1}{32}}{-\frac{3}{16}} = -\frac{16}{96} = -\frac{1}{6}$$

$$l_1(e) = \frac{[e-(e-\frac{1}{2})]}{[(e-\frac{1}{4})-(e-\frac{1}{2})]} \cdot \frac{[e-(e+\frac{1}{4})]}{[(e-\frac{1}{4})-(e+\frac{1}{4})]} \cdot \frac{[e-(e+\frac{1}{2})]}{[(e-\frac{1}{4})-(e+\frac{1}{2})]} = \frac{\frac{1}{2} \cdot -\frac{1}{4} \cdot -\frac{1}{2}}{\frac{1}{4} \cdot -\frac{1}{2} \cdot -\frac{3}{4}} = \frac{\frac{1}{16}}{\frac{3}{32}} = \frac{2}{3}$$

$$l_2(e) = \frac{[e-(e-\frac{1}{2})]}{[(e+\frac{1}{4})-(e-\frac{1}{2})]} \cdot \frac{[e-(e-\frac{1}{4})]}{[(e+\frac{1}{4})-(e-\frac{1}{4})]} \cdot \frac{[e-(e+\frac{1}{2})]}{[(e+\frac{1}{4})-(e+\frac{1}{2})]} = \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot -\frac{1}{2}}{\frac{3}{4} \cdot \frac{1}{2} \cdot -\frac{1}{4}} = \frac{-\frac{1}{16}}{-\frac{3}{32}} = \frac{2}{3}$$

$$l_3(e) = \frac{[e-(e-\frac{1}{2})]}{[(e+\frac{1}{2})-(e-\frac{1}{2})]} \cdot \frac{[e-(e-\frac{1}{4})]}{[(e+\frac{1}{2})-(e-\frac{1}{4})]} \cdot \frac{[e-(e+\frac{1}{4})]}{[(e+\frac{1}{2})-(e+\frac{1}{4})]} = \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot -\frac{1}{4}}{1 \cdot \frac{3}{4} \cdot \frac{1}{4}} = \frac{-\frac{1}{32}}{\frac{3}{16}} = -\frac{1}{6}$$

$$P_3(e) = l_0(e) \cdot 3.9203 + l_1(e) \cdot 5.9169 + 11.3611 \cdot l_2(e) + 14.855 \cdot l_3(e)$$
$$= \underline{\underline{8.3895}}$$

b) $I(e) = 2 \cdot \int_1^e x \cdot \ln(x^2)\,dx \rightarrow$ matlab $=$ 2 · int(x · log(x²), 1, exp(1)) $= 8.3891$

$I(e) =$ exakter wert für $a = e$

Absolute Fehler
$|I(e) - P_3(e)| = 4.439 \cdot 10^{-4}$

Relativer Fehler
$$\frac{|I(e) - P_3(e)|}{|I(e)|} = 5.291 \cdot 10^{-5}$$

c) G3_S3_Aufg3(@(x) 2·x·log(x²), 1, exp(1), 3) $= 8.3887 = I_{romb}$

Absolute Fehler
$|I(e) - I_{romb}| = 3.926 \cdot 10^{-4}$

Relativer Fehler
$$\frac{|I(e) - I_{romb}|}{|I(e)|} = 4.679 \cdot 10^{-5}$$

$\Rightarrow$ Romberg-Expolation schneidet besser ab.

2) Geg:

| | 1 | | 2 | | 3 | 4 | |
|---|---|---|---|---|---|---|---|
| Höhe über Meer [m] | 0 | 1250 | 2500 | 3750 | 5000 | 10000 | x |
| Atmosphärendruck [hPa] | 1013 | ***NaN*** | 747 | ***NaN*** | 540 | 226 | y |

Ges: Schätzung für NaN werte mittels Aitken-Neville

| x | $x_{i-j}$ | $x_{i-j+1}$ | $\dots$ | $x_i$ |
|---|---|---|---|---|
| y | $y_{i-j}$ | $y_{i-j+1}$ | $\dots$ | $y_i$ |

$$p_{i0} = y_i$$

$$p_{ij} = \frac{(x_i - x)p_{i-1,j-1} + (x - x_{i-j})p_{i,j-1}}{x_i - x_{i-j}}$$

$i = 1, \dots$ $\quad j = 1, \dots i$

Um 1250 zu schätzen $x = 1250$ einsetzen.

| X | Y | | | |
|---|---|---|---|---|
| 0 | $P_{00} = 1013$ | | | |
| 2500 | $P_{10} = 747$ | $P_{11} = \frac{(2500-1250)\cdot 1013 + (1250-0)\cdot 747}{2500-0} = 880$ | | |
| 5000 | $P_{20} = 540$ | $P_{21} = 850.5$ | $P_{22} = 872.625$ | |
| 10000 | $P_{30} = 226$ | $P_{31} = 775.5$ | $P_{32} = 863$ | $P_{33} = 871.42$ |

$x = 3750$

| X | Y | | | |
|---|---|---|---|---|
| 0 | $P_{00} = 1013$ | | | |
| 2500 | $P_{10} = 747$ | $P_{11} = 614$ | | |
| 5000 | $P_{20} = 540$ | $P_{21} = 643.5$ | $P_{22} = 636.1$ | |
| 10000 | $P_{30} = 226$ | $P_{31} = 618.5$ | $P_{32} = 639.3$ | $P_{33} = 637.3$ |