Serie 5

Aufgabe 2

$$\frac{dy}{dx} = \frac{x^2}{y} \Rightarrow f(x, y(x)) = \frac{x^2}{y}$$

a)

$x_0 = 0$
$x_1 = 0.7 + 0.7 = 0.7$
$x_2 = 0.7 + 0.7 = 1.4$
$x_3 = 1.4 + 0.7 = 2.1$

$y_0 = 2$
$y_1 = 2 + 0.7 \cdot \left(\frac{0}{2}\right) = 2$
$y_2 = 2 + 0.7\left(\frac{0.7^2}{2}\right) = 2.1715$
$y_3 = 2.1715 + 0.7 \cdot \left(\frac{1.4^2}{2.1715}\right) = 2.8033$

abs. Fehler $|y(x_i) - y_i|$

$|2 - 2| = 0$
$|2.0564 - 2| = 0.0564$
$|2.4144 - 2.1715| = 0.2429$
$|3.1897 - 2.8033| = 0.3864$

b)

$x_0 = 0$
$x_1 = 0.7$
$x_2 = 1.4$
$x_3 = 2.1$

$xh_0 = 0.35$
$xh_1 = 1.05$
$xh_2 = 1.75$
$xh_3 = 2.45$

$x_0 = 2$
$y_1 = 2 + 0.7 \cdot \frac{0.35^2}{2} = 2.0429$
$y_2 = 2.0429 + 0.7 \cdot \frac{1.05^2}{2.1268} = 2.4057$
$y_3 = 2.4057 + 0.7 \cdot \frac{1.75}{2.6909} = 3.2024$

$yh_0 = 2 + 0.35\left(\frac{0}{2}\right) = 2$
$yh_1 = 2.0429 + 0.35\left(\frac{0.7^2}{2.0429}\right) = 2.1268$
$yh_2 = 2.4057 + 0.35\left(\frac{1.4^2}{2.4057}\right) = 2.6909$
$yh_3 = 3.2024 + 0.35\left(\frac{2.1^2}{3.2024}\right) = 3.6844$

abs. Fehl.
$|2 - 2| = 0$
$|2.0564 - 2.0429| = 0.0135$
$|2.4144 - 2.4057| = 0.0087$
$|3.1857 - 3.2024| = 0.0127$

c)

$x_0 = 0$
$x_1 = 0.7$
$x_2 = 1.4$
$x_3 = 2.1$

$k_1 = 0$ $k_2 = 0.2450$
$k_3 = 0.2547$ $k_4 = 0.8710$
$k_1 = 0.7726$ $k_2 = 1.4566$

$y_0 = 2$
$y_1 = 2.0858$
$y_2 = 2.4728$
$y_3 = 3.2600$

abs. Fehl.
$|2 - 2| = 0$
$|2.0564 - 2.0858| = 0.0294$
$|2.4144 - 2.4728| = 0.0584$
$|3.1597 - 3.2600| = 0.0703$