Aufgaben 5.5 → wir lesen die Kettenregel rückwärts laufen

③

$$\int (3x+2)(3x^2+4x)^4\,dx$$

!ABLEITEN!

$u = 3x^2 + 4x$

$\frac{du}{dx} = u' = 6x+4$

$du = (6x+4)\,dx$

$\frac{du}{6x+4} = dx$

$\int (3x+2)\, u^4\, dx$

AUFLEITEN

$$\int (3x+2)\, u^4 \frac{du}{(6x+4)} = \frac{1}{2}\int u^4\, du = \frac{1}{2}\,\frac{u^5}{5} + c = \frac{1}{10}(3x^2+4x)^5 + c$$

Achtung

④

$$\int \sin 3x\, dx$$

$u = 3x \qquad \frac{du}{dx} = 3 \qquad \frac{du}{3} = dx$

$$= \frac{1}{3}\int \sin u\, du = -\frac{1}{3}\cos u\, du + c = -\frac{1}{3}\cos 3x + c$$

⑥

$$\int \sqrt{3-2s}\, ds = \int 1\cdot(3-2s)^{\frac{1}{2}}\, ds$$

$u = 3-2s \qquad \frac{du}{ds} = -2 \qquad \frac{du}{-2} = ds$

$$= -\frac{1}{2}\int \sqrt{u}\, du = -\frac{1}{2}\int u^{\frac{1}{2}}\, du = -\frac{1}{2}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{3}u^{\frac{3}{2}} + c = -\frac{1}{3}\cdot\sqrt[3]{(3-2s)^2}$$

7

$$\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\,dx$$

$u = 1+\sqrt{x}$ $\qquad \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $\qquad 2\sqrt{x}\,du = dx$

$$\int \frac{2\cancel{\sqrt{x}}\,du}{\cancel{\sqrt{x}}\cdot u^2} = \frac{2\,du}{u^2} = 2\int u^{-2}\,du = -2u^{-1}+c = -\frac{2}{u}+c = -\frac{2}{1+\sqrt{x}}+c$$

11

$$\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\,d\theta$$

$u = \frac{1}{\theta}$ $\qquad \frac{du}{d\theta} = -\frac{1}{\theta^2}$ $\qquad du = -\frac{1}{\theta^2}\,d\theta$

$$= -\int \sin u \cos u\,du$$

$v = \sin u$ $\qquad \frac{dv}{du} = \cos u$ $\qquad dv = \cos u \cdot du$

$$= -\int v\,dv = -\frac{v^2}{2}+c$$

$$= -\frac{\sin^2 u}{2}+c$$

$$= -\frac{\sin^2\left(\frac{1}{\theta}\right)}{2}+c$$

(12)

$$\int t^3 (1+t^4)^3 \, dt$$

$u = 1+t^4 \qquad \frac{du}{dt} = u' = 4t^3$

$$dt = \frac{du}{4t^3}$$

$$\int \underline{t^3}\, u^3 \frac{du}{4t^3} = \int t^3\, u^3 \cdot \underline{\frac{1}{4t^3}\; du}$$