Kapitel 6.3

$$L = \int_a^b \sqrt{1 + (y')^2}\, dx$$

①

$$y(x) = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$$

$(a+b)^2 = a^2 + 2ab + b^2$

$$y'(x) = \underbrace{\frac{1}{3}\cdot\frac{3}{2}(x^2+2)^{\frac{1}{2}}}_{\text{äussere Abl.}} \cdot \underbrace{(2x)}_{\text{innere Abl.}} = x\cdot\sqrt{x^2+2}$$

$$L = \int_0^3 \sqrt{1 + x^2(x^2+1)}\, dx = \int_0^3 \sqrt{(x^2+1)^2} = \int_0^3 (x^2+1)\, dx$$

$1 + x^4 + 2x^2$

$$= \left(\frac{x^3}{3} + x\right)\Big|_0^3 = 9 + 3 = \underline{12}$$

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③ $$L = \int_c^d \sqrt{1 + (x')^2}\, dy$$

$$x(y) = \frac{y^3}{3} + \frac{1}{4y} \longrightarrow \left[\frac{1}{4}\cdot\frac{1}{y}\right]' = \frac{1}{4}\left[y^{-1}\right]' = \frac{1}{4}(-1)\cdot y^{-2}$$

$$x'(y) = \frac{3y^2}{3} - \frac{1}{4y^2} \qquad = -\frac{1}{4y^2}$$

$(a-b)^2 = a^2 - 2ab + b^2$

$$L = \int_1^3 \sqrt{1 + y^4 - \frac{2\cdot\cancel{y^2}}{4\cancel{y^2}} + \frac{1}{16y^4}}\, dy$$

$\int y^{-2}\, dy = \frac{y^{-1}}{-1}$

$$L = \int_1^3 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}}\, dy = \int_1^3 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)}\, dy$$

$$L = \int_1^3 \left(y^2 + \frac{1}{4y^2}\right) dy = \left(\frac{y^3}{3} - \frac{1}{4y}\right)\Big|_1^3 = \underbrace{\frac{27}{3}}_{9} - \frac{1}{12} - \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= \frac{108 - 1 - 4 + 3}{12} = \frac{106}{12} = \underline{\frac{53}{6}}$$