Gruppe10_IT17tb_S8_Aufg1

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Aufgabe a)

$y^{(4)} + 1.1y''' - 0.1y'' - 0.3y = \sin x + 5$ mit $y(0) = y''(0) = y'''(0) = 0$ und $y'(0) = 2$

① $y^{(4)} = \sin(x) + 5 - 1{,}1\,y''' + 0{,}1\,y'' + 0{,}3y$

② $z_1(x) = y(x) \qquad z_3(x) = y''(x)$
$z_2(x) = y'(x) \qquad z_4(x) = y'''(x)$

③ $z_1'(x) = y'(x) \quad (= z_2(x))$
$z_2'(x) = y''(x) \quad (= z_3(x))$
$z_3'(x) = y'''(x) \quad (= z_4(x))$
$z_4'(x) = y^{(4)}(x)$

$= \sin(x) + 5 - 1{,}1\,y''' + 0{,}1\,y'' + 0{,}3y$

$= \sin(x) + 5 - 1{,}1\,z_4(x) + 0{,}1\,z_3(x) + 0{,}3\,z_1(x)$

④ $$z' = \begin{pmatrix} z_1' \\ z_2' \\ z_3' \\ z_4' \end{pmatrix} = \begin{pmatrix} z_2 = y'(x) \\ z_3 = y''(x) \\ z_4 = y'''(x) \\ \sin(x)+5-1{,}1z_4+0{,}1z_3+0{,}3z_1 \end{pmatrix} = \underline{f(x,z)} \qquad z(0) = z_0 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

Euler

$$z^{(1)} = z^{(0)} + hf(x_0, z^{(0)})$$
$$= z^{(0)} + 0{,}1 \begin{pmatrix} z_2^{(0)} \\ z_3^{(0)} \\ z_4^{(0)} \\ \sin(x_0)+5-1{,}1z_4^{(0)}+0{,}1z_3^{(0)}+0{,}3z_1^{(0)} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0{,}1 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0{,}2 \\ 2 \\ 0 \\ 0{,}5 \end{pmatrix}}}$$

Runge-Kutta

$$\triangleright\ k_1 = f(x_0, z^{(0)}) = \underline{\begin{pmatrix} 2 \\ 0 \\ 0 \\ 5 \end{pmatrix}}$$

$$\triangleright\ k_2 = f\left(x_0 + \frac{h}{2}, z^{(0)} + \frac{h}{2}k_1\right) = \underline{\begin{pmatrix} 2 \\ 0 \\ 0{,}25 \\ 4{,}8050 \end{pmatrix}}$$

$$\hookrightarrow \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0{,}05 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0{,}1 \\ 2 \\ 0 \\ 0{,}25 \end{pmatrix}$$

$$\triangleright\ k_3 = f\left(x_0 + \frac{h}{2}, z^{(0)} + \frac{h}{2}k_2\right) = \begin{pmatrix} 2 \\ 0{,}0125 \\ 0{,}2402 \\ 4{,}8120 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0{,}05 \begin{pmatrix} 2 \\ 0 \\ 0{,}25 \\ 4{,}8050 \end{pmatrix}$$

$(2{,}0012$

$$\begin{pmatrix} \vdots \\ 0 \\ 0 \end{pmatrix} + 0{,}05 \begin{pmatrix} \vdots \\ 0{,}25 \\ 4{,}8050 \end{pmatrix}$$

$$\triangleright\ k_4 = f\left(x_0 + h,\ \underbrace{z^{(0)} + h k_3}_{\begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0{,}1 \begin{pmatrix} 2 \\ 0{,}0125 \\ 0{,}2402 \\ 4{,}8170 \end{pmatrix}}\right) = \begin{pmatrix} 2{,}0013 \\ 0{,}0240 \\ 0{,}4817 \\ 4{,}6324 \end{pmatrix}$$

$$\Rightarrow z^{(1)} = z^{(0)} + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) = \underline{\underline{\begin{pmatrix} 0{,}2 \\ 2{,}0008 \\ 0{,}0244 \\ 0{,}4813 \end{pmatrix}}}$$

Aufgabe b)

$x^2y'' + xy' + (x^2 - n^2)y = 0$ mit $y(1) = y'(1) = 2$

(1) $x^2 y'' = -xy' - (x^2 - n^2)y$

$$y'' = \frac{-xy' - (x^2 - n^2)y}{x^2}$$

(2) $z_1(x) = y(x)$

$z_2(x) = y'(x)$

(3) $z_1'(x) = z_2(x) = y'(x)$

$$z_2'(x) = y''(x) = \frac{-xy' - (x^2 - n^2)y}{x^2}$$

$$= \frac{-x z_2(x) - (x^2 - n^2) z_1(x)}{x^2}$$

(4)

$$z' = \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} = \begin{pmatrix} z_2 = y'(x) \\ \dfrac{-x z_2 - (x^2 - n^2) z_1}{x^2} \end{pmatrix} \quad \text{mit } z(1) = z_0 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\frac{-\cancel{x} z_2}{\cancel{x^2}\,x} - \frac{\cancel{x^2} z_1}{\cancel{x^2}} + \frac{n^2 z_1}{x^2}$$

Euler mit $n^2 = 1$

$$z^{(1)} = z^{(0)} + h f(x_0, z^{(0)}) = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + 0{,}1 \begin{pmatrix} 2 \\ -\frac{2}{1} - 2 + \frac{2}{1} \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ 1{,}8 \end{pmatrix}}}$$

Runge-Kutta

$$\triangleright\ k_1 = f(x_0, z^{(0)}) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\triangleright\ k_2 = f\left(x_0 + \frac{h}{2},\ z^{(0)} + \frac{h}{2} \cdot k_1\right) = \begin{pmatrix} 1{,}9000 \\ -2{,}0048 \end{pmatrix}$$

$$\triangleright\ k_3 = f\left(x_0 + \frac{h}{2},\ z^{(0)} + \frac{h}{2} k_2\right) = \begin{pmatrix} 1{,}7996 \\ [illegible] \end{pmatrix}$$

$\triangleright\ k_3 = f\left(x_0 + \frac{h}{2}, z^{(0)} + \frac{h}{2} k_2\right) = \begin{pmatrix} 1{,}7996 \\ -2{,}0161 \end{pmatrix}$

$\triangleright\ k_4 = f\left(x_0 + h, z^{(0)} + h k_3\right) = \begin{pmatrix} 2{,}19 \\ 1{,}7996 \end{pmatrix}$

$\Rightarrow z_1 = \begin{pmatrix} 2{,}19 \\ 1{,}7994 \end{pmatrix}$