Übungsserie 5

2) geg: $\frac{dy}{dx} = \frac{x^2}{y} = f$ dem Intervall $0 < x < 2.1$ mit $y(0) = 2$ und $h = 0.7 = \frac{b-a}{n} \rightarrow \frac{2.1}{n} = 0.7 \rightarrow n = 3$

a) Euler - Verfahren

$x_{i+1} = x_i + h$

$x_0 = 0$

$x_1 = 0.7$

$x_2 = 1.4$

$x_3 = 2.1$

$y_{i+1} = y_i + h \cdot f(x_i + y_i)$

$y_0 = 2$ (weil geg.)

$y_1 = 2 + 0.7 \cdot \left(\frac{0}{2}\right) = 2$

$y_2 = 2 + 0.7 \cdot \left(\frac{0.7^2}{2}\right) = 2.1715$

$y_3 = 2.1715 + 0.7 \cdot \left(\frac{1.4^2}{2.1715}\right) = 2.8033$

abs. Fehler $= |y(x_i) - y_i|$

$y(x_i) = \sqrt{\frac{2x^3}{3} + 4}$ = exakte Lösung (geg.)

$|\sqrt{4} - 2| = 0$

$|2.0564 - 2| = 0.0564$

$|2.4144 - 2.1715| = 0.2429$

$|3.1697 - 2.8033| = 0.3864$

b) Mittelpunkt - Verfahren

$x_{i+1} = x_i + h$

$x_0 = 0$

$x_1 = 0.7$

$x_2 = 1.4$

$x_3 = 2.1$

$x_{\frac{h}{2}} = x_i + \frac{h}{2}$

$x_{\frac{h}{2}} = 0.35$

$x_{\frac{h}{2}} = 1.05$

$x_{\frac{h}{2}} = 1.75$

$y_{\frac{h}{2}} = y_i + \frac{h}{2} \cdot f(x_i, y_i)$

$y_{\frac{h}{2}} = 2 + 0.35 \cdot \left(\frac{0}{2}\right) = 2$

$y_{\frac{h}{2}} = 2.0429 + 0.35 \cdot \left(\frac{0.7^2}{2.0429}\right) = 2.1268$

$y_{\frac{h}{2}} = 2.4057 + 0.35 \cdot \left(\frac{1.4}{2.4057}\right) = 2.6909$

$y_{i+1} = y_i + h \cdot f(x_{\frac{h}{2}}, y_{\frac{h}{2}})$

$y_0 = 2$

$y_1 = 2 + 0.7 \cdot \left(\frac{0.35^2}{2}\right) = 2.0429$

$y_2 = 2.0429 + 0.7 \cdot \left(\frac{1.05^2}{2.1268}\right) = 2.4057$

$y_3 = 2.4057 + 0.7 \cdot \left(\frac{1.75}{2.6909}\right) = 3.2024$

abs. Fehler

$|2 - 2| = 0$

$|2.0564 - 2.0429| = 0.0135$

$|2.4144 - 2.4057| = 0.0087$

$|3.1697 - 3.2024| = 0.0127$

b) Modifiziertes Euler - Verfahren

$y_{i+1}^{euler} = y_i + h \cdot f(x_i + y_i)$

$y_1^{euler} = 2 + 0.7 \cdot \left(\frac{0}{2}\right) = 2$

$y_2^{euler} = 2.0857 + 0.7 \cdot \left(\frac{0.7^2}{2.0857}\right) = 2.2502$

$y_3^{euler} = 2.4728 + 0.7 \cdot \left(\frac{1.4^2}{2.4728}\right) = 3.0276$

$y_{i+1} = y_i + h \cdot \frac{1}{2}\left(f(x_i, y_i) + f(x_{i+1} + y_{i+1}^{Euler})\right)$

$y_0 = 2$

$y_1 = 2 + 0.7 \cdot \frac{1}{2} \cdot \left[\left(\frac{0}{2}\right) + \left(\frac{0.7^2}{2}\right)\right] = 2.0857$

$y_2 = 2.0857 + 0.7 \cdot \frac{1}{2} \cdot \left[\left(\frac{0.7^2}{2.0857}\right) + \left(\frac{1.4^2}{2.2502}\right)\right] = 2.4728$

$y_3 = 2.4728 + 0.7 \cdot \frac{1}{2} \cdot \left[\left(\frac{1.4^2}{2.4728}\right) + \left(\frac{2.1^2}{3.0276}\right)\right] = 3.26$

abs. Fehler

$|2 - 2| = 0$

$|2.0564 - 2.0857| = 0.0294$

$|2.4144 - 2.4728| = 0.0584$

$|3.1697 - 3.26| = 0.0703$