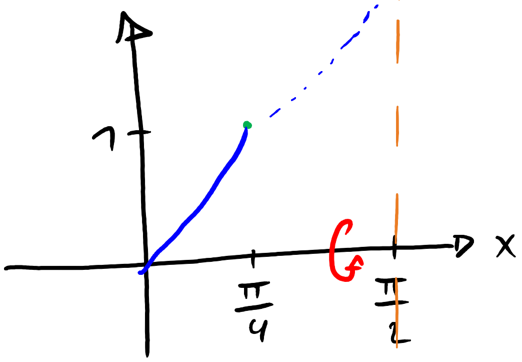
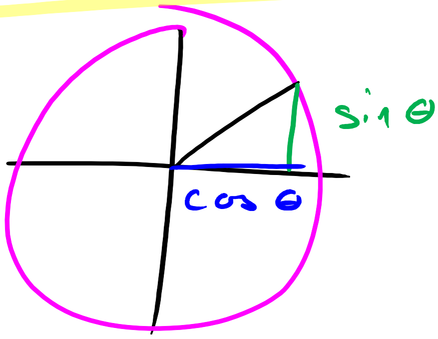
Kapitel 6.4

$$\underbrace{\text{Oberfläche}}_{O} = \int 2\pi \cdot \underbrace{\text{Radius}}_{y(x)} \cdot \underbrace{\sqrt{1+(y')^2}}_{dL}\, dx$$

① $y = \tan x = \dfrac{\sin x}{\cos x}$ } Steigung des Radius'





	x	sin x	cos x	$\frac{\sin x}{\cos x}$
0° =	0	0	1	0
45° =	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
90° =	$\frac{\pi}{2}$	1	0	"$\frac{1}{0}$" = ∞

$$y' = \left[\frac{\sin x}{\cos x}\right]' = \frac{\overbrace{\cos^2 x + \sin^2 x}^{\text{gibt immer } 1!}}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \qquad (\,)^2$$

$$O = 2\pi \int_0^{\frac{\pi}{4}} \tan x \cdot \sqrt{1 + \frac{1}{\cos^4 x}}$$

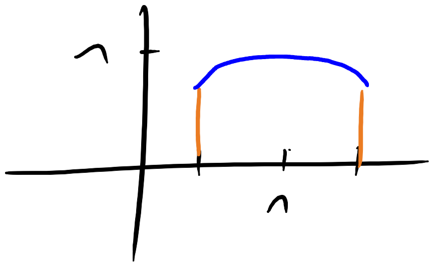
$\Rightarrow$ TR: 3.839077045

(14) $y = \sqrt{2x - x^2} = (2x - x^2)^{\frac{1}{2}}$

$$y' = \frac{1}{2}(2x - x^2)^{-\frac{1}{2}}(2 - 2x)$$

$$= \frac{1 - x}{\sqrt{2x - x^2}}$$

x	$\sqrt{2x - x^2}$
0	0
0.5	$\frac{3}{4}$
1	1
1.5	$\frac{3}{4}$
2	0



$$O = \int_{0.5}^{1.5} 2\pi \sqrt{2x - x^2} \cdot \sqrt{1 + \frac{(1-x)^2}{2x - x^2}}\, dx$$

$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

$$= 2\pi \int_{0.5}^{1.5} \sqrt{(2x - x^2)\left(1 + \frac{(1-x)^2}{2x - x^2}\right)}\, dx$$

$$= 2\pi \int_{0.5}^{1.5} \sqrt{2x - x^2 + (1-x)^2}\, dx$$

$$= 2\pi \int_{0.5}^{1.5} \sqrt{\cancel{2x} - \cancel{x^2} + 1 - \cancel{2x} + \cancel{x^2}}\, dx = 2\pi \int_{0.5}^{1.5} 1\, dx = 2\pi \Big|_{0.5}^{1.5} = 2\pi(1.5 - 0.5)$$

$$= \underline{\underline{2\pi}}$$

16 $x = \frac{y^3}{3}$ Rotation an y-Achse $0 \le y \le 1$

$$O = 2\pi \int_0^1 \frac{y^3}{3} \cdot \underbrace{\sqrt{1+y^4}}_{u = 1+y^4}\, dy$$

$$\frac{du}{dy} = 4y^3$$

$$dy = \frac{du}{4y^3}$$

$$= 2\pi \int \frac{\cancel{y^3}}{3} \sqrt{u}\, \frac{du}{\cancel{4y^3}} = \frac{\pi}{6} \int_{u(0)}^{u(1)} u^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{6} \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_1^2 = \frac{\pi}{6} \cdot \left[\frac{2^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\frac{3}{2}} \right]$$

$$= \frac{2\pi}{18} \left[2^{\frac{3}{2}} - 1 \right] = \frac{\pi}{9} \left[\sqrt{8} - 1 \right]$$

x	y
0	0
$\frac{1}{3 \cdot 64}$	$\frac{1}{4}$
$\frac{1}{24}$	$\frac{1}{2}$
$\frac{1}{3}$	1