3. a) $\frac{\partial f_1}{\partial x_1} = 5x_2 = 10$ $\quad \frac{\partial f_1}{\partial x_2} = 5x_1 = 5$

$\frac{\partial f_2}{\partial x_1} = 2x_1x_2^2 + 1 = 9$ $\quad \frac{\partial f_2}{\partial x_2} = 2x_1^2x_2 + 2 = 6$

$$Df(x_1, x_2) = \begin{pmatrix} 5x_2 & 5x_1 \\ 2x_1x_2^2+1 & 2x_1^2x_2+2 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 9 & 6 \end{pmatrix}$$

b) $\frac{\partial f_1}{\partial x_1} = \frac{2x_1}{x_1^2+x_2^2} = \frac{2}{5}$ $\quad \frac{\partial f_1}{\partial x_2} = \frac{2x_2}{x_1^2+x_2^2} = \frac{4}{5}$ $\quad \frac{\partial f_1}{\partial x_3} = 2x_3 = 6$

$\frac{\partial f_2}{\partial x_1} = 2x_1 = 2$ $\quad \frac{\partial f_2}{\partial x_2} = 2x_2 \cdot e^{x_2^2+x_3^2} = 4e^{13}$ $\quad \frac{\partial f_2}{\partial x_3} = 2x_3 \cdot e^{x_2^2+x_3^2} = 6e^{13}$

$\frac{\partial f_3}{\partial x_1} = \frac{2x_1}{(x_1^2+x_3^2)^2} = \frac{1}{25}$ $\quad \frac{\partial f_3}{\partial x_2} = 2x_2 = 4$ $\quad \frac{\partial f_3}{\partial x_3} = -\frac{2x_3}{(x_1^2+x_3^2)^2} = -\frac{3}{50}$

$$Df(x_1, x_2, x_3) = \begin{pmatrix} \frac{2}{5} & \frac{4}{5} & 6 \\ 2 & 4e^{13} & 6e^{13} \\ \frac{1}{25} & 4 & -\frac{3}{50} \end{pmatrix}$$

$$Df(x_1,x_2,x_3,x_4) = \begin{pmatrix} -\sin(x_2 x_3)\cdot\sin(x_1) & x_3\cdot\cos(x_2\cdot x_3)\cdot\cos(x_1) & x_2\cdot\cos(x_2\cdot x_3)\cdot\cos(x_1) & 0 \\ \cdot & -\sin(x_3\cdot x_4)\cdot\sin(x_2) & x_4\cdot\cos(x_3\cdot x_4)\cdot\cos(x_2) & x_3\cos(x_3\cdot x_4)\cdot\cos(x_2) \\ x_4\cdot\cos(x_1\cdot x_3)\cdot\cos(x_3) & 0 & -\sin(x_1\cdot x_4)\cdot\sin(x_3) & x_1\cdot\cos(x_1\cdot x_4)\cdot\cos(x_3) \\ x_2\cdot\cos(x_1\cdot x_4)\cdot\cos(x_4) & x_1\cdot\cos(x_1\cdot x_2)\cdot\cos(x_4) & 0 & -\sin(x_1\cdot x_2)\cdot\sin(x_4) \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 5{,}89e^{-33} & 3{,}75e^{-33} & 0 \\ 0 & -0{,}8415 & 3{,}31e^{-17} & 5{,}197e^{-17} \\ -9.78e^{-17} & 0 & -1 & 5{,}9e^{-33} \\ 3.31e^{-17} & 5{,}197e^{-17} & 0 & -0{,}8415 \end{pmatrix}$$

⇒ gerechnet mit Matlab
→ unter der Maschinengenauigkeit!

$$= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -0{,}8415 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -0{,}8415 \end{pmatrix}$$