Übungsserie 10

1. Geg: Stützpunkte:

x:	0	1	2	3
y:	2	1	2	2

Ges: Koeffizienten: a_i, b_i, c_i, d_i der kubische Polynome S_i $i=0,1,2$
mittels natürliche kubische Splinefunktion:

1.) $a_i = y_i$

$a_0 = 2 \qquad a_1 = 1 \qquad a_2 = 2 \qquad a_3 = 2$

2.) $h_i = x_{i+1} - x_i$

$h_0 = 1-0 = 1 \qquad h_1 = 2-1 = 1 \qquad h_2 = 3-2 = 1$

3.) $c_0 = 0, \qquad c_n = 0 \qquad n = i-1 = 3$

4.) Berechnen $c_1, \dots, c_{n-1}$

a) $i=1$:

$$2(h_0+h_1)\cdot c_1 + h_1 \cdot c_2 = 3\cdot\frac{y_2-y_1}{h_1} - 3\,\frac{y_1-y_0}{h_0}$$

$$4\cdot c_1 + c_2 = 3 + 3 = 6$$

b) $i=2$ bis $n-2$:

$$h_{i-1}\cdot c_{i-1} + 2\cdot(h_{i-1}+h_i)\cdot c_i + h_i\cdot c_{i+1} = 3\cdot\frac{y_{i+1}-y_i}{h_i} - 3\cdot\frac{y_i - y_{i-1}}{h_{i-1}}$$

Brauchen wir nicht weil $i = 2 = n-1$ (weiter bei c))

c) $i = n-1$

$$h_{n-2}\cdot c_{n-2} + 2\cdot(h_{n-2}+h_{n-1})\cdot c_{n-1} = 3\cdot\frac{y_n-y_{n-1}}{h_{n-1}} - 3\cdot\frac{y_{n-1}-y_{n-2}}{h_{n-2}}$$

$$c_1 + 4\cdot c_2 = -3$$

$$c_1 = -3 - 4c_2$$

$\Rightarrow$ In a) einsetzen:

$$4(-4\cdot c_2 - 3) + c_2 = 6$$

$$-16\cdot c_2 - 12 + c_2 = 6$$

$$-18 = 15\cdot c_2$$

$$c_2 = -\frac{6}{5} \qquad \Rightarrow c_1 = -3 + \frac{24}{5} = \frac{9}{5}$$

5) $b_i = \frac{y_{i+1}-y_i}{h_i} - \frac{h_i}{3}\cdot(c_{i+1} + 2c_i)$

$b_0 = -1 - \frac{1}{3}\cdot\left(\frac{9}{5}\right) = -\frac{8}{5} \qquad b_1 = 1 - \frac{1}{3}\left(-\frac{6}{5} + \frac{18}{5}\right) = \frac{1}{5} \qquad b_2 = \frac{4}{5}$

6) $d_i = \frac{1}{3\cdot h_i}\cdot(c_{i+1} - c_i)$

$d_0 = \frac{3}{5} \qquad d_1 = -1 \qquad d_2 = \frac{2}{5}$

7) Polynome bestimmen

$$S_i(x) = a_i + b_i\cdot(x-x_i) + c_i\cdot(x-x_i)^2 + d_i(x-x_i)^3$$

$$S_0(x) = 2 - \frac{8}{5}\cdot(x-0) + 0\cdot(x-0)^2 + \frac{3}{5}(x-0)^3$$

$$S_1(x) = 1 + \frac{1}{5}\cdot(x-1) + \frac{9}{5}\cdot(x-1)^2 - 1\,(x-1)^3$$

$$S_2(x) = 2 + \frac{4}{5}\cdot(x-2) + \frac{6}{5}\cdot(x-2)^2 + \frac{2}{5}(x-2)^3$$