1a) $y^{(4)} + 1.1y''' - 0.1y'' - 0.3y = \sin x + 5$

$$\vec{z}' = \begin{pmatrix} z_2 \\ z_3 \\ z_4 \\ \sin(x) + 5 - 1.1z_4 + 0.1z_3 + 0.3z_1 \end{pmatrix} = \vec{f}(x, \vec{z}) \qquad \vec{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

$$\vec{z}(0) = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{matrix}, \quad x_0 = 0, \quad h = 0.1$$

Euler:

$x_{i+1} = x_i - h$

$\vec{z}^{(i+1)} = \vec{z}^{(i)} + h \cdot \vec{f}(x_i, \vec{z}^{(i)})$

$$\Rightarrow \vec{z}^{(1)} = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0.1 \cdot \begin{pmatrix} 2 \\ 0 \\ 0 \\ \underbrace{\sin(0)}_{0} + 5 - \underbrace{1.1 \cdot 0}_{0} + \underbrace{0.1 \cdot 0}_{0} + \underbrace{0.3 \cdot 0}_{0} \end{pmatrix}$$

$$= \begin{matrix} 0 \\ 2 \\ 0 \\ 0 \end{matrix} + \begin{matrix} 0.2 \\ 0 \\ 0 \\ 0.5 \end{matrix} = \begin{pmatrix} 0.2 \\ 2 \\ 0 \\ 0.5 \end{pmatrix}$$

$\underline{RK}:$ $x_{i+1} = x_i + h$

$y_{i+1} = y_i + \frac{h}{6}(u_1 + 2u_2 + 2u_3 + u_4)$

$i=0:$ $\vec{u}_1 = \vec{f}(x_0, \vec{z}^{(0)})$

$$= \begin{pmatrix} 2 \\ 0 \\ 0 \\ \sin(0) + 5 - 0 + 0 + 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 5 \end{pmatrix}$$

$$u_2 = f\left(x_0 + \frac{h}{2}, \underbrace{\vec{z}^{(0)} + \frac{h}{2}\vec{u}_1}_{\begin{pmatrix} 0.1 \\ 2 \\ 0 \\ 0.25 \end{pmatrix}}\right)$$

einsetzen

$$k_2 = \begin{pmatrix} 2 \\ 0 \\ 0.25 \\ \underbrace{\sin(0.05) + 5 - 1.1 \cdot 0.25 + 0.1 \cdot 0 + 0.3 \cdot 0.1}_{} \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0.25 \\ 4.805 \end{pmatrix}$$

1a) $\underline{RK}$

$$k_3 = f\left(x_0 + \frac{h}{2}, \; \vec{z}^{(0)} + \frac{h}{2}\vec{u_2}\right)$$

$$= f\left(0.05, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0.05 \cdot \begin{pmatrix} 2 \\ 0 \\ 0.25 \\ 4.805 \end{pmatrix}\right)$$

$$= f\left(0.05, \begin{pmatrix} 0.1 \\ 2 \\ 0.0125 \\ 0.2402 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 0.0125 \\ 0.2402 \\ \sin(0.05 + 5 - (1.1 \cdot 0.2402) \\ +0.1 \cdot 0.0125 + \\ 0.3 \cdot 0.1) \end{pmatrix}$$

$$k_3 = \begin{pmatrix} 2 \\ 0.0125 \\ 0.2402 \\ 4.817 \end{pmatrix}$$

$$k_4 = f\left(0.1, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0.1 \cdot u_3\right)$$

$$= f\left(0.1, \begin{pmatrix} 0.2 \\ 2.00125 \\ 0.02402 \\ 0.4817 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2.00125 \\ 0.02402 \\ 0.4817 \\ \sin(0.1) + 5 - (1.1 \cdot 0.4817) + (0.1 \cdot 0.02402) + \\ 0.3 \cdot 0.2 \end{pmatrix}$$

$$\underset{\sim}{4.6363}$$

$$z_1 = z_0 + \frac{h}{6}(u_1 + 2u_2 + 2u_3 + u_4)$$

$$= \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + \frac{0.1}{6} \left(\begin{pmatrix} 2 \\ 0 \\ 0 \\ 5 \end{pmatrix} + 2 \cdot \begin{pmatrix} 2 \\ 0 \\ 0.25 \\ 4.805 \end{pmatrix} + 2 \cdot \begin{pmatrix} 2 \\ 0.0125 \\ 0.2402 \\ 4.817 \end{pmatrix} \right.$$

$$\left. + \begin{pmatrix} 2.00125 \\ 0.02402 \\ 0.4817 \\ 4.6323 \end{pmatrix} \right)$$

$$z_1 = \begin{pmatrix} 0.2 \\ 2.0008 \\ 0.0244 \\ 0.4813 \end{pmatrix}$$

1b) $\quad x^2 y'' + xy' + (x^2 - n^2)y = 0$

$\qquad$ mit $\quad y(1) = y'(1) = 2 \qquad\qquad n^2 = 1$

① $\quad y'' = \dfrac{-xy' + (x^2 - 1)y}{x^2}$

② $\quad \begin{array}{l} z_1 = y \\ z_2 = y' \end{array}$

③ $\quad z_1' = y'$

$\qquad z_2 = y'' = \dfrac{-xy' + (x^2 - 1)y}{x^2}$

$$\vec{z}' = \begin{pmatrix} z_2 \\ \dfrac{-xz_2 + (x^2 - 1)z_1}{x^2} \end{pmatrix}$$

Euler: $\qquad x_0 = 1$

$$\vec{z}(1) = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + 0.1 \left(\underbrace{\begin{array}{c} 2 \\ \dfrac{-1 + (1-1)\cdot 2}{1^2} \end{array}}_{\begin{pmatrix} 2 \\ -2 \end{pmatrix}} \right)$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0.2 \\ -0.2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2.2 \\ 1.8 \end{pmatrix}}}$$

$$\underline{RK} \qquad x_0 = 1 \qquad \vec{z}(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{k_1} = \vec{f}(1, \vec{z}^{(0)})$$

$$= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \qquad \begin{pmatrix} 2.1 \\ 1.9 \end{pmatrix}$$

$$\vec{k_2} = \vec{f}\left(1+0.05, \ \vec{z}^{(0)} + \frac{h}{2} \vec{k_1}\right)$$

$$= \frac{-1.05 \cdot 1.9 + (1.05^2 - 1) \cdot 2.1}{1.05^2}$$

$$\vec{k_2} = \begin{pmatrix} 1.9 \\ -2.0048 \end{pmatrix}$$

$$\vec{k_3} = \vec{f}\left(1.05, \ \vec{z}^{(0)} + \frac{h}{2} \vec{k_2}\right)$$

$$\begin{pmatrix} 2.095 \\ 1.8998 \end{pmatrix}$$

$$= \frac{-1.05 \cdot 1.8998 + (1.05^2 - 1) \cdot 2.095}{1.05^2}$$

$$= \begin{pmatrix} 1.8998 \\ -2.0041 \end{pmatrix}$$

$$u_4 = \left(1+0.1,\ \vec{z}^{(0)} + 0.1 \cdot \vec{u_3}\right)$$

$$= \left(1.1,\ \begin{pmatrix} 2.18958 \\ 1.79959 \end{pmatrix}\right)$$

$$= \frac{-1.1 \cdot 1.79959 + (1.1^2 - 1)\, 2.18958}{1.1^2}$$

$$u_4 = \begin{pmatrix} 1.79959 \\ -2.0160 \end{pmatrix}$$

$$z^{(1)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \cdot\ \frac{0.1}{6}\left(u_1 + 2u_2 + 2u_3 + u_4\right) =$$

$$\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{0.1}{6}\left(\begin{pmatrix} 2 \\ -2 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1.9 \\ -2.0048 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1.898 \\ -2.0041 \end{pmatrix}\right.$$

$$\left. + \begin{pmatrix} 1.79959 \\ -2.0160 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2.19 \\ 1.7994 \end{pmatrix}$$