Übungsserie 2

1)

h	D_{i0}	P_{i1}	D_{i2}	D_{i3}
0.1	0.975	0.9996	0.9999961	0.9999999819
0.05	0.988	0.9998	0.999999952	
0.025	0.994	0.99997		
0.0125	0.997			

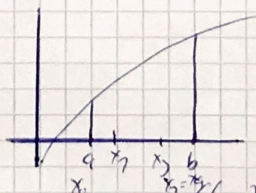| h	E_{i0}	E_{i1}	E_{i2}	F_{i3}
0.1	$2.49 \cdot 10^{-2}$	$3.94 \cdot 10^{-4}$	$3.64 \cdot 10^{-6}$	$1.8086 \cdot 10^{-8}$
0.5	$1.22 \cdot 10^{-2}$	$1.01 \cdot 10^{-4}$	$4.72 \cdot 10^{-7}$	
0.25	$6.19 \cdot 10^{-3}$	$2.56 \cdot 10^{-5}$		
0.125	$3.112 \cdot 10^{-3}$			

3)

$$T_f = \frac{f(a) + f(b)}{2} \cdot (b-a) \qquad\qquad T_f(h) = h\left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right)$$

wobei $[a,b]$ Inkrall mit $h = \frac{b-a}{n}$

Annahme: wenn $[a,b]$ in Subintervalle unterteilt wird, kann
$T_f(h)$ aus T_f hergeleitet werden.

Mit $\quad x_i = a + ih$
$\qquad x_0 = a$
$\qquad x_n = b$

$$\Rightarrow \quad \frac{f(x_0) + f(x_n)}{2} \cdot (x_n - x_0) + \frac{f(x_0) + f(x_1)}{2} \cdot h + \frac{f(x_1) + f(x_2)}{2} \cdot h$$

$$+ \frac{f(x_2) + f(x_3)}{2} \cdot h$$

$$= \frac{h}{2}\left(f(x_0) + f(x_1) + f(x_1) + f(x_2) + f(x_2) + f(x_3) + \dots \right)$$

$$= \frac{h}{2}\left(f(x_0) + 2 \cdot f(x_1) + 2 \cdot f(x_2) + 2 \cdot f(x_3) \dots + f(x_n) \right)$$

$$= \frac{h}{2} \cdot \left(f(x_0) + f(x_n) \right) + h \cdot \left(\underset{i=n}{\underbrace{f(x_1)}} + f(x_2) + f(x_3) + \dots + f(x_{n-1}) \right)$$

$$= h \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right)$$

4)

a)
$$t = \int_{v(t_0)}^{v(t)} \frac{m}{R(v)} \, dv$$

$m = 10 \, kg$

$v(t_0) = 20 \, m/s$

$v(t) = 5 \, m/s$

$n = 5$

$$t = \int_{20}^{5} \frac{10}{-v \cdot \sqrt{v}} \, dv = -10 \cdot \int_{20}^{5} v^{-3/2} \, dv$$

$$= +10 \cdot \left[-2v^{-1/2} \right]_{5}^{20} = 4.47216 \, s = I$$

$$R f(h) = 10 \cdot \left(\left(\tfrac{13}{2} \right)^{-3/2} + \left(\tfrac{19}{2} \right)^{-3/2} + \left(\tfrac{25}{2} \right)^{-3/2} + \left(\tfrac{31}{2} \right)^{-3/2} + \left(\tfrac{37}{2} \right)^{-3/2} \right)$$

$$= \underline{4.38235}$$

absolute Fehler: $\left| I - R f(h) \right| = 0.0898 = \underline{8.98 \cdot 10^{-2}}$

b) $f = v^{-3/2}$, $n = 5$, $h = 3$, $x_i = a + i \cdot h$, $x_0 = 5$

$$T f(h) = h \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right)$$

$$= 3 \cdot \left(\frac{x_0^{-3/2} + x_5^{-3/2}}{2} + x_1^{-3/2} + x_2^{-3/2} + x_3^{-3/2} + x_4^{-3/2} \right)$$

$$= \underline{4.658185}$$

$$\left| I - T f(h) \right| = 0.18602 \, s = \underline{1.8602 \cdot 10^{-1} \, s} = E$$