1 Question I

Proved in lab7.mlx file.

2 Question II

Note that $d=3$ in the following proves, for the d=1 case, let the magnitude in y,z of the vectors equal zero.

2.1 $\frac{\partial F}{\partial \rho}>0$ and $\frac{\partial^2 F}{\partial \rho^2}=0$

Proved in lab7.mlx file.

2.2 $F|_{\rho=0}\geqslant 0$

Notice that $F|_{\rho_*=0}\leqslant F|_{\rho_*>0}$, thus:

$$
\begin{aligned}
& F|_{\rho=0} \\
\geqslant & F|_{\rho=\rho_*=0} \\
= & (|B|^2 v-(v\cdot B)B)\cdot(-v_*)-(|B_*|^2 v_*-(v_*\cdot B_*)B_*)\cdot(-v_*) \\
& +((1-|v_*|^2)B_*+(v_*\cdot B_*)v_*)\cdot(-B)-((1-|v_*|^2)B_*+(v_*\cdot B_*)v_*)\cdot(-B_*) \\
& +\frac{(1+|v|^2)|B|^2-(v\cdot B)^2}{2}-\frac{(1+|v_*|^2)|B_*|^2-(v_*\cdot B_*)^2}{2} \\
& +\frac{(1-|v_*|^2)|B|^2+(v_*\cdot B_*)^2}{2}-\frac{(1-|v_*|^2)|B|^2+(v_*\cdot B_*)^2}{2} \\
= & (|B|^2 v-(v\cdot B)B)\cdot(-v_*)+(|B_*|^2|v_*|^2-(v_*\cdot B_*)^2) \\
& +((1-|v_*|^2)B_*+(v_*\cdot B_*)v_*)\cdot(-B)+((1-|v_*|^2)|B_*|^2+(v_*\cdot B_*)^2) \\
& +\frac{(1+|v|^2)|B|^2-(v\cdot B)^2}{2}-\frac{|B_*|^2}{2} \\
& +\frac{(1-|v_*|^2)|B_*|^2+(v_*\cdot B_*)^2}{2}-\frac{|B_*|^2}{2} \\
= & (|B|^2 v-(v\cdot B)B)\cdot(-v_*)+(|B_*|^2|v_*|^2) \\
& +((1-|v_*|^2)B_*+(v_*\cdot B_*)v_*)\cdot(-B)+((1-|v_*|^2)|B_*|^2) \\
& +\frac{(1+|v|^2)|B|^2-(v\cdot B)^2}{2}-|B_*|^2 \\
& +\frac{(1-|v_*|^2)|B_*|^2+(v_*\cdot B_*)^2}{2} \\
= & (|B|^2 v-(v\cdot B)B)\cdot(-v_*) \\
& +((1-|v_*|^2)B_*+(v_*\cdot B_*)v_*)\cdot(-B) \\
& +\frac{(1+|v|^2)|B|^2-(v\cdot B)^2}{2} \\
& +\frac{(1-|v_*|^2)|B_*|^2+(v_*\cdot B_*)^2}{2} \\
= & \frac{(1-|v_*|^2)|B_*-B|^2}{2}+\frac{|v_*-v|^2|B|^2}{2} \\
& +\frac{(v_*\cdot B_*)^2}{2}-(v_*\cdot B_*)(v_*\cdot B)-\frac{(v\cdot B)^2}{2}+(v\cdot B)(v_*\cdot B)
\end{aligned}
$$

$$\geqslant \frac{|v_* - v|^2|B|^2}{2} + \frac{[(v_* \cdot B_*) - (v_* \cdot B)]^2}{2} - \frac{[(v \cdot B) - (v_* \cdot B)]^2}{2}$$
$$= \frac{|v_* - v|^2|B|^2}{2} - \frac{[(v_* - v) \cdot B]^2}{2} + \frac{[(B_* - B) \cdot v_*]^2}{2} \geqslant 0$$

Now we can tell that

$$F|_{\rho=0} \geqslant F|_{\rho=\rho_*=0} \geqslant 0$$

As desired.

2.3 $\boldsymbol{F \geqslant 0}$

The result can be directly derived from 2.1 and 2.2 .

2.4 d=3

Since my previous work did not specify the dimension, this should automatically hold for all d.