AC Circuits

Nolan Tremelling

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Abstract

An experiment exploring the resulting voltage and current in alternating current (AC) circuit given changes in impedance.

1 Introduction

As explored in *Experiment 8, Capacitance and the Oscilloscope*, the behavior of passive linear components (resistors, capacitors, and inductors) is well-known when driven by a voltage and either direct current (DC) or alternating current (AC). These characteristics can be utilized to design many useful systems, such as amplifiers, oscillators, triggers, and much more. The usual analysis of circuits, which includes analyzing behaviors with techniques such as Kirchoff's Loop Rule, nodal and mesh analysis, and equivalent circuits holds in both the DC and AC domains.

Linear components which contain purely real impedance impart no change on the phase of the current passing through them. Further, if these components are *only* in series with the supply voltage, neither will they have an impact on the magnitude of the current. This reveals that, for resistors (which have a purely real impedance of $Z = R$), the resulting voltage is then $V_R = I(t) \times R = I_{max} R sin\omega t$.

Linear components which contain imaginary components of impedance impart a shift in current. This can be seen for both the capacitor, which has an impedance $Z = \frac{1}{j\omega C}$, and the inductor, with an impedance $Z = j\omega L$. This is further explained by the differential relationship to voltage and current, respectively, through the devices. For a capacitor, $I = C\frac{dV}{dt}$, where it is then found that the voltage across a capacitor as a function of time and frequency is given by $V_C = \frac{1}{C}\int I(t)dt = \frac{1}{\omega C} I_{max} sin(\omega t - \frac{\pi}{2})$. For an inductor, $V = L\frac{dI}{dt}$, where it is then found that the voltage across an inductor as a function of time and frequency is given by $V_L = \omega L I_{max} cos\omega t = \omega L I_{max} sin(\omega t + \frac{\pi}{2})$. Compared to the input current, the current in the capacitor is seen to lag with a phase shift of $\phi = -90°$ and the current in the inductor is seen to lead with a phase shift of $\phi = 90°$. Then, analyzing the extremes of frequency, that is $\lim_{\omega \to 0,\infty} V_C, V_L$, it is observed that at low frequencies (that is, DC), inductors act as shorts while capacitors act as open circuits, and at high frequencies, inductors act as open circuits while capacitors act as shorts.

The combination of these components is done through analysis of their impedances. These values are solved by combining impedances in parallel or series, applying nodal or mesh analysis, or other techniques. Regardless of the analysis techniques employed, the resulting phase shift of the circuit is found by combining the real and reactive parts of the circuit to find the total phase vector. For an RLC circuit, this is found to be the resulting magnitude of impedances, given by the real and imaginary parts. Ohm's Law dictates that we then multiply the observed impedance by the maximum current to find maximum voltage: $V_{max} = I_{max}\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$. Figure 1 illustrates the phasor analysis for an RLC circuit.

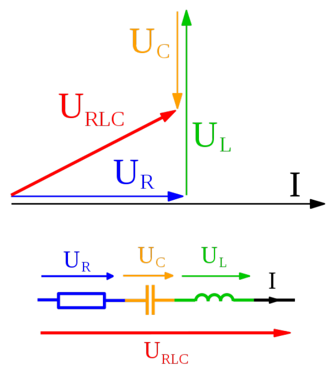

Figure 1: Phasor Diagram for an RLC Circuit.

Finally, it can be seen that the voltage drop across the resistor is maximized when the impedances of the inductor and capacitor are minimized. Unlike with the resistor, these impedances can be minimized because of their dependence on frequency. This frequency, denoted as the natural frequency, ω_0, is found when the reactance of the inductor is equal to that of the capacitor (for the series combination of LC, that is). Setting $\omega_0 L = \frac{1}{\omega_0 C}$, we find that $\omega_0 = \frac{1}{\sqrt{LC}}$. This frequency minimizes the inductances for both components, resulting in a peak in the voltage drop across the resistor. The size of the resistor sets the bandwidth of the peak, and is often defined as the range of frequencies for which the magnitude varies by less than $3dB$, alternatively known as full width at half maximum (FWHM). This is illustrated in Figure 2, where it is clear that the magnitude voltage across the resistor occurs at the frequency where $\omega = \frac{1}{\sqrt{LC}}$.

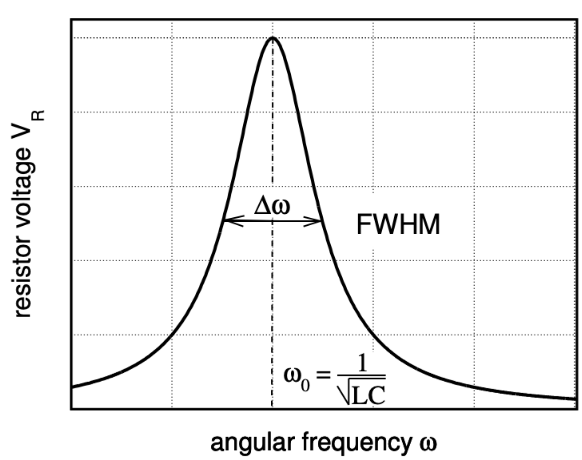

Figure 2: Frequency response of an RLC circuit

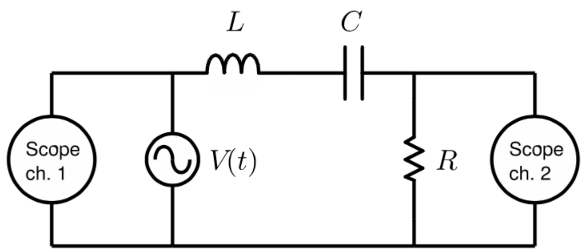

Figure 3: RLC Circuit with Input and Resistor Voltage Measurements

2 Method

This experiment makes use of a simple RLC circuit using a variable resistor, capacitor, and inductor all in series with one another. They are connected to a function generator, and an oscilloscope is used to probe the voltages in shunt with the function generator and the resistor. A schematic for this circuit is shown in Figure 3.

The function generator is set to produce a sinusoidal signal at 1 kHz with a peak-to-peak voltage (V_{pp}) of 20 V.

The resonance of the circuit is investigated using a multimeter to measure the resistance of the 150 mH inductor. With the inductor, capacitor, and the decade resistor box are connected in series with the function generator, the resistance is initially set to 50 Ω in the decade resistor box. The frequency of the function generator is swept to locate the circuit's resonance frequency, indicated by the peaking of the output signal. Observations are made for the peak-to-peak voltage at various frequencies around the resonance frequency. This process is repeated for different resistance settings of 10Ω and 500Ω, with the voltage amplitude recorded at each resistance.

The phase relationship between the driving voltage and the voltage across the resistor is analyzed at, above, and below the resonant frequency and is measured using the oscilloscope's cursor function. This involves recording the time for one full cycle of the driving signal and the time difference between the peak of the driving signal and the peak of the signal across the resistor. The phase difference ϕ is calculated using the formula $\phi = \frac{2\pi(t_R - t_d)}{T_d}$, where t_R and t_d are the time measurements of the resistor voltage and driving voltage, respectively.

For the phase shift analysis of the inductor, the frequency of the function generator is increased to a value well above the resonance frequency (in the range of 10 kHz or greater). At this frequency, the inductor's reactance dominates, determining the overall phase shift in the circuit. The phase shift is measured across the resistor using the same method as before. For the capacitor, the frequency is decreased to well below the resonance frequency (in the range of 10Hz). At this range, the capacitor's reactance becomes dominant, and the phase shift observed across the resistor reflects this.

3 Results and Discussion

To measure the resonance of the RLC circuit, the circuit in 3 is constructed with a $150mH$ inductor, a $500nF$ capacitor, and a resistor: three values are used, 10Ω, 50Ω, and 500Ω. An input voltage of $1KHz, 20V_{pp}$ is generated by the function generator and a multimeter is used to read the voltage across the resistor.

Given the values of the passive components, it is expected that an angular frequency, ω_0, of 581.07Hz will be observed. The resulting plot, showing the peak-to-peak output voltage for given frequencies is shown in Figure 4 and Figure 5, where the later is normalized. There is a range of frequencies for which the voltage peaks, so the peak frequency is determined as the average frequency at which the peak voltage occurs: 571.325Hz for the 10Ω resistor, 560.6Hz for the 50Ω resistor, and 532.975Hz for the 500Ω resistor. The relative accuracy for each resistor is then found to be $1.68\%, 3.52\%$, and 8.28%, showing a strong relationship to the expected value.

Analyzing the Full Width at Half Maximum (FWHM) for each resistor value provides insights into the bandwidth and sharpness of the resonance peak. The FWHM bandwidth for each resistor is found to be 912.5Hz, 1081.5Hz, and 1538.8Hz for the 10Ω, 50Ω, and 500Ω resistor, respectively. These values are found with large accuracy, as the oscilloscope and function generator are generally accurate to hundreths of a Hz in variation, so it is safe to say that these vary by, at worst, ± 1Hz. The variation in FWHM across different resistances suggests that the size of a resistor is directly proportional to its FWHM bandwidth, aligning well with the theoretical understanding that an increase in resistance typically broadens the resonance peak, indicating a wider range of frequencies over which the circuit can resonate.

Further, the effect of varying the resistance on the resonant frequency itself was observed. As resistance increased, the resonant frequency was found with a higher relative error rate. This can be attributed to variation in component values as well as the broader resonance peak.

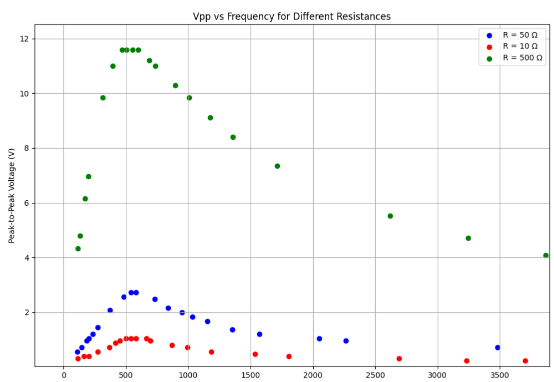

Figure 4: Output Voltage for an Input Range of Frequencies

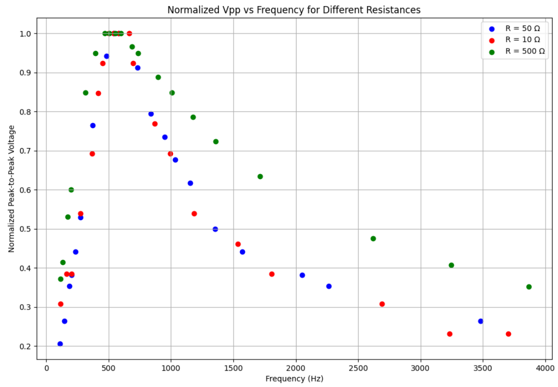

Figure 5: Output Voltage for an Input Range of Frequencies, Normalized

This process is repeated with the large inductor ring of "unknown" value. Plotting the output voltages in a similar manner produces Figure 6, where the measured resonance frequency is 1519.25Hz. Solving the resonant frequency equation for L gives a measured inductance of 0.02195H. This inductor ring was labeled with a value of 0.0200H, suggesting that there was a relative error of 4.51% from the reported value. The FWHM bandwidth is found to be 7692.8Hz in this case with, again, and uncertainty of just a few Hz.

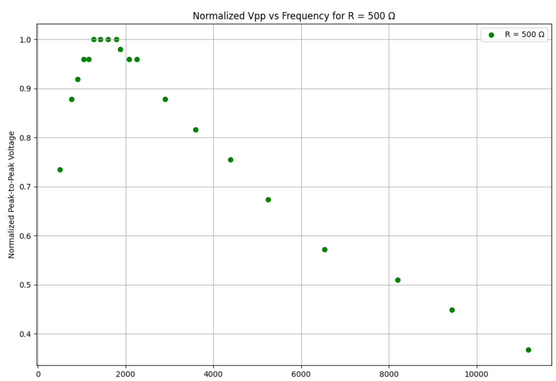

Figure 6: Output Voltage for an Input Range of Frequencies using an Unknown Inductor Value, Normalized

In addition to resonance behavior, the phase relationship between voltage and current plays a crucial role in understanding the dynamics of the RLC circuit. The phase shifts at various frequencies, especially around resonance and at extremes of high and low frequencies can be analyzed to gain insight into the circuit's behavior. These results are seen in Figure 7.

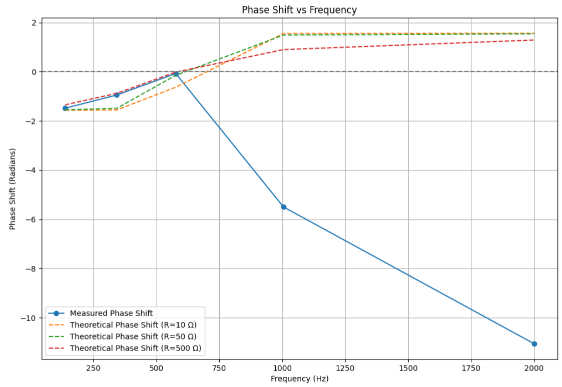

Figure 7: Phase Shift of RLC Circuit

The measured phase shifts were compared with theoretical values calculated using the formula $\tan(\phi) = \frac{\omega L - \frac{1}{\omega C}}{R}$. While there was general agreement between the measured and theoretical values, some discrepancies can be seen. These could be attributed to factors such as the internal resistances of the circuit components, the accuracy of the measurements, and the potential influence of the function generator's output impedance.

The phase difference between the voltage across the resistor and the driving voltage was measured at various frequencies. As expected, the phase shift values were negative near the resonance frequency, indicating that the voltage across the resistor was lagging behind the driving voltage. This lag can be attributed to the influence of the inductive and capacitive reactances in the circuit, which vary with frequency. Particularly, at frequencies close to resonance (138 Hz to 2000 Hz), the phase shifts ranged from -1.48 to -11.06 radians. These measurements align with the theoretical understanding of the phase behavior in an RLC circuit, where the phase shift is governed by the relative magnitudes of the inductive and capacitive reactances compared to the resistance. At a high frequency of 10,000 Hz, the phase shift for the inductor was measured to be 1.63 radians, indicating that the inductor voltage leads the driving voltage, as expected. This leading phase shift is a characteristic of inductive reactance, which increases with frequency. Conversely, at a low frequency of 20 Hz, the phase shift for the capacitor was -1.83 radians, demonstrating that the capacitor voltage lags the driving voltage. This behavior is typical of capacitive reactance, which decreases with frequency.

These observations made on the inductor and capacitor at different frequencies are explained by their fundamental properties. The inductor resists changes in current, more so at higher frequencies due to the generation of a stronger opposing electromagnetic field. Conversely, the capacitor's tendency to block DC signals but allow high-frequency AC signals can be attributed to its charging and discharging behavior.

4 Conclusion

This experiment explored RLC circuits and provided insights into the behavior of resonance and phase relationships under AC inputs. The resonant frequencies, FWHM, and phase shift matched the theoretical understanding of the circuits.