Experiment 1
Velocity, Acceleration, and *g*

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Abstract—An experimental analysis exploring techniques to measure and derive velocity, acceleration, and the gravitational constant, *g*.

1 Introduction

A brief experiment exploring experimental error in calculation for velocity, accleleration, and the gravitational constant.

2 Motion with Constant Velocity

A leveled air track with a rider can be used to observe motion under constant acceleration. In an ideal system, we would expect the velocity vs. time graph to appear as depicted in Figure **??**. In this scenario, the rider travels with a consistent acceleration until it reaches the end of the track. There, it undergoes a perfectly elastic collision with the rubber band, causing the velocity to change direction instantaneously while preserving its magnitude. In the ideal case, there is no loss in energy.

Fig. 1. An example of an ideal collision, where there is no loss observed.

In reality, a perfect system doesn't exist, and energy loss due to factors such as air resistance and imperfect elasticity mean that the velocity's magnitude after the collision will be slightly diminished. Using our initial and final velocity measurements, we can create a model that approximates the ideal case. This approach assumes that when the rider reaches the rubber band, it stops for a brief moment before for beginning its motion back towards its starting point. Real measurements are demonstrated in Figure 2.

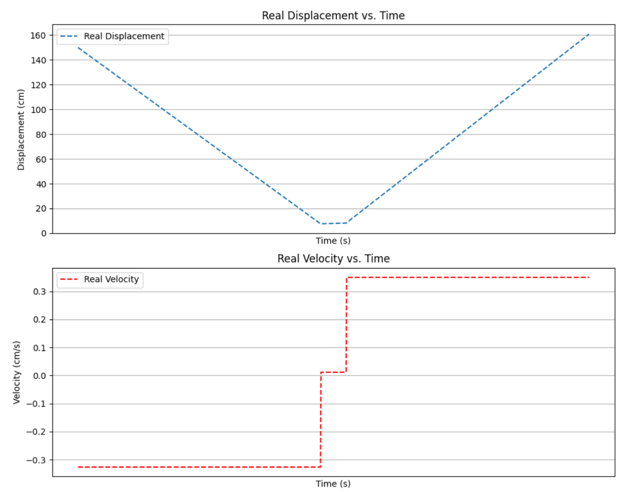

Fig. 2. An example of an ideal collision, where there is no loss observed.

2.1 Coefficient of Restitution

We can quantify the energy loss due to collision with the coefficient of restitution, given by e, where a perfectly elastic collision—showing no loss in energy—would be given by a coefficient of 1.

$$e = \frac{|\vec{v}_f|}{|\vec{v}_i|}$$

A Python script was used to analyze the collected initial and final velocities to find the value of e for each trial. These values were then used to find both the unweighted and weighted mean of e. The results of this analysis are presented in Table 1.

Analysis of the collected data reveals a number of observations. In particular, it can be noted that ($\bar{e}_w$) is only trivially smaller than ($\bar{e}$), however a significantly smaller standard error for the weighted mean suggests that it is a more accurate measurement than that of the unweighted mean. Figure 3 suggests a no correlation between the glider's initial velocity and the coefficient of restitution. This observation aligns with a theoretical understanding that elasticity and energy transfer during a collision are influenced by a material's physical properties, rather than the velocities at which they interact. The measurement of velocities has an inherent level of uncertainty. This is due to

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TABLE 1
Trial-wise Results and Calculated Values

Trial #	v final	v initial	Uncertainty v_f	Uncertainty v_i	e calculated	e uncertainty
1	-0.407	0.434	0.00024	0.00089	0.937788	0.002001
2	-0.166	0.180	0.00063	0.00100	0.922222	0.006205
3	-0.158	0.163	0.00075	0.00100	0.969325	0.007519
4	-0.353	0.377	0.00079	0.00100	0.936340	0.003250
5	-0.409	0.411	0.00050	0.00078	0.995134	0.002246
6	-0.487	0.525	0.00025	0.00056	0.927619	0.001098
7	-0.156	0.167	0.00080	0.00096	0.934132	0.007196
8	-0.322	0.355	0.00036	0.00093	0.907042	0.002584
9	-0.570	0.623	0.00083	0.00110	0.914928	0.002094
10	-0.240	0.264	0.00055	0.00092	0.909091	0.003792

Unweighted mean ($\bar{e}$)	0.9353620452145774
Standard deviation (σ)	0.027642286533999166
Standard error on the mean ($\bar{\sigma}_e$)	0.008741258518243878
Weighted mean ($\bar{e}_w$)	0.9330381560077968
Standard error on the weighted mean ($\bar{\sigma}_{ew}$)	0.0007288707736145523

a lack of precision of the measurement equipment: we are not sure of calibration, potential misalignment of sensors, and changes in airflow for the track. While this list is not exhaustive, a number of small variations in the measurement of velocities can compound and create dependent errors. Finally, we see in Figure 4 that there the averages tend to be fairly evenly distributed, although more tests may reveal that a few higher than usual trials have inflated the average.

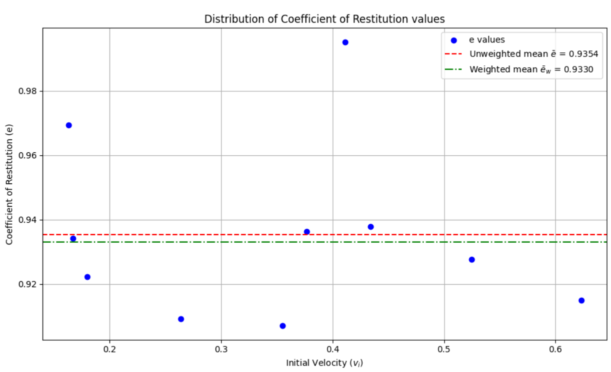

Fig. 3. No strong correlation appears between the coefficient of restitution and the initial velocity.

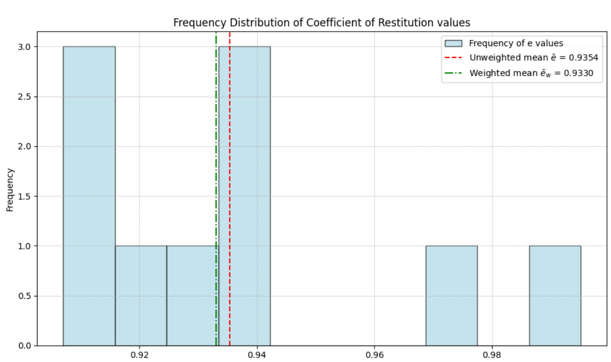

Fig. 4. Distribution of e around the average.

3 Gravitational Acceleration

By varying the height of the track and measuring the change in acceleration, we can estimate the value of gravitational acceleration, g. The results of this analysis is seen in Table 2, and we then calculate a slope (m): 0.09363 ± 0.00307, intercept (b): 0.00009 ± 0.00322, and estimated value for g: $14.0440 \pm 0.4606\ m/s^2$. Figure 5.

The calculated value for gravitational acceleration reveals a deviation of 9.21σ. This suggests a systematic error in data collection, likely die to error in data collection, likely due to variations in positioning of measurement tools. This is confirmed by a non-zero intercept b. Such systematic error is likely due to misalignment and poor calibration of measurement tools. Variation in the track height should reveal that the gravitational constant is just that, constant, however we see increased variation as the track's height is increased there is increased variability in the measurements. This illustrates an additional point of error: the air track is not frictionless, although it appears to have less friction in more horizontal configurations. These systematic errors(calibration issues, misalignment, air and track friction) introduce a number of possible causes for error, which seem to compound in the calculation of g.

An alternative method of measuring g would likely be much more accurate, such as the use of a pendulum, would likely be much more appropriate here. Such would allow for the calculation to be made with $T = 2\pi\sqrt{\frac{l}{g}}$, which is only reliant on the pendulum length, greatly reducing possible sources of error.

3.1 Friction Losses

It can be expected that loss due to friction is the largest cause of error in our analysis. From this, any deviation from this energy conservation highlights the presence of energy losses, primarily due to friction.

Knowing that in the absence of friction, the conservation of energy implies that half the square of the velocity, $\frac{1}{2}mv^2$, should be equal to the product of mass (m), gravitational acceleration (g), and the distance from the pivot point (l_2) multiplied by the sine of the angle ($\sin\theta$).

To quantify this deviation Δ is calculated for several trials as:

TABLE 2
Results for gravitational acceleration experiment

Height (h) in cm	**Mean Acceleration ($\bar{a_x}$) in m/s^2**	**Standard Error ($\sigma_{\bar{a_x}}$) in m/s^2**
0.12	0.011007	0.000269
0.24	0.022990	0.000203
0.36	0.033000	0.000548
0.48	0.046576	0.000395
0.60	0.055390	0.001394

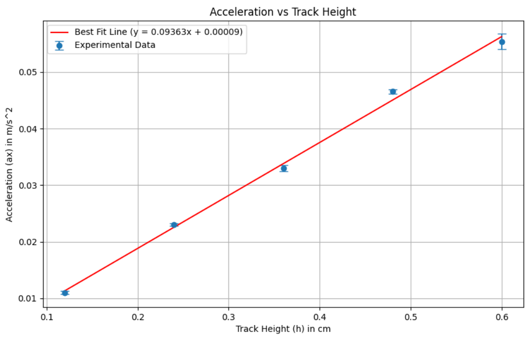

Fig. 5. Acceleration as a function of track height.

$$\Delta = \frac{v^2}{2axl_2} - 1$$

The calculated values for Δ across various trials in Table 3 highlight the effect of friction in the system.

References

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.

TABLE 3
Delta values for the trials

Trial #	**Delta**
1	-0.308612
2	-0.622064
3	-0.466572
4	-0.401643
5	-0.356808
6	-0.476777
7	-0.448693
8	-0.178378
9	-0.007586
10	-0.316653
11	-0.398773
12	-0.143593
13	0.448440
14	-0.154798
15	-0.145405
16	-0.334597
17	-0.197799
18	-0.085194
19	-0.237526
20	-0.194476
21	-0.292622
22	-0.267096
23	-0.203148
24	-0.299478
25	-0.354706
26	-0.341396
27	-0.318957
28	-0.379357
29	-0.322228
30	-0.360462
31	-0.302859
32	-0.096152
33	-0.254829
34	-0.266596
35	-0.240968
36	-0.241498
37	-0.180614
38	-0.212626
39	-0.241386
40	-0.254165
41	-0.205801
42	-0.204015
43	-0.173724
44	0.142498
45	-0.248631
46	-0.296728
47	-0.353534
48	-0.317023
49	-0.283247
50	-0.251581