Capacitance and the Oscilloscope

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Abstract

An experiment exploring the charging and discharging characteristics of capacitors in DC circuits.

1 Introduction

Capacitors are passive electrical components that store energy in an electric field. This is achieved through their construction, typically involving two parallel conductive plates. When a voltage is applied across these plates, an electric field develops between them, allowing the capacitor to store charge. The capacitance, denoted as C, is defined by the formula $C = \frac{Q}{V}$, where Q represents the charge stored by the capacitor, and V is the applied voltage. Therefore, the capacitance indicates how much charge the capacitor can hold for each unit of voltage.

Capacitors can be built in numerous geometries, where the dimensions and configurations used for the plates then affect the quantity of charge that can be stored.

A capacitor is charged when a voltage, ε, is established across the capacitor. The capacitor is a non-linear device, which results in a gradual charging time. To derive the current equation in a capacitor Kirchhoff's laws are used. Starting with the basic capacitor relation $C = \frac{Q}{V}$, a current, I, is defined as the rate of change of charge, Q, hence $I = \frac{dQ}{dt}$. Differentiating $Q = CV$ with respect to time, $I = C\frac{dV}{dt}$, indicating that the current in a capacitor is proportional to the rate of change of the voltage across it. Applying Kirchhoff's Voltage Law in a circuit with a capacitor, and using the relation $I = C\frac{dV}{dt}$, we can establish a connection between the voltage source, the capacitor, and other circuit elements, such as resistors, based on the circuit's configuration.

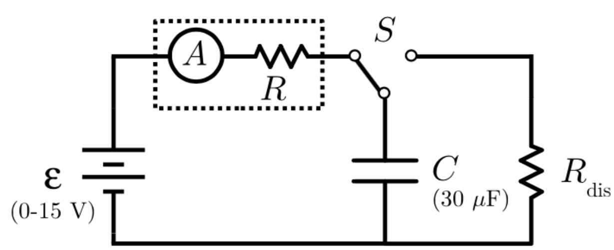

Figure 1: Ammeter Reading Charging Capacitor

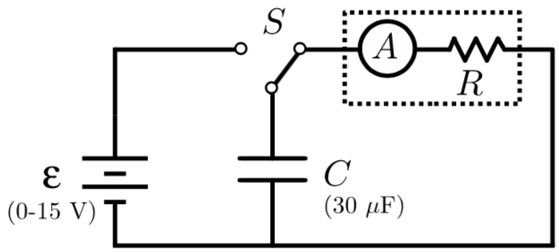

Figure 2: Ammeter Reading Discharging Capacitor

A resistor is placed in series with the capacitor to limit current flow. By considering a simple RC circuit with a resistor, R, and a capacitor, C, connected in series to a DC voltage source, ε. When the circuit is closed, a time-dependent current $I(t)$ flows through the circuit. Kirchhoff's Voltage Law gives $\varepsilon = IR + V_C$, where V_C is the voltage across the capacitor. Using the capacitor's fundamental relation, $V_C = \frac{Q}{C}$, and the fact that $I(t) = \frac{dQ}{dt}$, this is substituted into Kirchhoff's law to get $\varepsilon = IR + \frac{Q}{C}$. Differentiating and rearranging leads to the first-order linear differential equation $\frac{dI}{dt} + \frac{1}{RC}I = 0$. The solution to this differential equation under initial conditions is $I(t) = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}$, showing that current decays exponentially over time in an RC circuit. The same analysis is used for the discharging of a capacitor, but where the direction of current has changed, such that the current equation becomes $I(t) = -\frac{\varepsilon}{R}e^{-\frac{t}{RC}}$

2 Method

To measure the charging of a capacitor, an ammeter is placed in series with the battery, allowing for the entire current pull of the system to be measured accurately. An effective capacitance is then placed in shunt with a discharge resistors (where the effective capacitance is a bank of shunted capacitors each of $10\mu F$ in value). Readings off of the ammeter can then be taken to find the current through the capacitor as a function of time. The ammeter, which, naturally, has its own input impedance, will then dictate the charging time of the capacitor where the time constant is defined as $\tau = RC$.

To measure the discharging of the capacitor a similar method is used. The key difference, however, is that the ammeter is placed in series with the capacitor such that when the switch disconnects the voltage source from the capacitor, the capacitor discharges across the ammeter.

In a similar manner, the charging and discharging of the capacitor can be measured with an oscilloscope. By passing an input signal from a function generator into the RC circuit, the voltage across the capacitor can be analyzed to find the related charging and discharging characteristics. From this, measurements of the voltage and time can be made to verify voltage at a time constant.

It should be noted that, when working with an oscilloscope, one must be careful to not work with frequencies above the Nyquist rate, otherwise aliasing will occur and a distorted or nonsensical waveform will be detected. Thankfully, most modern oscilloscopes sample in the GHz range, which often exceeds the frequency of function generators. Additionally, it is very likely that we would be slew rate limited by our circuit before we reach the Nyquist rate. Nonetheless, considerations with the input frequency should not be overlooked.

3 Results and Discussion

In an RC circuit experiment with capacitances of 10 μF, 20 μF, and 30 μF, time constants (τ) for both charging and discharging phases were determined through linear regression analysis of $\ln(I)$ versus time, effectively linearizing the exponential current decay at a $5V$ potential. For the charging phase, the calculated τ values were 6.09 ± 0.29 s, 14.32 ± 0.48 s, and 18.89 ± 0.45 s, while for the discharging phase, they were 6.50 ± 0.28 s, 13.42 ± 0.48 s, and 20.04 ± 0.90 s, respectively. The ratios of these time constants closely aligned with theoretical expectations ($\tau_{30\mu F}/\tau_{20\mu F} = 3/2$, $\tau_{30\mu F}/\tau_{10\mu F} = 3$, $\tau_{20\mu F}/\tau_{10\mu F} = 2$), indicating strong correspondence with theory, albeit with slight deviations within 2 sigma. These deviations suggest potential minor systematic or random errors, possibly due to challenges in precise measurements given the short time intervals. Overall, the experiments confirmed the exponential decay behavior in RC circuits and reinforced the theoretical relationship between capacitance, resistance, and time constants, validating the experimental approach within its experimental limits.

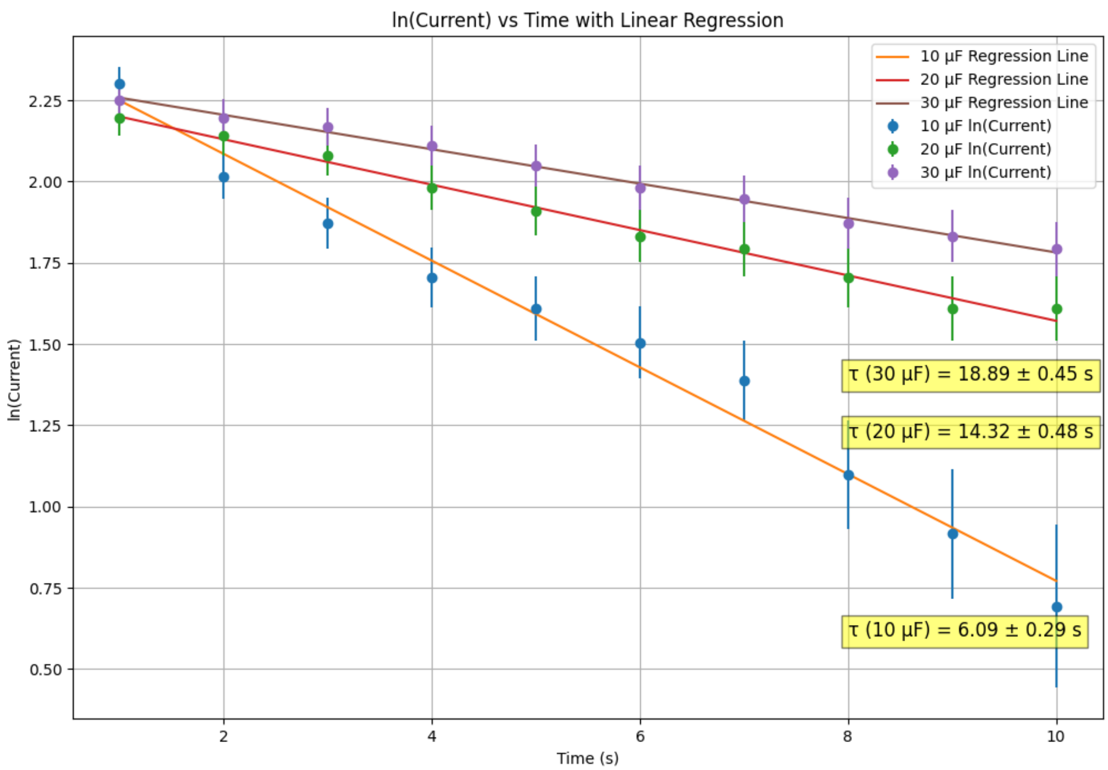

Figure 3: Charging Capacitor Logarithmic Scale Current vs. Time

In the RC circuit experiment involving capacitances of 10 μF, the measurements of τ from the digital oscilloscope were compared with theoretical estimates calculated using the RC product. The total resistance for these calculations included both the circuit component resistance and the 50 Ω output impedance of the function generator. The experimentally determined τ values, approximately 850 μs, 800 μs, and 810 μs at varying frequencies, closely aligned with the theoretical estimate of 850 microseconds. This agreement within uncertainties, and with the RC calculation,

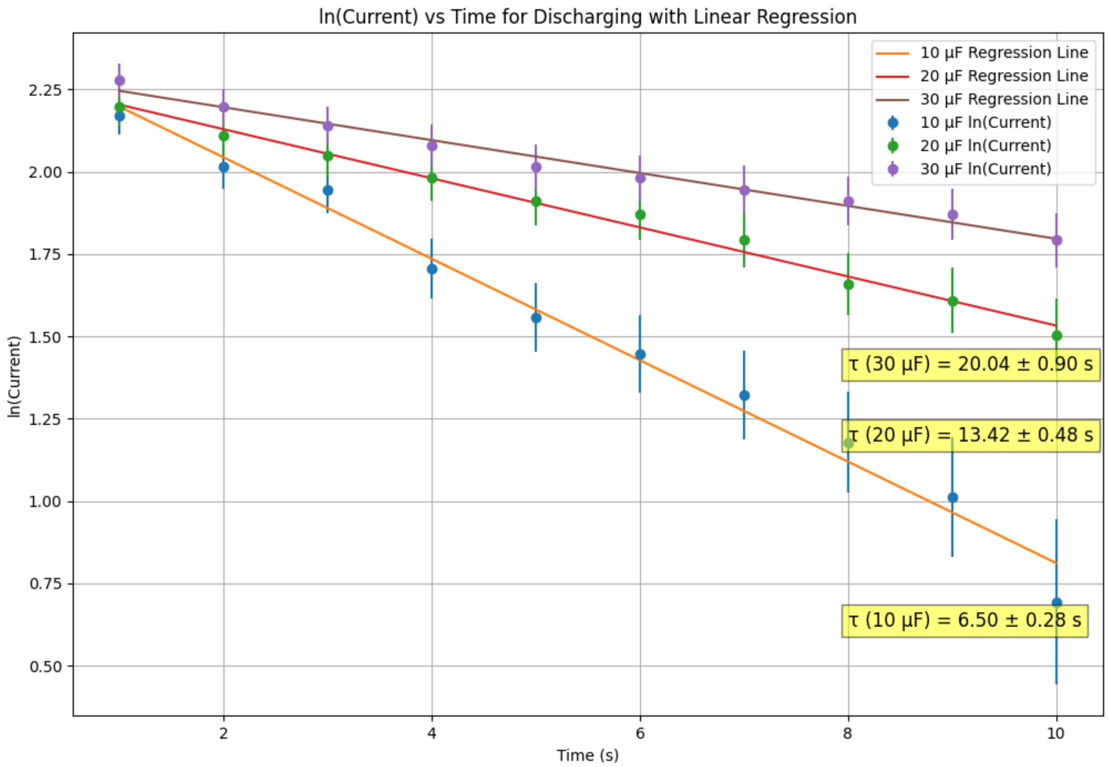

Figure 4: Discharging Capacitor Logarithmic Scale Current vs. Time

suggests a high degree of accuracy in the experimental setup. The potential 10% tolerance in circuit elements may contribute to some discrepancies, but these were not significant in this context.

Regarding the voltage behavior across the resistor and capacitor, the expectation is an exponential rise or decay, in line with the charging or discharging of the capacitor. The magnitude and shape of the voltage across these elements should reflect this exponential behavior, consistent with the observed time constants. This is demonstrated in Figure 5. The internal resistance of the oscilloscope, approximately 1 MΩ, is considerably higher than the resistance used in the circuit, which minimizes its influence on the measured τ. However, substituting the 10 kΩ resistor with a 1 MΩ resistor would drastically increase the time constant, altering the circuit's response significantly. At higher frequencies, such as 1 kHz, the ability of the capacitor to fully charge and discharge within each cycle might be compromised, potentially leading to less accurate measurements of τ. As this continues upwards, we'll reach the slew rate before hitting the Nyquist frequency and experiencing aliasing. The ideal frequency range for accurate τ measurements depends on the specific RC values, with lower frequencies generally providing more reliable results in standard RC circuits.

4 Conclusion

This experiment explored the charging and discharging characteristics of capacitors in RC circuits, using both an ammeter and an oscilloscope. The time constants (τ) for capacitances of 10 μF, 20

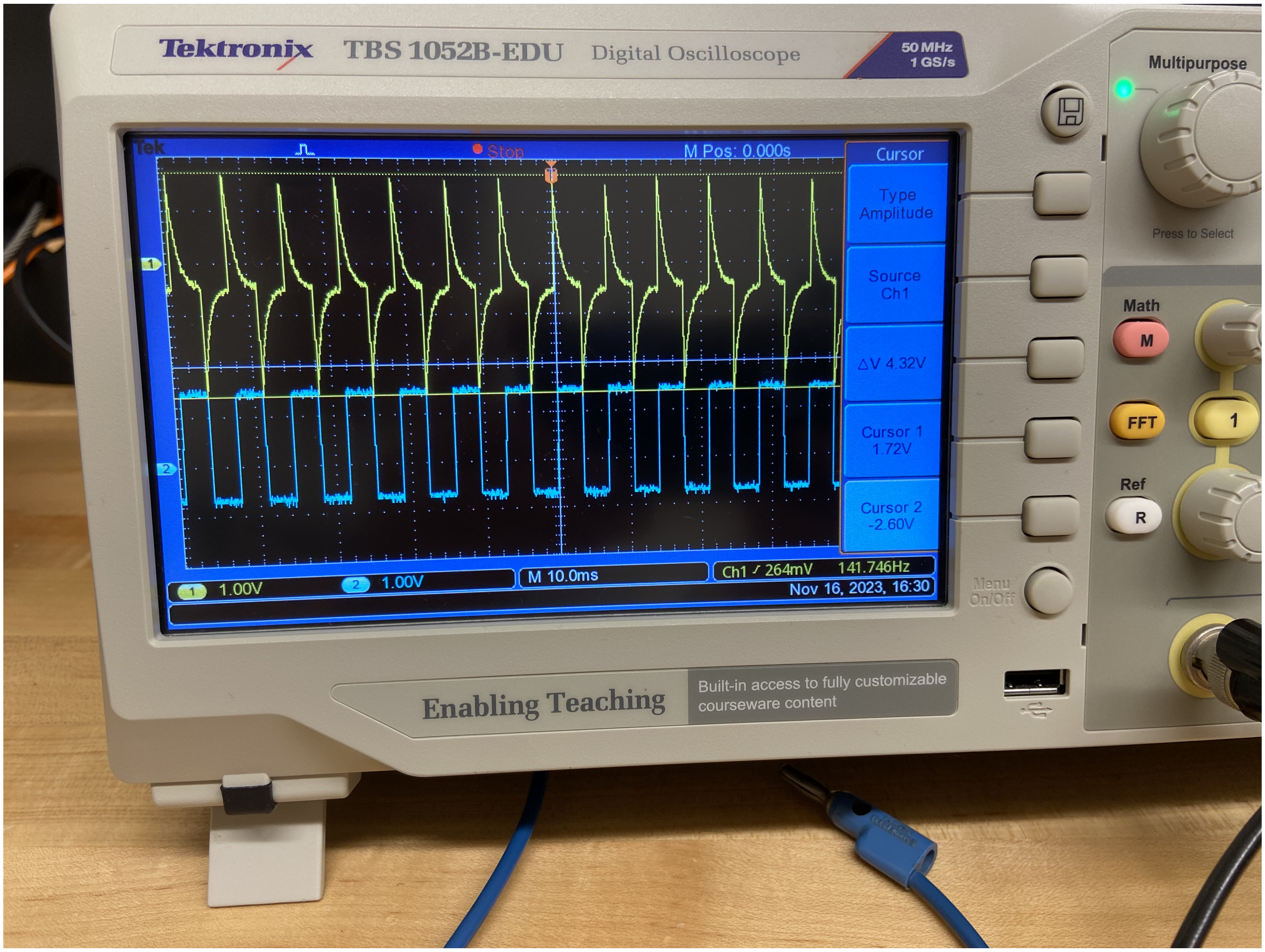

Figure 5: Output Voltage across the Capacitor with a Pulse Train

μF, and 30 μF were determined and compared with theoretical predictions based on the RC product. The close alignment of experimental τ values with theoretical calculations, within expected uncertainties and tolerances of up to 10%, confirmed the accuracy of the experimental setup and the validity of the theoretical model. The voltage behavior across the resistor and capacitor was consistent with the expected exponential rise and decay, validating the fundamental principles of capacitor charging and discharging in RC circuits.

The findings also underscore the need to consider frequency effects in such experiments. While the experimental frequencies used were within an appropriate range to accurately measure τ, increasing the frequency could potentially compromise the ability to fully charge and discharge the capacitor within each cycle. This underlines the importance of selecting appropriate frequencies for accurate measurements in RC circuits, especially when using oscilloscopes for time-dependent observations.