Speech Recognition

Assignment 2: Viterbi Algorithm

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1. Model Parameters

1. **States**: `good`, `neutral`, `bad` (a total of 3 states).
2. **Observations**: A, C, B, A, C (corresponding to the `observation` array as 0, 2, 1, 0, 2).
3. **State Transition Probability Matrix** `a`:
   - $a_{\text{good, good}} = 0.2$
   - $a_{\text{good, neutral}} = 0.3$
   - $a_{\text{good, bad}} = 0.5$
   - $a_{\text{neutral, good}} = 0.2$
   - $a_{\text{neutral, neutral}} = 0.2$
   - $a_{\text{neutral, bad}} = 0.6$
   - $a_{\text{bad, good}} = 0.0$
   - $a_{\text{bad, neutral}} = 0.2$
   - $a_{\text{bad, bad}} = 0.8$
4. **Emission Probability Matrix** `b`:

- $b_{\text{good, A}} = 0.7$
- $b_{\text{good, B}} = 0.2$
- $b_{\text{good, C}} = 0.1$
- $b_{\text{neutral, A}} = 0.3$
- $b_{\text{neutral, B}} = 0.4$
- $b_{\text{neutral, C}} = 0.3$
- $b_{\text{bad, A}} = 0.0$
- $b_{\text{bad, B}} = 0.1$
- $b_{\text{bad, C}} = 0.9$

2. Initialization Step

According to the initialization conditions, we assume an equal initial distribution across states (meaning each state's initial probability is $\frac{1}{3}$). We then calculate the initial probabilities for Day 1 (Monday, with observation A).

3. Viterbi algorithm

3.1. Day 1 (Monday, Observation A)

$$V_1^{\text{good}} = \frac{b_{\text{good}}(A)}{\text{STATE\_NUM}} = \frac{0.7}{3} \approx 0.23333333$$

$$V_1^{\text{neutral}} = \frac{b_{\text{neutral}}(A)}{\text{STATE\_NUM}} = \frac{0.3}{3} = 0.1$$

$$V_1^{\text{bad}} = \frac{b_{\text{bad}}(A)}{\text{STATE\_NUM}} = \frac{0.0}{3} = 0.0$$

The results are as follows:

	A
good	0.23333333
neutral	0.1
bad	0.0

3.2. Day 2 (Tuesday, Observation C)

Using the recursive formula, we compute each state probability V_2^{state}:

3.2.1. State Good

$$V_2^{\text{good}} = b_{\text{good}}(C) \cdot \max\left(V_1^{\text{good}} \cdot a_{\text{good, good}}, V_1^{\text{neutral}} \cdot a_{\text{neutral, good}}, V_1^{\text{bad}} \cdot a_{\text{bad, good}}\right)$$

$$= 0.1 \cdot \max\left(0.23333333 \cdot 0.2, 0.1 \cdot 0.2, 0.0 \cdot 0.0\right)$$

$$= 0.1 \cdot \max(0.04666667, 0.02, 0.0) = 0.1 \cdot 0.04666667 = 0.00466667$$

3.2.2. State Neutral

$$V_2^{\text{neutral}} = b_{\text{neutral}}(C) \cdot \max\left(V_1^{\text{good}} \cdot a_{\text{good, neutral}}, V_1^{\text{neutral}} \cdot a_{\text{neutral, neutral}}, V_1^{\text{bad}} \cdot a_{\text{bad, neutral}}\right)$$

$$= 0.3 \cdot \max(0.23333333 \cdot 0.3, 0.1 \cdot 0.2, 0.0 \cdot 0.2)$$

$$= 0.3 \cdot \max(0.07, 0.02, 0.0) = 0.3 \cdot 0.07 = 0.021$$

3.2.3. State Bad

$$V_2^{\text{bad}} = b_{\text{bad}}(C) \cdot \max\left(V_1^{\text{good}} \cdot a_{\text{good, bad}}, V_1^{\text{neutral}} \cdot a_{\text{neutral, bad}}, V_1^{\text{bad}} \cdot a_{\text{bad, bad}}\right)$$

$$= 0.9 \cdot \max(0.23333333 \cdot 0.5, 0.1 \cdot 0.6, 0.0 \cdot 0.8)$$

$$= 0.9 \cdot \max(0.11666667, 0.06, 0.0) = 0.9 \cdot 0.11666667 = 0.105$$

3.2.4. Day 2 Results

	A	C
good	0.23333333	0.00466667
neutral	0.1	0.021
bad	0.0	0.105

3.3. Day 3 (Wednesday, Observation B)

Using the recursive formula, we compute each state probability V_3^{state}:

3.3.1. State Good

$$V_3^{\text{good}} = b_{\text{good}}(B) \cdot \max\left(V_2^{\text{good}} \cdot a_{\text{good, good}}, V_2^{\text{neutral}} \cdot a_{\text{neutral, good}}, V_2^{\text{bad}} \cdot a_{\text{bad, good}}\right)$$

$$= 0.2 \cdot \max(0.00466667 \cdot 0.2, 0.021 \cdot 0.2, 0.105 \cdot 0.0)$$

$$= 0.2 \cdot \max(0.00093333, 0.0042, 0.0) = 0.2 \cdot 0.0042 = 0.00084$$

3.3.2. State Neutral

$$V_3^{\text{neutral}} = b_{\text{neutral}}(B) \cdot \max\left(V_2^{\text{good}} \cdot a_{\text{good, neutral}}, V_2^{\text{neutral}} \cdot a_{\text{neutral, neutral}}, V_2^{\text{bad}} \cdot a_{\text{bad, neutral}}\right)$$

$$= 0.4 \cdot \max(0.00466667 \cdot 0.3, 0.021 \cdot 0.2, 0.105 \cdot 0.2)$$

$$= 0.4 \cdot \max(0.0014, 0.0042, 0.021) = 0.4 \cdot 0.021 = 0.0084$$

3.3.3. State Bad

$$V_3^{\text{bad}} = b_{\text{bad}}(B) \cdot \max\left(V_2^{\text{good}} \cdot a_{\text{good, bad}}, V_2^{\text{neutral}} \cdot a_{\text{neutral, bad}}, V_2^{\text{bad}} \cdot a_{\text{bad, bad}}\right)$$

$$= 0.1 \cdot \max(0.00466667 \cdot 0.5, 0.021 \cdot 0.6, 0.105 \cdot 0.8)$$

$$= 0.1 \cdot \max(0.00233333, 0.0126, 0.084) = 0.1 \cdot 0.084 = 0.0084$$

3.3.4. Day 3 Results

	A	C	B
good	0.23333333	0.00466667	0.00084
neutral	0.1	0.021	0.0084
bad	0.0	0.105	0.0084

3.4. Day 4 (Thursday, Observation A)

3.4.1. State Good

$$V_4^{\text{good}} = b_{\text{good}}(A) \cdot \max\left(V_3^{\text{good}} \cdot a_{\text{good, good}}, V_3^{\text{neutral}} \cdot a_{\text{neutral, good}}, V_3^{\text{bad}} \cdot a_{\text{bad, good}}\right)$$

$$= 0.7 \cdot \max(0.00084 \cdot 0.2, 0.0084 \cdot 0.2, 0.0084 \cdot 0.0)$$

$$= 0.7 \cdot \max(0.000168, 0.00168, 0.0) = 0.7 \cdot 0.00168 = 0.001176$$

3.4.2. State Neutral

$$V_4^{\text{neutral}} = b_{\text{neutral}}(A) \cdot \max\left(V_3^{\text{good}} \cdot a_{\text{good, neutral}}, V_3^{\text{neutral}} \cdot a_{\text{neutral, neutral}}, V_3^{\text{bad}} \cdot a_{\text{bad, neutral}}\right)$$

$$= 0.3 \cdot \max(0.00084 \cdot 0.3, 0.0084 \cdot 0.2, 0.0084 \cdot 0.2)$$

$$= 0.3 \cdot \max(0.000252, 0.00168, 0.00168) = 0.3 \cdot 0.00168 = 0.000504$$

3.4.3. State Bad

$$V_4^{\text{bad}} = b_{\text{bad}}(A) \cdot \max\left(V_3^{\text{good}} \cdot a_{\text{good, bad}}, V_3^{\text{neutral}} \cdot a_{\text{neutral, bad}}, V_3^{\text{bad}} \cdot a_{\text{bad, bad}}\right)$$

$$= 0.0 \cdot \max(0.00084 \cdot 0.5, 0.0084 \cdot 0.6, 0.0084 \cdot 0.8) = 0.0$$

3.4.4. Day 4 Results

	A	C	B	A
good	0.23333333	0.00466667	0.00084	0.001176
neutral	0.1	0.021	0.0084	0.000504
bad	0.0	0.105	0.0084	0.0

3.5. Day 5 (Friday, Observation C)

3.5.1. State Good

$$V_5^{\text{good}} = b_{\text{good}}(C) \cdot \max\left(V_4^{\text{good}} \cdot a_{\text{good, good}}, V_4^{\text{neutral}} \cdot a_{\text{neutral, good}}, V_4^{\text{bad}} \cdot a_{\text{bad, good}}\right)$$

$$= 0.1 \cdot \max\left(0.001176 \cdot 0.2, 0.000504 \cdot 0.2, 0.0 \cdot 0.0\right)$$

$$= 0.1 \cdot \max\left(0.0002352, 0.0001008, 0.0\right) = 0.1 \cdot 0.0002352 = 0.00002352$$

3.5.2. State Neutral

$$V_5^{\text{neutral}} = b_{\text{neutral}}(C) \cdot \max\left(V_4^{\text{good}} \cdot a_{\text{good, neutral}}, V_4^{\text{neutral}} \cdot a_{\text{neutral, neutral}}, V_4^{\text{bad}} \cdot a_{\text{bad, neutral}}\right)$$

$$= 0.3 \cdot \max\left(0.001176 \cdot 0.3, 0.000504 \cdot 0.2, 0.0 \cdot 0.2\right)$$

$$= 0.3 \cdot \max\left(0.0003528, 0.0001008, 0.0\right) = 0.3 \cdot 0.0003528 = 0.00010584$$

3.5.3. State Bad

$$V_5^{\text{bad}} = b_{\text{bad}}(C) \cdot \max\left(V_4^{\text{good}} \cdot a_{\text{good, bad}}, V_4^{\text{neutral}} \cdot a_{\text{neutral, bad}}, V_4^{\text{bad}} \cdot a_{\ textbad,bad}\right)$$

$$= 0.9 \cdot \max\left(0.001176 \cdot 0.5, 0.000504 \cdot 0.6, 0.0 \cdot 0.8\right)$$

$$= 0.9 \cdot \max\left(0.000588, 0.0003024, 0.0\right) = 0.9 \cdot 0.000588 = 0.0005292$$

4. C++ Code

I wrote a c++ program to verify my answer. Here is my code:

```cpp
// Author: 252750 赵卓冰
#include <iostream>
using namespace std;

// Number of states
const int STATE_NUM = 3;
const int DAY_NUM = 5;
// Transition probabilities between any two states
const double a[STATE_NUM][STATE_NUM] = { {0.2, 0.3, 0.5},
                                         {0.2, 0.2, 0.6},
                                         {0.0, 0.2, 0.8 } };
// Emission probabilities
const double b[STATE_NUM][STATE_NUM] = { {0.7, 0.2, 0.1},
                                         {0.3, 0.4, 0.3},
                                         {0.0, 0.1, 0.9 } };
// A C B A C
const int observation[DAY_NUM] = { 0, 2, 1, 0, 2 };
// The table
double V[DAY_NUM][STATE_NUM] = { 0 };
int backtrace_pointer[DAY_NUM][STATE_NUM];

```

```
// Initiallization
void InitV() {
  int day = 0;
  int state = observation[day];
  for (int j = 0; j < STATE_NUM; ++j) {
    V[day][j] = b[j][state] / STATE_NUM;
  }
}

void DisplayTable() {
  cout << "Table result is:" << endl;
  cout << "             A                   C                   B                   A                   C" << endl;
  for (int j = 0; j < STATE_NUM; ++j) {
    if (0 == j) {
      printf("good:    ");
    }
    else if (1 == j) {
      printf("neutral: ");
    }
    else {
      printf("bad:     ");
    }
    for (int day = 0; day < DAY_NUM; ++day) {
      printf("%9.8lf    |    ", V[day][j]);
    }
    printf("\n");
  }
}

void DisplayBacktracePointer() {
  cout<< endl << "Backtrace pointer result is:" << endl;
  for (int j = 0; j < STATE_NUM; ++j) {
    for (int day = 0; day < DAY_NUM; ++day) {
      printf("%4d", backtrace_pointer[day][j]);
    }
    printf("\n");
  }
}

void ViterbiAlgorithm() {
  InitV();
  memset(backtrace_pointer, -1, sizeof(backtrace_pointer));
  for (int day = 1; day < DAY_NUM; ++day) {
    int state = observation[day];
    for (int j = 0; j < STATE_NUM; ++j) {
      double max_value = -1;
      for (int i = 0; i < STATE_NUM; ++i) {
        if (V[day - 1][i] * a[i][j] > max_value) {
          max_value = V[day - 1][i] * a[i][j];
          backtrace_pointer[day][j] = i; // update backtrace pointer
        }
```

```
        }
        V[day][j] = b[j][state] * max_value;
      }
    }
}

int main() {
    ViterbiAlgorithm();
    DisplayTable();
    DisplayBacktracePointer();
    return 0;
}
```

4.1. Result of Program

The result of the program is exactly the same as my calculation.

```
Microsoft Visual Studio 调试控制台
Table result is:
              A               C               B               A               C
good:     0.23333333  |  0.00466667  |  0.00084000  |  0.00117600  |  0.00002352  |
neutral:  0.10000000  |  0.02100000  |  0.00840000  |  0.00050400  |  0.00010584  |
bad:      0.00000000  |  0.10500000  |  0.00840000  |  0.00000000  |  0.00052920  |

Backtrace pointer result is:
  -1    0    1    1    0
  -1    0    2    1    0
  -1    0    2    2    0
```

5. Final Result

	A	C	B	A	C
good	0.23333333	0.00466667	0.00084	0.001176	0.00002352
neutral	0.1	0.021	0.0084	0.000504	0.00010584
bad	0.0	0.105	0.0084	0.0	0.0005292

The most likely path for the teacher's mood over the week is:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Assignment	A	C	B	A	C
Mood	good	bad	neutral	good	bad