Math3 (1)

Assignment 1

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[1] $6 \times \begin{bmatrix} \theta^2 & 2\theta-1 \\ 4 & 1/\theta \end{bmatrix} - \theta \begin{bmatrix} \theta^2-1 & 6 \\ 3/\theta & \theta^3+2\theta+1 \end{bmatrix}$

$= \begin{bmatrix} 6\theta^2 & 12\theta-6 \\ 24 & 6/\theta \end{bmatrix} - \begin{bmatrix} \theta^3-\theta & 6\theta \\ 3 & \theta^4+2\theta^2+\theta \end{bmatrix}$

$= \begin{bmatrix} 6\theta^2-\theta^3+\theta & 6\theta-6 \\ 21 & 6/\theta-\theta^4-2\theta^2-\theta \end{bmatrix}$

[2] $\begin{bmatrix} x-z \\ 3x+y+z \\ x+3y \end{bmatrix}$

[3] $= \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 8 \\ 16 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ 16 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Zero Matrix

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$\boxed{9}$ State that $(AB)^T = B^T A^T$.

$$\underset{3\times 4}{AB} = \underset{3\times 2}{\begin{bmatrix} t & t^2 \\ 1 & 2t \\ 1 & 0 \end{bmatrix}} \cdot \underset{2\times 4}{\begin{bmatrix} 3 & t & t+1 & 0 \\ t & 2t & t^2 & t^3 \end{bmatrix}}$$

$$= \begin{bmatrix} 3t+t^3 & t^2+3t^3 & t^2+t+t^4 & t^5 \\ 3+2t^2 & t+4t^2 & t+1+2t^3 & 2t^4 \\ 3 & t & t+1 & 0 \end{bmatrix}$$

$$\underset{4\times 3}{(AB)^T} = \begin{bmatrix} 3t+t^3 & 3+2t^2 & 3 \\ t^2+3t^3 & t+4t^2 & t \\ t^2+t+t^4 & t+1+2t^3 & t+1 \\ t^5 & t+1 & 0 \end{bmatrix}$$

$$B^T = \underset{4\times 2}{\begin{bmatrix} 3 & t \\ t & 2t \\ t+1 & t^2 \\ 0 & t^3 \end{bmatrix}} \cdot \quad A^T = \underset{2\times 3}{\begin{bmatrix} t & 1 & 1 \\ t^2 & 2t & 0 \end{bmatrix}}$$

$$B^T A^T = \begin{bmatrix} 3t+t^3 & 3+2t^2 & 3 \\ t^2+3t^3 & t+4t^2 & t \\ t^2+t+t^4 & t+1+2t^3 & t+1 \\ t^5 & t+1 & 0 \end{bmatrix} \therefore (AB)^T = B^T A^T$$

$\boxed{4}$

$$\left(\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}2&-1&1\\3&-2&1\\0&0&1\end{bmatrix}\right)\left(\begin{bmatrix}2&-1&1\\3&-2&1\\0&0&1\end{bmatrix}\begin{bmatrix}2&-1&1\\3&-2&1\\0&0&1\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right)$$

$$=\left(\begin{bmatrix}-1&1&-1\\-3&3&-1\\0&0&0\end{bmatrix}\right)\left(\begin{bmatrix}1&0&2\\0&1&2\\0&0&1\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right)$$

$$\begin{bmatrix}-1&1&-1\\-3&3&-1\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&2\\0&0&2\\0&0&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Zero Matrix

$\boxed{6}$ All of these are in RREF : D, H, N, R

$\boxed{7}$ All of these are upper triangular :-
E, F, L, M, H, N, K, R, T.

$\boxed{8}$ Yes, because all pivots of RREF force all entries below them to be zero

$\boxed{9}$ No, it is not guaranteed

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[10] Yes, can be both only if it is a diagonal matrix

$A = \begin{bmatrix} 5 & -1 \\ 6 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 4 \\ 3 & 1 & 5 \end{bmatrix}$ using adjugate

$A^{-1} = \frac{1}{|A|} \cdot adj(A)$; $|A| = 40 - (-6) = 46$

$A^{-1} = \frac{1}{46} \begin{bmatrix} 8 & +1 \\ -6 & 5 \end{bmatrix} = \begin{bmatrix} \frac{8}{46} & \frac{1}{46} \\ \frac{-6}{46} & \frac{5}{46} \end{bmatrix}$

$|B| = 1\underbrace{(-5-4)}_{-9} - 0 + 2\underbrace{(2+3)}_{5}$

$= -9 + 10 = 1$

$adj(B) = \begin{bmatrix} -9 & -2 & 5 \\ -2 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$

$\therefore adj(B) = \begin{bmatrix} -9 & 2 & 5 \\ 2 & -1 & -1 \\ 2 & 0 & -1 \end{bmatrix}^T$

$= \begin{bmatrix} -9 & 2 & 2 \\ 2 & -1 & 0 \\ 5 & -1 & -1 \end{bmatrix}$

$\therefore B^{-1} = 1 \cdot$ 11-15

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using Guass jordan elemination

A)

$$\left[\begin{array}{cc|cc} 5 & -1 & 1 & 0 \\ 6 & 8 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \div 5} \left[\begin{array}{cc|cc} 1 & \frac{-1}{5} & \frac{1}{5} & 0 \\ 6 & 8 & 0 & 1 \end{array}\right]$$

$$-6R_1 + R_2 \to R_2 \to \left[\begin{array}{cc|cc} 1 & \frac{-1}{5} & \frac{1}{5} & 0 \\ 0 & \frac{46}{5} & -\frac{6}{5} & 1 \end{array}\right]$$

$$\frac{5}{46} \times R_2 \to R_2 = \left[\begin{array}{cc|cc} 1 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{3}{23} & \frac{5}{46} \end{array}\right]$$

$$\frac{1}{5}R_2 + R_1 \to R_1 = \left[\begin{array}{cc|cc} 1 & 0 & \frac{4}{23} & \frac{1}{46} \\ 0 & 1 & -\frac{3}{23} & \frac{5}{46} \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{4}{23} & \frac{1}{46} \\ -\frac{3}{23} & \frac{5}{46} \end{bmatrix}$$

B)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 4 & 0 & 1 & 0 \\ 3 & 1 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1 \to R_2$

$R_3 - 3R_1 \to R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 1 & -1 & -3 & 0 & 1 \end{array}\right]$$

$-R_2 \to R_2$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -3 & 0 & 1 \end{array}\right]$$

$R_3 - R_2 \to R_2$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & -5 & 1 & 1 \end{array}\right]$$

$-R_3 \to R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 5 & -1 & -1 \end{array}\right]$$

$R_1 - 2R_3 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -9 & 2 & 2 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 5 & -1 & -1 \end{array}\right] \quad B^{-1} = \begin{bmatrix} -9 & 2 & 2 \\ 2 & -1 & 0 \\ 5 & -1 & -1 \end{bmatrix}$$