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1 (a)

$$\left[\begin{array}{ccc|c} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right] \quad R_3 \to R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{array}\right] \quad \begin{array}{l} R_1 \to \frac{1}{3} R_1 \\ R_2 \to R_2 / -2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -\frac{2}{3} \\ 0 & 1 & -\frac{3}{2} & -1/2 \\ 0 & -6 & 9 & 9 \end{array}\right] \quad R_3 \to R_3 + 6R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -\frac{2}{3} \\ 0 & 1 & -3/2 & -1/2 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{6} \end{array}\right] \quad 0 = 6$$

No solution

(b)

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \\ R_4 \to R_4 - 3R_1 \end{array}$$

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التاريخ 24/2/2023

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & -4 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right] \quad R_4 \rightarrow R_4 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & -4 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow \frac{1}{3} R_2 \\ \cancel{R_3 \rightarrow R_3 - R_2} \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -\frac{4}{3} & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right] \end{array} \quad R_3 \rightarrow R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -\frac{4}{3} & 0 \\ 0 & 0 & 0 & \frac{4}{3} & 0 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \rightarrow \frac{3}{4} R_3 \\ R_4 \rightarrow \frac{1}{4} R_4 \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} X & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -\frac{4}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$W = 0$, $y - 2Z - \frac{4}{3} W = 0$

$y - 2Z = 0$, $y = 2Z$

$X - y + 2Z - W = -1$

$\boxed{X = -1}$

infinite
# of solutions

**2** a) $\left[\begin{array}{cc|c} 4 & -8 & 12 \\ 3 & -6 & 9 \\ -2 & 4 & -6 \end{array}\right]$ $R_1 \to R_1/4$ $\to$ $\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 3 & -6 & 9 \\ -2 & 4 & -6 \end{array}\right]$

$R_2 \to R_2 - 3R_1$ $\to$ $\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 0 & 0 \\ -2 & 4 & -6 \end{array}\right]$ $R_3 \to R_3 + 2R_1$

$\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\quad x_1 - 2x_2 = 3$ $\quad$ Many solutions

$x_1 = 3 + 2x_2$ $\quad$ "infinite"

(b) $\left[\begin{array}{cccc|c} 0 & 10 & -4 & 1 & 1 \\ 1 & 4 & -1 & 1 & 2 \\ 3 & 2 & 1 & 2 & 5 \\ -2 & -8 & 2 & -2 & -4 \\ 1 & -6 & 3 & 0 & 1 \end{array}\right]$ $\to$ $\left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 3 & 2 & 1 & 2 & 5 \\ -2 & -8 & 2 & -2 & -4 \\ 1 & -6 & 3 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - 3R_1 \\ R_4 + 2R_1 \\ R_5 - R_1 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 0 & -10 & 4 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -10 & 4 & -1 & -1 \end{array}\right]$ $R_2 \to \frac{1}{10} R_2$ $\left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & -0.4 & 0.1 & 0.1 \\ 0 & -10 & 4 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -10 & 4 & -1 & -1 \end{array}\right] \begin{array}{l} R_1 - 4R_2 \\ \\ R_3 + 10R_2 \\ \\ R_5 + 10R_2 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0.6 & 0.6 & 1.6 \\ 0 & 1 & -0.4 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $\quad$ infinite solutions

Ⓒ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 2 & 0 & 2 & b \\ 0 & 3 & 3 & c \end{array}\right] R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 3 & 3 & c \\ 2 & 0 & 2 & b \end{array}\right] R_2/3$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & 1 & \frac{c}{3} \\ 2 & 0 & 2 & b \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_2 \\ \\ R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & a - \frac{c}{3} \\ 0 & 1 & 1 & \frac{c}{3} \\ 0 & 0 & 0 & b - 2a \end{array}\right]$

if $b - 2a = 0$ $\therefore$ $0 = 0$ infinite solutions

" " " $\neq 0$ $\therefore$ No solution

③ a) $\left[\begin{array}{ccc|c} 2 & -1 & -3 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right] R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c} -1 & 2 & -3 & 0 \\ 2 & -1 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right] -1R_1$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & -1 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 3 & -9 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right] R_2/3$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 + 2R_2 \\ \\ R_3 - 3R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 10 & 0 \end{array}\right] R_3/10$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \therefore Z = 0 \, , \, y - 3Z = 0 \, , \, y = 0 \, , \, X = 0$

b) $\left[\begin{array}{ccccc|c} 0 & 0 & 1 & 1 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \end{array}\right]$ Multiply by $-1$ $\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \end{array}\right]$ $R_3 \hookrightarrow -R_1+R_3$

$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \end{array}\right]$ $-2R_1+R_4 \to R_4$ $\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & -6 & 3 & 0 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & -6 & 3 & 0 \end{array}\right]$ $\begin{array}{l} \\ \\ R_3/-3 \\ R_4-3R_2 \end{array}$ $\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -9 & 0 & 0 \end{array}\right]$ $R_4+9R_3$

$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$Z_4 = 0$

$Z_3 + Z_5 = 0 \Rightarrow Z_3 = -Z_5$

$Z_1 + Z_2 - 2Z_3 + 3Z_4 - Z_5 = 0$

$\therefore Z_1 = -Z_2 - Z_5$

Making infinite solu.

4) $\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 5 \\ 4 & 1 & a^2-14 \end{bmatrix}$, we need to determine $|A|$

$|A| = 1 \cdot \begin{vmatrix} -1 & 5 \\ 1 & a^2-14 \end{vmatrix} - 2 \begin{vmatrix} 3 & 5 \\ 4 & a^2-14 \end{vmatrix} + (-3) \cdot \begin{vmatrix} 3 & -1 \\ 4 & 1 \end{vmatrix}$

$= -7a^2 + (9+124-21) = -7a^2 + 112 = -7(a^2-16) = -7(a-4)(a+4)$

$$\det(A) = -7(a-4)(a+4)\begin{cases} \to \text{if } a \neq 4, -4 \text{ then one Solution} \\ \to a=4 \text{ , Many solutions} \\ \to a=-4 \text{ , No solution} \end{cases}$$

⑤ $A = \begin{bmatrix} \lambda-3 & 1 \\ 1 & \lambda-3 \end{bmatrix}$ , $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\det(A) = (\lambda-3)(\lambda-3) - (1)(1) = (\lambda-3)^2 - 1 = 0$

$(\lambda-3)^2 = 1$

$\lambda - 3 = \pm 1$

$\therefore \lambda = 4, \lambda = 2$

$\therefore$ the system has noninterval solutions if and only if $\lambda = 2$, $\lambda = 4$

6 Let $a=\frac{1}{x}$ , $b=\frac{1}{y}$ , $c=\frac{1}{z}$

$\therefore a+2b-4c=1$

$2a+3b+8c=0$

$-a+9b+10c=5$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 1 \\ 2 & 3 & 8 & 0 \\ -1 & 9 & 10 & 5 \end{array}\right] \begin{array}{l} R_2-2R_1 \to R_2 \\ R_3+R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 1 \\ 0 & -1 & 16 & -2 \\ 0 & 11 & 6 & 6 \end{array}\right] \to -1 \cdot R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & -4 & 1 \\ 0 & 1 & -16 & 2 \\ 0 & 11 & 6 & 6 \end{array}\right] R_3-11R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 1 \\ 0 & 1 & -16 & 2 \\ 0 & 0 & 182 & -16 \end{array}\right]$$

$\to 182c=-16 \quad \therefore c=\frac{-8}{91}$

$\to b-16\left(\frac{-8}{91}\right)=2$

$\therefore b=\frac{54}{91}$

$\therefore a=\frac{-7}{13}$

then $x=-\frac{13}{7}$

$y=\frac{91}{54}$ $\qquad$ $z=-\frac{91}{8}$

7 $A^{-1}$ for

$$\begin{pmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix} \rightarrow 1$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{pmatrix} \rightarrow 2$$

1 $$\det(A) = 2\underbrace{\begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix}}_{\boxed{-3}} - 5\underbrace{\begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix}}_{\boxed{-3}} + 5\underbrace{\begin{vmatrix} -1 & -1 \\ 2 & 4 \end{vmatrix}}_{\boxed{-2}}$$

$|A| = -1$ $\therefore$ A is invertible

→ Using adjoint method:- $A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$

$$\text{adj}(A) = \begin{bmatrix} -3 & -3 & -2 \\ -5 & -4 & -2 \\ 5 & 5 & 3 \end{bmatrix} \xRightarrow{\text{cofactor}} \begin{bmatrix} -3 & 3 & -2 \\ 5 & -4 & 2 \\ 5 & -5 & 3 \end{bmatrix}^T$$

$$\text{adj}(A) = \begin{bmatrix} -3 & 5 & 5 \\ 3 & -4 & -5 \\ -2 & 2 & 3 \end{bmatrix} \times -1$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$$

→ Using Gauss-Jordan:-

$$\left[\begin{array}{ccc|ccc} 2 & 5 & 5 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \rightarrow \frac{1}{2} R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2.5 & 2.5 & 0.5 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \to R_2 \to R_2 + R_1 \\ \to R_3 \to R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2.5 & 2.5 & 0.5 & 0 & 0 \\ 0 & 1.5 & 2.5 & 0.5 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right] \quad R_2 \to \frac{2}{3} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 2.5 & 2.5 & 0.5 & 0 & 0 \\ 0 & 1 & \frac{5}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - \frac{5}{2} R_2 \\ R_3 \to R_2 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{5}{3} & -\frac{1}{3} & -\frac{5}{3} & 0 \\ 0 & 1 & \frac{5}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} & 1 \end{array}\right] \quad R_3 \to -3R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{5}{3} & -\frac{1}{3} & -\frac{5}{3} & 0 \\ 0 & 1 & \frac{5}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] \begin{array}{l} R_1 \to R_1 + \frac{5}{3} R_3 \\ R_2 \to R_2 - \frac{5}{3} R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & -5 \\ 0 & 1 & 0 & -3 & 4 & 5 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] \quad \therefore A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$$

② $A = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$ $\quad |A| = 2\begin{vmatrix} 1 & 0 \\ 3 & 6 \end{vmatrix} - 0\begin{vmatrix} 8 & 0 \\ -5 & 6 \end{vmatrix} + 0\begin{vmatrix} 8 & 1 \\ -5 & 3 \end{vmatrix}$

$|A| = 12$

$\text{adj}(A) = \begin{bmatrix} 6 & 48 & 29 \\ 0 & 12 & 6 \\ 0 & 0 & 2 \end{bmatrix} \xRightarrow{\text{cofactor}} \begin{bmatrix} 6 & -48 & 29 \\ 0 & 12 & -6 \\ 0 & 0 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 6 & 0 & 0 \\ -48 & 12 & 0 \\ 29 & -6 & 2 \end{bmatrix} \cdot \frac{1}{12} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -4 & 1 & 0 \\ \frac{29}{12} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix}$

Using Gauss-jordan :-

$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 8 & 1 & 0 & 0 & 1 & 0 \\ -5 & 3 & 6 & 0 & 0 & 1 \end{array}\right] \rightarrow R_1 \rightarrow \frac{1}{2}R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.5 & 0 & 0 \\ 8 & 1 & 0 & 0 & 1 & 0 \\ -5 & 3 & 6 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - 8R_1 \\ R_3 \rightarrow R_3 - 5R_1 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 3 & 6 & \frac{29}{2} & 0 & 1 \end{array}\right] \quad R_3 \rightarrow R_3 - 3R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 6 & \frac{29}{2} & -3 & 1 \end{array}\right] \quad R_3 \rightarrow \frac{1}{6} \cdot R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & 0 \\ 0 & 0 & 1 & \frac{29}{12} & -\frac{1}{2} & \frac{1}{6} \end{array}\right] \quad A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -4 & 1 & 0 \\ \frac{29}{12} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

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→ using Cramer's Rule

$\boxed{8}$ (a) $A = \begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$|A| = 13 \quad , |A_{x_1}| = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} = 13 \quad , |A_{x_2}| = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix}$

$= 26$

$x_1 = \frac{|A_{x_1}|}{|A|} = \frac{13}{13} = 1$

$x_2 = \frac{|A_{x_2}|}{|A|} = \frac{26}{13} = 2$

→ using inverse Method :- $A^{-1} = \frac{1}{\det(A)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{13} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 7 \end{bmatrix} \quad , X = A^{-1} \cdot B = \frac{1}{13} \underbrace{\underbrace{\begin{bmatrix} 1 & 2 \\ -3 & 7 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} 3 \\ 5 \end{bmatrix}}_{2\times1}}$

$X = \frac{1}{13} \cdot \begin{bmatrix} 13 \\ 26 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\therefore \; x_1 = 1 \quad , \quad x_2 = 2$

$$\boxed{AX=B}$$

(b) $A=\begin{bmatrix}1 & -3 & 1\\ 2 & -1 & 0\\ 4 & 0 & -3\end{bmatrix}$, $X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$, $B=\begin{bmatrix}4\\ -2\\ 0\end{bmatrix}$

$$|A| = 1\cdot\underbrace{\begin{vmatrix}-1 & 0\\ 0 & -3\end{vmatrix}}_{3} + 3\underbrace{\begin{vmatrix}2 & 0\\ 4 & -3\end{vmatrix}}_{-6} + 1\begin{vmatrix}2 & -1\\ 4 & 0\end{vmatrix} = -11$$

$$|A_{x_1}| = \begin{vmatrix}4 & -3 & 1\\ -2 & -1 & 0\\ 0 & 0 & -3\end{vmatrix} = 30 \quad \therefore x_1 = -\frac{30}{11}$$

$$|A_{x_2}| = \begin{vmatrix}1 & 4 & 1\\ 2 & -2 & 0\\ 4 & 0 & -3\end{vmatrix} = 38 \quad \therefore x_2 = -\frac{38}{11}$$

$$|A_{x_3}| = \begin{vmatrix}1 & -3 & 4\\ 2 & -1 & -2\\ 4 & 0 & 0\end{vmatrix} = 40 \quad \therefore x_3 = -\frac{40}{11}$$

Using inverse Method:

$$A^{-1} = \frac{1}{|A|}\cdot \operatorname{adj}(A)$$

$$\operatorname{adj}(A) = \underbrace{\begin{bmatrix}3 & -6 & 4\\ -9 & -7 & 12\\ 1 & -2 & 5\end{bmatrix}}_{\text{Minor of All}} \xrightarrow[\text{cofactors}]{\text{after}} \begin{bmatrix}3 & 6 & 4\\ -9 & -7 & -12\\ 1 & 2 & 5\end{bmatrix}^{T}$$

$$A^{-1} = \begin{bmatrix}3 & -9 & 1\\ 6 & -7 & 2\\ 4 & -12 & 5\end{bmatrix}\cdot -\frac{1}{11} = \begin{bmatrix}-\frac{3}{11} & \frac{9}{11} & -\frac{1}{11}\\ -\frac{6}{11} & \frac{7}{11} & -\frac{2}{11}\\ -\frac{4}{11} & \frac{12}{11} & -\frac{5}{11}\end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} -3/11 & 9/11 & -1/11 \\ -6/11 & 7/11 & -2/11 \\ -4/11 & 12/11 & -5/11 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

$$\underbrace{3\times3 \qquad\qquad 3\times1}$$

$$= \begin{bmatrix} -30/11 \\ -38/11 \\ -40/11 \end{bmatrix} \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \\ \rightarrow x_3 \end{matrix}$$

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$\boxed{g}$ $|A| = 0 \begin{vmatrix} -\sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix} - 0 \begin{vmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{vmatrix} + 1 \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

$= \cos^2\theta + \sin^2\theta = 1$, then $A$ is invertible for All Values of $\theta$

$$A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj}(A)$$

$$\operatorname{adj}(A) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \xLeftarrow{\text{cofactors}} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot 1$$

$$AX = B$$

16) $A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 1 & 1 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 1 \\ -3 \\ 3 \end{bmatrix}$

$$y = \frac{\det(A_y)}{\det(A)}$$

$$\det(A) = 4\begin{vmatrix} 7 & -1 & 1 \\ 3 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & -1 & 1 \\ 7 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 7 & 1 \\ 7 & 3 & 8 \\ 1 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 7 & -1 \\ 7 & 3 & -5 \\ 1 & 1 & 1 \end{vmatrix}$$

$$4\left(7\underbrace{\begin{vmatrix} -5 & 8 \\ 1 & 2 \end{vmatrix}}_{-18} + 1\underbrace{\begin{vmatrix} 3 & 8 \\ 1 & 2 \end{vmatrix}}_{-2} + 1\underbrace{\begin{vmatrix} 3 & -5 \\ 3 & 1 \end{vmatrix}}_{12}\right)$$

$$-1\left[3\begin{vmatrix} -5 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 7 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 7 & -5 \\ 1 & 1 \end{vmatrix}\right]$$

$$+1\left(3\begin{vmatrix} 3 & 8 \\ 1 & 2 \end{vmatrix} - 7\begin{vmatrix} 7 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 7 & 3 \\ 1 & 1 \end{vmatrix}\right)$$

$$-1\left(3\begin{vmatrix} 3 & -5 \\ 1 & 1 \end{vmatrix} - 7\begin{vmatrix} 7 & -5 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 7 & 3 \\ 1 & 1 \end{vmatrix}\right)$$

$$\Rightarrow -480 + 36 - 44 + 44 \Rightarrow -424 \neq 0$$

$$\Delta y = \begin{vmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{vmatrix}$$

$$4\begin{vmatrix} 1 & -1 & 1 \\ -3 & -5 & 8 \\ 3 & 1 & 2 \end{vmatrix} - 6\begin{vmatrix} 3 & -1 & 1 \\ 7 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 & 1 \\ 7 & -3 & 8 \\ 1 & 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 & -1 \\ 7 & -3 & -5 \\ 1 & 3 & 1 \end{vmatrix}$$

$$4\left[1\begin{vmatrix} -5 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} -3 & 8 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} -3 & -5 \\ 3 & 1 \end{vmatrix}\right]$$

$$-6\left[3\begin{vmatrix} -5 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 7 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 7 & -5 \\ 1 & 1 \end{vmatrix}\right]$$

$$+1\left[3\begin{vmatrix} -3 & 8 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 7 & 8 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 7 & -3 \\ 1 & 3 \end{vmatrix}\right]$$

$$-1\left[3\begin{vmatrix} -3 & -5 \\ 3 & 1 \end{vmatrix} - 1\begin{vmatrix} 7 & -5 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 7 & -3 \\ 1 & 3 \end{vmatrix}\right]$$

$$\Rightarrow -144 + 216 - 72 = 0 \qquad \therefore y = \frac{|A_y|}{|A|} = \frac{0}{-424} = 0$$