第七章

赛题：Camping along the Big Long River

Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar-powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river. Currently, X trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park managers have been asked to allow more trips to travel down the river. They want to determine how they might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other words, how many more boat trips could be added to the Big Long River's rafting season? The river managers have hired you to advise them on ways in which to develop the best schedule and on ways in which to determine the carrying capacity of the river, remembering that no two sets of campers can occupy the same site at the same time. In addition to your one page summary sheet, prepare a one page memo to the managers of the river describing your key findings.

游客到大长河(225 英里)可以领略风光和令人兴奋的白色激流。这条河是进不去登山者的,因此观光只能用的方式是沿河旅行,这需要几天的露营。所有的沿河旅行都始于第一站并在下游 225 英里远的最终出口退出。观光者或者乘坐桨动力船只以平均每小时 4 英里速度旅行,或者乘坐摩托船以平均每小时 8 英里旅行。这趟沿河行程可能有 6 至 18 个夜晚需在河边露营。政府管理当局负责管理这条河,使得每一个旅行者都可享受一份野营体验,且在河岸可以最少地接触其他的船只。目前,在每年在六个月的旅游季节中有 X 个旅行团队沿着大长河旅行,一年其余部分的天气太冷不适合做这种河流旅行。有 Y 个营地差不多均匀分布在大长河河道两岸上。因为沿河漂流受欢迎程度上升,有关方要求公园管理者允许更多的旅行团队沿河旅行。他们想确定怎样可以安排一种最优的混合旅行方式,可以是不同的持续时间(以在河边过夜为单位)和不同的旅行方式进(马达或桨),可以

最大限度地利用好营地。换句话说,在漂流季节可能容纳多少旅行团队?河流管理者雇用你给他们提供建议,安排一个最佳行程方式和测算河流的承载能力,记住没有两队露营者在同一时间占据同一露营地点。除了你的一页摘要,请准备一页备忘录,对河流管理人员描述你主要的发现。

第八章

论文 1: Optimal Rafting Schedule for River Manager

by Tong Yang, Bowen Du and Li Dong

Summary

River rafting has gained more and more popularity nowadays. The river manager is searching a schedule for more trips and less contact. Their profit directly depends on the number of tourists and affected by campsites. To obtain a maximum profit, they should decide an optimal number of trips during the 6-month period and that of campsites offered. In the meantime, less contact should be considered when rafting and choosing the campsite.

A Probability Model is proposed to describe the stochastic behavior of deciding on boats and distances proceeded each day. The relation between trips number and campsite number is analyzed when it's in the "saturation" state. In this state, the number of boats existing on the river equals that of campsites each day by using the fluid profile equivalence method. The river is treated as a pipeline and a profile is drawn from it. The tourist flow traveling across the profile each day is identical to the largest trips number. Some constraints make campsite number determined. For example, the contact number when rafting is below a given value. In the end, the largest trips number is 928 and the campsite number is 38. Then random circle method is adopted to search the optimal trips number. However, the method centers on the state of non-saturation and the contact on the campsites. The campsites are arranged in circular figure and trips of maximum number are randomly placed at these campsites. Then the next campsites are appointed to each trip according to certain probability. Simulations are made 500 thousand times for each time to reduce trips number. The average probability of contact among these trips corresponding to different trips numbers can be worked out. Given acceptable standards, the optimal trips number is 781 and campsite number is 38.

Next, the modified online algorithm is utilized to draft a reasonable plan for tourists to choose unoccupied campsites. This algorithm focuses only on the previous state of campsites being occupied. And the probability of the available campsites for tourists in the

next state can be worked out and tourists are given a list of ranked campsites to choose based on the probability of contact. As for the trip schedule of the river manager, the total number of trips of different durations and propulsion is proposed. The river manager can also separate off season from busy season and adjust the number of trips with different durations launching in one day flexibly. For example, they can substitute three 6-days trips for one 18-days trip at the busy season.

§ 8.1 Introduction

River rafting has gained more and more popularity nowadays. Through the trip, the tourists can enjoy exhilarating rapids, scenic views and camping experience among the unique beauty. A well-equipped outfit for rafting is a must and tourists can choose different combination. For example, they can choose oar-powered rubber rafts or motorized boats based on their expected tour duration and specific preference. Also a set of camping outfit is needed in that the long trip may last a couple of days or weeks. The campsite is built along the riverside and tourists can adjust their pace to find unoccupied campsites to stay the night[1].

However, the supply capacity can't meet the rising demand well in the light of existing launch schedule and campsite layout. The river managers want more trips to go on rafting every year and gain more profits. In order to raise the degree of tourist satisfaction, they also have to consider the way to minimize the contact between different groups on both the river and campsite. Then, they need an optimal schedule to accomplish the above-mentioned goals. A classical and well-studied example is the management of the Grand Canyon river trip. The Colorado River in Grand Canyon is the famous destination for rafting, and Catherine A. Roberts along with her partners has developed a simulator for the control of the tourist flow there. Given the desired length of trip, the simulator can give advice on duration of rafting, attraction sites and the choice of campsites.

In this problem, the river manager needs to develop the optimal schedule for deciding the duration, propulsion and launch timetable. The goal is to maximize the profit of the manager and the constraints are less contact, larger possibility of finding unoccupied campsites and fuller use of the campsite in one day. Most importantly, they need to determine an optimal number of trips during the open period and optimal number of campsites. The river manager's profit directly depends on the number of tourists and campsites.

We mainly propose Probability and optimization Model to determine the value of X and Y. With X and Y known, we develop a feasible schedule for the river manager. In this course, we use some methods such as random circle method, modified online algorithm and so on.

Schedules like the number of trips launching each day, proportion of different durations and propulsion are also given for reference. Finally we further relax some assumptions, improve previous models to approach the practical situation and develop some flexible schedules for river manager to balance. For example, we can separate slack season from boom season and adjust the number of trips launching in one day according to tourist flow each day.

§ 8.2 Restatement of the Problem

Alongside this 225-miles-long river, there are Y campsites for tourists to stay the night. In practice, one campsite can only accommodate one group. Because the trip may last about 6～18 nights, these tourists must choose 6 - 18 campsites from the total during their stay along this river. In First Launch, the tourists can take oar-powered rubber rafts with an average speed of 4 mph or motorized boats which travel on average 8 mph. Once they choose the specific outfit, they can decide the time they raft on the river and the distance they proceed in one day spontaneously. These decisions are out control of the river manager and fully stochastic. They only know every group is sure to exit the river in the range of 6 - 18 nights. In this situation, the tourist can only visit the river during a six-month period each year (suppose 30 days in one month). During this period, the river can only accommodate X trips for rafting and camping. The river manager wants more trips to go on rafting and less contact between different groups. So they need to decide the X value that is maximum and feasible as much as possible. It should be noted that X is subjected to the amount of campsites. By experience, X can be approximately proportional to Y in some degree. The choice of Y, which is the measurement of the river capacity, is also very significant. With X and Y determined, the river manager needs to develop the optimal schedule for deciding the duration, propulsion and launch timetable. The goal they pursue is to maximize their profit given the constraints of ensuring minimum contact between different groups and less probability for tourists finding no campsites to stay.

§ 8.3 Problem Analysis

We suppose that the behavior of tourists is stochastic and the choice of duration, propulsion and campsites obeys a certain type of probability distribution. In order to determine the parameters of the distribution such as mean value and variance, we need to collect historical data bound up with rafting along the river and process these data statistically. We can obtain the probability of choosing propulsion and the distribution in the choice of duration for tourists with different propulsion on average. We also need to know how far tourists proceed with certain duration in one day. According to the assumption, the tourist can choose the distance with freedom and the distances in one day obey the normal distribution. We should work out the mean value and its variance. In this course, we need to utilize 3σ principle to determine variance and the inverse of duration as the mean value. We obtain the probability of different proportion among the total journey. Coupled with the probability of choosing propulsion, we can get the distribution of the average distance in one day for tourists with different duration.

The relation of X and Y can also be determined in the extreme situation where the boats staying on the river occupy all campsites in one day. Suppose the number of the tourists reaches a peak, which equals the number of available campsites. We treat the river as a pipeline and draw a profile from it. Assume the tourist flow is in the state of saturation among enough scope at both sides of the profile. So we think the tourist flow traveling across the profile each day identical with the maximum tourist number. Then we can get the relation between X and Y.

Because the motorized boat travels faster than oar-powered rafts, the contact during the raft must be the situation of the former overtaking the latter. So we focus on the motorized boats launching each day and obtain the contact for the trips launching in this day according to these trips' relative motion. We assume the mass of trips as continuous fluid which can be divided into 2 levels. The top level is the flow of motorized boats and the bottom one is the flow of oar-powered rafts. The density of these two levels can be worked out. Then we can obtain the contact in one day for trips launching in the same day on average statistically.

In order to determine X and Y, we must use the optimization method. Our goal is to maximize the profit with the constraints of contact on the river or at the campsite below a desired value, the using degree of the campsites maximum and the convenience degree of finding available campsites for one trip above a specific value. By this method, we narrow the range of Y and decide a value to our satisfaction and standard. By using the relation between X and Y, we can determine the maximum value of X. Then we should find an optimal number of trips launching in the period. We can use random circle algorithm to simulate the actual tourist flow. At the beginning, we arrange X trips randomly into $Y+1$ points. Then we appoint destinations for each trip according to the given probability and count the number of contact in position. Now, we can work out the proportion of contact trip among x. Go on like this for many times and get the average value as the probability of contact on the campsites. Later, we alter the value of X and also work out the contact probability. Choose the optimal contact probability and its corresponding X value is what we want.

With X and Y known, we can develop the best schedule to make full use of the campsites and attract more trips on the river. The schedule covers the decision of how many trips launch in one day, the proportion of different durations and different propulsion.

We can further relax some assumptions to approach the practical situation. For example, we can consider the distinction of slack season and boom season so as to adjust the number of trips launching in one day.

§ 8.4 Assumptions

- The campsites are distributed fairly uniformly alongside the river corridor, which means equal intervals between adjacent campsites;
- If a tourist chooses motorized boat, the time he choose through the whole trip obey the Poisson distribution with λ as the mean value. If the tourist chooses oar-powered rafts, the time obey the same distribution with a different mean value;
- The distance groups of different duration proceed in one day obeys the normal distribution with different mean values and variance;
- At each campsite, the tourist can't alter the type of propulsion;
- The tourist must launch at the entrance and exit at the final. In other word, they can only complete the journey;
- During the raft on the river, the tourist can choose campsite anywhere only if the campsite hasn't been occupied by others;
- Once the tourist set off, they are free to make their decision about the distance each day, the campsite choice and so on. That means the choice of different groups is independently distributed and free from other's intervene;
- There are no campsites at the Launch and Exit;
- The maximum time on the river is confined to 10 h;
- Tourists depart from the launch at the same time in one day.

§ 8.5 Model and Solution

As mentioned in the Problem Analysis sector, we need to collect historical data bound up with rafting along the river and process these data statistically. Some data are listed below:

Table 8.1 Launches of Trips[2]

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	23	92	90	92	91	37	0	425
Non-Motor	16	31	30	28	28	37	11	181
Total	39	123	120	120	119	74	11	606
Percent	6.4%	20.3%	19.8%	19.8%	19.6%	12.2%	1.8%	

Table 8.2 The Number of Passengers [2]

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	492	3139	3219	3251	2929	924	0	13954
Non-Motor	352	787	846	851	739	705	183	4463
Total	844	3926	4065	4102	3668	1629	183	18417
Percent	4.6%	21.3%	22.1%	22.3%	19.9%	8.8%	1.0%	

We can obtain the probability of choosing propulsion.

Table 8.3 Total Days of Trips [2]

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	3014	15996	17022	16788	14794	5076	0	72690
Non-Motor	457	7248	7052	6642	6011	8288	2618	41316
Total	6471	23244	24074	23430	20805	13364	2618	114006
Percent	5.7%	20.4%	21.1%	20.6%	18.2%	11.7%	2.3%	

Table 8.4 The Probability of Choosing

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	0.590	0.748	0.750	0.767	0.765	0.5	0	0.701
Non-Motor	0.410	0.252	0.250	0.233	0.235	0.5	1	0.299

Through calculation, we get the probability of choosing motorized boats as 0.7 and oar-powered rafts as 0.3.

$$P_m = 0.7, \quad P_o = 0.3. \tag{8.1}$$

If the tourist chooses motorized boat, the time they make through the whole trip obey the Poisson distribution with λ as the mean value. Through calculation, we can determine λ as 5.21.

If the tourist chooses oar-powered rafts, the time obey the same distribution with a different mean value. Through calculation, we determine λ as 9.25.

Table 8.5 The Times of Choosing

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	6.126	5.096	5.288	5.164	5.051	5.494	NaN	5.209
Non-Motor	9.821	9.210	8.336	7.805	8.134	11.756	14.306	9.258

Then, we can get the distribution functions below:

$$p(k) = \frac{\lambda}{k!} e^{-\lambda}, \tag{8.2}$$

$$p_1(k) = \frac{5.21^k}{k!} e^{-5.21}, \tag{8.3}$$

$$p_2(k) = \frac{9.26^k}{k!} e^{-9.26}. \tag{8.4}$$

In order to make the sum of probability of different durations equal 1, we should normalize the probability. Then we get the probability of choosing one certain duration when using different propulsion.

$$p_m(k) = \frac{p_1(k)}{\sum_{i=6}^{18} p_1(i)}, \tag{8.5}$$

$$p_o(k) = \frac{p_2(k)}{\sum_{i=6}^{18} p_2(i)}. \tag{8.6}$$

Coupled with the probability of choosing motorized or oar-powered boats, we get the average value:

$$p(k) = P_m \cdot p_m(k) + P_o \cdot p_o(k). \tag{8.7}$$

We list the result in the next table:

Table 8.6 **The Average Probability of Choosing**

days	6	7	8	9	10	11	12
motor	0.3605	0.2683	0.1747	0.1012	0.0527	0.0250	0.0108
oar	0.0930	0.1230	0.1424	0.1465	0.1357	0.1142	0.0881
average	0.2803	0.2247	0.1650	0.1148	0.0776	0.0517	0.0340
days	13	14	15	16	17	18	
motor	0.0043	0.0016	0.0006	0.0002	0.0001	0.0000	
oar	0.0628	0.0415	0.0256	0.0148	0.0081	0.0042	
average	0.0219	0.0136	0.0081	0.0046	0.0025	0.0013	

According to the assumption, the tourist can choose the distance with freedom and the distances in one day obey the normal distribution[2]. We should work out the mean value and its variance. Suppose one tourist choose i-days trip, the average distances each day is $1/i$ of the total journey,

$$a_i = \frac{1}{i}. \tag{8.8}$$

We know the maximum time for rafting

$$t_{\max} = 10\ \mathrm{h}. \tag{8.9}$$

The average speed:

$$\bar{v} = P_m \cdot v_m + P_o \cdot v_o = 6.8\ (\text{miles/h}). \tag{8.10}$$

So we can get the proportion of the maximum distances each day (the total as 1):

$$l_{\max} = \frac{t_{\max}\bar{v}}{l} = \frac{10\ \text{h} \times 6.8\ \text{miles/h}}{225\ \text{miles}} \approx 0.30. \tag{8.11}$$

We need to utilize 3σ principle to determine variance:

$$3\sigma_i = 0.30 - \frac{1}{i}, \tag{8.12}$$

$$\sigma_i = 0.10 - \frac{1}{3i} \quad (i = 6, 7, 8, \cdots, 18). \tag{8.13}$$

Then, we get the probability function of the distances each day for i-days trips:

$$\varphi_i(t) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(t-a_i)^2}{2\sigma_i^2}}. \tag{8.14}$$

Coupled with the probability of choosing propulsion, we can get the distribution of the average distance in one day for tourists with different duration.

$$\varphi(t) = \sum_{i=6}^{18} p(i)\varphi_i(t). \tag{8.15}$$

The relation of X and Y can also be determined in the extreme situation where the boats staying on the river occupy all campsites in one day. Suppose the number of the tourists reaches a peak, which equals the number of available campsites. We treat the river as a pipeline and draw a profile from it. Assume the tourist flow is in the state of saturation among enough scope at both sides of the profile. So we think the tourist flow traveling across the profile each day identical with the maximum tourist number. That satisfies the equation:

$$\int_0^{+\infty} (Y+1)\frac{\mathrm{d}s}{1} \cdot \left(1 - \int_0^s \varphi(t)\mathrm{d}t\right) = \frac{x}{180}. \tag{8.16}$$

We can get the relation expression as:

$$X = 23.8(Y+1). \tag{8.17}$$

Optimization

Our goal is to maximize the river manager's profit and raise the satisfaction degree of tourists as much as possible. We will analyze the optimization problem in the next section.

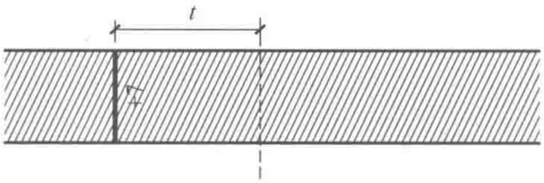

Figure 8.1 Passenger Flow

We suppose the tourist stops at the campsite nearest to the place where they are most likely to stop. We know the average speed is $\bar{v}=$

6. 8 miles/h and the maximum hour for rafting is $t_{\max}=10$ h.

We try to estimate different values of Y and then get the acceptable range of Y according to related optimization method. It's found that the value of 38 can match well up with the range. Now, we give the calculation process setting Y as 38, get the reasonable range and verify our initial outcome.

So when we have 38 points which symbolize 38 campsites along the river, the point tourists can arrive at can be calculated as follows:

$$N=\frac{t_{\max}\cdot \bar{v}}{\frac{l}{Y+1}}=\frac{10\times 6.8}{\frac{225}{38+1}}=11.8. \quad (8.18)$$

We get the result that the tourists can get to the 12th points if they raft for 10h. As the picture below shows, we can work out the probability that one trip starting from one point arrives at the following 12 points by using the density function:

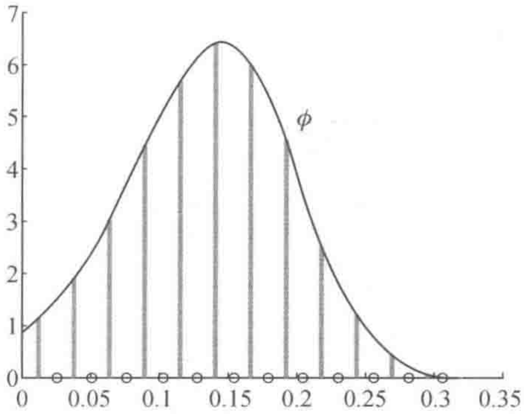

Figure 8. 2 The Density Function

Assuming i as the number of points away from the starting point, then we can get the probability of camping at this point:

$$q(i)=\int_{\frac{2i-1}{2}\frac{1}{Y+1}}^{\frac{2i+1}{2}\frac{1}{Y+1}}\varphi(x)\mathrm{d}x. \quad (8.19)$$

Table 8. 7 The Probability of Camping

i	1	2	3	4	5	6
$q(i)$	0. 0397	0. 0641	0. 0956	0. 1301	0. 1572	0. 1618
i	7	8	9	10	11	12
$q(i)$	0. 1367	0. 0919	0. 0484	0. 0199	0. 0065	0. 0017

Constraints

1. When the number of boats existing on the river equals that of campsites each day, the probability of one campsite being occupied: For one certain day, if we know every trips' starting point, we can get the probability of the points being occupied in the night. We assume that one campsite may be occupied by X trips. The probability of one trip stopping at this point is set as $q_i (i=1, 2, \cdots, x)$. Then the probability of this point being occupied is:

$$P=1-\prod_{i=1}^{x}(1-q_i). \quad (8.20)$$

Through the calculation above, we can get the probability of points being occupied in one certain day. Considering the full use of the points, we think one trip start from each point and set $x=12$. So we can get:

$$q_i = q(i). \tag{8.21}$$

$$P = 1 - \prod_{i=1}^{x}(1-q(i)). \tag{8.22}$$

In this situation, P is only related to Y. We get the value of P with different Y and list them below:

Table 8.8 The Probability of One Campsite Being Occupied

Y	30	31	32	33	34	35	36	37	38
P	0.6429	0.6421	0.6414	0.6408	0.6402	0.6396	0.6390	0.6385	0.6380
Y	39	40	41	42	43	44	45	46	47
P	0.6375	0.6371	0.6367	0.6363	0.6359	0.6355	0.6351	0.6348	0.6345
Y	48	49	50	51	52	53	54	55	56
P	0.6341	0.6338	0.6335	0.6333	0.6330	0.6327	0.6325	0.6323	0.6320

2. When the number of boats existing on the river equals that of campsites each day, the probability of one campsite being occupied by at least 2 trips:

$$P' = \sum_{i>j} q_i \cdot q_j = \sum_{i>j} q(i) \cdot q(j). \tag{8.23}$$

3. When the number of boats existing on the river equals that of campsites each day, the time of the tourist in one trip finding an unoccupied campsite with the probability of 0.9:

Table 8.9 The Probability of One Campsite Being Occupied By At Least 2 Trips

Y	30	31	32	33	34	35	36	37	38
P'	0.3810	0.3837	0.3863	0.3887	0.3910	0.3932	0.3952	0.3971	0.3990
Y	39	40	41	42	43	44	45	46	47
P'	0.4007	0.4024	0.4040	0.4055	0.4069	0.4082	0.4095	0.4108	0.4119
Y	48	49	50	51	52	53	54	55	56
P'	0.4130	0.4140	0.4150	0.4162	0.4171	0.4179	0.4190	0.4198	0.4206

Under this situation, we assume that the probability of each campsite being occupied approximately obeys binary distribution. So we can work out the probability of the tourist in one trip finding at least an unoccupied campsite k:

$$k = 1 - P^{t\frac{\bar{x}}{t}(Y+1)}. \tag{8.24}$$

We get:

$$t = \frac{l}{\bar{v}} \cdot \frac{1}{Y+1} \log_P (1-k). \tag{8.25}$$

We set $k=0.9$, and represent the relation of t and Y in the form of table:

Table 8.10　The Time of The Probability of 0.9

Y	30	31	32	33	34	35	36	37	38
t	5.3603	5.1618	4.9963	4.8309	4.6985	4.5662	4.4338	4.3015	4.1691
Y	39	40	41	42	43	44	45	46	47
t	4.0699	3.9706	3.8713	3.7721	3.6728	3.6066	3.5074	3.4412	3.3750
Y	48	49	50	51	52	53	54	55	56
t	3.2757	3.2096	3.1434	3.0772	3.0441	2.9779	2.9118	2.8456	2.8125

4. We assume the mass of trips as continuous fluid which can be divided into 2 levels. The top of the picture denotes the starting state and the bottom of it denotes the ending state in one day. The density of these two levels can be denoted as

$$\rho_m = 0.7(Y+1), \quad \rho_o = 0.3(Y+1). \tag{8.26}$$

We can get average distances of one trip according to density function φ:

$$\bar{s} = \int_0^{+\infty} \varphi(x) \cdot X \mathrm{d}x = 0.1318. \tag{8.27}$$

Because of the speed of motorized boats being twice that of oar-powered rafts and these two levels' density relation, we can get:

$$\begin{cases} s_m = 2s_o, \\ \bar{s} = 0.7\, s_m + 0.3\, s_o. \end{cases} \tag{8.28}$$

And then, it gives:

$$\begin{cases} s_m = 0.1550, \\ \bar{s} = 0.0775. \end{cases} \tag{8.29}$$

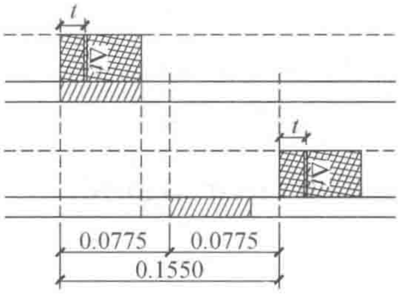

Figure 8.3　The contact of trips

We can work out the contact number of trips launching in the same day in the situation where the number of boats existing on the river equals that of campsites each day in one night.

$$C = \int_0^{\frac{X}{180(Y+1)}} \mathrm{d}t \cdot 0.7(Y+1) \cdot \frac{s_m}{2} \cdot 0.3(Y+1) \approx \frac{X^2}{2.63 \times 10^5}. \tag{8.30}$$

Above is the description of four constraints, and now calculation is made as follows:

1. The contact number of trips launching in one day is below 4 in this day:

$$C=\int_0^{\frac{x}{180(Y+1)}} dt \cdot 0.7(Y+1) \cdot \frac{s_m}{2} \cdot 0.3(Y+1) < 4. \tag{8.31}$$

Table 8.11 The Contact Number

Y	30	31	32	33	34	35	36	37	38
C	2.0698	2.2055	2.3454	2.4898	72.6384	2.7913	2.9485	3.1100	3.2759
Y	39	40	41	42	43	44	45	46	47
C	3.4460	3.6205	3.7992	3.9823	4.1697	4.3614	4.5574	4.7577	4.9623
Y	48	49	50	51	52	53	54	55	56
C	5.1712	5.3844	5.6019	5.8238	6.0499	6.2804	6.5151	6.7542	6.9976

2. The time of tourists finding an unoccupied campsite in the probability of 0.9 is below 5 h:

$$t=\frac{l}{\bar{v}} \cdot \frac{1}{Y+1} \log_P(1-k) < 5. \tag{8.32}$$

3. The average number of trips camped at each campsite is larger than 20:

$$r=\frac{X}{Y} > 20. \tag{8.33}$$

4. The probability of contact on the campsites is below 0.4:

$$P'=\sum_{i>j} q(i) \cdot q(j) < 0.4. \tag{8.34}$$

Considering the four constraints above, we confirm Y as 38.

According to the relation between X and Y solved above, the maximum X can be set as 928 and Y as 38.

Random circle algorithm

The above-mentioned discussion is under the assumption of saturation (the number of boats existing on the river equals that of campsites each day). But that is the extreme situation and we don't want it to occur[3, 4]. When it fails to reach saturation, the problems of contact and the convenience of finding an unoccupied campsite will be less[5]. In order to manage trips reasonably, we research into the situation of non-saturation and analyze the contact on the campsites[6][7].

We adopt the random simulation on the computer. In order to simulate the actual tourist flow, we arrange the campsites in circular shape. According to the outcome above, Y is set as 38. So, there are 39 points in the figure symbolizing campsites, launch and exit site.

When it is in the non-saturation state, $X<38$. At the beginning, we arrange X trips randomly into 39 points. Then we appoint destinations for each trip according to the probability given by equation (8.19) and count the number of contact in position. Now, we can work out the proportion of contact trip among x. Go on like this for many times and get the average value as the probability of contact on the campsites. Later, the value of X is altered and the contact probability is also worked out.

For example, X is set as 35 and then 35 trips are appointed at 35 initial positions in the figure randomly. The figure is shown below:

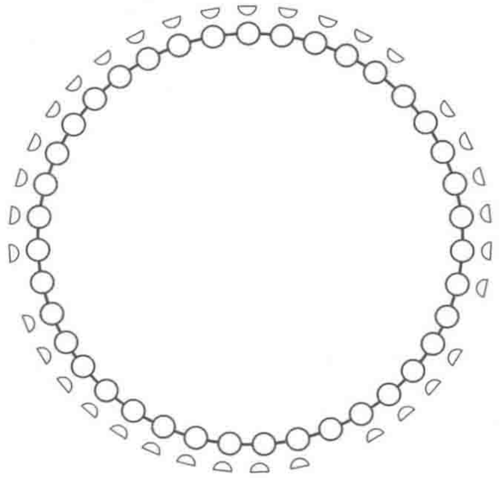

Figure 8.4 The Initial State of Random Circle

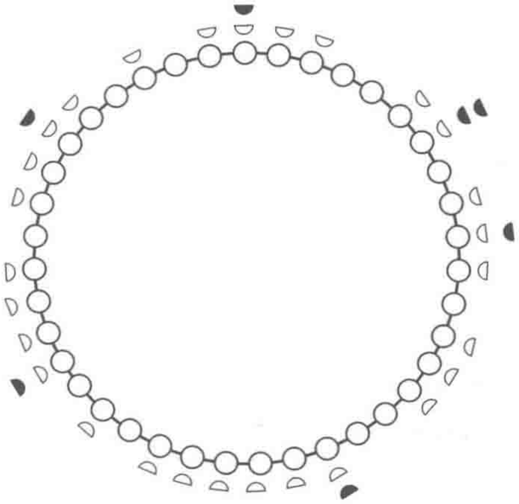

Figure 8.5 The Next State of Random Circle

Then, we choose the destinations of each trip according to corresponding probability, the result is shown below:

Now, we get the number of contact trips 7 and the probability is 0.200. Below is the table covering the average number of contact trips and the corresponding probability with different X value. As for every X value, we go on simulating for 500000 times.

We can accept the result that the probability of contact is less than 0.2. Considering the data in the table, we choose $x=32$ and get the total trips during the open period $X = 32/38 \times 928 = 781$. The average contact number is 2 each day, which is to our satisfaction.

Table 8.12 The Probability of Contact On The Campsite

X	38	37	36	35	34	33	32
number	9.38	8.89	8.41	7.96	7.50	7.06	6.64
probability	0.247	0.240	0.234	0.227	0.221	0.214	0.208
X	31	30	29	28	27	26	25
number	6.23	5.83	5.45	5.08	4.72	4.37	4.04
probability	0.201	0.194	0.187	0.181	0.175	0.168	0.162

According to the result above, we make $X=781$. First, we assign the number of motorized boats and oar-powered rafts by the ratio 0.7 : 0.3. The number of motorized boats $X_m = P_m \cdot X = 546.7$. The number of oar-powered rafts $X_o = P_o \cdot X = 234.3$. Then, we can work out the total number of different durations by their Poisson distribution (see the table below).

Table 8.13 The Schedule of Trips

Days	6	7	8	9	10	11	12	13	14	15	16	17	18
Motor trips	197	146	96	55	29	14	6	2	1	0	0	0	0
Oar trips	22	29	33	34	32	27	21	15	10	6	3	2	1

The schedule above can act as reference for the manager and they can make a tiny adjustment flexibly according to tourist flow. According to the campsites occupied in each case, we have the equation below:

$$N_i \cdot i = N_j \cdot j,$$
$$N_i = j/i \cdot N_j.$$

N_i denotes the number of campsites i-days trip occupies during the journey. For example, if the river manager wants to decrease one 18-days trip and increase three 6-days trips. Then we get: $N_6 = 18/6\ N_{18} = 3$.

That means the river manager can schedule three more 6-days trip by decreasing one 18-days trip.

Modified online algorithm

Based on the campsites from which trips start, we make analysis from the last to the first. As for each trip starting from one point, we determine the range that it may exist in the night, which covers 12 points. In the range, we rank the campsite according to the probability of being occupied by one certain trip. The list represents the order of campsites we advise the tourist to choose in the night. Every mark will be on the list regardless of its probability. The mark is the prior point we advise tourists to choose in the night. In this way can we ensure each trip can find unoccupied campsite to large degree.

For example, in one certain day, trips start from the 3rd, 5th, 7th, 11th, 14th, 22nd, 25th, 34th points, the picture of density function is shown below:

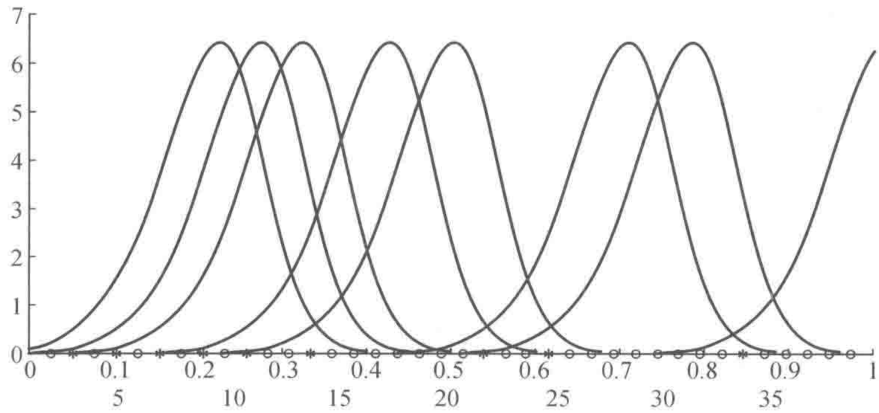

Figure 8.6 The Probability of Campsites Being Occupied

Then we advise the tourists according to the list below, and the rank symbolizes the priority of the next campsite.

In order to avoid contact on the campsite, advice about choosing campsites should be given to tourists each day. The state of campsites being occupied should be collected each day over the report from tourists finding certain campsite. Then the probability of contact on every campsite will be worked out according to our method. The river manager can give tourists a list of prior campsites to choose based on the probability of contact. Tourists can adjust their distances in one day according to our advice.

§ 8.6 Evaluation of the Model

Strength

1. The essence of the model is probability description and optimization method. The probability model describes the stochastic behavior of tourists well and avoids the sophisticated discussion about different situations. The traditional optimization method matches up well with the probability model. By optimizing our goal and constraints, we narrow the range of Y. Considering practical constraints as many as possible, we determine an appropriate value of Y.

Table 8.14 The campsites suggested

label	34										
campsites	35	36	37	38	exit						
label	25										
campsites	34	36	37	33	26	32	27	31	28	30	29	35
label	22											
campsites	24	25	23	33	26	32	27	31	28	30	29	34
label	14											
campsites	25	23	22	26	21	20	19	18	16	17	15	24
label	11											
campsites	23	22	21	20	19	18	16	17	15	14	13	12
label	7											
campsites	19	8	18	16	17	15	14	9	13	10	11	12
label	5											
campsites	6	7	8	16	17	15	14	9	13	10	11	12
label	3											
campsites	4	5	7	8	15	14	9	13	10	11	12	6

2. By using modified real-time online algorithm, we can give valuable advice on tourists about which campsite can be chosen next night. This can avoid the worst situation where some tourists can't find an empty campsite as much as possible.

3. The random circle method helps us determine the optimal value of X to a certain extent. By simulating stochastic process of matching trips and campsites, we minimize contact in one day. This gives us a statistically feasible solution.

Weakness

1. We can't give a clear schedule about launching number, proportion of different durations and propulsion each day. The method can only gives out the total number of trips of different durations and different propulsion. It needs the manager to balance and cope with possible situations;

2. The assumption of probability distribution of different behavior is derived from the congeneric data of other river for rafting. Maybe it deviates from the practical situation.

Improvement

1. We can separate slack season from boom season and adjust the number of trips launching in one day according to tourist flow each day;

2. We may form an index to measure the using degree of campsites and add another constraint to optimization model, which may improve the accuracy of the value of X and Y.

§ 8.7 Conclusions

The optimal number of trips launching during the open period is 781 and the campsite number is 38. The number of ships launching each day can be the same or varies in the range of 4 - 6. Propulsion can be chosen by tourists at random. The average contact number is 2 each day, which is to our satisfaction. The number of trips with different durations is listed in the form of Table 8. 14.

In order to avoid contact on the campsite, advice about choosing campsites should be given to tourists each day. The state of campsites being occupied should be collected each day over the report from tourists finding certain campsite. Then the probability of contact on every campsite will be worked out according to our method. The river manager can give tourists a list of prior campsites to choose based on the probability of contact. Tourists can adjust their distances in one day according to our advice. The example is listed in the form of chart at the end.

The river manager can also separate off season from busy season and adjust the number of trips with different durations launching in one day flexibly. For example, the river manager can substitute three 6-days trips for one 18-days trip at the busy season.

Table 8.15 The Schedule of Trips

Days	6	7	8	9	10	11	12	13	14	15	16	17	18
Motor trips	197	146	96	55	29	14	6	2	1	0	0	0	0
Oar trips	22	29	33	34	32	27	21	15	10	6	3	2	1

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[7] Joanna A. Bieri and Catherine A. Roberts "Using the Grand Canyon River Trip Simulator to Test New Launch Schedules on the Colorado River", Association for Women in Science, 2000, 29(3), 6-10.

§ 8.8 Memo

To: The river manager
From: Team 13772
Subject: Schedule for rafting
Date: 14 February 2012

We are presenting our own schedule for rafting trip to attract more tourists to raft on the river and ensure less contact. The number of ships launching each day can vary in the range of 4-6. Propulsion can be chosen by tourists at random. Then the optimal number of trips launching during the open period is 781 and the campsite number is 38. The average contact number is 2 each day, which is to our satisfaction. In the schedule, the number of trips increases with durations decreasing. And the ratio between motorized boats and oar-powered ones is 9 : 1 in the 6-days trip, 5 : 1 in the 7-days trip and so on. The schedule of trips with different durations can be obtained from us.

In order to avoid contact on the campsite, the state of campsites being occupied should be collected each day over the report from tourists finding certain campsite. Then the probability of contact on every campsite will be worked out according to our method. We can give tourists a list of prior campsites to choose based on the probability of contact. Tourists can adjust their distances in one day according to our advice.

We can also separate off season from busy season and adjust the number of trips with different durations launching in one day flexibly. For example, we can substitute three 6-days trips for one 18-days trip at the busy season.

If you have any question, please feel free to contact us.

§8.9 点评

陈雄达

这篇竞赛论文“Optimal rafting schedule for river manager”做的是 2012 年美国数学建模竞赛的 B 题，得一等奖。

这是一篇优秀的竞赛论文。全文结构合理，语言较顺。

第一，结构。这篇论文的结构规范完整。全文总体上思路清晰，过渡合理，给评阅者一个很好的印象。文章一共分为 7 段：引言、背景介绍、问题分析、假设、主要模型和解答、模型评价及总结论。

第二，内容。本文的工作是把一个本来关系复杂的问题分割成几个相对独立的小问题，再把它们有机地联系起来。首先参考一些资料获取露营中桨和马达的比例，估计露营天数，分析讨论这二者之间的关系。接着作为概率事件，讨论游客的每天行程，以数值模拟的方式估计大长河截面的游客流，从而给出所需的营地数量。

第三，特点。本文的特色是给出了一个随机圆圈的算法，用它来模拟大长河露营的情况，其中的示意图非常直观。

本文中的优缺点描述中也提到，本文无法给出一个确切的露营安排的描述，因为许多因素都是以概率的方式来处理的。尽管如此，还是应该给出一个典型的例子，使得读者可以看出一个一般的运营日是怎样管理的，以及如何面对其中的各种要素。

综上，尽管还有地方需要改进，这还是一篇非常优秀的竞赛论文。