第七章

赛题：Camping along the Big Long River

Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar-powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river. Currently, X trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park managers have been asked to allow more trips to travel down the river. They want to determine how they might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other words, how many more boat trips could be added to the Big Long River's rafting season? The river managers have hired you to advise them on ways in which to develop the best schedule and on ways in which to determine the carrying capacity of the river, remembering that no two sets of campers can occupy the same site at the same time. In addition to your one page summary sheet, prepare a one page memo to the managers of the river describing your key findings.

游客到大长河(225 英里)可以领略风光和令人兴奋的白色激流。这条河是进不去登山者的,因此观光只能用的方式是沿河旅行,这需要几天的露营。所有的沿河旅行都始于第一站并在下游 225 英里远的最终出口退出。观光者或者乘坐桨动力船只以平均每小时 4 英里速度旅行,或者乘坐摩托船以平均每小时 8 英里旅行。这趟沿河行程可能有 6 至 18 个夜晚需在河边露营。政府管理当局负责管理这条河,使得每一个旅行者都可享受一份野营体验,且在河岸可以最少地接触其他的船只。目前,在每年在六个月的旅游季节中有 X 个旅行团队沿着大长河旅行,一年其余部分的天气太冷不适合做这种河流旅行。有 Y 个营地差不多均匀分布在大长河河道两岸上。因为沿河漂流受欢迎程度上升,有关方要求公园管理者允许更多的旅行团队沿河旅行。他们想确定怎样可以安排一种最优的混合旅行方式,可以是不同的持续时间(以在河边过夜为单位)和不同的旅行方式进(马达或桨),可以

最大限度地利用好营地。换句话说,在漂流季节可能容纳多少旅行团队？河流管理者雇用你给他们提供建议,安排一个最佳行程方式和测算河流的承载能力,记住没有两队露营者在同一时间占据同一露营地点。除了你的一页摘要,请准备一页备忘录,对河流管理人员描述你主要的发现。

第八章

论文 1：Optimal Rafting Schedule for River Manager

by Tong Yang, Bowen Du and Li Dong

Summary

River rafting has gained more and more popularity nowadays. The river manager is searching a schedule for more trips and less contact. Their profit directly depends on the number of tourists and affected by campsites. To obtain a maximum profit, they should decide an optimal number of trips during the 6-month period and that of campsites offered. In the meantime, less contact should be considered when rafting and choosing the campsite.

A Probability Model is proposed to describe the stochastic behavior of deciding on boats and distances proceeded each day. The relation between trips number and campsite number is analyzed when it's in the "saturation" state. In this state, the number of boats existing on the river equals that of campsites each day by using the fluid profile equivalence method. The river is treated as a pipeline and a profile is drawn from it. The tourist flow traveling across the profile each day is identical to the largest trips number. Some constraints make campsite number determined. For example, the contact number when rafting is below a given value. In the end, the largest trips number is 928 and the campsite number is 38. Then random circle method is adopted to search the optimal trips number. However, the method centers on the state of non-saturation and the contact on the campsites. The campsites are arranged in circular figure and trips of maximum number are randomly placed at these campsites. Then the next campsites are appointed to each trip according to certain probability. Simulations are made 500 thousand times for each time to reduce trips number. The average probability of contact among these trips corresponding to different trips numbers can be worked out. Given acceptable standards, the optimal trips number is 781 and campsite number is 38.

Next, the modified online algorithm is utilized to draft a reasonable plan for tourists to choose unoccupied campsites. This algorithm focuses only on the previous state of campsites being occupied. And the probability of the available campsites for tourists in the

next state can be worked out and tourists are given a list of ranked campsites to choose based on the probability of contact. As for the trip schedule of the river manager, the total number of trips of different durations and propulsion is proposed. The river manager can also separate off season from busy season and adjust the number of trips with different durations launching in one day flexibly. For example, they can substitute three 6-days trips for one 18-days trip at the busy season.

§ 8.1 Introduction

River rafting has gained more and more popularity nowadays. Through the trip, the tourists can enjoy exhilarating rapids, scenic views and camping experience among the unique beauty. A well-equipped outfit for rafting is a must and tourists can choose different combination. For example, they can choose oar-powered rubber rafts or motorized boats based on their expected tour duration and specific preference. Also a set of camping outfit is needed in that the long trip may last a couple of days or weeks. The campsite is built along the riverside and tourists can adjust their pace to find unoccupied campsites to stay the night[1].

However, the supply capacity can't meet the rising demand well in the light of existing launch schedule and campsite layout. The river managers want more trips to go on rafting every year and gain more profits. In order to raise the degree of tourist satisfaction, they also have to consider the way to minimize the contact between different groups on both the river and campsite. Then, they need an optimal schedule to accomplish the above-mentioned goals. A classical and well-studied example is the management of the Grand Canyon river trip. The Colorado River in Grand Canyon is the famous destination for rafting, and Catherine A. Roberts along with her partners has developed a simulator for the control of the tourist flow there. Given the desired length of trip, the simulator can give advice on duration of rafting, attraction sites and the choice of campsites.

In this problem, the river manager needs to develop the optimal schedule for deciding the duration, propulsion and launch timetable. The goal is to maximize the profit of the manager and the constraints are less contact, larger possibility of finding unoccupied campsites and fuller use of the campsite in one day. Most importantly, they need to determine an optimal number of trips during the open period and optimal number of campsites. The river manager's profit directly depends on the number of tourists and campsites.

We mainly propose Probability and optimization Model to determine the value of X and Y. With X and Y known, we develop a feasible schedule for the river manager. In this course, we use some methods such as random circle method, modified online algorithm and so on.

Schedules like the number of trips launching each day, proportion of different durations and propulsion are also given for reference. Finally we further relax some assumptions, improve previous models to approach the practical situation and develop some flexible schedules for river manager to balance. For example, we can separate slack season from boom season and adjust the number of trips launching in one day according to tourist flow each day.

§ 8.2 Restatement of the Problem

Alongside this 225-miles-long river, there are Y campsites for tourists to stay the night. In practice, one campsite can only accommodate one group. Because the trip may last about 6～18 nights, these tourists must choose 6 - 18 campsites from the total during their stay along this river. In First Launch, the tourists can take oar-powered rubber rafts with an average speed of 4 mph or motorized boats which travel on average 8 mph. Once they choose the specific outfit, they can decide the time they raft on the river and the distance they proceed in one day spontaneously. These decisions are out control of the river manager and fully stochastic. They only know every group is sure to exit the river in the range of 6 - 18 nights. In this situation, the tourist can only visit the river during a six-month period each year (suppose 30 days in one month). During this period, the river can only accommodate X trips for rafting and camping. The river manager wants more trips to go on rafting and less contact between different groups. So they need to decide the X value that is maximum and feasible as much as possible. It should be noted that X is subjected to the amount of campsites. By experience, X can be approximately proportional to Y in some degree. The choice of Y, which is the measurement of the river capacity, is also very significant. With X and Y determined, the river manager needs to develop the optimal schedule for deciding the duration, propulsion and launch timetable. The goal they pursue is to maximize their profit given the constraints of ensuring minimum contact between different groups and less probability for tourists finding no campsites to stay.

§ 8.3 Problem Analysis

We suppose that the behavior of tourists is stochastic and the choice of duration, propulsion and campsites obeys a certain type of probability distribution. In order to determine the parameters of the distribution such as mean value and variance, we need to collect historical data bound up with rafting along the river and process these data statistically. We can obtain the probability of choosing propulsion and the distribution in the choice of duration for tourists with different propulsion on average. We also need to know how far tourists proceed with certain duration in one day. According to the assumption, the tourist can choose the distance with freedom and the distances in one day obey the normal distribution. We should work out the mean value and its variance. In this course, we need to utilize 3σ principle to determine variance and the inverse of duration as the mean value. We obtain the probability of different proportion among the total journey. Coupled with the probability of choosing propulsion, we can get the distribution of the average distance in one day for tourists with different duration.

The relation of X and Y can also be determined in the extreme situation where the boats staying on the river occupy all campsites in one day. Suppose the number of the tourists reaches a peak, which equals the number of available campsites. We treat the river as a pipeline and draw a profile from it. Assume the tourist flow is in the state of saturation among enough scope at both sides of the profile. So we think the tourist flow traveling across the profile each day identical with the maximum tourist number. Then we can get the relation between X and Y.

Because the motorized boat travels faster than oar-powered rafts, the contact during the raft must be the situation of the former overtaking the latter. So we focus on the motorized boats launching each day and obtain the contact for the trips launching in this day according to these trips' relative motion. We assume the mass of trips as continuous fluid which can be divided into 2 levels. The top level is the flow of motorized boats and the bottom one is the flow of oar-powered rafts. The density of these two levels can be worked out. Then we can obtain the contact in one day for trips launching in the same day on average statistically.

In order to determine X and Y, we must use the optimization method. Our goal is to maximize the profit with the constraints of contact on the river or at the campsite below a desired value, the using degree of the campsites maximum and the convenience degree of finding available campsites for one trip above a specific value. By this method, we narrow the range of Y and decide a value to our satisfaction and standard. By using the relation between X and Y, we can determine the maximum value of X. Then we should find an optimal number of trips launching in the period. We can use random circle algorithm to simulate the actual tourist flow. At the beginning, we arrange X trips randomly into $Y+1$ points. Then we appoint destinations for each trip according to the given probability and count the number of contact in position. Now, we can work out the proportion of contact trip among x. Go on like this for many times and get the average value as the probability of contact on the campsites. Later, we alter the value of X and also work out the contact probability. Choose the optimal contact probability and its corresponding X value is what we want.

With X and Y known, we can develop the best schedule to make full use of the campsites and attract more trips on the river. The schedule covers the decision of how many trips launch in one day, the proportion of different durations and different propulsion.

We can further relax some assumptions to approach the practical situation. For example, we can consider the distinction of slack season and boom season so as to adjust the number of trips launching in one day.

§ 8.4 Assumptions

- The campsites are distributed fairly uniformly alongside the river corridor, which means equal intervals between adjacent campsites;
- If a tourist chooses motorized boat, the time he choose through the whole trip obey the Poisson distribution with λ as the mean value. If the tourist chooses oar-powered rafts, the time obey the same distribution with a different mean value;
- The distance groups of different duration proceed in one day obeys the normal distribution with different mean values and variance;
- At each campsite, the tourist can't alter the type of propulsion;
- The tourist must launch at the entrance and exit at the final. In other word, they can only complete the journey;
- During the raft on the river, the tourist can choose campsite anywhere only if the campsite hasn't been occupied by others;
- Once the tourist set off, they are free to make their decision about the distance each day, the campsite choice and so on. That means the choice of different groups is independently distributed and free from other's intervene;
- There are no campsites at the Launch and Exit;
- The maximum time on the river is confined to 10 h;
- Tourists depart from the launch at the same time in one day.

§ 8.5 Model and Solution

As mentioned in the Problem Analysis sector, we need to collect historical data bound up with rafting along the river and process these data statistically. Some data are listed below:

Table 8.1 **Launches of Trips**[2]

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	23	92	90	92	91	37	0	425
Non-Motor	16	31	30	28	28	37	11	181
Total	39	123	120	120	119	74	11	606
Percent	6.4%	20.3%	19.8%	19.8%	19.6%	12.2%	1.8%	

Table 8.2 The Number of Passengers [2]

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	492	3139	3219	3251	2929	924	0	13954
Non-Motor	352	787	846	851	739	705	183	4463
Total	844	3926	4065	4102	3668	1629	183	18417
Percent	4.6%	21.3%	22.1%	22.3%	19.9%	8.8%	1.0%	

We can obtain the probability of choosing propulsion.

Table 8.3 Total Days of Trips [2]

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	3014	15996	17022	16788	14794	5076	0	72690
Non-Motor	457	7248	7052	6642	6011	8288	2618	41316
Total	6471	23244	24074	23430	20805	13364	2618	114006
Percent	5.7%	20.4%	21.1%	20.6%	18.2%	11.7%	2.3%	

Table 8.4 The Probability of Choosing

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	0.590	0.748	0.750	0.767	0.765	0.5	0	0.701
Non-Motor	0.410	0.252	0.250	0.233	0.235	0.5	1	0.299

Through calculation, we get the probability of choosing motorized boats as 0.7 and oar-powered rafts as 0.3.

$$P_m = 0.7, \quad P_o = 0.3. \tag{8.1}$$

If the tourist chooses motorized boat, the time they make through the whole trip obey the Poisson distribution with λ as the mean value. Through calculation, we can determine λ as 5.21.

If the tourist chooses oar-powered rafts, the time obey the same distribution with a different mean value. Through calculation, we determine λ as 9.25.

Table 8.5 The Times of Choosing

	Apr	May	Jun	Jul	Aug	Sep	Oct	Total
Motor	6.126	5.096	5.288	5.164	5.051	5.494	NaN	5.209
Non-Motor	9.821	9.210	8.336	7.805	8.134	11.756	14.306	9.258

Then, we can get the distribution functions below:

$$p(k) = \frac{\lambda}{k!} e^{-\lambda}, \tag{8.2}$$

$$p_1(k) = \frac{5.21^k}{k!} e^{-5.21}, \tag{8.3}$$

$$p_2(k) = \frac{9.26^k}{k!} e^{-9.26}. \tag{8.4}$$

In order to make the sum of probability of different durations equal 1, we should normalize the probability. Then we get the probability of choosing one certain duration when using different propulsion.

$$p_m(k) = \frac{p_1(k)}{\sum_{i=6}^{18} p_1(i)}, \tag{8.5}$$

$$p_o(k) = \frac{p_2(k)}{\sum_{i=6}^{18} p_2(i)}. \tag{8.6}$$

Coupled with the probability of choosing motorized or oar-powered boats, we get the average value:

$$p(k) = P_m \cdot p_m(k) + P_o \cdot p_o(k). \tag{8.7}$$

We list the result in the next table:

Table 8.6 **The Average Probability of Choosing**

days	6	7	8	9	10	11	12
motor	0.3605	0.2683	0.1747	0.1012	0.0527	0.0250	0.0108
oar	0.0930	0.1230	0.1424	0.1465	0.1357	0.1142	0.0881
average	0.2803	0.2247	0.1650	0.1148	0.0776	0.0517	0.0340
days	13	14	15	16	17	18	
motor	0.0043	0.0016	0.0006	0.0002	0.0001	0.0000	
oar	0.0628	0.0415	0.0256	0.0148	0.0081	0.0042	
average	0.0219	0.0136	0.0081	0.0046	0.0025	0.0013	

According to the assumption, the tourist can choose the distance with freedom and the distances in one day obey the normal distribution[2]. We should work out the mean value and its variance. Suppose one tourist choose i-days trip, the average distances each day is $1/i$ of the total journey,

$$a_i = \frac{1}{i}. \tag{8.8}$$

We know the maximum time for rafting

$$t_{\max} = 10 \text{ h}. \tag{8.9}$$

The average speed:

$$\bar{v} = P_m \cdot v_m + P_o \cdot v_o = 6.8\ (\text{miles/h}). \tag{8.10}$$

So we can get the proportion of the maximum distances each day (the total as 1):

$$l_{\max} = \frac{t_{\max}\bar{v}}{l} = \frac{10\ \text{h} \times 6.8\ \text{miles/h}}{225\ \text{miles}} \approx 0.30. \tag{8.11}$$

We need to utilize 3σ principle to determine variance:

$$3\sigma_i = 0.30 - \frac{1}{i}, \tag{8.12}$$

$$\sigma_i = 0.10 - \frac{1}{3i} \quad (i = 6, 7, 8, \cdots, 18). \tag{8.13}$$

Then, we get the probability function of the distances each day for i-days trips:

$$\varphi_i(t) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(t-a_i)^2}{2\sigma_i^2}}. \tag{8.14}$$

Coupled with the probability of choosing propulsion, we can get the distribution of the average distance in one day for tourists with different duration.

$$\varphi(t) = \sum_{i=6}^{18} p(i)\varphi_i(t). \tag{8.15}$$

The relation of X and Y can also be determined in the extreme situation where the boats staying on the river occupy all campsites in one day. Suppose the number of the tourists reaches a peak, which equals the number of available campsites. We treat the river as a pipeline and draw a profile from it. Assume the tourist flow is in the state of saturation among enough scope at both sides of the profile. So we think the tourist flow traveling across the profile each day identical with the maximum tourist number. That satisfies the equation:

$$\int_0^{+\infty} (Y+1)\frac{\mathrm{d}s}{1} \cdot \left(1 - \int_0^s \varphi(t)\,\mathrm{d}t\right) = \frac{x}{180}. \tag{8.16}$$

We can get the relation expression as:

$$X = 23.8(Y+1). \tag{8.17}$$

Optimization

Our goal is to maximize the river manager's profit and raise the satisfaction degree of tourists as much as possible. We will analyze the optimization problem in the next section.

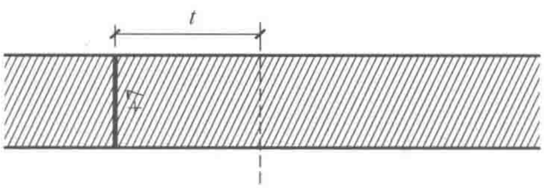

Figure 8.1 Passenger Flow

We suppose the tourist stops at the campsite nearest to the place where they are most likely to stop. We know the average speed is $\bar{v}=$

6.8 miles/h and the maximum hour for rafting is $t_{\max} = 10$ h.

We try to estimate different values of Y and then get the acceptable range of Y according to related optimization method. It's found that the value of 38 can match well up with the range. Now, we give the calculation process setting Y as 38, get the reasonable range and verify our initial outcome.

So when we have 38 points which symbolize 38 campsites along the river, the point tourists can arrive at can be calculated as follows:

$$N = \frac{t_{\max} \cdot \bar{v}}{\frac{l}{Y+1}} = \frac{10 \times 6.8}{\frac{225}{38+1}} = 11.8. \quad (8.18)$$

We get the result that the tourists can get to the 12th points if they raft for 10h. As the picture below shows, we can work out the probability that one trip starting from one point arrives at the following 12 points by using the density function:

Assuming i as the number of points away from the starting point, then we can get the probability of camping at this point:

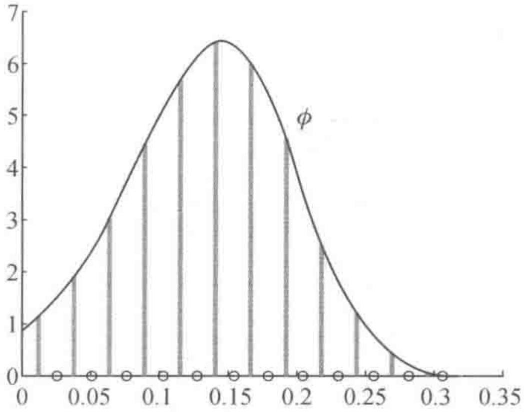

Figure 8.2　The Density Function

$$q(i) = \int_{\frac{2i-1}{2}\frac{1}{Y+1}}^{\frac{2i+1}{2}\frac{1}{Y+1}} \varphi(x)\,\mathrm{d}x. \quad (8.19)$$

Table 8.7　The Probability of Camping

i	1	2	3	4	5	6
$q(i)$	0.0397	0.0641	0.0956	0.1301	0.1572	0.1618
i	7	8	9	10	11	12
$q(i)$	0.1367	0.0919	0.0484	0.0199	0.0065	0.0017

Constraints

1. When the number of boats existing on the river equals that of campsites each day, the probability of one campsite being occupied: For one certain day, if we know every trips' starting point, we can get the probability of the points being occupied in the night. We assume that one campsite may be occupied by X trips. The probability of one trip stopping at this point is set as q_i ($i = 1, 2, \cdots, x$). Then the probability of this point being occupied is:

$$P = 1 - \prod_{i=1}^{x} (1 - q_i). \quad (8.20)$$

Through the calculation above, we can get the probability of points being occupied in one certain day. Considering the full use of the points, we think one trip start from each point and set $x=12$. So we can get:

$$q_i = q(i). \tag{8.21}$$

$$P = 1 - \prod_{i=1}^{x}(1-q(i)). \tag{8.22}$$

In this situation, P is only related to Y. We get the value of P with different Y and list them below:

Table 8.8 The Probability of One Campsite Being Occupied

Y	30	31	32	33	34	35	36	37	38
P	0.6429	0.6421	0.6414	0.6408	0.6402	0.6396	0.6390	0.6385	0.6380
Y	39	40	41	42	43	44	45	46	47
P	0.6375	0.6371	0.6367	0.6363	0.6359	0.6355	0.6351	0.6348	0.6345
Y	48	49	50	51	52	53	54	55	56
P	0.6341	0.6338	0.6335	0.6333	0.6330	0.6327	0.6325	0.6323	0.6320

2. When the number of boats existing on the river equals that of campsites each day, the probability of one campsite being occupied by at least 2 trips:

$$P' = \sum_{i>j} q_i \cdot q_j = \sum_{i>j} q(i) \cdot q(j). \tag{8.23}$$

3. When the number of boats existing on the river equals that of campsites each day, the time of the tourist in one trip finding an unoccupied campsite with the probability of 0.9:

Table 8.9 The Probability of One Campsite Being Occupied By At Least 2 Trips

Y	30	31	32	33	34	35	36	37	38
P'	0.3810	0.3837	0.3863	0.3887	0.3910	0.3932	0.3952	0.3971	0.3990
Y	39	40	41	42	43	44	45	46	47
P'	0.4007	0.4024	0.4040	0.4055	0.4069	0.4082	0.4095	0.4108	0.4119
Y	48	49	50	51	52	53	54	55	56
P'	0.4130	0.4140	0.4150	0.4162	0.4171	0.4179	0.4190	0.4198	0.4206

Under this situation, we assume that the probability of each campsite being occupied approximately obeys binary distribution. So we can work out the probability of the tourist in one trip finding at least an unoccupied campsite k:

$$k = 1 - P^{t\frac{\bar{x}}{t}(Y+1)}. \tag{8.24}$$

We get：

$$t=\frac{l}{\bar{v}}\cdot\frac{1}{Y+1}\log_P(1-k). \tag{8.25}$$

We set $k=0.9$, and represent the relation of t and Y in the form of table：

Table 8.10 The Time of The Probability of 0.9

Y	30	31	32	33	34	35	36	37	38
t	5.3603	5.1618	4.9963	4.8309	4.6985	4.5662	4.4338	4.3015	4.1691
Y	39	40	41	42	43	44	45	46	47
t	4.0699	3.9706	3.8713	3.7721	3.6728	3.6066	3.5074	3.4412	3.3750
Y	48	49	50	51	52	53	54	55	56
t	3.2757	3.2096	3.1434	3.0772	3.0441	2.9779	2.9118	2.8456	2.8125

4. We assume the mass of trips as continuous fluid which can be divided into 2 levels. The top of the picture denotes the starting state and the bottom of it denotes the ending state in one day. The density of these two levels can be denoted as

$$\rho_m=0.7(Y+1),\quad \rho_o=0.3(Y+1). \tag{8.26}$$

We can get average distances of one trip according to density function φ：

$$\bar{s}=\int_0^{+\infty}\varphi(x)\cdot X\mathrm{d}x=0.1318. \tag{8.27}$$

Because of the speed of motorized boats being twice that of oar-powered rafts and these two levels' density relation, we can get：

$$\begin{cases} s_m=2s_o, \\ \bar{s}=0.7\,s_m+0.3\,s_o. \end{cases} \tag{8.28}$$

And then, it gives：

$$\begin{cases} s_m=0.1550, \\ \bar{s}=0.0775. \end{cases} \tag{8.29}$$

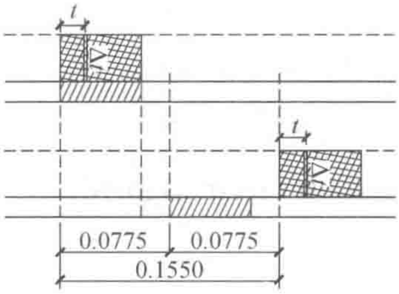

Figure 8.3 The contact of trips

We can work out the contact number of trips launching in the same day in the situation where the number of boats existing on the river equals that of campsites each day in one night.

$$C=\int_0^{\frac{x}{180(Y+1)}}\mathrm{d}t\cdot 0.7(Y+1)\cdot\frac{s_m}{2}\cdot 0.3(Y+1)\approx\frac{X^2}{2.63\times 10^5}. \tag{8.30}$$

Above is the description of four constraints, and now calculation is made as follows：

1. The contact number of trips launching in one day is below 4 in this day：

$$C=\int_{0}^{\frac{x}{180(Y+1)}} dt \cdot 0.7(Y+1) \cdot \frac{s_m}{2} \cdot 0.3(Y+1) < 4. \tag{8.31}$$

Table 8.11 **The Contact Number**

Y	30	31	32	33	34	35	36	37	38
C	2.0698	2.2055	2.3454	2.4898	72.6384	2.7913	2.9485	3.1100	3.2759
Y	39	40	41	42	43	44	45	46	47
C	3.4460	3.6205	3.7992	3.9823	4.1697	4.3614	4.5574	4.7577	4.9623
Y	48	49	50	51	52	53	54	55	56
C	5.1712	5.3844	5.6019	5.8238	6.0499	6.2804	6.5151	6.7542	6.9976

2. The time of tourists finding an unoccupied campsite in the probability of 0.9 is below 5 h:

$$t=\frac{l}{\bar{v}} \cdot \frac{1}{Y+1} \log_P(1-k) < 5. \tag{8.32}$$

3. The average number of trips camped at each campsite is larger than 20:

$$r=\frac{X}{Y} > 20. \tag{8.33}$$

4. The probability of contact on the campsites is below 0.4:

$$P'=\sum_{i>j} q(i) \cdot q(j) < 0.4. \tag{8.34}$$

Considering the four constraints above, we confirm Y as 38.

According to the relation between X and Y solved above, the maximum X can be set as 928 and Y as 38.

Random circle algorithm

The above-mentioned discussion is under the assumption of saturation (the number of boats existing on the river equals that of campsites each day). But that is the extreme situation and we don't want it to occur[3, 4]. When it fails to reach saturation, the problems of contact and the convenience of finding an unoccupied campsite will be less[5]. In order to manage trips reasonably, we research into the situation of non-saturation and analyze the contact on the campsites[6][7].

We adopt the random simulation on the computer. In order to simulate the actual tourist flow, we arrange the campsites in circular shape. According to the outcome above, Y is set as 38. So, there are 39 points in the figure symbolizing campsites, launch and exit site.

When it is in the non-saturation state, $X<38$. At the beginning, we arrange X trips randomly into 39 points. Then we appoint destinations for each trip according to the probability given by equation (8.19) and count the number of contact in position. Now, we can work out the proportion of contact trip among x. Go on like this for many times and get the average value as the probability of contact on the campsites. Later, the value of X is altered and the contact probability is also worked out.

For example, X is set as 35 and then 35 trips are appointed at 35 initial positions in the figure randomly. The figure is shown below:

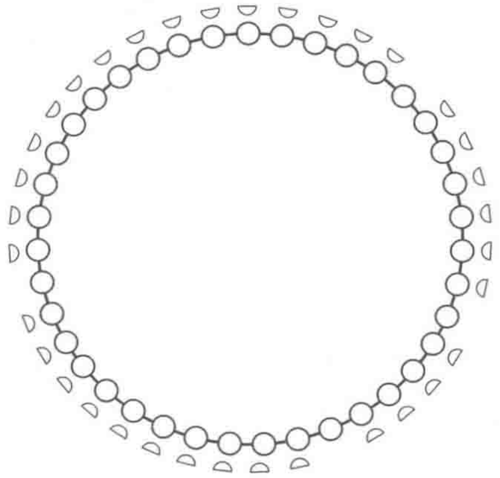

Figure 8.4 The Initial State of Random Circle

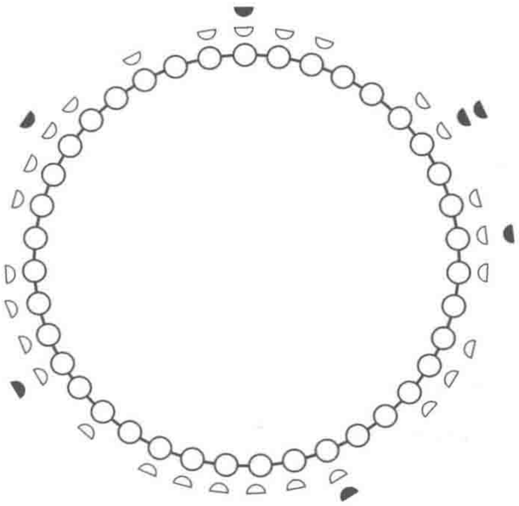

Figure 8.5 The Next State of Random Circle

Then, we choose the destinations of each trip according to corresponding probability, the result is shown below:

Now, we get the number of contact trips 7 and the probability is 0.200. Below is the table covering the average number of contact trips and the corresponding probability with different X value. As for every X value, we go on simulating for 500000 times.

We can accept the result that the probability of contact is less than 0.2. Considering the data in the table, we choose $x=32$ and get the total trips during the open period $X = 32/38 \times 928 = 781$. The average contact number is 2 each day, which is to our satisfaction.

Table 8.12 The Probability of Contact On The Campsite

X	38	37	36	35	34	33	32
number	9.38	8.89	8.41	7.96	7.50	7.06	6.64
probability	0.247	0.240	0.234	0.227	0.221	0.214	0.208
X	31	30	29	28	27	26	25
number	6.23	5.83	5.45	5.08	4.72	4.37	4.04
probability	0.201	0.194	0.187	0.181	0.175	0.168	0.162

According to the result above, we make $X=781$. First, we assign the number of motorized boats and oar-powered rafts by the ratio 0.7 : 0.3. The number of motorized boats $X_m = P_m \cdot X = 546.7$. The number of oar-powered rafts $X_o = P_o \cdot X = 234.3$. Then, we can work out the total number of different durations by their Poisson distribution (see the table below).

Table 8.13 The Schedule of Trips

Days	6	7	8	9	10	11	12	13	14	15	16	17	18
Motor trips	197	146	96	55	29	14	6	2	1	0	0	0	0
Oar trips	22	29	33	34	32	27	21	15	10	6	3	2	1

The schedule above can act as reference for the manager and they can make a tiny adjustment flexibly according to tourist flow. According to the campsites occupied in each case, we have the equation below:

$$N_i \cdot i = N_j \cdot j,$$
$$N_i = j/i \cdot N_j.$$

N_i denotes the number of campsites i-days trip occupies during the journey. For example, if the river manager wants to decrease one 18-days trip and increase three 6-days trips. Then we get: $N_6 = 18/6\ N_{18} = 3$.

That means the river manager can schedule three more 6-days trip by decreasing one 18-days trip.

Modified online algorithm

Based on the campsites from which trips start, we make analysis from the last to the first. As for each trip starting from one point, we determine the range that it may exist in the night, which covers 12 points. In the range, we rank the campsite according to the probability of being occupied by one certain trip. The list represents the order of campsites we advise the tourist to choose in the night. Every mark will be on the list regardless of its probability. The mark is the prior point we advise tourists to choose in the night. In this way can we ensure each trip can find unoccupied campsite to large degree.

For example, in one certain day, trips start from the 3rd, 5th, 7th , 11th, 14th, 22nd, 25th, 34th points, the picture of density function is shown below:

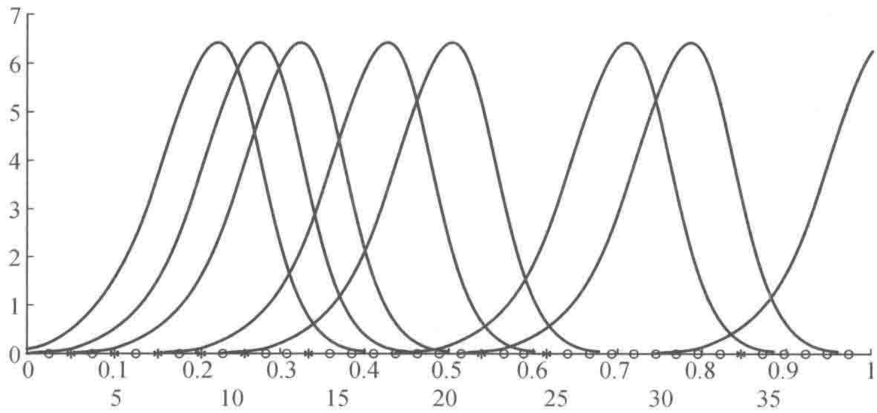

Figure 8.6 The Probability of Campsites Being Occupied

Then we advise the tourists according to the list below, and the rank symbolizes the priority of the next campsite.

In order to avoid contact on the campsite, advice about choosing campsites should be given to tourists each day. The state of campsites being occupied should be collected each day over the report from tourists finding certain campsite. Then the probability of contact on every campsite will be worked out according to our method. The river manager can give tourists a list of prior campsites to choose based on the probability of contact. Tourists can adjust their distances in one day according to our advice.

§ 8.6　Evaluation of the Model

Strength

1. The essence of the model is probability description and optimization method. The probability model describes the stochastic behavior of tourists well and avoids the sophisticated discussion about different situations. The traditional optimization method matches up well with the probability model. By optimizing our goal and constraints, we narrow the range of Y. Considering practical constraints as many as possible, we determine an appropriate value of Y.

Table 8.14　　**The campsites suggested**

label	34										
campsites	35	36	37	38	exit						
label	25										
campsites	34	36	37	33	26	32	27	31	28	30	29	35
label	22											
campsites	24	25	23	33	26	32	27	31	28	30	29	34
label	14											
campsites	25	23	22	26	21	20	19	18	16	17	15	24
label	11											
campsites	23	22	21	20	19	18	16	17	15	14	13	12
label	7											
campsites	19	8	18	16	17	15	14	9	13	10	11	12
label	5											
campsites	6	7	8	16	17	15	14	9	13	10	11	12
label	3											
campsites	4	5	7	8	15	14	9	13	10	11	12	6

2. By using modified real-time online algorithm, we can give valuable advice on tourists about which campsite can be chosen next night. This can avoid the worst situation where some tourists can't find an empty campsite as much as possible.

3. The random circle method helps us determine the optimal value of X to a certain extent. By simulating stochastic process of matching trips and campsites, we minimize contact in one day. This gives us a statistically feasible solution.

Weakness

1. We can't give a clear schedule about launching number, proportion of different durations and propulsion each day. The method can only gives out the total number of trips of different durations and different propulsion. It needs the manager to balance and cope with possible situations;

2. The assumption of probability distribution of different behavior is derived from the congeneric data of other river for rafting. Maybe it deviates from the practical situation.

Improvement

1. We can separate slack season from boom season and adjust the number of trips launching in one day according to tourist flow each day;

2. We may form an index to measure the using degree of campsites and add another constraint to optimization model, which may improve the accuracy of the value of X and Y.

§ 8.7 Conclusions

The optimal number of trips launching during the open period is 781 and the campsite number is 38. The number of ships launching each day can be the same or varies in the range of 4 - 6. Propulsion can be chosen by tourists at random. The average contact number is 2 each day, which is to our satisfaction. The number of trips with different durations is listed in the form of Table 8. 14.

In order to avoid contact on the campsite, advice about choosing campsites should be given to tourists each day. The state of campsites being occupied should be collected each day over the report from tourists finding certain campsite. Then the probability of contact on every campsite will be worked out according to our method. The river manager can give tourists a list of prior campsites to choose based on the probability of contact. Tourists can adjust their distances in one day according to our advice. The example is listed in the form of chart at the end.

The river manager can also separate off season from busy season and adjust the number of trips with different durations launching in one day flexibly. For example, the river manager can substitute three 6-days trips for one 18-days trip at the busy season.

Table 8.15　　The Schedule of Trips

Days	6	7	8	9	10	11	12	13	14	15	16	17	18
Motor trips	197	146	96	55	29	14	6	2	1	0	0	0	0
Oar trips	22	29	33	34	32	27	21	15	10	6	3	2	1

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[7] Joanna A. Bieri and Catherine A. Roberts "Using the Grand Canyon River Trip Simulator to Test New Launch Schedules on the Colorado River", Association for Women in Science, 2000, 29(3), 6-10.

§ 8.8 Memo

To: The river manager
From: Team 13772
Subject: Schedule for rafting
Date: 14 February 2012

We are presenting our own schedule for rafting trip to attract more tourists to raft on the river and ensure less contact . The number of ships launching each day can vary in the range of 4-6. Propulsion can be chosen by tourists at random. Then the optimal number of trips launching during the open period is 781 and the campsite number is 38. The average contact number is 2 each day, which is to our satisfaction. In the schedule, the number of trips increases with durations decreasing. And the ratio between motorized boats and oar-powered ones is 9 : 1 in the 6-days trip, 5 : 1 in the 7-days trip and so on. The schedule of trips with different durations can be obtained from us.

In order to avoid contact on the campsite, the state of campsites being occupied should be collected each day over the report from tourists finding certain campsite. Then the probability of contact on every campsite will be worked out according to our method. We can give tourists a list of prior campsites to choose based on the probability of contact. Tourists can adjust their distances in one day according to our advice.

We can also separate off season from busy season and adjust the number of trips with different durations launching in one day flexibly. For example, we can substitute three 6-days trips for one 18-days trip at the busy season.

If you have any question, please feel free to contact us.

§8.9 点评

陈雄达

这篇竞赛论文"Optimal rafting schedule for river manager"做的是2012年美国数学建模竞赛的B题,得一等奖。

这是一篇优秀的竞赛论文。全文结构合理,语言较顺。

第一,结构。这篇论文的结构规范完整。全文总体上思路清晰,过渡合理,给评阅者一个很好的印象。文章一共分为7段:引言、背景介绍、问题分析、假设、主要模型和解答、模型评价及总结论。

第二,内容。本文的工作是把一个本来关系复杂的问题分割成几个相对独立的小问题,再把它们有机地联系起来。首先参考一些资料获取露营中桨和马达的比例,估计露营天数,分析讨论这二者之间的关系。接着作为概率事件,讨论游客的每天行程,以数值模拟的方式估计大长河截面的游客流,从而给出所需的营地数量。

第三,特点。本文的特色是给出了一个随机圆圈的算法,用它来模拟大长河露营的情况,其中的示意图非常直观。

本文中的优缺点描述中也提到,本文无法给出一个确切的露营安排的描述,因为许多因素都是以概率的方式来处理的。尽管如此,还是应该给出一个典型的例子,使得读者可以看出一个一般的运营日是怎样管理的,以及如何面对其中的各种要素。

综上,尽管还有地方需要改进,这还是一篇非常优秀的竞赛论文。

第九章

论文 2：Camping Model along the Big Long River

by Yuan Zhang, Fan Zhang and Tianxing Yan

Summary

It is a type of popular entertainment to raft and camp along a long river. Three models are presented in this paper to solve the problem relevant to river rafting management and camping schedule.

First, assume that the all passengers fully accept the arrangement of the managers, both in the duration and propulsion. By mathematical analysis, we prove that value X (total number of trips) is the maximum correspond to the fixed value Y, when all passengers have opted for the journey of six days. The drawback of this model is that passengers must obey the restriction that only 6-days-trip is available, which is impractical.

Further, assume that each passenger can stay at one campsite for no more than one night, and each passenger has its own schedule in advance, a model of numerical experiments is created, with which a group of data is accessible including number of the potential conflicts and usage counts of each campsite. By using cubic curve fitting and creating a fitting evaluation, the optimal value of Y can be found, which is 39. Then we analyze the influence that the percentage of short trips has on the river rafting system both in the potential conflict frequency and the average frequency of each campsite being used.

Moreover, model three is an improvement of model two. We introduce a schedule test process, thus eliminating the schedule that will choose the same site at the same time as other groups of trips. This possibility simulation model, then through cubic curve fitting and multi-objective programming, we also conclude that $Y=39$ is an optimal number of campsites. Besides, after run this model for 500 times and choose the maximum of X among these data created, we find that when $Y=39$, $X=520$ is a local optimal mix of trips.

In the end, we write a one page memo to the managers describing our key findings. We mainly tell them our method of getting the schedule plan of maximum trips, as well as

pointing that different management should be taken in a busy season and an off-season.

Keywords: schedule management, probability simulation, camp sites, affection analysis.

§ 9.1 Introduction

In the last 20 years, the demand for white water experiences has increased, especially for the self-outfitted public. Nowadays it is always a type of popular entertainment to raft and camp along a long river. Visitors can enjoy scenic views and exciting white water rapids. However, as the rise in popularity of river rafting, it is becoming more and more necessary for the river managers to provide more campsites and make appropriate schedules for passengers.

In 1989 the National Park Service (NPS) approved a Colorado River Management Plan based on the Grand Canyon Wilderness Recommendation and findings from a comprehensive research program. Later, in 1999, a team of faculty and students from the University of Arizona's School of Renewable Natural Resources and Northern Arizona University's Department of Mathematics and Statistics have created the Grand Canyon River Trip simulator(GCRTsim). But the algorithm of their model is very complex, which adopts artificial intelligence.

Given that there is the Big Long River whose length is 225 miles downstream for passengers to raft. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. Passengers take either oar-powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish. In order to enjoy a wilderness experience, every group tries to have minimal contact with other ones of boats on the river. Currently, X trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Besides, no two sets of campers can occupy the same site at the same time.

One of our tasks is to determine how to schedule an optimal mix of trips of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. Also, we need to determine the carrying capacity of the river, which is reflected by the value of Y, the number of camp sites. At last, we are to write a letter to the managers of the river to give some suggestions.

In order to accomplish these missions, we build three models, which provide solutions to the problems either in theoretical analysis or model simulation.

Model one: By mathematical analysis, we prove that value X (total number of trips) is the maximum correspond to the fixed value Y, when all passengers have opted for the journey of six days and all comply with the specific arrangements.

Model two: assume that each passenger can stay at one campsite for no more than one night, and each passenger has its own schedule in advance, a model of numerical experiments is created, with which a group of data is accessible including number of the potential conflicts and usage counts of each campsite. By using cubic curve fitting and creating a fitting evaluation, the optimal value of Y can be found, which is $Y=39$.

Model three: model three is an improvement of model two. We introduce a schedule test process, thus eliminating the schedule that will choose the same site at the same time as other groups of trips.

§ 9.2 Models

In order to simply the problem, in the help of Ref[1] and life experience, we make the following assumptions, which are the foundation of our three models.

- Every group travels at most ten hours during one day. As a result, a group which takes motorized boats can travel 80 miles at most each day, while a group which takes oar-powered rubbers travel 40 miles at most.
- Every group must find a campsite to camp during the night. A group can't raft during the nighttime.
- Every group can stay at one certain campsite for no more than one night. Note: In fact, if one group camper stays at one certain campsite for more than one night, we can assume that the group travel 0 miles in day time.
- The distance between two adjacent campsites is more than 3 miles.

Model One

Our purpose is to make X as large as possible, and keep the plan simple.

Assumption

- Every month is 30 days.
- For every trip, the longest drive time is 10 hours per day.
- We could control the number of trips sent every day.
- We could control every trip's plan and the boat they use.
- Every trip should be 6 – 18 nights strictly.

Algorithm

We will divide all the trips into two types, the normal trip and the extra trip. Note that trip launched in the last 5 day of the six months will have no way to reach the Final Exit (they have to stay at least 6 night on the river) so we will only send trips in 175 days.

Normal trip

We divide the whole river in to 6 sectors uniformly. In each sector there are at least $\left[\frac{Y}{6}\right]$ camp sites. We number the camp site in every sector. Every day every trip move to the next sector. And the first sector's camp sites will be filled with new trip members. So every day we could send out $\left[\frac{Y}{6}\right]$ trip teams. And because every trip team must stay at least 6 nights, the last 5 days we will not be able to send trip teams. So this could afford $X'=175\left[\frac{Y}{6}\right]$ trips. If all the trips use the same kind of boat, and go to the next sector's same number camp site, the contact will be smallest, which is zero.

Extra trip

We also note that there should be $6\left\{\frac{Y}{6}\right\}$ empty camp sites according to the plan.

If $6\left\{\frac{Y}{6}\right\}<2$ there is no way to use them without revising the normal trip, so the number of extra trip will be 0. In extra trip, we arrange the trip in the following:

1. Only move to the camp site which belongs to the extra trip.
2. Move to the next camp site.
3. If they cannot move (e. g. end trip in 5th day), they stay in this camp site.
4. If there is an empty camp site within First Launch's reach, send a new trip team.

In this way every six day extra trip could afford $6\left\{\frac{Y}{6}\right\}$ trips, we call this a trip cycle. In 175 days there are 29 cycles and a day left (the 175th day). So the total amount that extra trip could afford is $X''=\begin{cases} 0, & 6\left\{\frac{Y}{6}\right\}<2, \\ 174\left\{\frac{Y}{6}\right\}+1, & 6\left\{\frac{Y}{6}\right\}\geqslant 2. \end{cases}$ trips.

In conclusion, the total amount of X of this algorithm is

$$X=\begin{cases} 175\left[\frac{Y}{6}\right], & 6\left\{\frac{Y}{6}\right\}<2, \\ 29Y+1+\left[\frac{Y}{6}\right], & 6\left\{\frac{Y}{6}\right\}\geqslant 2. \end{cases}$$

Analysis

In the whole 180 days, the Y camp sites offer $180Y$ times to camp. It takes every trip team 6-18 camping times to go through the river, so there are at most $30Y$ trip teams. In fact, some of the 180 camping times are not accessible (e. g. the camping time offered by last camp site in 1st day). So the maximum of X is less than $30Y$. Then we have the difference $X_{\max}-X<30Y-X$, which equals to $30Y-175\left[\frac{Y}{6}\right]$, $\left(\left\{\frac{Y}{6}\right\}<2\right)$ or $Y-1-\left[\frac{Y}{6}\right]$, $\left(\left\{\frac{Y}{6}\right\}\geqslant 2\right)$. Both of them is very close to the maximum.

Model Two

Symbols

- X the number of trips traveling down the Big Long River each year during a six month period in total;

- Y the number of camp sites on the Big Long River;
- Nu the maximum number of trips that can start at First Launch each, which is be connected with Y and the actual numbers of passengers;
- $comp(l,i,j)$ the state record matrix for the trips, l represents the serial number of the trip, i represent the date of the trip during its rafting schedule(date from 1st day to the 180th day), j represents the ordinal number of the camp sites where the trip stops to camp;
- p the percentage of the short trips (6～12 days), and $(1-p)$ represents the long trips (13-18 days).

Algorithm

In fact, though we just gave the maximum schedule plan and prove it to be the maximum limit, it is hardly possible to executive that plan. Because different passengers have different schedules, and the demands for raft serve of different time (from the first day of the first month to the last day of the sixth month) may differ from day to day. In order to take more details into consideration and make our model more close to the actual life, we build a possibility simulation model to simulate the actual situation of the life.

In the possibility simulation model, besides the assumptions made above, we adopt the following rules to describe the behavior of passengers as well as the number of raft serve demand every day.

1. A person who decides to go on a rafting trip must first decide how many days he will spend on the river, thus choose a suitable schedule. For each trip, we use the computer to create a random number *len*, ranging from 6 to 18, to represent the mathematical expectation duration of it.

2. when a trip is choosing a campsite, the current conditions of the river and the individual trip play a role, as does the campsite's historical popularity.

It is a complex decision-made process, the best way to solve it is fuzzy logic theory. But it is difficult to adopt that theory in our relatively simple model. In order to simply the process, we also use the computer to create *len* random numbers ranging from 0 to Y, to represent the serial numbers of the campsites that the group will choose to camp. Of course these random numbers after sorted should meet the condition that any difference of two adjacent is no more than Nu. As to the condition that no two sets of campers can occupy the same site at the same time, we will take into consideration in different ways respectively in the following two possibility simulation models.

3. The number of trips added to the river everyday is also random.

In order to determine the optimal Y (the number of campsites), we first don't restrict the rule that no two sets of campers can occupy the same site at the same time in our simulation. It means that more than two sets of campers can occupy the same site at the same time in their schedule plan. If it did happen, we call it one conflict, and for every value of Y, we count the total number of conflicts and compute its average on each trip, named conflict frequency per trip on average, regarding it as one of the factors to determine whether the value of Y is appropriate.

In fact, it is reasonable to statistics the conflict level on average. Suggested that every passenger has made his own plan on when and where to stop to camp, if there are not enough camp sites, some of them want to occupy the same site at the same time, conflicts will happen. In order to improve the serve quality, enough camp sites should be provided.

However, every campsite costs money to maintain. Besides, too much density of campsites will have bad influence on the environment of the river corridor. So the frequency of each campsite being used per day is also a factor to determine Y. We statistic the value—the average frequency of each campsite being used per day.

Solution

The pseudo code of this possibility simulation model is as Figure 9.1:

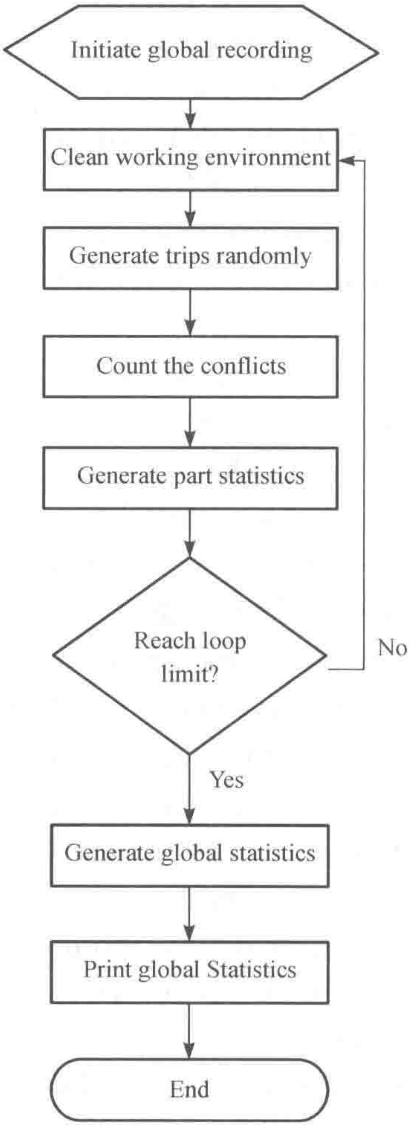

Figure 9.1 Simulation Model

When the number of trips added to the river everyday is random ranging from 0 to 7, this scatter reflects the average frequency of each campsite being used per day (represented by counts in this figure) varying with the number of campsites Y. The result is gotten after the program is run 20 times. The blue curve is got through cubic curve fitting. It reflects that the average frequency of each campsite being used per day reduces as Y increases when the number of trips added per day changes in a fixed range.

Use cubic curve to fit those scattered points, we get as in Figure 9.2:

$$f_1 = counts = -0.00001073 \times Y^3 + 0.0019367 \times Y^2 - 0.12383 \times Y + 3.2955.$$

The optimal average frequency of each campsite being used per day should close to 1, so that the resource can be made fully use of.

When the number of trips added to the river everyday is random ranging from 0 to 7, the potential conflict frequency per trip encounters on average (represented by $conflict/X$ in this figure) varying with the number of campsites Y. The result is gotten after the program is run 20 times. The blue curve is got through cubic curve fitting. It also reflects that conflict frequency per trip on average reduces as the number of campsites increase.

Use cubic curve to fit those scattered points, we get as in Figure 9.3:

$$f_2 = conflicts/X = -0.000056383 \times Y^3 + 0.010135 \times Y^2 - 0.64358 \times Y + 16.987.$$

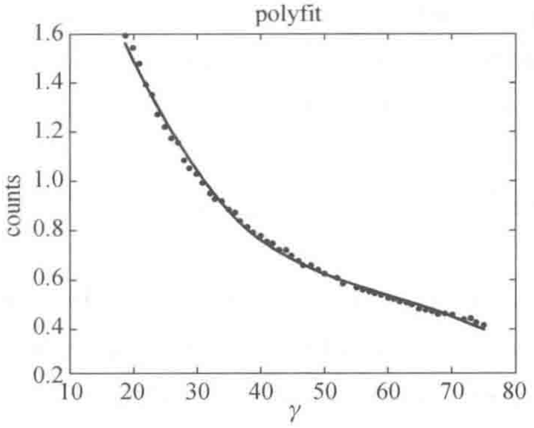

Figure 9.2 Counts

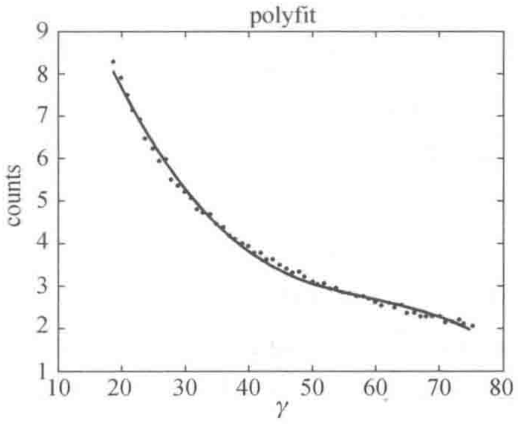

Figure 9.3 Conflicts/X

For passengers, if conflict/X is smaller, they will be more satisfactory. For managers, if the average frequency of each campsite being used per day is closer to 1, more resource can be saved. Obviously, we can figure out that the optimal Y should between 30 and 50. Now, we face a problem of multi-objective optimization.

We can define an evaluation function to search the relatively optimal Y.

The evaluation function $F(Y)$ can be defined as follows:

$$F(Y) = \frac{f_1}{f_2} = \frac{-0.00001073 \times Y^3 + 0.0019367 \times Y^2 - 0.12383 \times Y + 3.2955}{-0.000056383 \times Y^3 + 0.010135 \times Y^2 - 0.64358 \times Y + 16.987},$$

$$Y \in [30, 50].$$

Use matlab to draw the figure of the function, we can get the following graph:

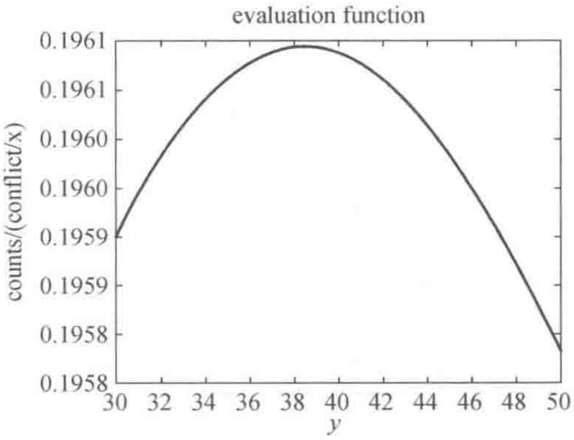

Figure 9.4 Function $F(Y)$

From Figure 9.4, we can see that when $Y = 39$, $F(Y)$ obtain the maximum.

Therefore, through the analysis of the average frequency of each campsite being used per day and the potential conflict frequency per trip encounters on average, we can conclude that the optimal number of campsites to both make full use of resource and satisfy those potential passengers is

$$Y = 39.$$

Further analysis of the possibility simulation model

Basically, we can divide those trips into two types—the short trips (6-12 days), and the long trips (13～18 days). If we use p to represent percentage of the short trips, and $1-p$ for the long ones. What influence it will have if p changes from 0 to 1? We can use this model to simulate the influence that p has on the average frequency of each campsite being used per day (f_1) and the potential conflict frequency per trip encounters on average (f_2). The significance to do this work is that we can make suggestions on what influence it will have that if we change the constitution of trips (mainly on the constitution of duration). The sensitivity analysis result is as Table 9.1.

In this table, $Y=39$, the number of trips added to the river every day ranges randomly from 0 to 14(when $Y=39$, $Nu=14$).

Table 9.1 demonstrates the changes in p will have obvious influence on the average frequency of each campsite being used per day and the potential conflict frequency per trip encounters on average. It tells us that, we can control the percentage of long trips every day to make the whole river system functions well.

In the busy season, river managers can permit more short trips but not long trips to avoid large scale of discontent of passengers in finding the same campsite at the same time. While in the off-season, more long trips can be permitted to improve the average frequency of each campsite used per day.

Table 9.1 **Conflicts changes with p**

p	$conflict/total$	$conflict/X$
0	0.96237	1.068697
0.1	0.9962	1.149208
0.2	0.94093	1.039328
0.3	0.93127	1.027764
0.4	0.89267	0.948295
0.5	0.85735	0.882428
0.6	0.86136	0.866031
0.7	0.83084	0.820572
0.8	0.79033	0.733562
0.9	0.78815	0.715751
1	0.74644	0.628305

Model Three

Model two is the further improvement of model one.

In this model, we made several changes in the assumptions based those in Model One.

Assumption

- no two sets of campers can occupy the same site at the same time;

- every group can stay at one certain campsite for more than one night.

Now, we give some introduction of our algorithm.

Algorithm of our model

This model is made for this situation: A group of passengers take their schedule plan to the First Launch of Big Long River. The workers of the park agency put their schedule into the current state matrix of the river in the rule of first come first served. If the schedule of anyone has conflicts (mainly referring that two sets of campers will have to occupy the same site at the same time) with the one who comes earlier (including the days before) than him or he, he or her will not got the permission to Big Long River rafting that day. In not, he will got the permission. We call this process the schedule test. Each day, the maximum number of groups being tested is certain, for example $2Nu$ or $3Nu$.

Result of simulation

Firstly, when Y changes from 19 to 74, we do 200 times simulations for each Y. Suppose that everyday there are 50 groups being tested, which means that the amount of passenger is abundant. We collect the maximum X from the 200 groups of data for each Y. We obtain the Table 9. 2.

Table 9. 2 Result of simulations(fac stands for "frequency of average contact")

Y	max X	total	contact	fac	Y	max X	total	contact	fac
19	205	2034	440. 4	2. 2746	37	342	3387	1575. 8	4. 8184
20	211	2082	470. 4	2. 3471	38	352	3495	1650. 4	4. 9297
21	217	2196	524. 1	2. 5031	39	359	3567	1775	5. 1041
22	234	2293	615. 9	2. 762	40	361	3632	1849. 5	5. 2699
23	232	2289	627. 9	2. 7549	41	364	3696	1895	5. 3072
24	245	2408	687. 8	2. 9325	42	383	3808	2066. 3	5. 575
25	252	2498	772. 3	3. 1545	43	383	3867	2122. 3	5. 7128
26	260	2628	830. 4	3. 3131	44	387	3859	2232. 5	5. 8906
27	267	2673	870. 6	3. 4114	45	395	3982	2300. 3	5. 9599
28	274	2729	954. 4	3. 5827	46	406	4103	2381. 7	6. 064
29	277	2801	996. 2	3. 6848	47	416	4126	2546. 8	6. 3444
30	290	2913	1087. 5	3. 8982	48	425	4187	2617. 4	6. 3568
31	298	3010	1182. 7	4. 0915	49	419	4231	2675. 2	6. 5104
32	305	3039	1206. 5	4. 0996	50	439	4399	2803. 3	6. 7099
33	315	3125	1289. 5	4. 2683	51	437	4348	2867. 8	6. 721
34	321	3193	1386. 7	4. 4753	52	451	4495	2898. 1	6. 757
35	322	3233	1430. 5	4. 5317	53	449	4486	3130. 8	7. 0635
36	341	3377	1547. 4	4. 739	54	454	4647	3167. 1	7. 1466

(Continued)

Y	max X	total	contact	fac	Y	max X	total	contact	fac
55	470	4719	3291.9	7.3034	66	539	5330	4462.9	8.5584
56	467	4738	3363.2	7.3509	67	551	5481	4636.4	8.7315
57	482	4769	3551.6	7.5466	68	553	5567	4694.1	8.8185
58	489	4878	3604	7.6328	69	549	5526	4858.8	8.9539
59	486	4902	3658.4	7.7131	70	561	5611	5001.8	9.1573
60	502	5038	3730.4	7.7337	71	569	5689	5076.7	9.1972
61	499	4981	3936.6	8.0361	72	575	5732	5184.1	9.3111
62	513	5145	4097.1	8.2146	73	577	5810	5359	9.4613
63	517	5135	4150.9	8.288	74	582	5831	5404.9	9.49
64	518	5150	4253.2	8.3911	75	602	5991	5670.4	9.7614
65	533	5335	4430.9	8.5151					

Using cubic curve fitting we can obtain as in Figure 9.5 and 9.6:

$$\max X = 0.00019016 \times Y^3 - 0.043909 \times Y^2 + 9.6889 \times Y + 33.097,$$
$$\text{contact} = 0.00000035581 \times Y^3 - 0.00039601 \times Y^2 + 0.1676 \times Y - 0.81895.$$

As the coefficients of Y^3 and Y^2 is very small, we can approximately regard that the maximum X and the frequency of average contact (contact) has a linear relationship respectively with Y.

Then if the amount of passengers being tested each day is abundant,

$$f_3 = 9.6889 \times Y + 33.097,$$
$$f_4 = 0.1676 \times Y - 0.81895.$$

In order to make X as much as possible while contact as litter as possible, we can adopt the efficacy coefficient method. Now we hope to find an optimal value of Y between 26 and 50.

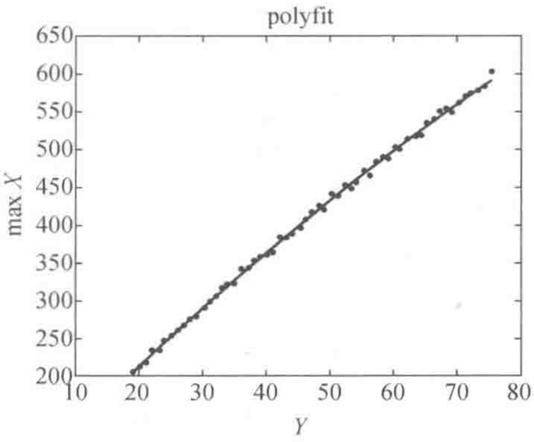

Figure 9.5 maximum X

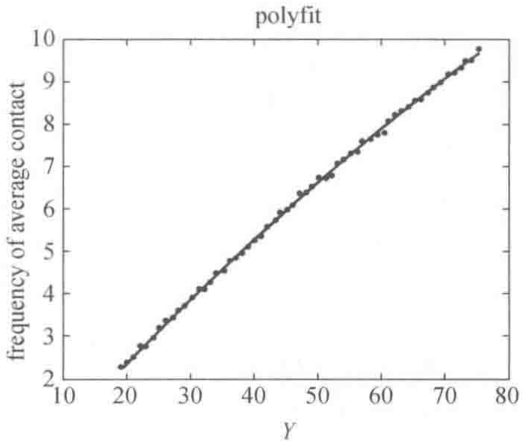

Figure 9.6 Frequency of average contacts

Now the purpose is

$$\min\{f'_3 = -f_3 = -9.6889 \times Y - 33.907\},$$
$$\min\{f_4 = 0.1676 \times Y - 0.81895\}, \quad Y \in [26, 50], Y \in \mathbf{N}^+.$$

We define

$$d_3(Y) = \frac{f'_{3\max} - f'_3(Y)}{f'_{3\max} - f'_{3\min}} = \frac{f'_3(26) - f'_3(Y)}{f'_3(26) - f'_{3\min}},$$

$$d_4(Y) = \frac{f_{4\max} - f_4(Y)}{f_{4\max} - f'_{4\min}} = \frac{f_4(50) - f_4(Y)}{f_4(50) - f_4(26)}.$$

max evaluation function $= d_4(Y) \cdot d_3(Y)$. The graph is as follows:

when $Y=39$, the evaluation function has the maximum.

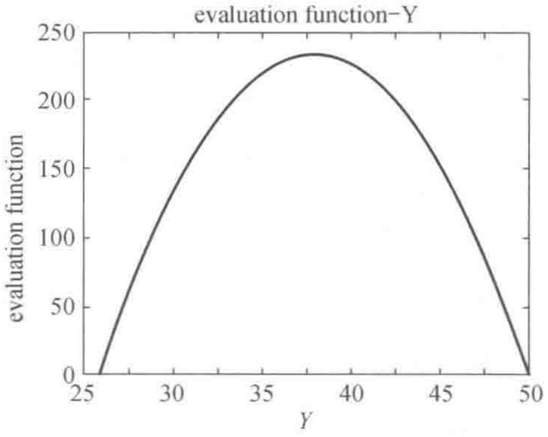

Figure 9.7　Function $d_4(Y) \cdot d_3(Y)$

So, in order to make X as much as possible while contact as litter as possible, the optimal Y for that multiple objective programming is $Y=39$.

So we obtain the same optimal value of Y as we did in model two, which is a good verification that $Y=39$ is indeed an optimal value for the river rafting system.

Set $Y=39$, run this model for 500 times and choose the maximum of X among these data created, a local optimal mix of trips is obtained:

$$Y = 39, \quad X = 520.$$

As the schedule matrix is as big as 520×39, we just represent the first 20 trips in Table 3.

Note: if we run this model for thousands of times, we may obtain a larger X when $Y=39$. But the more times it runs, the longer it will spend.

Further discussion about this model:

1. We can change the number of trips being tested every day to make it more in line with the real. For example, in the busy season of river rafting the number of trips being tested every day can be large enough, while in the off-season, the number of trips being tested every day can be smaller.

2. We can divide those trips into two types-long trip and short trip, and change the percentage of them to see how the constitution of trips duration influence the total system, thus tell the river managers how to make optimal managements and what influence that different policies will have.

3. We can adjust the length of the circular day and the initial state to make most models suitable to any other situations.

§ 9.3 Memo to the Managers

TO: MANAGERS OF THE RIVE

Table 9. 3 **20 Trips**

trip	the serial number of campsites									
1	1	2	3	4	5	6	7	8	9	10
2	9	13	14	17	18	24	26	27	33	37
3	1	2	3	4	5	6	7	8	9	
4	3	5	9	19	20	30	33	36	36	
5	1	2	3	4	5	6				
6	10	20	22	23	27	28				
7	1	2	3	4	5	6	7			
8	4	7	21	27	31	34	37			
9	1	2	3	4	5	6	7	8	9	10
10	1	2	8	14	24	26	30	34	35	36
11	1	2	3	4	5	6	7	8	9	10
12	11	12	13	20	22	23	25	30	31	33
13	1	2	3	4	5	6	7	8	9	10
14	2	4	7	15	16	20	20	21	27	29
15	1	2	3	4	5	6				
16	5	19	26	26	33	38				
17	1	2	3	4	5	6	7	8	9	10
18	7	8	10	11	11	17	17	18	25	32
19	1	2	3	4	5	6	7			
20	13	14	20	21	28	31	35			

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§ 9.4 To Managers of the River

SUBJECT：DEVELOPING THE BEST SCHEDULE OF THE BIG LONG RIVER

The Big Long River is a good place for passengers to have fun. The managers of the river want to add more boat trips to the Big Long River within its carrying capacity. They also expect that every trip can enjoy a wilderness experience with minimal contact with other groups of boats on the river.

To meet the demand, the schedule of the river should be developed and the number of camp sites should be optimized.

According to the research we have done to the Big Long River, a computer program is designed to list every feasible schedule so that the most appropriate number of camp sites and the best schedule can be found.

After the comparison, we finally decide the best schedule which includes 39 camp sites. This schedule can both add many more trips to the river and make every trip with less contact with others on the river within its carrying capacity. It is tested a lot times to make sure the number of trips is the highest with 39 camp sites. Passengers can chose different plans from the schedule to make sure they would enjoy their trips. And enough camp sites make sure the emergency can be solved easily.

According to the different expectation of the length of the trips, we also imitate the different situations from the aspect of tourist to prove that the flexibility and practicability of our schedule is the best.

§ 9.5 点评

周羚君

本文研究了徒步、漂流旅行区的营地设置问题。作者从静态模型出发，得到一个初步的结论方案，之后采用模拟结合数值拟合的方法，得到一个动态的模型。本文表述清晰、语言准确，是一篇很好的科技论文。不足之处在于，在摘要中，不能清楚地体现关键性的方法。

第十章

论文 3：Camping along the Big Long River

by Chuandeng Wei，Yangqing Liu and Zhen Chen

Summary

With the aim of utilize the scenic views in the best way possible, it's a real-life phenomenon today to make arrangement of visiting program for attracting more people. The river managers need to make a concrete schedule and an effective plan to determine number of added boat trips. Provided the number of trips can be predicted, our goals are to determine the reasonable number of campsites to get more trips, higher utilization rate and fewer contacts. For determined number of campsites, this article tries to make a plan to attract more trips and find the carrying capacity of the river.

Firstly, according to similar background data of drifting, this model initially distributes all trips which range from 6 to 18 days into 3 groups, namely 6～8 days' trips, 9～13 days' trips and 14-18 days' trips. It's prescribed that trips in a same group will use the same campsites, but no two sets of campers occupy the same site at the same time. Considering these 3 groups' cycling time is almost close to each other, so for a fixed number of campsites, all campsites are distributed into 3 parts fairly using a program. With the help of Method of Mean Difference, one can figure out the distribution of campsites. In this way, the upper limit of boat trips is calculated. All visitors are classified into 3 groups in probability by Cluster Analysis. Simulation program produce random sequence on basis of probability in order to get the real number of boat trips.

Next, number of contacts among boats can be counted by Comparative Sequence Method.

Finally, this plan analyzes three factors: campsites utilization, boat trips and contacts and establish comprehensive evaluation index by Analytical Hierarchy Process which could judge the rationality of this model. Through Analytical Hierarchy Process, different number of campsites can be tested and help manager to get the best value.

According to Analytical Hierarchy Process, the model can help get a reasonable number of campsites, which can direct mangers to choose the number of campsites. When

the number of campsites is determined, the model can give the best schedule and determine the carrying capacity of the Big Long River and with the help of starting timetable, and number of added boat trips can be calculated.

Through various situations, the model presents the conclusions that when the number of trips is less than 400, 18 campsites are reasonable, otherwise 35 campsites are reasonable. 18 campsites can provide about 270 trips and 35 campsites can provide about 450 trips in 6 months.

The end of the article shows that this plan is not optimal and can be improved. Under some fair assumptions and our suggested solution, is provided and easy to use, includes a detailed timetable and the description of visiting key points. And also, it is broad enough to accommodate various situations.

Keywords: Comparative Sequence, Mean Difference, AHP, Cluster Analysis

§ 10.1 Introduction

A 225-mile-river trip require camping which range from 6 to 18 nights. Passengers take either oar- powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph.

Originally, we presume that X trips travel down the river each year during a six month period and Y camp sites on the river, distributed fairly uniformly throughout the river corridor. Differently, we need to let more trips in now, as a result, to schedule an optimal mix of trips so as to utilize the campsites in the best way possible with minimal contact with other groups of boats on the river, we should determine the schedule fairly, of varying duration and propulsion. And, no two sets of campers can occupy the same site at the same time.

In our model, we determine that

- How many more boat trips could be added to the Big Long River's rafting season.
- An optimal timetable of boat trips in the season.
- A piece of suggestion of determining the carrying capacity of the river.

§ 10.2 Assumptions

1. During the trip, groups which continue their trip leave their campsite at the same time.
2. Every boat will pull in as schedule.
3. Managers of the sites can make prediction about number of visitors next year.
4. Every camp site can be occupied by only one group.
5. In every camp site, visitors can choose the drifting way: oar or motor.
6. At night, visitors must stay at sites, and cannot to proceed.

7. Visitors have decided how many days they will enjoy before they start their trips.

8. Influence of weather is omitted.

9. When pull-in at one sites, visitor can choose a 2-day-stay to enjoy the scenery or proceed.

§ 10.3 Analysis and Solution of the Model

Background

Through gathering historical data of visiting situation in 2011 and 2012, we make a table to show intuitively the visiting situations.

Table 10.1 **historical data**

number of visiting days	6	7	8	9	10	11	12
number of visiting groups in 2011	2	15	10	13	1	0	0
number of visiting groups in 2012	4	18	5	19	0	0	0
number of visiting days	13	14	15	16	17	18	
number of visiting groups in 2011	4	4	13	5	1	1	
number of visiting groups in 2012	2	4	17	4	0	0	

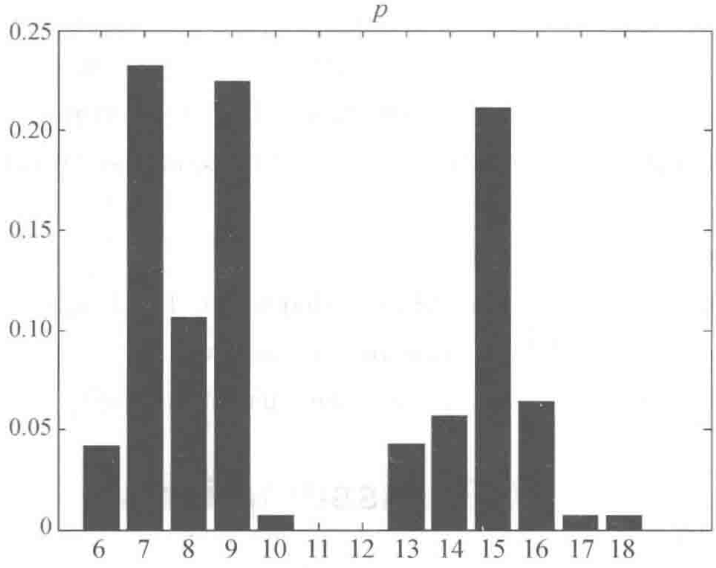

Figure 10.1 Historical probability

Figure 10.1 indicates the result that the average of 2011 and 2012 probability through normalization. The vertical coordinate is weight p, and the sum of all weights is 1. We get the results.

$$p=\begin{matrix} 0.0419 & 0.2320 & 0.1067 & 0.2243 & 0.0072 & 0 & 0 \\ 0.0427 & 0.0564 & 0.2106 & 0.0636 & 0.0072 & 0.0072 & \end{matrix}$$

So we classify them into

Table 10.2　　Classify of three groups

Group	Number of days	Percentage
A group	6，7，8	0.3806
B group	9，10，11，12，13	0.3306
C group	14，15，16，17，18	0.2886

The advantage is

1. Numbers of visitors in every group is nearly close.

2. In each group，number of visiting days is close to each other，it's convenient to arrange the schedule.

Establishment of model

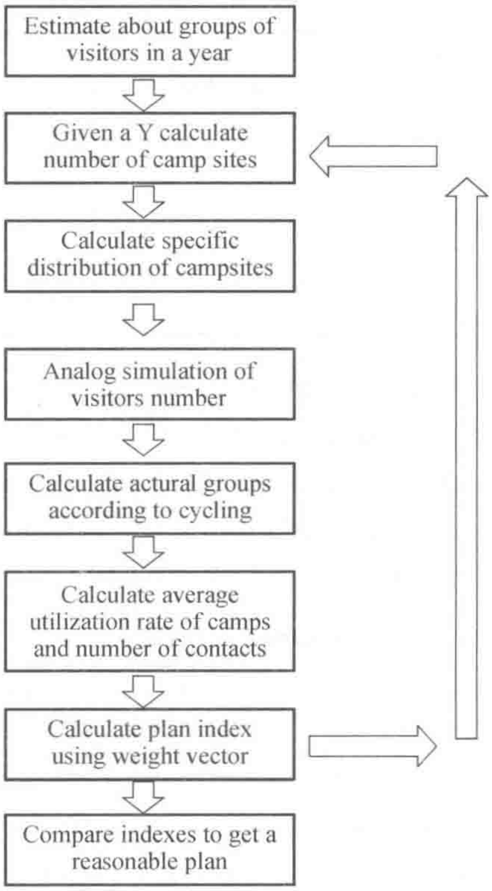

Figure 10.2　Flow sheet

Distribution of campsites

The purpose is distributing Y campsites to 3 visiting groups：A，B，C. Because number of visitors in these groups is nearly close，so we should do our best to guarantee cycles of boats is nearly close too. The cycle of boats depends on visiting time and shared number of campsites. For instance, the longest visiting days for A is 8 days, if the shared number of campsites is 5, the pull-in period in these campsites is 1, 1, 2, 2, 2. As a result, the cycle of boats for A group is 2 days. In this process, it is easy to find out that groups whose number of visiting days is bigger in principle of closer cycle of boats, it should have more campsites. If number of campsites for A, B, C is a, b, c, then the less difference between $8/a$, $13/b$, $18/c$ is, the better this plan is. After programming, we get the concrete Y, corresponded a, b, c and cycle of boats for each group.

In this way, we get the timetable of boats. For instance, $Y=20$, we get the timetables and camp sites of 2 groups.

Table 10. 3 **Distribute of campsites**

	A	B	C
Number of camp sites	4	7	9
Cycle of boat trips	2	2	2

Table 10. 4 **Time table**

Days	1	2	3	4	5	6	7	8	9	10	11
A	1		1		1		1		1		1
B	1		1		1		1		1		1
C	1		1		1		1		1		1

So we figure out that the biggest times of cycling "A" group in 6 months, in other words, the maximum capacity of visitors in A group. Based on the demand proportion of visitors on 6–8 days' visiting, we can calculate the maximum capacity on 6–8 days' visiting. In the same way, we can get data of other groups. If the demand proportion of one way is too low, we can omit it, like the 10, 11, 12, 17 and 18.

Table 10. 5 **Maximum capacity**

Visiting days	6	7	8	9	10	11	12	13	14	15	16	17	18
Maximum capacity	18	54	18	67	0	0	0	23	18	54	18	0	0

From the analysis above, we need to give a concrete plan of distribution. Considering the distance of travel every day is limited, we should make sure that distance everyday is certain.

In case of A group, we assume the total length of this trip is 1 and A group get a campsite, so the average journey is $1/(1+a)$, so the locations of campsites are known.

In the same way, we get B and C groups' location.

We sort the campsites and distribute Y campsites to those groups.

For instance, we assume A, B, C group get 3, 4, 5 campsites, the locations of A group are {0. 25, 0. 5, 0. 75}, of B group are {0. 2, 0. 4, 0. 6, 0. 8}, of C group are

{0.167, 0.334, 0.501, 0.668, 0.835}, we get the locations. Conversely, if we make sure there are 12 campsites in all, we will know their corresponded groups. In this way, we finish the distribution of campsites.

Cluster Analysis and Analog Simulation

Cluster Analysis

One important way for human to know the world is classify. Original classify depended on accumulated experience and related knowledge, with the rapid development of technology, demand for classify becomes higher, that's why Numerical classification emerges. Cluster analysis is an important part in Numerical classification.

According to Figure 1, we classify the data in probability using Cluster analysis.

Firstly, we put data through regularization by

$$x_{ij}^{*} = \frac{x_{ij} - x_j(\min)}{x_j(\max) - x_j(\min)} \quad (i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m).$$

After the variation, we calculate the distance coefficient matrix by

$$d_{ij} = \left[\frac{1}{n}\sum_{i=1}^{m}(x_{ij} - x_{sj})^2\right]^{\frac{1}{2}}.$$

In the process of cluster analysis, we choose elements that have minimal distance coefficient, classify their corresponded numbers of days. It's obvious that there are 3 groups: {6, 14}, {10, 17, 18}, {11, 12}.

Next, we calculate the weighted average of elements in a same group to get a new Regularized numerical table.

Table 10.6 Regularized numerical table

Number of visiting days	6,14	7	8	9	10,17,18	11,12	13	15	16
2011	0.2667	1	0.6667	0.8667	0.0667	0	0.2667	0.8667	0.3333
2012	0.2105	0.9474	0.2632	1	0	0	0.1053	0.8947	0.2105

According to distance coefficient, we can get the distance coefficient matrix as in Table 10.7.

Table 10.7 Distance coefficient matrix

Days	6	7	8	9	10	11	13	15	16
6	0	0.3465	0.1345	0.3305	0.0968	0.1133	0.0351	0.3033	0.0222
7	0.3465	0	0.2537	0.0478	0.4433	0.4592	0.3722	0.0478	0.3312
8	0.1345	0.2537	0	0.2545	0.2184	0.2389	0.1433	0.2208	0.1125
9	0.3305	0.0478	0.2545	0	0.4269	0.4411	0.3591	0.0351	0.3176

(Continued)

Days	6	7	8	9	10	11	13	15	16
10	0.0968	0.4433	0.2184	0.4269	0	0.0222	0.0753	0.4001	0.1133
11	0.1133	0.4592	0.2389	0.4411	0.0222	0	0.0956	0.4152	0.1314
13	0.0351	0.3722	0.1433	0.3591	0.0753	0.0956	0	0.3305	0.0415
15	0.3033	0.0478	0.2208	0.0351	0.4001	0.4152	0.3305	0	0.2892
16	0.0222	0.3312	0.1125	0.3176	0.1133	0.1314	0.0415	0.2892	0

We choose the minimum to proceed cluster analysis and get the result that there turns up 2 groups: {6, 14, 16} {10, 11, 12, 17, 18}. After we repeat the procedures the above, we get tables 10.8 and 10.9.

Table 10.8 **Regularized numerical table**

Number of visiting days	6, 14, 16	7	8	9	10, 11, 12, 17, 18	13	15
2011	0.2889	1	0.6667	0.8667	0.0400	0.2667	0.8667
2012	0.2105	0.9474	0.2632	1	0	0.1053	0.8947

Table 10.9 **Distance coefficient matrix**

Days	6	7	8	9	10	13	15
6	0	0.3870	0.1442	0.3698	0.1232	0.0407	0.3385
7	0.3870	0	0.2877	0.0542	0.5098	0.4221	0.0542
8	0.1442	0.2877	0	0.2886	0.2569	0.1625	0.2504
9	0.3698	0.0542	0.2886	0	0.4904	0.4072	0.0398
10	0.1232	0.5098	0.2569	0.4904	0	0.0945	0.4604
13	0.0407	0.4221	0.1625	0.4072	0.0945	0	0.3748
15	0.3385	0.0542	0.2504	0.0398	0.4604	0.3748	0

We choose the minimum to proceed cluster analysis and get the result that there turns up 3 groups: {6, 13, 14, 16}, {10, 11, 12, 17, 18}, {9, 15}. After we repeat the procedures the above several times, we get

Table 10.10 **Regularized numerical table**

Days	6, 8, 13, 14, 16	7, 9, 15	10, 11, 12, 17, 18
2011	0.4500	0.9111	0.0400
2012	0.2500	0.9474	0

In this way, we distribute visiting days into 3 groups and define them from the perspective of visitors' number as hot line, usual line, unpopular line.

Analog simulation

Our model can simulate the condition of visiting using Method of Analog Simulation to guide the construct of plan and test of our model.

According to classify of Cluster Analysis, we can classify them as hot line(h), usual line(n), unpopular line(c). We use similar historical data to display the probability of 3 lines after Normalization.

$$P_h = 0.2223; \quad P_n = 0.0623; \quad P_c = 0.0043.$$

We assume that there come N groups' visitors in 6 months, and display N random numbers in section (0,1). We can determine that the random numbers are visiting days.

Table 10.11 **Corresponded value table**

Days	6	7	8	9	10	11	12
Corresponded value	0.0623	0.2846	0.3469	0.5692	0.5735	0.5779	0.5822
Days	13	14	15	16	17	18	
Corresponded value	0.6445	0.7067	0.9291	0.9914	0.9957	1.0000	

For instance, if the random value is 0.5000, in the section (0.3469, 0.5692), it means the visitor choose a 9-day-visit. For N random numbers that can give statistics about 6～18 days' visiting, we can proceed to establish the model in next step.

Number of contacts

Conditions in day time are complex and random, so we depend on pull-in conditions to judge number of contacts. To determine the number, we assume u and v group enjoy their trip at the same day, if u's start campsite is in the lower reaches of the river than v's, but the condition in pull-in camp site is opposite, it means u and v have a contact. To simplify the calculation process, we assume that

1. All boat trips in A group choose 7-day-trip.
2. All boat trips in B group choose 9-day-trip.
3. All boat trips in C group choose 15-day-trip.

In A group, the proportion of 6, 7 and 8 days' trip is 1∶1∶3, it means 7-day-trip is the mean visiting type. In the same way, 9-day-trip is the mean visiting type in B group; 15-day-trip is the mean visiting type in C group. So we can guarantee the rationality of our plan.

Then we calculate number of contact according to the starting timetable.

For fixed Y, we can get the camp sites in each group by the principal the above. Through analyzing the rationality of pull-in conditions, we get the rational schedule. Based on the schedule, we can list the sequence of camp sites in any groups. In the difference between two lists of sequence, if any adjacent elements' sign is different, then they have a contact.

For instance, we assume there are 25 camp sites, A group has {1, 3, 5, 8, 12, 16, 21}, B has {2, 4, 9, 13, 15, 17, 18, 20, 24}, and the 7-day-trip and 9-day-trip start at the same time, the theoretical sequence is in Table 10.12.

Table 10.12 **Theoretical sequence**

Times(day)	1	2	3	4	5	6	7	8	9	10
A group	1	3	5	8	12	16	21	26	26	26
B group	2	4	9	13	15	17	18	20	24	26

The number 26 means it has reached the end.

Figure 3 shows the campsites the 2 groups used. The difference sequence is

$$[-1, -1, -4, -5, -3, -1, 3, 6, 2, 0],$$

it means they have a contact.

On the other hand, we can see the same result in the Figure 10.3.

If they don't start at the same time, we can fill in an 0 between the second sequence, meaning it is at the starting point.

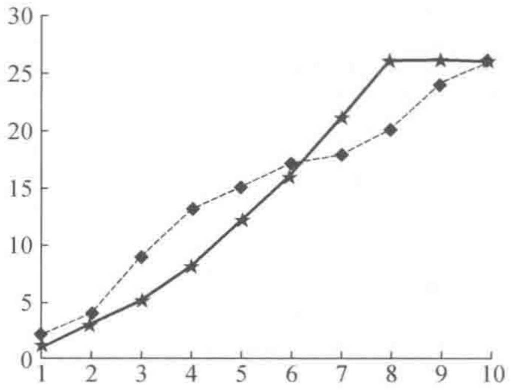

Figure 10.3 time vs campsites

To sum up, the number of contact is the number of adjacent elements' sign becoming different. Furthermore, we can count the number of all contacts in our plan.

The establishment of comprehensive evaluation index

In our model, we use the Analytic hierarchy process to establish the comprehensive evaluation index.

Symbol description

- Target level—A level; criterion level—B level; plan level—C level;
- Average utilization rate of camps—R;
- Number of open days—n;
- Number of camps—Y;
- Number of visiting days in different way—k;
- Number of k-day-visiting teams—x_k;
- Number of accepted teams—X;
- Number of boats' contact—M.

To schedule an optimal mix of trips so as to utilize the campsites in the best way possible (A), we take account of three indexes below.

- The average utilization rate of camps (B1);
- The number of visitors (B2);
- Encounter times of motors or oars (B3).

Establish hierarchical structure model:

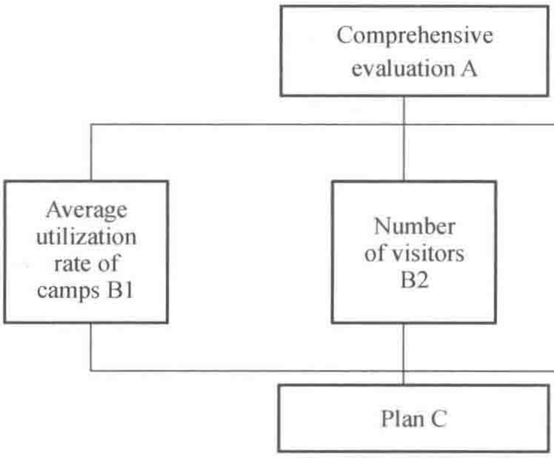

Figure 10.4 Hierarchical structure model

Explanation of B level

To utilize the campsites in the best way possible, each camp must be occupied every night.

$$R = \frac{\sum_{k=6}^{18} k \times x_k}{n \times Y}.$$

(1) The higher R is, the more reasonable C is. With this river trip becomes more and more popular, the park managers have been asked to allow more trips to travel down the river.

(2) The higher X is, the more reasonable C is. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river.

(3) The lower M is, the more reasonable C is.

Structure of pair comparison matrix

Using paired comparison method and 1-9 comprison of scale to construct B level's 3× 3 pair comparison matrix. Compare these 3 indexes, confirming their importance using relative scale. For the indexes, we arrange the judgment matrix A—B, the effect can be indicated as

A	B1	B2	B3
B1	1	1/3	5
B2	3	1	7
B3	1/5	1/7	1

So, the matrix is

$$\begin{pmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{pmatrix}$$

We use the consistency test, defining consistency index CI, consistency ratio CR.

$$CR = \frac{CI}{RI}$$

We get that X's biggest characteristic root is $b=3.0649$, the corresponding feature vector is

$$\begin{pmatrix} 0.3928 \\ 0.9140 \\ 0.1013 \end{pmatrix}$$

After the normalization, we get

$$w = \begin{pmatrix} 0.2790 \\ 0.6491 \\ 0.0719 \end{pmatrix}$$

So $CI=0.0325$, $RI=0.58$, $CR=0.0559<0.1$, w can be the weight vector, which means regarding 3 aspects of weight, $B2>B1>B3$.

In conclusion, for R, X, M, considering M is a negative index, so the compressive evaluation index can be represented as

$$g = 0.2790 \times R + 0.6491 \times X - 0.0719 \times M.$$

The higher g is, the more reasonable this plan is.

§ 10.4 Analysis of the Results

For different estimated visitor groups, we give our plan using our model. For determined pre-estimated number of group, with Y becomes higher, X becomes higher too, but the span is bigger than Y's. M rises as R falls. The variation of comprehensive evaluation index g is complex. But in fixed range, it has obvious maximum value and minimum value. According to actual situation, it's easy to pick up the best Y to make g value highest.

Table 10.13 **Results for $r=200$**

Y	15	18	20	22	25	30
X	177	195	197	197	197	197
R	69.63	64.14	58.44	53.13	46.76	38.96
M	889	441	621	799	797	2644
g	70.3984	112.7617	99.5276	85.2479	83.6144	−51.3611

Table 10.14 Results for $r=300$

Y	15	17	18	20	22	25	30	35
X	180	237	267	267	267	267	267	283
R	71.11	83.73	88.27	79.44	72.22	63.56	52.96	48.98
M	889	441	441	621	799	797	2644	2288
g	72.7586	145.4895	166.2291	150.8236	136.011	133.7386	−2.0181	32.8535

Table 10.15 Results for $r=400$

Y	15	18	20	25	30	34	35	37	40
X	180	270	270	270	270	270	286	395	395
R	71.11	88.82	79.94	63.96	53.3	47.03	65.60	63.58	58.81
M	889	441	621	797	2644	2652	2288	2654	2654
g	72.76	168.33	152.91	135.80	0.0241	−2.300	104.35	83.31	81.98

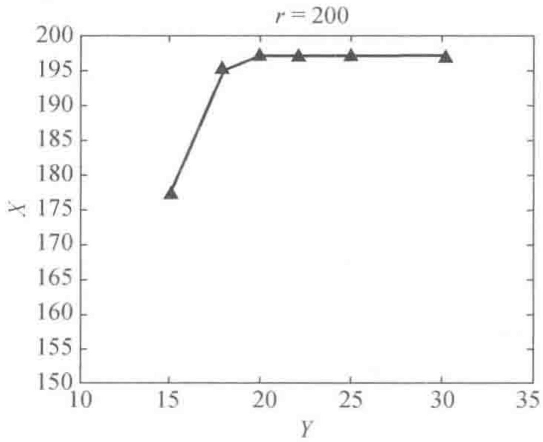

Figure 10.5 Result($r=200$, X vs. Y)

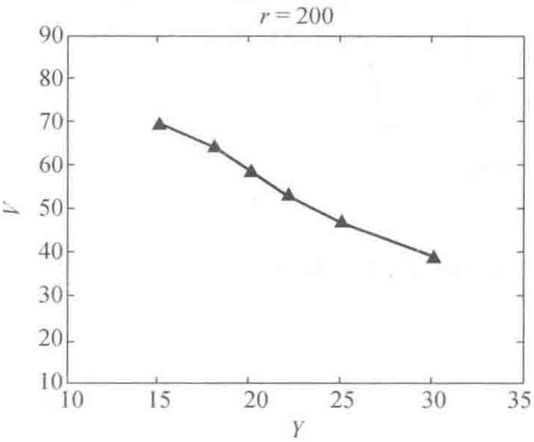

Figure 10.6 Result($r=200$, V vs. Y)

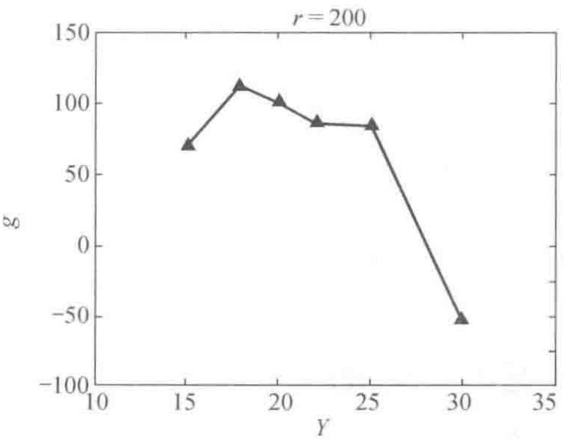

Figure 10.7 Result($r=200$, g vs. Y)

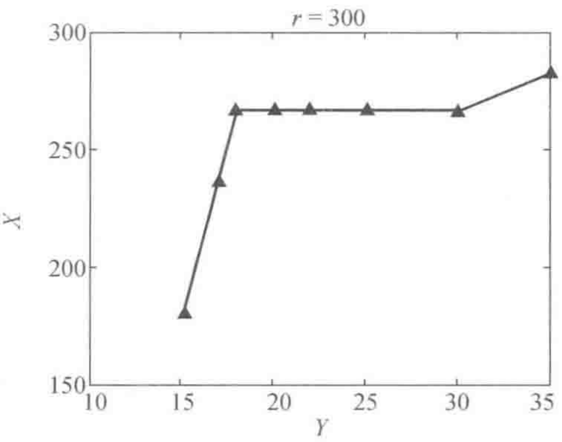

Figure 10.8 Result($r=300$, X vs. Y)

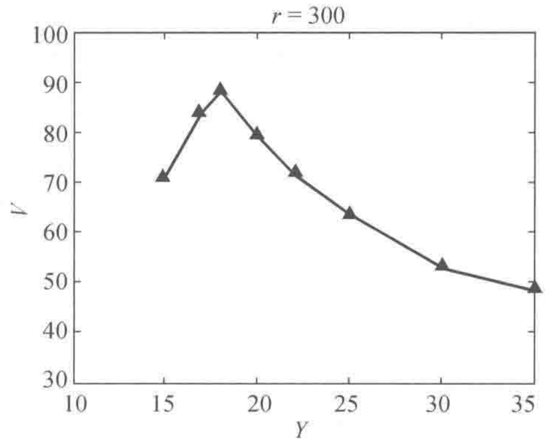

Figure 10.9 Result($r=200$, V vs. Y)

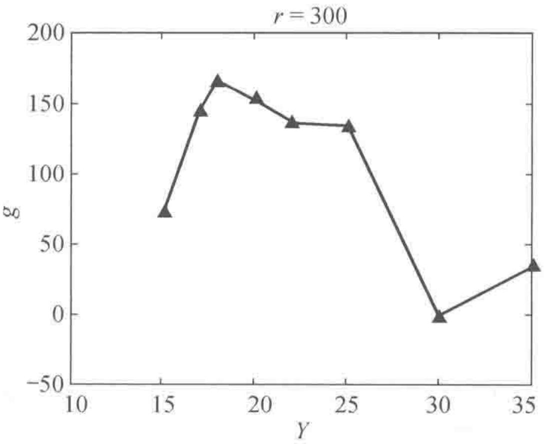

Figure 10.10 Result($r=300$, g vs. Y)

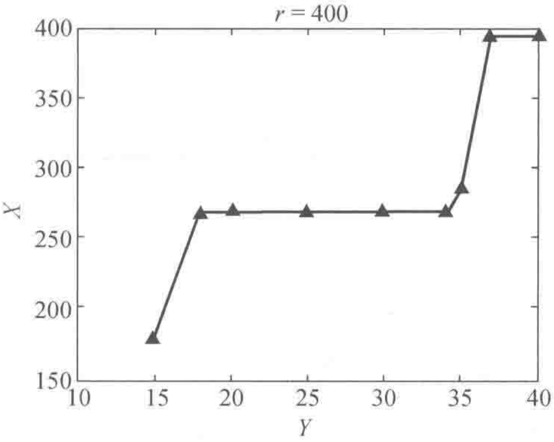

Figure 10.11 Result($r=400$, X vs. Y)

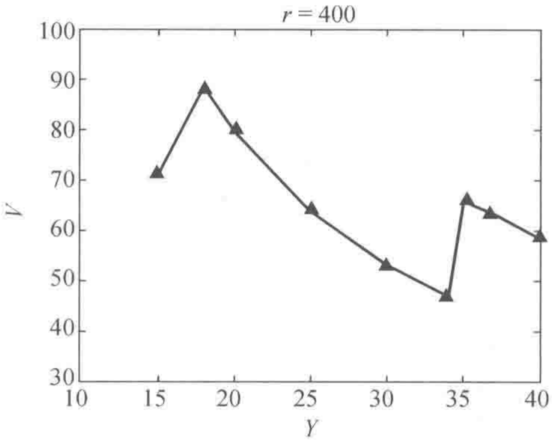

Figure 10.12 Result($r=400$, V vs. Y)

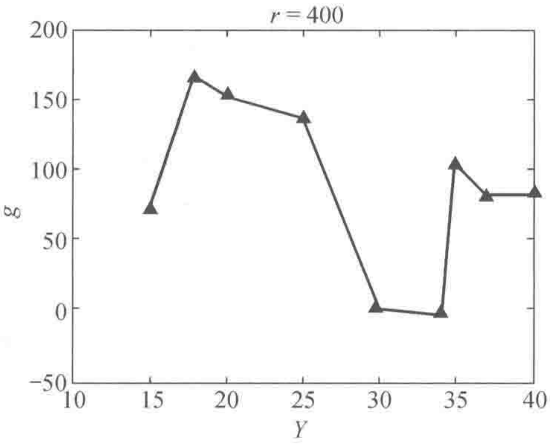

Figure 10.11 Result($r=400$, g vs. Y)

§ 10.5 Model Evaluation

Fixed drifting route make mangers arrange as many camp sites as possible and convenient to manage. Besides, visitors are distributed into 3 groups: A, B and C, and camp sites are fixed, this arrangement can reduce conflicts and number of contacts. Furthermore, analytical Hierarchy Process gives us a chance to take account of many conditions integrally. In this way, we can determine the best number of camp sites in the end.

But we have disadvantages, like visitors' freedom is omitted, time in each sites is limited. On the other hand, the best number of camp sites next year needs accurate prediction number. Anyway, we have more advices, for instance, if some sites are popular, managers can add more camp in the neighborhood. It can satisfy the demand and increase average utilization rate of camps.

References

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§ 10.6 Key Findings

With the aim of utilize the scenic views in the best way possible, it's a real-life phenomenon today to make arrangement of visiting program for attracting more people. The river managers need to make a concrete schedule and an effective plan to determine number of added boat trips. Provided the number of trips can be predicted, our goals are to determine the reasonable number of campsites to get more trips, higher utilization rate and fewer contacts. For determined number of campsites, this article tries to make a plan to attract more trips.

There are some results from the model.

Under some estimates through results, if estimate about trips' number is 200, reasonable number of campsites is 18 and number of trips is 195; if estimate about trips' number is 300, reasonable number of campsites is 18 and number of trips is 267; if estimate about trips' number is 400, reasonable number of campsites is 18 or 35 and number of trips is 270 or 386. And if given a predicted number of visitors, our model can help choose number of camp sites and give a rational number.

From the result, the average contact with other group is 2-3. The average utilization rate of sites is 60%~90%, so increasing numbers of campsites may not be able to help. So managers should take more conditions into consideration. If number of sites is fixed, our model can calculate a quite reasonable X. The fixed campsites ensure visitors can view the scenery as much as possible, avoiding conditions that visitors only focus on the drifting; it is profitable for the whole plan.

Furthermore, to attract more visitors, managers can make some fixed visiting route to limit the freedom of visitors. So the boats cycle can be more regular and efficient. Regular cycle have advantages to build unified plan to increase the profit. In other hand, our model takes full consideration of the average utilization rate of camps, which means managers can save costs and increase profits. Fewest contacts bring more fun and chances to get close to nature to visitors, which mean this river can draw more visitors.

Our model is broad enough to accommodate to other conditions, like similar visiting route, similar sites, even other similar commercial activities.

More importantly, the model includes a detailed timetable and the description of important points. It is explicit for visitors to read and know the whole process. It provides convenience for both manager and visitors. The detailed timetable is crucial for visitors to determine the number of days for visiting. The description of important points ensures visitors safety and good mood when drifting.

With the prosperous of visiting industry, number of visitors must be raised. So related sites may need to be redistributed. Managers should focus on the variation of visiting industry and adjust plans to it; this requires reasonable predictions about condition.

§10.7 点评

殷俊锋

本题是沿河流漂流露营的旅行方案设计问题,看上去是一个规划问题,但需要结合季节、旅行者习惯、露营地数目和漂流交通工具等因素进行综合考虑。因此,也是一个仁者见仁、智者见智的数学建模题。

首先,根据以往两年的数据,本文使用统计和聚类分析的办法将旅行方案分成三个时间长短不同的旅行方案套餐,能符合绝大部分游客的需要。根据旅行方案的时间长短,根据距离、河流和地理信息等再来决定露营地点和交通方式。最后,运用层次分析法针对露营地利用率、船只利用率和游客数目这三个因素对提出的旅行方案进行评价和分析,结合不同的露营地数目可以对旅行方案调整、优化和改进。

本文的优点是将一个复杂的模型分解成若干简单模型进行考虑,譬如,虽然是多因素多目标的规划,本文选择旅行管理部门利益最大化,先根据数理统计决定符合大多数消费者的热门旅行方案,再根据热门旅行方案选择露营地和交通方式。文中提出了模型评价方法,可以根据每年游客消费的信息对露营地数目、摩托艇数目和旅行方案进行调整。

该模型的缺点是牺牲了游客的自由度,不利于新的旅游景点和项目的开发;也没有考虑游客需要宽松、舒适和静谧的环境,单目标最优化容易跌入过度开发的陷阱。

论文的写作中逻辑非常清晰,对模型的评价也非常客观且切中要点,运用大量的数据和图表支撑自己的观点,是一篇短小精悍且具有说服力的论文。