ORE Methodology

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Contents

Chapter 1

Introduction

This document contains a brief summary of the methods used in ORE, in particular the risk analytics in ORE. It may be read in parallel to the ORE User Guide, the parameterisation instructions and examples provided there.

We hope that this document provides a starting point for model validation documentation that may have to be compiled by organisations that use ORE "in anger".

The pricing methods applied in ORE are described separately in the ORE Product Catalogue.

Chapter 2

Exposure Simulation

2.1 Risk Factor Evolution

2.1.1 Cross Asset Model

ORE applies the cross asset model described in detail in [14] to evolve the market through time. So far the evolution model in ORE supports IR and FX risk factors for any number of currencies, Equity and Inflation as well as Credit. Extensions to full simulation of Commodity is planned.
The Cross Asset Model is based on the Linear Gauss Markov model (LGM) for interest rates, lognormal FX and equity processes, Dodgson-Kainth model for inflation, LGM or Extended Cox-Ingersoll-Ross model (CIR++) for credit, and a single-factor log-normal model for commodity curves. We identify a single *domestic* currency; its LGM process, which is labelled z_0; and a set of n foreign currencies with associated LGM processes that are labelled z_i, $i = 1, \ldots, n$.

We denote the equity spot price processes with state variables s_j and the index of the denominating currency for the equity process as $\phi(j)$. The dividend yield corresponding to each equity process s_j is denoted by q_j.

Following [14], 13.27 - 13.29 we write the inflation processes in the domestic LGM measure with state variables $z_{I,k}$ and $y_{I,k}$ for $k = 1, \ldots, K$ and the credit processes in the domestic LGM measure with state variables $z_{C,k}$ and $y_{C,k}$ for $k = 1, \ldots, K$ and single factor (drift-free) commodity processes in the domestic LGM measure with state variables c_l for $l = 1, \ldots, L$. If we consider n foreign exchange rates for converting foreign currency amounts into the single domestic currency by multiplication, x_i,

$i = 1, \dots, n$, then the cross asset model is given by the system of SDEs

$$
\begin{aligned}
dz_0 &= \alpha_0 \, dW_0^z \\
dz_i &= \gamma_i \, dt + \alpha_i \, dW_i^z, \qquad i > 0 \\
\frac{dx_i}{x_i} &= \mu_i \, dt + \sigma_i \, dW_i^x, \qquad i > 0 \\
\frac{ds_j}{s_j} &= \mu_j^S \, dt + \sigma_j^S \, dW_j^S \\
dz_{I,k} &= \alpha_{I,k}(t) dW_k^I \\
dy_{I,k} &= \alpha_{I,k}(t) H_{I,k}(t) dW_k^I \\
dz_{C,k} &= \alpha_{C,k}(t) dW_k^C \\
dy_{C,k} &= H_{C,k}(t) \alpha_{C,k}(t) dW_k^C \\
dc_l &= \mu_l^c dt + \sigma_l^c e^{\kappa^c t} dW_l^c
\end{aligned}
$$

$$
\begin{aligned}
\gamma_i &= -\alpha_i^2 \, H_i - \rho_{ii}^{zx} \, \sigma_i \, \alpha_i + \rho_{i0}^{zz} \, \alpha_i \, \alpha_0 \, H_0 \\
\mu_i &= r_0 - r_i + \rho_{0i}^{zx} \, \alpha_0 \, H_0 \, \sigma_i \\
\mu_j^S &= (r_{\phi(j)}(t) - q_j(t) + \rho_{0j}^{zs} \alpha_0 H_0 \sigma_j^S - \epsilon_{\phi(j)} \rho_{j\phi(j)}^{sx} \sigma_j^S \sigma_{\phi(j)}) \\
r_i &= f_i(0,t) + z_i(t) \, H_i'(t) + \zeta_i(t) \, H_i(t) \, H_i'(t), \quad \zeta_i(t) = \int_0^t \alpha_i^2(s) \, ds \\
\mu_l^c &= \rho_{0c}^{zl} \alpha_0 H_0 \sigma_l^c e^{\kappa_l^c t} - \epsilon_{\phi(l)} \rho_{l\phi(l)}^{cx} \sigma_{\phi(l)}^x \sigma_l^c e^{\kappa_l^c t}
\end{aligned}
$$

$$
dW_a^\alpha \, dW_b^\beta = \rho_{ij}^{\alpha\beta} \, dt, \qquad \alpha, \beta \in \{z, x, S, I, C, c\}, \qquad a, b \text{ suitable indices}
$$

where we have dropped time dependencies for readability, $f_i(0,t)$ is the instantaneous forward curve in currency i, and ϵ_i is an indicator such that $\epsilon_i = 1 - \delta_{0i}$, where δ is the Kronecker delta.

Parameters $H_i(t)$ and $\alpha_i(t)$ (or alternatively $\zeta_i(t)$) are LGM model parameters which determine, together with the stochastic factor $z_i(t)$, the evolution of numeraire and zero bond prices in the LGM model:

$$
N(t) = \frac{1}{P(0,t)} \exp\left\{ H_t \, z_t + \frac{1}{2} H_t^2 \, \zeta_t \right\} \tag{2.1}
$$

$$
P(t,T,z_t) = \frac{P(0,T)}{P(0,t)} \, \exp\left\{ -(H_T - H_t) \, z_t - \frac{1}{2} \left(H_T^2 - H_t^2 \right) \, \zeta_t \right\}. \tag{2.2}
$$

Note that the LGM model is closely related to the Hull-White model in T-forward measure [14].

The parameters $H_{I,k}(t)$ and $\alpha_{I,k}(t)$ determine together with the factors $z_{I,k}(t), y_{I,k}(t)$ the evolution of the spot Index $I(t)$ and the forward index $\hat{I}(t,T) = P_I(t,T)/P_n(t,T)$ defined as the ratio of the inflation linked zero bond and the nominal zero bond,

$$
\begin{aligned}
\hat{I}(t,T) &= \frac{\hat{I}(0,T)}{\hat{I}(0,t)} e^{(H_{I,k}(T) - H_{I,k}(t)) z_{I,k}(t) + \tilde{V}(t,T)} \\
I(t) &= I(0) \hat{I}(0,t) e^{H_{I,k}(t) z_{I,k}(t) - y_{I,k}(t) - V(0,t)}
\end{aligned}
$$

with, in case of domestic currency inflation,

$$
\begin{aligned}
V(t,T) &= \frac{1}{2}\int_t^T (H_{I,k}(T)-H_{I,k}(s))^2\alpha_{I,k}^2(s)ds \\
&\quad -\rho_{0,k}^{zI} H_0(T)\int_t^T (H_{I,k}(t)-H_{I,k}(s))\alpha_0(s)\alpha_{I,k}(s)ds \\
\tilde{V}(t,T) &= V(t,T)-V(0,T)-V(0,t) \\
&= -\frac{1}{2}(H_{I,k}^2(T)-H_{I,k}^2(t))\zeta_{I,k}(t,0) \\
&\quad +(H_{I,k}(T)-H_{I,k}(t))\zeta_{I,k}(t,1) \\
&\quad +(H_0(T)H_{I,k}(T)-H_0(t)H_{I,k}(t))\zeta_{0I}(t,0) \\
&\quad -(H_0(T)-H_0(t))\zeta_{0I}(t,1) \\
V(0,t) &= \frac{1}{2}H_{I,k}^2(t)\zeta_{I,k}(t,0)-H_{I,k}(t)\zeta_{I,k}(t,1)+\frac{1}{2}\zeta_{I,k}(t,2) \\
&\quad -H_0(t)H_{I,k}(t)\zeta_{0I}(t,0)+H_0(t)\zeta_{0I}(t,1) \\
\zeta_{I,k}(t,k) &= \int_0^t H_{I,k}^k(s)\alpha_{I,k}^2(s)ds \\
\zeta_{0I}(t,k) &= \rho_{0,k}^{zI}\int_0^t H_{I,k}^k(t)\alpha_0(s)\alpha_{I,k}(s)ds
\end{aligned}
$$

and for foreign currency inflation in currency $i>0$, with

$$
\tilde{V}(t,T) = V(t,T)-V(0,T)+V(0,T)
$$

and

$$
\begin{aligned}
V(t,T) &= \frac{1}{2}\int_t^T (H_{I,k}(T)-H_{I,k}(s))^2\alpha_{I,k}(s)ds \\
&\quad -\rho_{0,k}^{zI}\int_t^T H_0(s)\alpha_0(s)(H_{I,k}(T)-H_{I,k}(s)\alpha_{I,k}(s))ds \\
&\quad -\rho_{i,k}^{zI}\int_t^T (H_i(T)-H_i(s))\alpha_i(s)(H_{I,k}(T)-H_{I,k}(s))\alpha_{I,k}(s)ds \\
&\quad +\rho_{i,k}^{xI}\int_t^T \sigma_i(s)(H_{I,k}(T)-H_{I,k}(s))\alpha_{I,k}(s)ds
\end{aligned}
$$

Commodity

Each commodity component models the commodity price curve as

$$
\frac{dF(t,T)}{F(t,T)} = \alpha(T)\sigma\, e^{-\kappa\,(T-t)}\, dW(t) \tag{2.3}
$$

where $\alpha(T) := \exp(b(T))$ is the time dependent [1] multiplier to capture seasonality effect observed in the market for both commodity future price curves and option volatilities. This model is a single-factor version of the Gabillon (1991) model that is e.g. described in [14]. It can also be seen as the Schwartz (1997) model formulated in terms of forward curve dynamics. The extension to the full Gabillon model with two factors and time-dependent multiplier[2]

$$\frac{dF(t,T)}{F(t,T)} = \alpha(t)\,\left(\sigma_S\, e^{-\kappa\,(T-t)}\, dW_S(t) + \sigma_L\,\left(1 - e^{-\kappa\,(T-t)}\right)\, dW_L(t)\right) \tag{2.4}$$

for richer dynamics of the curve and accurate calibration to options will follow.

The commodity components' Wiener processes can be correlated. However, the integration of commodity components into the overall CAM assumes zero correlations between commodities and non-commodity drivers for the time being.

To propagate the one-factor model, we can use an artificial (Ornstein-Uhlenbeck) spot price process

$$dX(t) = -\kappa\, X(t)\, dt + \sigma(t)\, dW(t), \qquad X(0) = 0$$
$$X(t) = X(s)\, e^{-\kappa(t-s)} + \int_s^t \sigma\, e^{-\kappa(t-u)}\, dW(u)$$

with

$$F(t,T) = F(0,T)\,\exp\left(X(t)\, e^{b(T)-\kappa\,(T-t)} - \frac{1}{2}\,(V(0,T) - V(t,T))\right)$$
$$V(t,T) = e^{2(b(T)-\kappa T)} \int_t^T \sigma^2\, e^{2\kappa u}\, du.$$

Note that

$$\mathbb{V}[\ln F(T,T)] = \mathbb{V}[X(T)]$$

is the variance that is used in the pricing of a Futures Option which in turn is used in the calibration of the Schwartz model.

Alternatively, one can use the drift-free state variable $Y(t) = e^{\kappa t} X(t)$ with

$$dY(t) = \sigma\, e^{\kappa\, t}\, dW(t).$$

Both choices of state dynamics are possible in ORE.

2.1.2 Analytical Moments of the Risk Factor Evolution Model

We follow [14], chapter 16. The expectation of the interest rate process z_i conditional on $\mathcal{F}_{t_0}$ at $t_0 + \Delta t$ is

[1] See section 7.2 in Andersen [27] for the discussion on dependence of seasonality adjustment to calendar days and expiry of future contracts.

[2] Andersen [27] worked on a two factor set up, where the first factor affects the short-end of the futures curve and has the form the $e^{b(T)}$, and the second factor has an additional term containing $e^{a(T)} h_\infty$ for long futures maturities.

$$
\begin{aligned}
\mathbb{E}_{t_0}[z_i(t_0+\Delta t)] &= z_i(t_0) + \mathbb{E}_{t_0}[\Delta z_i], \qquad \text{with} \quad \Delta z_i = z_i(t_0+\Delta t) - z_i(t_0) \\
&= z_i(t_0) - \int_{t_0}^{t_0+\Delta t} H_i^z\,(\alpha_i^z)^2\,du + \rho_{0i}^{zz}\int_{t_0}^{t_0+\Delta t} H_0^z\,\alpha_0^z\,\alpha_i^z\,du \\
&\quad -\epsilon_i\rho_{ii}^{zx}\int_{t_0}^{t_0+\Delta t} \sigma_i^x\,\alpha_i^z\,du
\end{aligned}
$$

where ϵ_i is zero for $i=0$ (domestic currency) and one otherwise.

The expectation of the FX process x_i conditional on $\mathcal{F}_{t_0}$ at $t_0+\Delta t$ is

$$
\begin{aligned}
\mathbb{E}_{t_0}[\ln x_i(t_0+\Delta t)] &= \ln x_i(t_0) + \mathbb{E}_{t_0}[\Delta \ln x_i], \qquad \text{with} \quad \Delta \ln x_i = \ln x_i(t_0+\Delta t) - \ln x_i(t_0) \\
&= \ln x_i(t_0) + (H_0^z(t) - H_0^z(s))\,z_0(s) - (H_i^z(t) - H_i^z(s))\,z_i(s) \\
&\quad + \ln\left(\frac{P_0^n(0,s)}{P_0^n(0,t)}\frac{P_i^n(0,t)}{P_i^n(0,s)}\right) \\
&\quad -\frac{1}{2}\int_s^t (\sigma_i^x)^2\,du \\
&\quad +\frac{1}{2}\left((H_0^z(t))^2\zeta_0^z(t) - (H_0^z(s))^2\zeta_0^z(s) - \int_s^t (H_0^z)^2(\alpha_0^z)^2\,du\right) \\
&\quad -\frac{1}{2}\left((H_i^z(t))^2\zeta_i^z(t) - (H_i^z(s))^2\zeta_i^z(s) - \int_s^t (H_i^z)^2(\alpha_i^z)^2\,du\right) \\
&\quad +\rho_{0i}^{zx}\int_s^t H_0^z\,\alpha_0^z\,\sigma_i^x\,du \\
&\quad -\int_s^t (H_i^z(t) - H_i^z)\,\gamma_i\,du, \qquad \text{with} \quad s = t_0, \quad t = t_0+\Delta t
\end{aligned}
$$

with

$$
\gamma_i = -H_i^z\,(\alpha_i^z)^2 + H_0^z\,\alpha_0^z\,\alpha_i^z\,\rho_{0i}^{zz} - \sigma_i^x\,\alpha_i^z\,\rho_{ii}^{zx}
$$

The expectation of the Inflation processes $z_{I,k}, y_{I,k}$ conditional on $\mathcal{F}_{t_0}$ at any time $t > t_0$ is equal to $z_{I,k}(t_0)$ resp. $y_{I,k}(t_0)$ since both processes are drift free.

The expectation of the equity processes s_j conditional on $\mathcal{F}_{t_0}$ at $t_0 + \Delta t$ is

$$
\begin{aligned}
\mathbb{E}_{t_0}[\ln s_j(t_0+\Delta t)] &= \ln s_j(t_0) + \mathbb{E}_{t_0}[\Delta \ln s_j], \qquad \text{with} \quad \Delta \ln s_j = \ln s_j(t_0+\Delta t) - \ln s_j(t_0) \\
&= \ln s_j(t_0) + \ln\left[\frac{P_{\phi(j)}(0,s)}{P_{\phi(j)}(0,t)}\right] - \int_s^t q_j(u)du - \frac{1}{2}\int_s^t \sigma_j^S(u)\sigma_j^S(u)du \\
&\quad + \rho_{0j}^{zs}\int_s^t \alpha_0(u)H_0(u)\sigma_j^S(u)du - \epsilon_{\phi(j)}\rho_{j\phi(j)}^{sx}\int_s^t \sigma_j^S(u)\sigma_{\phi(j)}(u)du \\
&\quad + \frac{1}{2}\left(H_{\phi(j)}^2(t)\zeta_{\phi(j)}(t) - H_{\phi(j)}^2(s)\zeta_{\phi(j)}(s) - \int_s^t H_{\phi(j)}^2(u)\alpha_{\phi(j)}^2(u)du\right) \\
&\quad + (H_{\phi(j)}(t) - H_{\phi(j)}(s))z_{\phi(j)}(s) + \epsilon_{\phi(j)}\int_s^t \gamma_{\phi(j)}(u)(H_{\phi(j)}(t) - H_{\phi(j)}(u))du
\end{aligned}
$$

The expectation of the commodity process c_l conditional on $\mathcal{F}_{t_0}$ at $t_0 + \Delta t$ is

$$
\mathbb{E}_{t_0}[c_l(t_0+\Delta t] = c_l(0) + \rho_{0l}^{zc}\int_0^t H_0(u)\alpha_0(u)\sigma_l^c e^{\kappa_l^c u}du - \epsilon_{\phi(l)}\rho_{l\phi(l)}^{cx}\int_0^t \sigma_{\phi(l)}^x(u)\sigma_l^c e^{\kappa_l^c u}du
$$

The IR-IR covariance over the interval $[s,t] := [t_0, t_0+\Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
\mathrm{Cov}[\Delta z_a, \Delta \ln x_b] &= \rho_{0a}^{zz}\int_s^t (H_0^z(t) - H_0^z)\,\alpha_0^z\,\alpha_a^z\,du \\
&\quad - \rho_{ab}^{zz}\int_s^t \alpha_a^z\,(H_b^z(t) - H_b^z)\,\alpha_b^z\,du \\
&\quad + \rho_{ab}^{zx}\int_s^t \alpha_a^z\,\sigma_b^x\,du.
\end{aligned}
$$

The IR-FX covariance over the interval $[s,t] := [t_0, t_0+\Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
\mathrm{Cov}[\Delta z_a, \Delta \ln x_b] &= \rho_{0a}^{zz}\int_s^t (H_0^z(t) - H_0^z)\,\alpha_0^z\,\alpha_a^z\,du \\
&\quad - \rho_{ab}^{zz}\int_s^t \alpha_a^z\,(H_b^z(t) - H_b^z)\,\alpha_b^z\,du \\
&\quad + \rho_{ab}^{zx}\int_s^t \alpha_a^z\,\sigma_b^x\,du.
\end{aligned}
$$

The FX-FX covariance over the interval $[s,t] := [t_0, t_0+\Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
\mathrm{Cov}[\Delta \ln x_a, \Delta \ln x_b] &= \int_s^t (H_0^z(t) - H_0^z)^2\,(\alpha_0^z)^2\,du \\
&\quad -\rho_{0a}^{zz} \int_s^t (H_a^z(t) - H_a^z)\,\alpha_a^z\,(H_0^z(t) - H_0^z)\,\alpha_0^z\,du \\
&\quad -\rho_{0b}^{zz} \int_s^t (H_0^z(t) - H_0^z)\,\alpha_0^z\,(H_b^z(t) - H_b^z)\,\alpha_b^z\,du \\
&\quad +\rho_{0b}^{zx} \int_s^t (H_0^z(t) - H_0^z)\,\alpha_0^z \sigma_b^x\,du \\
&\quad +\rho_{0a}^{zx} \int_s^t (H_0^z(t) - H_0^z)\,\alpha_0^z\,\sigma_a^x\,du \\
&\quad -\rho_{ab}^{zx} \int_s^t (H_a^z(t) - H_a^z)\,\alpha_a^z \sigma_b^x,\,du \\
&\quad -\rho_{ba}^{zx} \int_s^t (H_b^z(t) - H_b^z)\,\alpha_b^z\,\sigma_a^x\,du \\
&\quad +\rho_{ab}^{zz} \int_s^t (H_a^z(t) - H_a^z)\,\alpha_a^z\,(H_b^z(t) - H_b^z)\,\alpha_b^z\,du \\
&\quad +\rho_{ab}^{xx} \int_s^t \sigma_a^x\,\sigma_b^x\,du
\end{aligned}
$$

The IR-INF covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
\mathrm{Cov}[\Delta z_a, \Delta z_{I,b}] &= \rho_{ab}^{zI} \int_s^t \alpha_a(s)\alpha_{I,b}(s)ds \\
\mathrm{Cov}[\Delta z_a, \Delta y_{I,b}] &= \rho_{ab}^{zI} \int_s^t \alpha_a(s)H_{I,b}(s)\alpha_{I,b}(s)ds
\end{aligned}
$$

The FX-INF covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
\mathrm{Cov}[\Delta x_a, \Delta z_{I,b}] &= \rho_{0b}^{zI} \int_s^t \alpha_0(s)(H_0(t) - H_0(s))\alpha_{I,b}(s)ds \\
&\quad -\rho_{ab}^{zI} \int_s^t \alpha_a(s)(H_a(t) - H_a(s))\alpha_{I,b}(s)ds \\
&\quad +\rho_{ab}^{xI} \int_s^t \sigma_a(s)\alpha_{I,b}(s)ds \\
\mathrm{Cov}[\Delta x_a, \Delta y_{I,b}] &= \rho_{0b}^{zI} \int_s^t \alpha_0(s)(H_0(t) - H_0(s))H_{I,b}(s)\alpha_{I,b}(s)ds \\
&\quad -\rho_{ab}^{zI} \int_s^t \alpha_a(s)(H_a(t) - H_a(s))H_{I,b}(s)\alpha_{I,b}(s)ds \\
&\quad +\rho_{ab}^{xI} \int_s^t \sigma_a(s)H_{I,b}(s)\alpha_{I,b}(s)ds
\end{aligned}
$$

The INF-INF covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
\mathrm{Cov}[\Delta z_{I,a}, \Delta z_{I,b}] &= \rho^{II}_{ab} \int_s^t \alpha_{I,a}(s)\alpha_{I,b}(s) ds \\
\mathrm{Cov}[\Delta z_{I,a}, \Delta y_{I,b}] &= \rho^{II}_{ab} \int_s^t \alpha_{I,a}(s) H_{I,b}(s)\alpha_{I,b}(s) ds \\
\mathrm{Cov}[\Delta y_{I,a}, \Delta y_{I,b}] &= \rho^{II}_{ab} \int_s^t H_{I,a}(s)\alpha_{I,a}(s) H_{I,b}(s)\alpha_{I,b}(s) ds
\end{aligned}
$$

The equity/equity covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
Cov\left[\Delta ln[s_i], \Delta ln[s_j]\right] &= \rho^{zz}_{\phi(i)\phi(j)} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))(H_{\phi(j)}(t) \\
&\quad - H_{\phi(j)}(u))\alpha_{\phi(i)}(u)\alpha_{\phi(j)}(u) du \\
&\quad + \rho^{zs}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)\sigma^S_j(u) du \\
&\quad + \rho^{zs}_{\phi(j)i} \int_s^t (H_{\phi(j)}(t) - H_{\phi(j)}(u))\alpha_{\phi(j)}(u)\sigma^S_i(u) du \\
&\quad + \rho^{ss}_{ij} \int_s^t \sigma^S_i(u)\sigma^S_j(u) du
\end{aligned}
$$

The equity/FX covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
Cov\left[\Delta ln[s_i], \Delta ln[x_j]\right] &= \rho^{zz}_{\phi(i)0} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))(H_0(t) - H_0(u))\alpha_{\phi(i)}(u)\alpha_0(u) du \\
&\quad - \rho^{zz}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))(H_j(t) - H_j(u))\alpha_{\phi(i)}(u)\alpha_j(u) du \\
&\quad + \rho^{zx}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)\sigma_j(u) du \\
&\quad + \rho^{sz}_{i0} \int_s^t (H_0(t) - H_0(u))\alpha_0(u)\sigma^S_i(u) du \\
&\quad - \rho^{sz}_{ij} \int_s^t (H_j(t) - H_j(u))\alpha_j(u)\sigma^S_i(u) du \\
&\quad + \rho^{sx}_{ij} \int_s^t \sigma^S_i(u)\sigma_j(u) du
\end{aligned}
$$

The equity/IR covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
Cov\left[\Delta ln[s_i], \Delta z_j\right] &= \rho^{zz}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)\alpha_j(u) du \\
&\quad + \rho^{sz}_{ij} \int_s^t \sigma^S_i(u)\alpha_j(u) du
\end{aligned}
$$

The equity/inflation covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$
\begin{aligned}
Cov\left[\Delta ln[s_i], \Delta z_{I,j}\right] &= \rho^{zI}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)\alpha_{I,j}(u)du \\
&\quad + \rho^{sI}_{ij} \int_s^t \sigma^S_i(u)\alpha_{I,j}(u)du \\
Cov\left[\Delta ln[s_i], \Delta y_{I,j}\right] &= \rho^{zI}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)H_{I,j}(u)\alpha_{I,j}(u)du \\
&\quad + \rho^{sI}_{ij} \int_s^t \sigma^S_i(u)H_{I,j}(u)\alpha_{I,j}(u)du
\end{aligned}
$$

The expectation of the Credit processes $z_{C,k}, y_{C,k}$ conditional on $\mathcal{F}_{t_0}$ at any time $t > t_0$ is equal to $z_{C,k}(t_0)$ resp. $y_{C,k}(t_0)$ since both processes are drift free.

The credit/credit covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$
\begin{aligned}
Cov\left[\Delta z_{C,a}, \Delta z_{C,b}\right] &= \rho^{CC}_{ab} \int_s^t \alpha_{C,a}(u)\alpha_{C,b}(u)du \\
Cov\left[\Delta z_{C,a}, \Delta y_{C,b}\right] &= \rho^{CC}_{ab} \int_s^t \alpha_{C,a}(u)H_{C,b}(u)\alpha_{C,b}(u)du \\
Cov\left[\Delta y_{C,a}, \Delta y_{C,b}\right] &= \rho^{CC}_{ab} \int_s^t \alpha_{C,a}(u)H_{C,a}(u)\alpha_{C,b}(u)H_{C,b}(u)du
\end{aligned}
$$

The IR/credit covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$
\begin{aligned}
Cov\left[\Delta z_a, \Delta z_{C,b}\right] &= \rho^{zC}_{ab} \int_s^t \alpha_a(u)\alpha_{C,b}(u)du \\
Cov\left[\Delta z_a, \Delta y_{C,b}\right] &= \rho^{zC}_{ab} \int_s^t \alpha_a(u)H_{C,b}(u)\alpha_{C,b}(u)du
\end{aligned}
$$

The FX/credit covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$
\begin{aligned}
\mathrm{Cov}[\Delta x_a, \Delta z_{C,b}] &= \rho^{zC}_{0b} \int_s^t \alpha_0(s)(H_0(t) - H_0(s))\alpha_{C,b}(s)ds \\
&\quad - \rho^{zC}_{ab} \int_s^t \alpha_a(s)(H_a(t) - H_a(s))\alpha_{C,b}(s)ds \\
&\quad + \rho^{xC}_{ab} \int_s^t \sigma_a(s)\alpha_{C,b}(s)ds \\
\mathrm{Cov}[\Delta x_a, \Delta y_{C,b}] &= \rho^{zC}_{0b} \int_s^t \alpha_0(s)(H_0(t) - H_0(s))H_{C,b}(s)\alpha_{C,b}(s)ds \\
&\quad - \rho^{zC}_{ab} \int_s^t \alpha_a(s)(H_a(t) - H_a(s))H_{C,b}(s)\alpha_{C,b}(s)ds \\
&\quad + \rho^{xC}_{ab} \int_s^t \sigma_a(s)H_{C,b}(s)\alpha_{C,b}(s)ds
\end{aligned}
$$

The inflation/credit covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$
\begin{aligned}
\mathrm{Cov}[\Delta z_{I,a}, \Delta z_{C,b}] &= \rho^{IC}_{ab} \int_s^t \alpha_{I,a}\alpha_{C,b}(u)du \\
\mathrm{Cov}[\Delta z_{I,a}, \Delta y_{C,b}] &= \rho^{IC}_{ab} \int_s^t \alpha_{I,a}H_{C,b}(u)\alpha_{C,b}(u)du \\
\mathrm{Cov}[\Delta y_{I,a}, \Delta z_{C,b}] &= \rho^{IC}_{ab} \int_s^t \alpha_{I,a}H_{I,a}(u)\alpha_{C,b}(u)du \\
\mathrm{Cov}[\Delta y_{I,a}, \Delta y_{C,b}] &= \rho^{IC}_{ab} \int_s^t \alpha_{I,a}H_{I,a}(u)\alpha_{C,b}(u)H_{C,b}(u)du
\end{aligned}
$$

The equity/credit covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$
\begin{aligned}
Cov\left[\Delta ln[s_i], \Delta z_{C,j}\right] &= \rho^{zC}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)\alpha_{C,j}(u)du \\
&\quad + \rho^{sC}_{ij} \int_s^t \sigma^S_i(u)\alpha_{C,j}(u)du \\
Cov\left[\Delta ln[s_i], \Delta y_{C,j}\right] &= \rho^{zC}_{\phi(i)j} \int_s^t (H_{\phi(i)}(t) - H_{\phi(i)}(u))\alpha_{\phi(i)}(u)H_{C,j}(u)\alpha_{C,j}(u)du \\
&\quad + \rho^{sC}_{ij} \int_s^t \sigma^S_i(u)H_{C,j}(u)\alpha_{C,j}(u)du
\end{aligned}
$$

The commodity/commodity covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
Cov\left[\Delta c_i, \Delta c_j\right] = \rho^{cc}_{ij} \int_s^t \sigma^c_i(u)e^{\kappa^c_i u}\sigma^c_j(u)e^{\kappa^c_j u}du
$$

The commodity/IR covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
Cov\left[\Delta c_i, \Delta z_j\right] = \rho^{cz}_{ij} \int_s^t \sigma^c_i e^{\kappa^c_i u}(u)\alpha_j(u)du
$$

The commodity/FX covariance over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) is

$$
\begin{aligned}
Cov\left[\Delta c_i, \Delta ln[x_j]\right] &= \rho^{cz}_{i0} \int_s^t (H_0(t) - H_0(u))\alpha_0(u)\sigma^c_i e^{\kappa^c_i u}du \\
&\quad - \rho^{cz}_{ij} \int_s^t (H_j(t) - H_j(u))\alpha_j(u)\sigma^c_i e^{\kappa^c_i u}du \\
&\quad + \rho^{cx}_{ij} \int_s^t \sigma^c_i e^{\kappa^c_i u}\sigma^x_j(u)du
\end{aligned}
$$

The commodity/inflation covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$\begin{aligned}
Cov\left[\Delta c_i, \Delta z_{I,j}\right] &= \rho^{cI}_{ij} \int_s^t \sigma^c_i e^{\kappa^c_i u} \alpha_{I,j}(u) du \\
Cov\left[\Delta c_i, \Delta y_{I,j}\right] &= \rho^{cI}_{ij} \int_s^t \sigma^c_i e^{\kappa^c_i u} H_{I,j}(u) \alpha_{I,j}(u) du
\end{aligned}$$

The commodity/credit covariances over the interval $[s,t] := [t_0, t_0 + \Delta t]$ (conditional on $\mathcal{F}_{t_0}$) are as follows:

$$\begin{aligned}
Cov\left[\Delta c_i, \Delta z_{C,j}\right] &= \rho^{cC}_{ij} \int_s^t \sigma^c_i e^{\kappa^c_i u} \alpha_{C,j}(u) du \\
Cov\left[\Delta c_i, \Delta y_{C,j}\right] &= \rho^{cC}_{ij} \int_s^t \sigma^c_i e^{\kappa^c_i u} H_{C,j}(u) \alpha_{C,j}(u) du
\end{aligned}$$

2.1.3 Change of Measure

We can change measure from LGM to the T-Forward measure by applying a shift transformation to the H parameter of the domestic LGM process, as explained in [14] and shown in Example 12. This does not involve amending the system of SDEs above.

In the following we show how to move from the LGM to the Bank Account measure when we start with the Cross Asset Model in the LGM measure. This description and the implementation in ORE is limited so far to the cross currency case.

First note that the stochastic Bank Account (BA) can be written

$$B(t) = \frac{1}{P(0,t)} \exp\left(\int_0^t (H_t - H_s)\,\alpha_s\,dW^B_s + \frac{1}{2}\int_0^t (H_t - H_s)^2\,\alpha^2_s\,ds\right)$$

with Wiener processes in the BA measure. We can express this in terms of the domestic LGM's state variable $z(t)$ and an auxiliary random variable $y(t)$

$$B(t) = \frac{1}{P(0,t)} \exp\left(H(t)\,z(t) - y(t) + \frac{1}{2}\left(H^2(t)\,\zeta_0(t) + \zeta_2(t)\right)\right)$$

with

$$\begin{aligned}
dz(t) &= \alpha(t)\,dW^B(t) - H(t)\,\alpha^2(t)\,dt \\
dy(t) &= H(t)\,\alpha(t)\,dW^B(t) \\
\zeta_n(t) &= \int_0^t \alpha^2(s)\,H^n(s)\,ds
\end{aligned}$$

Note the drift of LGM state variable $z(t)$ in the BA measure and the auxiliary state variable $y(t)$ which is driven by the same Wiener process as $z(t)$. The instantaneous correlation of dz and dy is one, but the terminal correlation of $z(t)$ and $y(t)$ is less than one because of their different volatility functions. This is all we need to switch measure to BA in a pure domestic currency case.

To change measure in the cross currency case we need to make changes to the SDE beyond adding an auxiliary state variable y and adding a drift to the domestic LGM state. Let us write down the SDEs in the LGM and BA measure with respective drift terms that ensure martingale properties.

SDE in the LGM measure

$$\begin{aligned} dz_0 &= \alpha_0\, dW_0^z \\ dz_i &= \left(-\alpha_i^2\, H_i - \rho_{ii}^{zx}\, \sigma_i\, \alpha_i + \rho_{i0}^{zz}\, \alpha_i\, \alpha_0\, H_0\right)\, dt + \alpha_i\, dW_i^z \\ d\ln x_i &= \left(r_0 - r_i - \frac{1}{2}\sigma_i^2 + \rho_{0i}^{zx}\, \alpha_0\, H_0\, \sigma_i\right)\, dt + \sigma_i\, dW_i^x \end{aligned}$$

SDE in the BA measure

$$\begin{aligned} dy_0 &= \alpha_0\, H_0\, d\widetilde{W}_0^z \\ dz_0 &= -\alpha_0^2\, H_0\, dt + \alpha_0\, d\widetilde{W}_0^z \\ dz_i &= \left(-\alpha_i^2\, H_i - \rho_{ii}^{zx}\, \sigma_i\, \alpha_i\right)\, dt + \alpha_i\, d\widetilde{W}_i^z \\ d\ln x_i &= \left(r_0 - r_i - \frac{1}{2}\sigma_i^2\right)\, dt + \sigma_i\, d\widetilde{W}_i^x, \qquad r_i = f_i(0,t) + z_i(t)\, H_i'(t) + \zeta_i(t)\, H_i(t)\, H_i'(t) \end{aligned}$$

Blue terms are added, red terms are removed when moving from LGM to BA.

These drift term changes lead to the following changes in conditional expectations

$$\begin{aligned} \mathbb{E}[\Delta y_0] =& 0 \\ \mathbb{E}[\Delta z_0] =& -\int_s^t H_0\, \alpha_0^2\, du \\ \mathbb{E}[\Delta z_i] =& -\int_s^t H_i\, \alpha_i^2\, du - \rho_{ii}^{zx} \int_s^t \sigma_i^x\, \alpha_i\, du + \rho_{0i}^{zz} \int_s^t H_0\, \alpha_0\, \alpha_i\, du \\ \mathbb{E}[\Delta \ln x] =& \left(H_0(t) - H_0(s)\right) z_0(s) - \left(H_i(t) - H_i(s)\right)\, z_i(s) \\ &+ \ln\left(\frac{P_0^n(0,s)}{P_0^n(0,t)}\frac{P_i^n(0,t)}{P_i^n(0,s)}\right) \\ &- \frac{1}{2}\int_s^t (\sigma_i^x)^2\, du \\ &+ \frac{1}{2}\left(H_0^2(t)\, \zeta_0(t) - H_0^2(s)\, \zeta_0(s) - \int_s^t H_0^2 \alpha_0^2\, du\right) \\ &- \frac{1}{2}\left(H_i^2(t)\, \zeta_i(t) - H_i^2(s)\, \zeta_i(s) - \int_s^t H_i^2 \alpha_i^2\, du\right) \\ &+ \rho_{0i}^{zx} \int_s^t H_0\, \alpha_0\, \sigma_i^x\, du \\ &- \int_s^t \left(H_i(t) - H_i\right) \gamma_i\, du \qquad \text{with} \qquad \gamma_i = -\alpha_i^2\, H_i - \rho_{ii}^{zx}\, \sigma_i\, \alpha_i + \rho_{i0}^{zz}\, \alpha_i\, \alpha_0\, H_0 \\ &+ \int_s^t \left(H_0(t) - H_0\right)\, \gamma_0\, du \qquad \text{with} \qquad \gamma_0 = -H_0\, \alpha_0^2 \end{aligned}$$

and the following additional variances and covariances

$$
\begin{aligned}
\mathrm{Var}[\Delta y_0] &= \int_s^t \alpha_0^2 \, H_0^2 \, du \\
\mathrm{Cov}[\Delta y_0, \Delta z_i] &= \rho_{0i}^{zz} \int_s^t \alpha_0 \, H_0 \, \alpha_i \, du \\
\mathrm{Cov}[\Delta y_0, \Delta \ln x_i] &= \int_s^t (H_0(t) - H_0) \, \alpha_0^2 \, H_0 \, du \\
&\quad - \rho_{0i}^{zz} \int_s^t \alpha_0 \, H_0 \, (H_i(t) - H_i) \; \alpha_i \, du \\
&\quad + \rho_{0i}^{zx} \int_s^t \alpha_0 \, H_0 \, \sigma_i^x \, du
\end{aligned}
$$

Example 36 illustrates the effect of the choice of measure on exposure simulations.

2.2 Exposures

In ORE we use the following exposure definitions

$$EE(t) = EPE(t) = \mathbb{E}^N \left[\frac{(NPV(t) - C(t))^+}{N(t)} \right] \tag{2.5}$$

$$ENE(t) = \mathbb{E}^N \left[\frac{(-NPV(t) + C(t))^+}{N(t)} \right] \tag{2.6}$$

where $NPV(t)$ stands for the netting set NPV and $C(t)$ is the collateral balance[3] at time t. Note that these exposures are expectations of values discounted with numeraire N (in ORE the Linear Gauss Markov model's numeraire) to today, and expectations are taken in the measure associated with numeraire N. These are the exposures which enter into unilateral CVA and DVA calculation, respectively, see next section. Note that we sometimes label the expected exposure (2.5) EPE, not to be confused with the Basel III Expected Positive Exposure below.

Basel III defines a number of exposures each of which is a 'derivative' of Basel's Expected Exposure:

Expected Exposure

$$EE_B(t) = \mathbb{E}[\max(NPV(t) - C(t), 0)] \tag{2.7}$$

Expected Positive Exposure

$$EPE_B(T) = \frac{1}{T} \sum_{t<T} EE_B(t) \cdot \Delta t \tag{2.8}$$

Effective Expected Exposure, recursively defined as running maximum

$$EEE_B(t) = \max(EEE_B(t - \Delta t), EE_B(t)) \tag{2.9}$$

[3] $C(t) > 0$ means that we have *received* collateral from the counterparty

Effective Expected Positive Exposure

$$EEPE_B(T) = \frac{1}{T} \sum_{t<T} EEE_B(t) \cdot \Delta t \tag{2.10}$$

The last definition, Effective EPE, is used in Basel documents since Basel II for Exposure At Default and capital calculation. Following [2, 3] the time averages in the EPE and EEPE calculations are taken over *the first year* of the exposure evolution (or until maturity if all positions of the netting set mature before one year).

To compute $EE_B(t)$ consistently in a risk-neutral setting, we compound (2.5) with the deterministic discount factor $P(t)$ up to horizon t:

$$EE_B(t) = \frac{1}{P(t)} \, EE(t)$$

Finally, we define another common exposure measure, the *Potential Future Exposure* (PFE), as a (typically high) quantile α of the NPV distribution through time, similar to Value at Risk but at the upper end of the NPV distribution:

$$PFE_\alpha(t) = \left(\inf \{x | F_t(x) \geq \alpha\}\right)^+ \tag{2.11}$$

where F_t is the cumulative NPV distribution function at time t. Note that we also take the positive part to ensure that PFE is a positive measure even if the quantile yields a negative value which is possible in extreme cases.

2.3 Exposures using American Monte Carlo

The exposure analysis implemented in ORE that is used in the bulk of the examples in this user guide, mostly vanilla portfolios, is divided into two independent steps:

1. in a first step a list of NPVs (or a "NPV cube") is computed. The list is indexed by the trade ID, the simulation time step and the scenario sample number. Each entry of the cube is computed using the same pricers as for the T0 NPV calculation by shifting the evaluation date to the relevant time step of the simulation and updating the market term structures to the relevant scenario market data. The market data scenarios are generated using a *risk factor evolution model* which can be a cross asset model, but also be based on e.g. historical simulation.

2. in a second step the generated NPV cube is passed to a post processor that aggregates the results to XVA figures of different kinds.

We label this approach in the following as the *classic* exposure analysis.

The AMC module in ORE allows to replace the first step by a different approach which works faster in particular for exotic deals. The second step remains the same. The risk factor evolution model coincides with the pricing models for the single trades in this approach and is always a cross asset model operated in a pricing measure.

For AMC the entries of the NPV cube are now viewed as conditional NPVs at the simulation time given the information that is generated by the cross asset model's driving stochastic process up to the simulation time. The conditional expectations are then computed using a regression analysis of some type. In our current implementation this is chosen to be a parametric regression analysis.

The regression models are calibrated per trade during a training phase and later on evaluated in the simulation phase. The set of paths in the two phases is in general different w.r.t. their number, time step structure, and generation method (Sobol, Mersenne Twister) and seed. Typically the regressand is the (deflated) dirty *path* NPV of the trade in question, or also its underlying NPV or an option continuation value (to take exercise decisions or represent the physical underlying for physical exercise rights). The regressor is typically the model state. Certain exotic features that introduce path-dependency (e.g. a TaRN structure) may require an augmentation of the regressor though (e.g. by the already accumulated amount in case of the TaRN).

The path NPVs are generated at their *natural event dates*, like the fixing date for floating rate coupons or the payment date for fixed cashflows. This reduces the requirements for the cross asset model to provide closed form expressions for the numeraire and conditional zero bonds only.

Since the evaluation of the regression functions is computationally cheap the overall timings of the NPV cube generation are generally smaller compared to the classic approach, in particular for exotic deals like Bermudan Swaptions.

From a methodology point of view an important difference between the classic and the AMC exposure analysis lies in the model consistency: While the conditional NPVs computed with AMC are by construction consistent with the risk factor evolution model driving the XVA simulation, the scenario NPVs in the classic approach are in general not consistent in this sense unless the market scenarios are fully implied by the cross asset model. Here "fully implied" means that not only rate curves, but also market volatility and correlation term structures like FX volatility surfaces, Swaption volatilities or CMS correlation term structures as well as other parameters used by the single trade pricers have to be deduced from the cross asset model, e.g. the mean reversion of the Hull White 1F model and a suitable model volatility feeding into a Bermudan Swaption pricer.

We note that the generation of such implied term structures can be computationally expensive even for simple versions of a cross asset model like one composed from LGM IR and Black-Scholes FX components etc., and even more so for more exotic component flavours like Cheyette IR components, Heston FX components etc.

In the current implementation only a subset of all ORE trade types can be simulated using AMC while all other trade types are still simulated using the classic engine. The separation of the trades and the joining of the resulting classic and AMC cubes is automatic. The post processing step is run on the joint cube from the classic and AMC simulations as before.

Trade types supported by AMC so far:

1. Swap
2. CrossCurrencySwap

3. FxOption
4. BermudanSwaption
5. MultiLegOption

2.3.1 AMC valuation engine and AMC pricing engines

The `AMCValuationEngine` is responsible for generating a NPV cube for a portfolio of AMC enabled trades and (optionally) to populate a `AggregationScenarioData` instance with simulation data for post processing, very similar to the classic `ValuationEngine` in ORE.

The AMC valuation engine takes a cross asset model defining the risk factor evolution. This is set up identically to the cross asset model used in the `CrossAssetModelScenarioGenerator`. Similarly the same parameters for the path generation (given as a `ScenarioGeneratorData` instance) are used, so that it is guaranteed that both the AMC engine and the classic engine produce the same paths, hence can be combined to a single cube for post processing. It is checked, that a non-zero seed for the random number generation is used.

The portfolio is build against an engine factory with specific AMC pricing engine configurations. The AMC engine builders are retrieved from `getAmcEngineBuilders()` and are special in that unlike usual engine builders they take two parameters

1. the cross asset model which serves as a risk factor evolution model in the AMC valuation engine
2. the date grid used within the AMC valuation engine

For technical reasons, the configuration also contains configurations for `CapFlooredIborLeg`, `CapFlooredInterpolatedIborLeg` and `CMS` because those are used within the trade builders (more precisely the leg builders called from these) to build the trade. The configuration can be the same as for T0 pricing for them, it is actually not used by the AMC pricing engines.

The AMC engine builders build a smaller version of the global cross asset model only containing the model components required to price the specific trade. Note that no deal specific calibration of the model is performed.

The AMC pricing engines perform a T0 pricing and - as a by-product - can be used as usual T0 pricing engines if a corresponding engine builder is supplied, see Example 39 (Exposure Simulation using American Monte Carlo).

In addition the AMC pricing engines perform the necessary calculations to yield conditional NPVs on the given global simulation grid. How these calculations are performed is completely the responsibility of the pricing engines, although some common framework for many trade types is given by a base engine, see 2.3.2. This way the approximation of conditional NPVs on the simulation grid can be tailored to each product and also each single trade, with regards to

1. the number of training paths and the required date grid for the training (e.g. containing all relevant coupon and exercise event dates of a trade)
2. the order and type of regression basis functions to be used

3. the choice of the regressor (e.g. a TaRN might require a regressor augmented by the accumulated coupon amount)

The AMC pricing engines then provide an additional result labelled `amcCalculator` which is a class implementing the `AmcCalculator` interface which consists of two methods: The method `simulatePath()` takes a `MultiPath` instance representing one simulated path from the global risk factor evolution model and returns an array of conditional, deflated NPVs for this path. The method `npvCurrency()` returns the currency c of the calculated conditional NPVs. This currency can be different from the base currency b of the global risk factor evolution model. In this case the conditional NPVs are converted to the global base currency within the AMC valuation engine by multiplying them with the conversion factor

$$\frac{N_c(t)X_{c,b}(t)}{N_b(t)} \tag{2.12}$$

where t is the simulation time, $N_c(t)$ is the numeraire in currency c, $N_b(t)$ is the numeraire in currency b and $X_{c,b}(t)$ is the FX rate at time t converting from c to b.

The technical criterion for a trade to be processed within the AMC valuation is engine is that a) it can be built against the AMC engine factory described above and b) it provides an additional result `amcCalculator`. If a trade does not meet these criteria it is simulated using the classic valuation engine. The logic that does this is located in the override of the method `XvaAnalyticImpl::runAnalytic()`.

The AMC valuation engine can also populate an aggregation scenario data instance. This is done only if necessary, i.e. only if no classic simulation is performed anyway. The numeraire and fx spot values produced by the AMC valuation engine are identical to the classic engine. Index fixings are close, but not identical, because the AMC engine used the T0 curves for projection while the classic engine uses scenario simulation market curves, which are not exactly matching those of the T0 market. In this sense the AMC valuation engine produces more precise values compared to the classic engine.

2.3.2 The multileg option AMC base engine and derived engines

Example 39 (Exposure Simulation using American Monte Carlo) provides an overview of the implemented AMC engine builders. These builders use the following QuantExt pricing engines

1. `McLgmSwapEngine` for single currency swaps
2. `McCamCurrencySwapEngine` for cross currency swaps
3. `McCamFxOptionEngine` for fx options
4. `McLgmSwaptionEngine` for Bermudan swaptions
5. `McMultiLegOptionEngine` for Multileg option

All these engine are based on a common McMultiLegBaseEngine which does all the computations. For this each of the engines sets up the following protected member variables (serving as parameters for the base engine) in their calculate() method:

1. leg_: a vector of QuantLib::Leg
2. currency_: a vector of QuantLib::Currency corresponding to the leg vector
3. payer_: a vector of +1.0 or −1.0 double values indicating receiver or payer legs
4. exercise_: a QuantLib::Exercise instance describing the exercise dates (may be nullptr, if the underlying represents the deal already)
5. optionSettlement_: a Settlement::Type value indicating whether the option is settled physically or in cash

A call to McMultiLegBaseEngine::calculate() will set the result member variables

1. resultValue_: T0 NPV in the base currency of the cross asset model passed to the pricing engine
2. underlyingValue_: T0 NPV of the underlying (again in base ccy)
3. *amcCalculator_: the AMC calculator engine to be used in the AMC valuation engine

The specific engine implementations should convert the resultValue_ to the npv currency of the trade (as defined by the (ORE) trade builder) so that they can be used as regular pricing engine consistently within ORE. Note that only the additional amcCalculator result is used by the AMC valuation engine, not any of the T0 NPVs directly.

2.3.3 Limitations and Open Points

This sections lists known limitations of the AMC simulation engine.

Trade Features

Some trade features are not yet supported by the multileg option engine:

1. exercise flows (like a notional exchange common to cross currency swaptions) are not supported

Flows Generation (for DIM Analysis)

At the current stage the AMC engine does not generate flows which are required for the DIM analysis in the post processor.

State interpolation for exercise decisions

During the simulation phase exercise times of a specific trade are not necessarily part of the simulated time grid. Therefore the model state required to take the exercise decision has in to be interpolated in general on the simulated path. Currently this is done using a simple linear interpolation while from a pure methodology point of view a

Brownian Bridge would be preferable. In our tests we do not see a big impact of this approximation though.

Basis Function Selection

Currently the basis function system is generated by specifying the type of the functions and the order, see Example 39 (Exposure Simulation using American Monte Carlo). The number of independent variables varies by product type and details. Depending on the number of independent variables and the order the number of generated basis functions can get quite big which slows down the computation of regression coefficients. It would be desirable to have the option to filter the full set of basis functions, e.g. by explicitly enumerating them in the configuration, so that a high order can be chosen even for products with a relatively large number of independent variables (like e.g. FX Options or Cross Currency Swaps).

2.3.4 Outlook: Trade Compression

For vanilla trades where the regression is only required to produce the NPV cube entries (and not to take exercise decisions etc.) it is not strictly necessary to do the regression analysis on a single trade level[4]. Although in the current implementation there is no direct way to do the regression analysis on whole (sub-)portfolios instead of single trades, one can represent such a subportfolio as a single technical trade (e.g. as a single swap or multileg option trade) to achieve a similar result. This might lead to better performance than the usual single trade calculation. However one should also try to keep the regressions as low-dimensional as possible (for performance and accuracy reasons) and therefore define the sub-portfolios by e.g. currency, i.e. as big as possible while at the same time keeping the associated model dimension as small as possible.

[4]except single trade exposures are explicitly required of course

Chapter 3

Value Adjustments

3.1 CVA and DVA

Using the expected exposures in 2.2 unilateral discretised CVA and DVA are given by [14]

$$CVA = \sum_i PD(t_{i-1}, t_i) \times LGD \times EPE(t_i) \tag{3.1}$$

$$DVA = \sum_i PD_{Bank}(t_{i-1}, t_i) \times LGD_{Bank} \times ENE(t_i) \tag{3.2}$$

where

$$\begin{aligned} EPE(t) &\text{ expected exposure (2.5)} \\ ENE(t) &\text{ expected negative exposure (2.6)} \\ PD(t_i, t_j) &\text{ counterparty probability of default in } [t_i; t_j] \\ PD_{Bank}(t_i, t_j) &\text{ our probability of default in } [t_i; t_j] \\ LGD &\text{ counterparty loss given default} \\ LGD_{Bank} &\text{ our loss given default} \end{aligned}$$

Note that the choice t_i in the arguments of $EPE(t_i)$ and $ENE(t_i)$ means we are choosing the *advanced* rather than the *postponed* discretization of the CVA/DVA integral [8]. This choice can be easily changed in the ORE source code or made configurable.

Moreover, formulas (3.1, 3.2) assume independence of credit and other market risk factors, so that PD and LGD factors are outside the expectations. With the extension of ORE to credit asset classes and in particular for wrong-way-risk analysis, CVA/DVA formulas is generalised and is applicable to calculations with dynamic credit

$$CVA^{dyn} = \sum_i \mathbb{E}^N \left[\frac{PD^{dyn}(t_{i-1}, t_i) \times PE(t_i)}{N(t)} \right] \times LGD \tag{3.3}$$

$$DVA^{dyn} = \sum_i \mathbb{E}^N \left[\frac{PD^{dyn}_{Bank}(t_{i-1}, t_i) \times NE(t_i)}{N(t)} \right] \times LGD_{Bank} \tag{3.4}$$

where

$PE(t)$ random variables representing positive exposure at $t : (NPV(t) - C(t))^+$
$NE(t)$ random variables representing negative exposure at $t : (-NPV(t) + C(t))^+$
$PD^{dyn}(t_i, t_j)$ random variables representing counterparty probability of default in $[t_i; t_j]$
$PD^{dyn}_{Bank}(t_i, t_j)$ random variables representing our probability of default in $[t_i; t_j]$
LGD counterparty loss given default
LGD_{Bank} our loss given default

3.2 FVA

Any exposure (uncollateralised or residual after taking collateral into account) gives rise to funding cost or benefits depending on the sign of the residual position. This can be expressed as a Funding Value Adjustment (FVA). A simple definition of FVA can be given in a very similar fashion as the sum of unilateral CVA and DVA which we defined by (3.1,3.2), namely as an expectation of exposures times funding spreads:

$$\begin{aligned} FVA &= \underbrace{\sum_{i=1}^{n} f_l(t_{i-1}, t_i)\, \delta_i\, \mathbb{E}^N \left\{ S_C(t_{i-1})\, S_B(t_{i-1})\, [-NPV(t_i) + C(t_i)]^+\, D(t_i) \right\}}_{\text{Funding Benefit Adjustment (FBA)}} \\ &\quad - \underbrace{\sum_{i=1}^{n} f_b(t_{i-1}, t_i)\, \delta_i\, \mathbb{E}^N \left\{ S_C(t_{i-1})\, S_B(t_{i-1})\, [NPV(t_i) - C(t_i)]^+\, D(t_i) \right\}}_{\text{Funding Cost Adjustment (FCA)}} \qquad (3.5) \end{aligned}$$

where

$D(t_i)$ stochastic discount factor, $1/N(t_i)$ in LGM
$NPV(t_i)$ portfolio value at time t_i
$C(t_i)$Collateral account balance at time t_i
$S_C(t_j)$ survival probability of the counterparty
$S_B(t_j)$ survival probability of the bank
$f_b(t_j)$ borrowing spread for the bank relative to OIS flat
$f_l(t_j)$ lending spread for the bank relative to OIS flat

For details see e.g. Chapter 14 in Gregory [11] and the discussion in [14].

The reasoning leading to the expression above is as follows. Consider, for example, a single partially collateralised derivative (no collateral at all or CSA with a significant threshold) between us (the Bank) and counterparty 1 (trade 1).

We assume that we enter into an offsetting trade with (hypothetical) counterparty 2 which is perfectly collateralised (trade 2). We label the NPV of trade 1 and 2 $NPV_{1,2}$ respectively (from our perspective, excluding CVA). Then $NPV_2 = -NPV_1$. The respective collateral amounts due to trade 1 and 2 are C_1 and C_2 from our perspective.

Because of the perfect collateralisation of trade 2 we assume $C_2 = NPV_2$. The imperfect collateralisation of trade 1 means $C_1 \neq NPV_1$. The net collateral balance from our perspective is then $C = C_1 + C_2$ which can be written $C = C_1 + C_2 = C_1 + NPV_2 = -NPV_1 + C_1$.

- If $C > 0$ we receive net collateral and pay the overnight rate on this notional amount. On the other hand we can invest the received collateral and earn our lending rate, so that we have a benefit proportional to the lending spread f_l (lending rate minus overnight rate). It is a benefit assuming $f_l > 0$. $C > 0$ means $-NPV_1 + C_1 > 0$ so that we can cover this case with "lending notional" $[-NPV_1 + C_1]^+$.
- If $C < 0$ we post collateral amount $-C$ and receive the overnight rate on this amount. Amount $-C$ needs to be funded in the market, and we pay our borrowing rate on it. This leads to a funding cost proportional to the borrowing spread f_b (borrowing rate minus overnight). $C < 0$ means $NPV_1 - C_1 > 0$, so that we can cover this case with "borrowing notional" $[NPV_1 - C_1]^+$. If the borrowing spread is positive, this term proportional to $f_b \times [NPV_1 - C_1]^+$ is indeed a cost and therefore needs to be subtracted from the benefit above.

Formula (3.5) evaluates these funding cost components on the basis of the original trade's or portfolio's NPV. Perfectly collateralised portfolios hence do not contribute to FVA because under the hedging fiction, they are hedged with a perfectly collateralised opposite portfolio, so any collateral payments on portfolio 1 are cancelled out by those of the opposite sign on portfolio 2.

3.3 COLVA

When the CSA defines a collateral compounding rate that deviates from the overnight rate, this gives rise to another value adjustment labeled COLVA [14]. In the simplest case the deviation is just given by a constant spread Δ:

$$COLVA = \mathbb{E}^N \left[\sum_i -C(t_i) \cdot \Delta \cdot \delta_i \cdot D(t_{i+1}) \right] \tag{3.6}$$

where $C(t)$ is the collateral balance[1] at time t and $D(t)$ is the stochastic discount factor $1/N(t)$ in LGM. Both $C(t)$ and $N(t)$ are computed in ORE's Monte Carlo framework, and the expectation yields the desired adjustment.

Replacing the constant spread by a time-dependent deterministic function in ORE is straight forward.

3.4 Collateral Floor Value

A less trivial extension of the simple COLVA calculation above, also covered in ORE, is the case where the deviation between overnight rate and collateral rate is stochastic itself. A popular example is a CSA under which the collateral rate is the overnight rate *floored at zero*. To work out the value of this CSA feature one can take the difference

[1] see 2.2, $C(t) > 0$ means that we have *received* collateral from the counterparty

of discounted margin cash flows with and without the floor feature. It is shown in [14] that the following formula is a good approximation to the collateral floor value

$$\Pi_{Floor} = \mathbb{E}^N \left[\sum_i -C(t_i) \cdot (-r(t_i))^+ \cdot \delta_i \cdot D(t_{i+1}) \right] \tag{3.7}$$

where r is the stochastic overnight rate and $(-r)^+ = r^+ - r$ is the difference between floored and 'un-floored' compounding rate.
Taking both collateral spread and floor into account, the value adjustment is

$$\Pi_{Floor,\Delta} = \mathbb{E}^N \left[\sum_i -C(t_i) \cdot ((r(t_i) - \Delta)^+ - r(t_i)) \cdot \delta_i \cdot D(t_{i+1}) \right] \tag{3.8}$$

3.5 Dynamic Initial Margin and MVA

The introduction of Initial Margin posting in non-cleared OTC derivatives business reduces residual credit exposures and the associated value adjustments, **CVA** and **DVA**.

On the other hand, it gives rise to additional funding cost. The value of the latter is referred to as Margin Value Adjustment (**MVA**).
To quantify these two effects one needs to model Initial Margin under future market scenarios, i.e. Dynamic Initial Margin (**DIM**). Potential approaches comprise

- Monte Carlo VaR embedded into the Monte Carlo simulation
- Regression-based methods
- Delta VaR under scenarios
- ISDA's Standard Initial Margin (SIMM) under scenarios

We skip the first option as too computationally expensive for ORE.

3.5.1 Regression Approach

In ORE releases up to version 12 we have focussed on a relatively simple regression approach as in [15, 19]. Consider the netting set values $NPV(t)$ and $NPV(t+\Delta)$ that are spaced one margin period of risk Δ apart. Moreover, let $F(t, t+\Delta)$ denote cumulative netting set cash flows between time t and $t+\Delta$, converted into the NPV currency. Let $X(t)$ then denote the netting set value change during the margin period of risk excluding cash flows in that period:

$$X(t) = NPV(t+\Delta) + F(t, t+\Delta) - NPV(t)$$

ignoring discounting/compounding over the margin period of risk. We actually want to determine the distribution of $X(t)$ conditional on the 'state of the world' at time t, and pick a high (99%) quantile to determine the Initial Margin amount for each time t.
Instead of working out the distribution, we content ourselves with estimating the conditional variance $\mathbb{V}(t)$ or standard deviation $S(t)$ of $X(t)$, assuming a normal distribution and scaling $S(t)$ to the desired 99% quantile by multiplying with the usual factor $\alpha = 2.33$ to get an estimate of the Dynamic Initial Margin DIM:

$$\mathbb{V}(t) = \mathbb{E}_t[X^2] - \mathbb{E}_t^2[X], \qquad S(t) = \sqrt{\mathbb{V}(t)}, \qquad DIM(t) = \alpha\, S(t)$$

We further assume that $\mathbb{E}_t[X]$ is small enough to set it to the expected value of $X(t)$ across all Monte Carlo samples X at time t (rather than estimating a scenario dependent mean). The remaining task is then to estimate the conditional expectation $\mathbb{E}_t[X^2]$. We do this in the spirit of the Longstaff Schwartz method using regression of $X^2(t)$ across all Monte Carlo samples at a given time. As a regressor (in the one-dimensional case) we could use $NPV(t)$ itself. However, we rather choose to use an adequate market point (interest rate, FX spot rate) as regression variable x, because this is generalised more easily to the multi-dimensional case. As regression basis functions we use polynomials, i.e. regression functions of the form $c_0 + c_1\,x + c_2\,x^2 + ... + c_n\,x^n$ where the order n of the polynomial can be selected by the user. Choosing the lowest order $n = 0$, we obtain the simplest possible estimate, the variance of X across all samples at time t, so that we apply a single $DIM(t)$ irrespective of the 'state of the world' at time t in that case. The extension to multi-dimensional regression is also implemented in ORE. The user can choose several regressors simultaneously (e.g. a EUR rate, a USD rate, USD/EUR spot FX rate, etc.) in order order to cover complex multi-currency portfolios.

Given the DIM estimate along all paths, we can next work out the Margin Value Adjustment [14] in discrete form

$$MVA = \sum_{i=1}^{n} (f_b - s_I)\,\delta_i\,S_C(t_i)\,S_B(t_i) \times \mathbb{E}^N\left[DIM(t_i)\,D(t_i)\right]. \tag{3.9}$$

with borrowing spread f_b as in the FVA section 3.2 and spread s_I received on initial margin, both spreads relative to the cash collateral rate.

3.5.2 VaR under Scenarios: Dynamic Parametric VaR

Because of the limitations of the regression approach, it needs benchmarking/validation. In [19] we have applied a dynamic parametric VaR method for that purpose covering

- Delta VaR
- Delta Gamma Normal VaR
- Delta Gamma VaR (Cornish-Fisher)

This has been added to ORE with release 13 and is implemented for the small range of products that are discussed in [19], i.e.

- Swaps
- Cross Currency Swaps
- European Swaptions
- FX Forwards
- FX Options

where relevant sensitivities can be computed analytically under scenarios which feed into the parametric VaR calculation. The covariance structure of the VaR model is implied from the calibrated cross asset model (rather than externally provided), because the primary motivation of the method was benchmarking of the regression approach, in particular to check the performance of regression methods in option portfolios.

The usage of Dynamic Parametric VaR as Initial Margin proxy is demonstrated in Example 13 (Dynamic Initial Margin and MVA), compared to Regression IM.

3.6 KVA

3.6.1 CCR

The KVA is calculated for the Counterparty Credit Risk Capital charge (CCR) following the IRB method concisely described in [12], Appendix 8A. It is following the Basel rules by computing risk capital as the product of alpha weighted exposure at default, worst case probability of default at 99.9 and a maturity adjustment factor also described in the Basel annex 4. The risk capital charges are discounted with a capital discount factor and summed up to give the total CCR KVA after being multiplied with the risk weight and a capital charge (following the RWA method).

Basel II internal rating based (IRB) estimate of worst case probability of default: large homogeneous pool (LHP) approximation of Vasicek (1997), KVA regulatory probability of default is the worst case probability of default floored at 0.03 (the latter is valid for corporates and banks, no such floor applies to sovereign counterparties):

$$PD_{99.9\%} = \max\left(floor, N\left(\frac{N^{-1}(PD) + \sqrt{\rho}N^{-1}(0.999)}{\sqrt{1-\rho}}\right) - PD\right)$$

N is the cumulative standard normal distribution,

$$\rho = 0.12\frac{1-e^{-50PD}}{1-e^{-50}} + 0.24\left(1 - \frac{1-e^{-50PD}}{1-e^{-50}}\right)$$

Maturity adjustment factor for RWA method capped at 5, floored at 1:

$$MA(PD, M) = \min\left(5, \max\left(1, \frac{1+(M-2.5)B(PD)}{1-1.5B(PD)}\right)\right)$$

where $B(PD) = (0.11852 - 0.05478\ln(PD))^2$ and M is the effective maturity of the portfolio (capped at 5):

$$M = \min\left(5, 1 + \frac{\sum\limits_{t_k > 1yr} EE_B(t_k)\Delta t_k B(0,t_k)}{\sum\limits_{t_k \le 1yr} EEE_B(t_k)\Delta t_k B(0,t_k)}\right)$$

where $B(0, t_k)$ is the risk-free discount factor from the simulation date t_k to today, Δt_k is the difference between time points, $EE_B(t_k)$ is the expected (Basel) exposure at time t_k and $EEE_B(t_k)$ is the associated effective expected exposure.

Expected risk capital at t_i:

$$RC(t_i) = EAD(t_i) \times LGD \times PD_{99.9\%} \times MA(PD, M)$$

where

- $EAD(t_i) = \alpha \times EEPE(t_i)$
- $EEPE(t_i)$ is estimated as the time average of the running maximum of $EPE(t)$ over the time interval $t_i \leq t \leq t_i + 1$
- α is the multiplier resulting from the IRB calculations (Basel II defines a supervisory alpha of 1.4, but gives banks the option to estimate their own α,subject to a floor of 1.2).
- the maturity adjustment MA is derived from the EPE profile for times $t \geq t_i$

KVA_{CCR} is the sum of the expected risk capital amount discounted at *capital discount rate* r_{cd} and compounded at rate given by the product of *capital hurdle* h and *regulatory adjustment* a:

$$KVA_{CCR} = \sum_i RC(t_i) \times \frac{1}{(1 + r_{cd})^{\delta(t_{i-1}, t_i)}} \times \delta(t_{i-1}, t_i) \times h \times a$$

assuming Actual/Actual day count to compute the year factions δ.

In ORE we compute KVA CCR from both perspectives - "our" KVA driven by EPE and the counterparty default risk, and similarly "their" KVA driven by ENE and our default risk.

3.6.2 BA-CVA

This section briefly summarizes the calculation of a capital value adjustment associated with the CVA capital charge (in the basic approach, BA-CVA) as introduced in Basel III [3, 5, 6]. ORE implements the *stand-alone* capital charge $SCVA$ for a netting set and computes a KVA for it[2]. In the basic approach, the stand-alone capital charge for a netting set is given by

$$SCVA = RW_c \cdot M \cdot EEPE \cdot DF$$

with

- supervisory risk weight RW_c for the counterparty;
- effective netting set maturity M as in section 3.6 (for a bank using IMM to calculate EAD), but without applying a cap of 5;

[2]In the reduced version of BA-CVA, where hedges are not recognized, the total BA-CVA capital charge across all counterparties c is given by

$$K = \sqrt{\left(\rho \sum_c SCVA_c\right)^2 + (1 - \rho^2) \sum_c SCVA_c^2}$$

with supervisory correlation $\rho = 0.5$ to reflect that the credit spread risk factors across counterparties are not perfectly correlated. Each counterparty $SCVA_c$ is given by a sum over all netting sets with this counterparty.

- supervisory discount DF for the netting set which is equal to one for banks using IMM to calculate $EEPE$ and $DF = (1 - \exp(-0.05\,M))/(0.05\,M)$ for banks not using IMM to calculate $EEPE$.

The associated capital value adjustment is then computed for each netting set's stand-alone CVA charge as above

$$KVA_{BA-CVA} = \sum_i SCVA(t_i) \times \frac{1}{(1+r_{cd})^{\delta(t_{i-1},t_i)}} \times \delta(t_{i-1}, t_i) \times h \times a$$

with

$$SCVA(t_i) = RW_c \cdot M(t_i) \cdot EEPE(t_i) \cdot DF$$

where we derive both M and EEPE from the EPE profile for times $t \geq t_i$.

In ORE we compute KVA BA-CVA from both perspectives - "our" KVA driven by EPE and the counterparty risk weight, and similarly "their" KVA driven by ENE and our risk weight.
Note: Banks that use the BA-CVA for calculating CVA capital requirements are allowed to cap the maturity adjustment factor $MA(PD, M)$ in section 3.6 at 1 for netting sets that contribute to CVA capital, if using the IRB approach for CCR capital.

3.7 Collateral (Variation Margin) Model

The collateral model implemented in ORE is based on the evolution of collateral account balances along each Monte Carlo path taking into account thresholds, minimum transfer amounts and independent amounts defined in the CSA, as well as margin periods of risk.

ORE computes the collateral requirement (aka *Credit Support Amount*) through time along each Monte Carlo path

$$CSA(t_m) = \begin{cases} \max(0, NPV(t_m) + IA - TH_{rec}), & NPV(t_m) + IA \geq 0 \\ \min(0, NPV(t_m) + IA + TH_{pay}), & NPV(t_m) + IA < 0 \end{cases} \tag{3.10}$$

where

- $NPV(t_m)$ is the value of the netting set as of time t_m from our perspective,
- TH_{rec} is the threshold exposure below which we do not require collateral, likewise $_{pay}$ is the threshold that applies to collateral posted to the counterparty,
- IA is the sum of all collateral independent amounts attached to the underlying portfolio of trades (positive amounts imply that we have received a net inflow of independent amounts from the counterparty), assumed here to be cash.

As the collateral account already has a value of $C(t_m)$ at time t_m, the collateral shortfall is simply the difference between $C(t_m)$ and $CSA(t_m)$. However, we also need to account for the possibility that margin calls issued in the past have not yet been settled (for instance, because of disputes). If $M(t_m)$ denotes the net value of all outstanding margin calls at t_m, and $\Delta(t)$ is the difference

$$\Delta(t) = CSA(t_m) - C(t_m) - M(t_m)$$

between the *Credit Support Amount* and the current and outstanding collateral, then the actual margin *Delivery Amount* $D(t_m)$ is calculated as follows:

$$D(t_m) = \begin{cases} \Delta(t), & |\Delta(t)| \geq MTA \\ 0, & |\Delta(t)| < MTA \end{cases} \tag{3.11}$$

where MTA is the minimum transfer amount.

Consider the upper case of (3.10): If the initial value of the netting set is zero ($NPV(t_0) = 0$) and if $TH_{rec} = 0$, but the combined $IA > 0$, then the Credit Support Amount equals the Independent Amount, $CSA(t_0) = IA$. If moreover the initial collateral balance is zero (because the Independent Amount has not been received yet), then $\Delta(t_0) = CSA(t_0) = IA$, and the delivery amount $D(t_0)$ also matches the IA (assuming this exceeds the MTA), so that the next call leads to the transfer of the Independent Amount to us. For a positive $TH_{rec} > 0$, the transfer to us is reduced accordingly. In that case we can view the Independent Amount as an offset to the threshold.

Consider the lower case of (3.10): If the netting set value is negative from our perspective and in absolute terms larger than the IA, then the Credit Support Amount is just the negative difference $CSA = -|NPV| + IA + TH_{pay}$ so that we need to post collateral, but only the amount beyond the combined threshold $IA + TH_{pay}$.

Margin Period of Risk

After a counterparty defaults, it takes time to close out the portfolio. During this time period the portfolio value will change upon market conditions, therefore the portfolio's close-out value is subject to market risk, which is referred also as the close-out risk and the corresponding close-out period is called as the *Margin Period of Risk* (MPoR).

Therefore, when a loss on the defaulted counterparty is realised at time t_d, the last time the collateral could be received is $t_d - \tau$, where τ denotes the MPoR. That is, the collateral at time t_d is determined by the collateral value at $t_d - \tau$, namely $CSA(t_d - \tau)$, see equation 3.10.

In ORE, we have two approaches to incorporate MPoR in the exposure simulations:

- *Close-out Approach*: Simulating on an auxiliary close-out grid additional to the default time grid.
- *Lagged Approach*: Simulating only on a default time grid and delaying the margin calls on the grid.

In the *Close-out Approach*, we use an auxiliary "close-out" grid in addition to the main simulation grid (see the user guide's simulation parameterisation section). The main simulation grid is used to compute "default values" which feed into the collateral balance $C(t)$ filtered by MTA and Threshold etc. The auxiliary "close-out" grid, offset from the main grid by the MPoR, is used to compute the delayed close-out values $V(t)$ associated with default time t[3]. The difference between $V(t)$ and $C(t)$ causes a residual

[3] We note that in ORE when the exposure of an uncollateralised netting-set or a single trade without considering the netting-set is calculated, then the default value is calculated at the main simulation grid, not on the close-out grid.

exposure $[V(t) - C(t)]^+$ even if minimum transfer amounts and thresholds are zero, see for example [9]. This approach allows a detailed modelling of what happens in the close-out period by calculating the close-out values in different ways. ORE currently supports two options:

- the close-out value can be computed as of default date, by just evolving the market from default date to close-out date ("sticky date"), or
- the close-out value can be computed as of close-out date, by evolving both valuation date and market over the close-out period ("actual date"), i.e., the portfolio ages and cash flows might occur in the close-out period causing spikes in the evolution of exposures.

The option "sticky date" is more aggressive in that it avoids any exposure evolution spikes due to contractual cashflows that occur in the close-out period after default, the only exposure effect is due to market evolution over the period. The "actual date" option is more conservative in that it includes the effect of all contractual cash flows in the close-out period, in particular outgoing cashflows at any time in the period which cause an exposure jump upwards. A more detailed framework for collateralised exposure modelling is introduced in the article [16], indicating a potential route for extending ORE.

On the other hand, in the *Lagged Approach* the simulation is conducted only on a default time grid. The collateral values are calculated, by delaying the delivery amounts between default times, specified by the *Margin Period of Risk* (MPoR) which leads to residual exposure.

In table 3.1, we present a toy example to illustrate how the delayed margin calls lead to residual exposures. In this example, we assume that the default time grid is equally-spaced with time steps that match the MPoR (which is 1M). Further, we assume zero threshold and MTA. At the initial time, the delivery amount is 2.00, which is the difference between the initial value of the portfolio and the default value at 1M. If this amount were settled immediately, then the collateral value would have been 10 and hence the residual exposure would have been zero at 1M. The delay of the delivery amount by MPoR implies a collateral value of 8.00 until 1M and hence a residual exposure of 2.

Time Grid	Default Value	Delivery Amount	Delivery Amount Delayed	Collateral Value	NPV
0	8.00	2.00	True		
1M	10.00	5.00	True	8.00	10.00
2M	15.00	-3.00	True	10.00	15.00
3M	12.00	-3.00	True	15.00	12.00
4M	9.00	5.00	True	12.00	9.00
5M	14.00	6.00	True	9.00	14.00
6M	20.00			14.00	20.00

Table 3.1: Toy example for delayed margin calls.

Some remarks and observations:

- *Lagged Approach* has the disadvantage that we need to use equally-spaced time grids with time steps that match the MPoR. In the above example, let us assume that the MPoR is 2W. Then, delaying the first delivery amount by 2W would still imply a collateral value of 10.00 at 1M and hence a zero residual exposure.
- In *Lagged Approach* approach, we support three calculation (settlement) types where the delay of the *Delivery Amount* depends on its sign. The above example corresponds to a "symmetric" calculation type where both positive and negative delivery amounts are settled with delay, see the user guide's Parameterisation section for other calculation types.
- In ORE, the *Close-out Approach* is the preferred method -and the *Lagged Approach* is the legacy method- to incorporate MPoR in the collateral model.

3.8 Exposure Allocation

XVAs and exposures are typically computed at netting set level. For accounting purposes it is typically required to *allocate* XVAs from netting set to individual trade level such that the allocated XVAs add up to the netting set XVA. This distribution is not trivial, since due to netting and imperfect correlation single trade (stand-alone) XVAs hardly ever add up to the netting set XVA: XVA is sub-additive similar to VaR. ORE provides an allocation method (labeled *marginal allocation* in the following) which slightly generalises the one proposed in [10]. Allocation is done pathwise which first leads to allocated expected exposures and then to allocated CVA/DVA by inserting these exposures into equations (3.1,3.2). The allocation algorithm in ORE is as follows:

- Consider the netting set's discounted NPV after taking collateral into account, on a given path at time t:

$$E(t) = D(0,t)\,(NPV(t) - C(t))$$

- On each path, compute contributions A_i of the latter to trade i as

$$A_i(t) = \begin{cases} E(t) \times NPV_i(t)/NPV(t), & |NPV(t)| > \epsilon \\ E(t)/n, & |NPV(t)| \leq \epsilon \end{cases}$$

  with number of trades n in the netting set and trade i's value $NPV_i(t)$.

- The EPE fraction allocated to trade i at time t by averaging over paths:

$$EPE_i(t) = \mathbb{E}\left[A_i^+(t)\right]$$

By construction, $\sum_i A_i(t) = E(t)$ and hence $\sum_i EPE_i(t) = EPE(t)$.
We introduced the *cutoff* parameter $\epsilon > 0$ above in order to handle the case where the netting set value $NPV(t)$ (almost) vanishes due to netting, while the netting set 'exposure' $E(t)$ does not. This is possible in a model with nonzero MTA and MPoR. Since a single scenario with vanishing $NPV(t)$ suffices to invalidate the expected exposure at this time t, the cutoff is essential. Despite introducing this cutoff, it is obvious that the marginal allocation method can lead to spikes in the allocated

exposures. And generally, the marginal allocation leads to both positive and negative EPE allocations.

As a an example for a simple alternative to the marginal allocation of EPE we provide allocation based on today's single-trade CVAs

$$w_i = CVA_i / \sum_i CVA_i.$$

This yields allocated exposures proportional to the netting set exposure, avoids spikes and negative EPE, but does not distinguish the 'direction' of each trade's contribution to EPE and CVA.

Chapter 4

Market Risk

4.1 Sensitivity Analysis

ORE's sensitivity analysis framework uses "bump and revalue" to compute Interest Rate, FX, Inflation, Equity and Credit sensitivities to

- Discount curves (in the zero rate domain)
- Index curves (in the zero rate domain)
- Yield curves including e.g. equity forecast yield curves (in the zero rate domain)
- FX Spots
- FX volatilities
- Swaption volatilities, ATM matrix or cube
- Cap/Floor volatility matrices (in the caplet/floorlet domain)
- Default probability curves (in the "zero rate" domain, expressing survival probabilities $S(t)$ in term of zero rates $z(t)$ via $S(t) = \exp(-z(t) \times t)$ with Actual/365 day counter)
- Equity spot prices
- Equity volatilities, ATM or including strike dimension
- Zero inflation curves
- Year-on-Year inflation curves
- CDS volatilities
- Base correlation curves

Apart from first order sensitivities (deltas), ORE computes second order sensitivities (gammas and cross gammas) as well. Deltas are computed using up-shifts and base values as

$$\delta = \frac{f(x + \Delta) - f(x)}{\Delta},$$

where the shift Δ can be absolute or expressed as a relative move Δ_r from the current level, $\Delta = x\,\Delta_r$. Gammas are computed using up- and down-shifts

$$\gamma = \frac{f(x+\Delta) + f(x-\Delta) - 2\,f(x)}{\Delta^2},$$

cross gammas using up-shifts and base values as

$$\gamma_{cross} = \frac{f(x+\Delta_x, y+\Delta_y) - f(x+\Delta_x, y) - f(x, y+\Delta_y) + f(x,y)}{\Delta_x\,\Delta_y}.$$

From the above it is clear that this involves the application of 1-d shifts (e.g. to discount zero curves) and 2-d shifts (e.g. to Swaption volatility matrices). The structure of the shift curves/matrices does not have to match the structure of the underlying data to be shifted, in particular the shift "curves/matrices" can be less granular than the market to be shifted. Figure 4.1 illustrates for the one-dimensional case how shifts are applied.

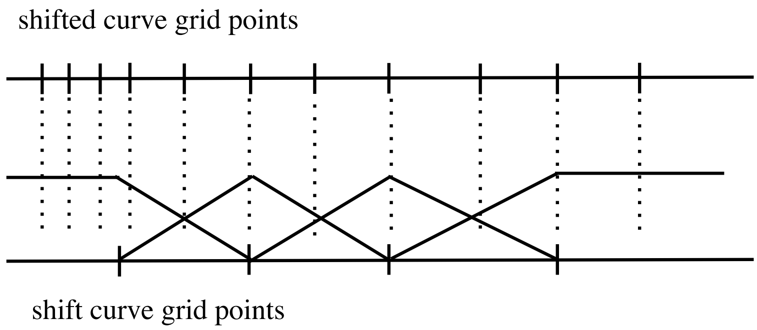

Figure 4.1: 1-d shift curve (bottom) applied to a more granular underlying curve (top).

Shifts at the left and right end of the shift curve are extrapolated flat, i.e. applied to all data of the original curve to the left and to the right of the shift curve ends. In between, all shifts are distributed linearly as indicated to the left and right up to the adjacent shift grid points. As a result, a parallel shift of the all points on the shift curve yields a parallel shift of all points on the underlying curve.
The two-dimensional case is covered in an analogous way, applying flat extrapolation at the boundaries and "pyramidal-shaped" linear interpolation for the bulk of the points.

The details of the computation of sensitivities to implied volatilities in strike direction can be summarised as follows, see also table 4.1 for an overview of the admissible configurations and the results that are obtained using them.

For *Swaption Volatilities*, the initial market setup can be an ATM surface only or a full cube. The simulation market can be set up to simulate ATM only or to simulate the full cube, but the latter choice is only possible if a full cube is set up in the initial market. The sensitivity set up must match the simulation setup with regards to the strikes (i.e. it is ATM only if and only if the simulation setup is ATM only, or it must contain exactly the same strike spreads relative to ATM as the simulation setup). Finally, if the initial market setup is a full cube, and the simulation / sensitivity setup is to simulate ATM only, then sensitivities are computed by shifting the ATM volatility w.r.t. the given shift size and type and shifting the non-ATM volatilities by the same absolute amount as the ATM volatility.

For *Cap/Floor Volatilities*, the initial market setup always contains a set of fixed strikes, i.e. there is no distinction between ATM only and a full surface. The same holds for the simulation market setup. The sensitivity setup may contain a different strike grid in this case than the simulation market. Sensitivity are computed per expiry and per strike in every case.

For *Equity Volatilities*, the initial market setup can be an ATM curve or a full surface. The simulation market can be set up to simulate ATM only or to simulate the full surface, where a full surface is allowed even if the initial market setup in an ATM curve only. If we have a full surface in the initial market and simulate the ATM curve only in the simulation market, sensitivities are computed as in the case of Swaption Volatilities, i.e. the ATM volatility is shifted w.r.t. the specified shift size and type and the non-ATM volatilities are shifted by the same absolute amount as the ATM volatility. If the simulation market is set up to simulate the full surface, then all volatilities are shifted individually using the specified shift size and type. In every case the sensitivities are aggregated on the ATM bucket in the sensitivity report.

For *FX Volatilities*, the treatment is similar to Equity Volatilities, except for the case of a full surface definition in the initial market and an ATM only curve in the simulation market. In this case, the pricing in the simulation market is using the ATM curve only, i.e. the initial market's smile structure is lost.

For *CDS Volatilities* only an ATM curve can be defined.

In all cases the smile dynamics is "sticky strike", i.e. the implied vol used for pricing a deal does not change if the underlying spot price changes.

Type	Init Mkt. Config.	Sim. Mkt Config.	Sensitivity Config.	Pricing	Sensitivities w.r.t.
Swaption	ATM	Simulate ATM only	Shift ATM only	ATM Curve	ATM Shifts
Swaption	Cube	Simulate Cube	Shift Smile Strikes	Full Cube	Smile Strike Shifts[a]
Swaption	Cube	Simulate ATM only	Shift ATM only	Full Cube	ATM Shifts[b]
Cap/Floor	Surface	Simulate Surface	Shift Smile Strikes	Full Surface	Smile Strike Shifts
Equity	ATM	Simulate ATM only	Shift ATM only	ATM Curve	ATM Shifts
Equity	ATM	Simulate Surface	Shift ATM only	ATM Curve	Smile Strike Shifts[c]
Equity	Surface	Simulate ATM only	Shift ATM only	Full Surface	ATM Shifts[b]
Equity	Surface	Simulate Surface	Shift ATM only	Full Surface	Smile Strike Shifts[c]
FX	ATM	Simulate ATM only	Shift ATM only	ATM Curve	ATM Shifts
FX	ATM	Simulate Surface	Shift ATM only	ATM Curve	Smile Strike Shifts[c]
FX	Surface	Simulate ATM only	Shift ATM only	ATM Curve	ATM Shifts
FX	Surface	Simulate Surface	Shift ATM only	Full Surface	Smile Strike Shifts[c]
CDS	ATM	Simulate ATM only	Shift ATM only	ATM Curve	ATM Shifts

Table 4.1: Admissible configurations for Sensitivity computation in ORE

[a]smile strike spreads must match simulation market configuration
[b]smile is shifted in parallel
[c]result sensitivities are aggregated on ATM

4.2 Par Sensitivity Analysis

The "raw" sensitivities in ORE are generated in a computationally convenient domain (such as zero rates, caplet/floorlet volatilities, integrated hazard rates, inflation zero

rates). These raw sensitivities are typically further processed in risk analytics such as VaR measures. On the other hand, for hedging purposes one is rather interested in sensitivities with respect to fair rates of hedge instruments such as Forward Rate Agreements, Swaps, flat Caps/Floors, CDS, Zero Coupon Inflation Swaps.
It is possible to generate par sensitivities from raw sensitivities using the chain rule as follows, and this is the approach taken in ORE. Recall for example the fair swap rate c for some maturity as a function of zero rates z_i in a single curve setting:

$$c = \frac{1 - e^{-z_n\,t_n}}{\sum_{i=1}^n \delta_i\, e^{-z_i\,t_i}}$$

More realistically, a given fair swap rate might be a function of the zero rates spanning the discount and index curves in the chosen currency. In a multi currency curve setting, that swap rate might even be a function of the zero rates spanning a foreign (collateral) currency discount curve, foreign and domestic currency index curves. Generally, we can write any fair par rate c_i as function of raw rates z_j,

$$c_i \equiv c_i(z_1, z_2, ..., z_n)$$

This function may not be available in closed form, but numerically we can evaluate the sensitivity of c_i with respect to changes in all raw rates,

$$\frac{\partial c_i}{\partial z_j}.$$

These sensitivities form a *Jacobi* matrix of derivatives. Now let V denote some trade's price. Its sensitivity with respect a raw rate change $\partial V/\partial z_k$ can then be expressed in terms of sensitivities w.r.t. par rates using the chain rule

$$\frac{\partial V}{\partial z_j} = \sum_{i=1}^n \frac{\partial V}{\partial c_i}\,\frac{\partial c_i}{\partial z_j},$$

or in vector/matrix form

$$\nabla_z V = C \cdot \nabla_c V, \qquad C_{ji} = \frac{\partial c_i}{\partial z_j}.$$

Given the raw sensitivity vector $\nabla_z V$, we need to invert the Jacobi matrix C to obtain the par rate sensitivity vector

$$\nabla_c V = C^{-1} \cdot \nabla_z V.$$

We then compute the Jacobi matrix C by

- setting up par instruments with links to all required term structures expressed in terms of raw rates
- "bumping" all relevant raw rates and numerically computing the par instrument's fair rate shift for each bump
- thus filling the Jacobi matrix with finite difference approximations of the partial derivatives $\partial c_i/\partial z_j$.

The par rate conversion supports the following par instruments:

- Deposits
- Forward rate Agreements
- Interest Rate Swaps (fixed vs. ibor)
- Overnight Index Swaps
- Tenor Basis Swaps (ibor vs. ibor)
- Overnight Index Basis Swaps (ibor vs. OIS)
- FX Forwards
- Cross Currency Basis Swaps
- Credit Default Swaps
- Caps/Floors

4.3 Economic P&L

The economic P&L of a portfolio denotes the change in its economic value over a time period t_1 to t_2. The economic value evolution during the period is due to three components

- the change in present value from period start to end
- incoming and outgoing cash flows
- accumulated cost of funding required to set up the portfolio initially

In the following, we consider a portfolio consisting of assets in various currencies. We decompose the portfolio into parts each denominated in a different currency and value each sub-portfolio in its currency. We denote the sub-portfolio values at time t in the respective currency $P_1(t), P_2(t), \ldots$. Instruments with cash flows in more than one currency are decomposed into single-currency instruments and assigned into the related sub-portfolio. The total portfolio value expressed in base currency (e.g. EUR) is

$$P(t) = \sum_c P_c(t)\, X_c(t) \tag{4.1}$$

where X_c is the exchange rate that converts an amount in currency c into an amount in base currency by multiplication. All prices $P_c(t)$ denote *dirty* market values (or theoretical values where market values are not available) at time t.

In the following we consider three points in time,

- t_0: the time just before the first actual cash flow has appeared in the portfolio under consideration, possibly years ago
- t_1: the beginning of the period for which we want to determine P&L
- t_2: the end of the period for which we want to determine P&L

Original P&L

The original P&L is the portfolio's P&L from portfolio inception t_0. In this case the portfolio value at t_0 is

$$P(t_0) = 0,$$

and the P&L up to time t_2 is given by the portfolio value at t_2 plus the balance of currency accounts that collect incoming and outgoing cash flows and are compounded up to time t_2:

$$\pi(0, t_2) = P(t_2) + \sum_c X_c(t_2)\, B_c(t_2) \tag{4.2}$$

where

$$B_c(t_2) = \sum_{j=0}^{I(t_2)-1} F_c(\tau_j)\, C_c(\tau_j, t_2), \quad C_c(\tau_j, t_2) = \prod_{k=I(\tau_j)}^{I(t_2)-1} (1 + r_c(\tau_k)\delta_k), \tag{4.3}$$

sums and products are taken over daily time steps τ_j and

$I(t)$	is the day's index associated with time t
$F_c(\tau_j)$	is the net cash flow in currency c on date/time τ_j, possibly zero
$r_c(\tau_j)$	is the Bank's overnight funding and investment rate in currency c for interest period $[\tau_j,\, \tau_{j+1}]$ (overnight)
δ_j	is the related day count fraction for period $[\tau_j,\, \tau_{j+1}]$

The balances B_c can also be constructed iteratively

$$\begin{aligned} B_c(\tau_{j+1}) &= B_c(\tau_j)(1 + r_c(\tau_j)\delta_j) + F_c(\tau_{j+1}) \\ j &= 0, 1, 2, \ldots \\ B_c(\tau_0) &= 0. \end{aligned}$$

The P&L for a period of interest $[t_1;\, t_2]$ is then computed by taking the difference

$$\begin{aligned} \pi(t_1, t_2) &= \pi(0, t_2) - \pi(0, t_1) \\ &= P(t_2) - P(t_1) + \sum_c (X_c(t_2)\, B_c(t_2) - X_c(t_1)\, B_c(t_1)) \end{aligned} \tag{4.4}$$

One can show that

$$B_c(t_2) = B_c(t_1)\, C_c(t_1, t_2) + \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\, C_c(\tau_j, t_2) \tag{4.5}$$

which separates the contribution to $B_c(t_2)$ from cash flows in period $[t_1; t_2]$ (right-most sum) and contributions from realized P&L and cost of funding of previous periods accumulated in $B_c(t_1)$. We can now insert (4.5) into (4.4) to eliminate $B_c(t_2)$ and obtain

$$\begin{aligned} \pi(t_1, t_2) &= P(t_2) - P(t_1) + \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\, C_c(\tau_j, t_2) \\ &\quad + \sum_c B_c(t_1)\, \{X_c(t_2)\, C_c(t_1, t_2) - X_c(t_1)\} \end{aligned} \tag{4.6}$$

Cost of Carry

Separating actual cash flows and prices from compounding effects yields

$$\pi(t_1,t_2) = P(t_2) - P(t_1) + \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j) + CC(t_1,t_2)$$

where the cost of carry term is

$$\begin{aligned} CC(t_1,t_2) &= \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\ \left(C_c(\tau_j,t_2) - 1\right) \\ &\quad + \sum_c B_c(t_1)\ \left\{X_c(t_2)\, C_c(t_1,t_2) - X_c(t_1)\right\} \end{aligned} \tag{4.7}$$

Period P&L after Sell-Down

At time t_1, we can write the original P&L (equation 4.2) in respective currencies

$$\pi_c(t_1) = P_c(t_1) + B_c(t_1), \qquad \pi(t_1) = \sum_c X_c(t_1)\, \pi_c(t_1).$$

Inserting this into (4.7),

$$\begin{aligned} CC(t_1,t_2) &= \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\ \left(C_c(\tau_j,t_2) - 1\right) \\ &\quad + \sum_c \left(\pi_c(t_1) - P_c(t_1)\right)\ \left\{X_c(t_2)\, C_c(t_1,t_2) - X_c(t_1)\right\}, \end{aligned}$$

shows that there is a contribution to $\pi(t_1,t_2)$, via the cost of carry, due to compounding and FX effects on previous periods' P&L result.

We now take the view that the portfolio is liquidated at time t_1, so that the account balance equals the P&L at t_1. We further assume that this balance is then removed ("sell down" of P&L) and transferred into a separate portfolio, the Bank's equity[1]. The same portfolio is thereafter set up again so that the currency account balance turns into a liability $B_c(t_1) = -P_c(t_1)$, and the total starting balance is $B(t_1) = -P(t_1)$. In contrast to the previous section, this changes the balance at time t_1 suddenly and without relation to an actual cash flow.

This raises the question how the artificial initial balance is funded subsequently, in currency for each sub-portfolio or in base currency only. This choice may vary by portfolio, depend on the actual currencies in which the Bank can source funding, depend on the location/economy in which the portfolio is run, which currency is a reasonable benchmark, etc.

[1] Equity is in turn managed and most likely invested into financial instruments other than a Bank account

Funding in Currency

In this section we take the view that each sub-portfolio is funded in currency so that we start with opening balances $B_c(t_1) = -P_c(t_1)$.

Inserting the artificial opening balances at t_1 into (4.6) yields

$$\begin{aligned}
\pi_2(t_1,t_2) &= P(t_2) - P(t_1) + \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\, C_c(\tau_j,t_2) \\
&\quad - \sum_c P_c(t_1)\ \{X_c(t_2)\, C_c(t_1,t_2) - X_c(t_1)\} \\
&= P(t_2) + \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\, C_c(\tau_j,t_2) \\
&\quad - \sum_c P_c(t_1)\, X_c(t_2)\, C_c(t_1,t_2)
\end{aligned} \tag{4.8}$$

Note that only exchange rates at t_2 enter into the expression.

Cost of Carry

Separating actual cash flows and prices from compounding effects yields

$$\pi_2(t_1,t_2) = P(t_2) + \sum_c X_c(t_2) \left\{ -P_c(t_1) + \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j) \right\} + CC_2(t_1,t_2)$$

where the cost of carry term is

$$\begin{aligned}
CC_2(t_1,t_2) &= \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\ (C_c(\tau_j,t_2) - 1) \\
&\quad - \sum_c X_c(t_2)\, P_c(t_1)\ (C_c(t_1,t_2) - 1)
\end{aligned}$$

Funding in Base Currency

In this section we assume that the setup cost for the portfolio is converted into base currency at t_1 and funded subsequently in base currency. This means we insert artificial initial balances $B_c(t_1) = 0$ except for the base currency account $B(t_1) = -P(t_1)$. Inserting this opening balance at t_1 into (4.6) now yields

$$\begin{aligned}
\pi_3(t_1,t_2) &= P(t_2) + \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\, C_c(\tau_j,t_2) \\
&\quad - P(t_1)\, C(t_1,t_2)
\end{aligned} \tag{4.9}$$

where $C(t_1,t_2)$ is the compounding factor in base currency.

Cost of Carry

Separating actual cash flows and prices from compounding effects yields now

$$\pi_3(t_1,t_2) = P(t_2) - P(t_1) + \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j) + CC_3(t_1,t_2)$$

where

$$\begin{aligned} CC_3(t_1,t_2) &= \sum_c X_c(t_2) \sum_{j=I(t_1)}^{I(t_2)-1} F_c(\tau_j)\,(C_c(\tau_j,t_2)-1) \\ &\quad -P(t_1)\,(C(t_1,t_2)-1) \end{aligned}$$

FX Effect

The difference between (4.8) and (4.2) is

$$\pi_2(t_1,t_2) - \pi_3(t_1,t_2) = P(t_1)\,C(t_1,t_2) - \sum_c P_c(t_1)\,X_c(t_2)\,C_c(t_1,t_2).$$

The expected value of this difference at period start t_1 is zero, but the retrospectively realized difference at period end t_2 is nonzero in general.

4.4 Risk Hypothetical P&L

In the following we briefly describe approaches to generating P&L vectors that feed into several subsequent sections for the purpose of computing risk measures such as Value at Risk or for backtesting a market risk model.

These P&L's are different from the economic P&L introduced in section 4.3 above, but rather *risk-hypothetical* due to the application of some historical market moves to the current market which gives rise to a valuation change.

Consider a history of market risk factors $X_i(j)$ where $i \in \{1,\dots,n\}$ identifies the risk factor and $j \in \{1,\dots,m\}$ corresponds to a time t_j on which the risk factor was observed. The times are assumed to be equally spaced, with $t_{j+1} - t_j$ corresponding to 1 business day w.r.t. a given calendar. To generate an overlapping k-day PL, define the k day return at time t_j to be

$$r_i(j) = R_{T_i}(X_i(j), X_i(j+k)) \tag{4.10}$$

for $j = 1,\dots,m-k$. Note that the case of non-overlapping k day returns fits in with straightforward modifications of the scheme described here. R defines the return value for two observations of the same factor, which is one of the following

$$R_A(x,y) = y - x \tag{4.11}$$

$$R_R(x,y) = y/x - 1 \tag{4.12}$$

$$R_L(x,y) = \log(y/x) \tag{4.13}$$

where the subscript stands for absolute (A), relative (R) and lognormal (L) returns, respectively. Note that the relative and lognormal returns are not defined for $x = 0$, and we consider a data point with $X_i(j) = 0$ and for which we compute relative or lognormal returns to be an error in the data that needs to be corrected or excluded from the analysis. Also note that $R_R \approx R_L$ for small values of $y/x - 1$, the difference $R_R - R_L$ approaching zero when y/x approaches 1.

Now assume t_m to be the reference date (e.g. for the value at risk calculation) and

$$X(m) = \{X_i(m)\}_{i=1,\ldots,n} \tag{4.14}$$

the market factor values on the reference date.

Full Revaluation P&L

For a given portfolio denote its NPV at t_m by $\nu(X(m))$. Then we can compute a *full revaluation PL* vector

$$\pi_F = \{\pi_F(j)\}_{j=1,\ldots,m-k} \tag{4.15}$$

as

$$\pi_F(j) = \nu(X'(m,j)) - \nu(X(m)) \tag{4.16}$$

by pricing the portfolio under each perturbed market factor vector

$$X'(m,j) = \{X_i'(m,j)\}_{i=1,\ldots,n} \tag{4.17}$$

which is defined by

$$X_i'(m,j) = a_{T_i}(X_i(m), R_{T_i}(X_i(j), X_i(j+k))) \tag{4.18}$$

with the return application function

$$a_A(x,r) = x + r \tag{4.19}$$
$$a_R(x,r) = x(1+r) \tag{4.20}$$
$$a_L(x,r) = xe^r \tag{4.21}$$

and return types $T_i \in \{A, R, L\}$, dependent on the particular factor X_i. Table 4.2 shows a possible choice of return types for the different risk factors (in ORE notation). Note, that the factors Discount Curve, Index Curve and Survival Probability are discount factors resp. survival probabilities that are converted to zero rate resp. hazard rate shifts by taking the log. Also note, that the factors Recovery Rate and Basis Correlation are bounded (a recovery rate must be in $[0,1]$ while the base correlation must be in $[-1,1]$), so that after a shift is applied, the result has to be capped / floored appropriately to ensure valid scenario values.

Sensitivity based P&L

As an alternative to the full revaluation PL in (4.15) we can approximate this PL using a Taylor expansion of $\nu(X'(m,j))$ viewed as a function of the returns R_{T_i}[2] around the expansion point $(0,0,\ldots,0)$ generating a *sensitivity based PL*,

$$\pi_S = \{\pi_S(j)\}_{j=1,\ldots,m-k} \tag{4.22}$$

with

$$\begin{aligned}\pi_S(j) = &\sum_{i=1}^{n} D^i_{T_i}\nu(X(m))R_{T_i}(X_i(m),X'_i(m,j))+ \\ &\frac{1}{2}\sum_{i,l=1}^{n} D^{i,l}_{T_i,T_l}\nu(X(m))R_{T_i}(X_i(m),X'_i(m,j))R_{T_l}(X_l(m),X'_l(m,j)),\end{aligned} \tag{4.23}$$

where we use sensitivities up to second order. Here $D^i_{T_i}$ denotes a first or second order derivative operator, depending on the market factor specific shift type $T_i \in \{A,R,L\}$, i.e.

$$D^i_A f(x) = \frac{\partial f(x)}{\partial x_i}, \tag{4.24}$$

$$D^i_R f(x) = D^i_L f(x) = x_i\frac{\partial f(x)}{\partial x_i} \tag{4.25}$$

and using the short hand notation

$$D^{i,l}_{T_i,T_l} f(x) = D^i_{T_i} D^l_{T_l} f(x). \tag{4.26}$$

These first and second order sensitivities may be computed analytically, or (more common) as finite difference approximations ("bump and revalue" approximations), see section 4.1. To clarify the relationship of (4.24) and a finite difference scheme for derivatives computation in a bit more detail we note that for a absolute shift $h > 0$

$$\frac{f(x+h)-f(x)}{h} \to f'(x) \tag{4.27}$$

for $h \to 0$ by definition of f' while for a relative shift

$$\frac{f(x(1+h))-f(x)}{h} = x\frac{f(x(1+h))-f(x)}{xh} \to xf'(x) \tag{4.28}$$

for $h \to 0$ and for a log shift

$$\frac{f(xe^h)-f(x)}{h} \to xf'(x) \tag{4.29}$$

[2]i.e. we view ν as a function of the second argument of a_{T_i} in 4.18

using e.g. L'Hospital's rule, so that

- both a relative and a log shift bump and revalue sensitivity approximate the same value $xf'(x)$ in the limit for $h \to 0$,
- an absolute shift sensitivity can be transformed into a relative / log shift sensitivity (in the limit for $h \to 0$) by multiplying with the risk factor value x, and vice versa.

We also note that the usual way of bumping continuously compounded zero rates to compute a Discount Curve or Index Curve sensitivity by h^* is equivalent to (4.29) with $h = h^* t$, where t is the maturity of the respective rate. Therefore in practice a log return of discount factors can not directly be combined with a sensitivity expressed in zero rate shifts, but has to be scaled by $1/t$ before doing so.

Since the number of second order derivatives can be quite big in realistic setups with hundreds or even thousands of market factors, in practice only part of the second order derivatives might be fed into (4.23) assuming the rest to be zero.

Note that the types T_i used to generate the historical returns (4.18) can be different from those used in the Taylor expansion (4.23). It is important though that the same types T_i are used for the derivatives operators $D^i_{T_i}$ and the returns R_{T_i} in (4.23).

A number of configurations are hard-coded into ORE depending on whether raw sensitivities, backtesting sensitivities or CRIF sensitivities are being called. These configurations are displayed in 4.2 - note that there is currently no distinction made in ORE between raw sensitivities and backtest sensitivities.

ORE Risk Factor	Backtest Sensitivities		CRIF Sensitivities	
	Return Type	Shift Size	Return Type	Shift Size
Discount Curve	A	0.01%	A	0.01%
Index Curve	A	0.01%	A	0.01%
Yield Curve	A	0.01%	A	0.01%
Dividend Yield	A	0.01%	A	0.01%
Equity Forecast Curve	A	0.01%	A	0.01%
Swaption Volatility*	R	1%	A	0.01%
Optionlet Volatility*	R	1%	A	0.01%
FX Spot**	R	1%	R	0.1%
FX Volatility	R	1%	A	1%
Equity Spot	R	1%	R	1%
Equity Volatility	R	1%	A	1%
Yield Volatility	R	1%	R	1%
Survival Probability	A	0.01%	A	0.01%
CDS Volatility	R	1%	R	1%
Correlation	R	1%	-	-
Base Correlation	A	1%	A	1%
Zero Inflation Curve	A	0.01%	A	0.01%
YoY Inflation Curve	A	0.01%	A	0.01%
Zero Inflation CF Vol	R	1%	R	1%
YoY Inflation CF Vol	R	1%	R	1%
Commodity Curve	R	1%	R	1%
Commodity Volatility	R	1%	A	1%
Security Spread	A	0.01%	-	-

Table 4.2: Sensitivity return type configuration for raw, backtest and CRIF sensitivities

*We predominantly use normal IR volatilities with an absolute shift of 0.01%. For lognormal IR volatilities an absolute shift of 1% applies. Also, notice that "Optionlet Volatility" is the ORE name for "Cap / Floor Volatility".

**During a CRIF run, the FX spot delta is computed using the central difference approximation by default. A smaller shift size of 0.1% is used because of this. Furthermore, for FX vanilla European and American options, there are analytic formulae available for FX deltas. These are implemented in ORE during a CRIF run.

Note - the values A and R here refer to the absolute and relative shifts defined in (4.24).

4.5 Value at Risk

4.5.1 Historical Simulation VaR

The historical simulation VaR is defined to be a p-quantile of the empirical distribution generated by the full revaluation PL vector $\pi_F = \{\pi_F(j)\}_j$. Here, with "generated" we mean that we weigh each $\pi_F(j)$ with the same probability $1/J$, where J denotes the number of elements in the vector.

4.5.2 Historical Simulation Taylor VaR

Similarly, the historical simulation Taylor VaR is defined to be the p-quantile of the empirical distribution generated by the sensitivity based PL vector π_S (call side), resp. $-\pi_S$ (post side).

4.5.3 Parametric VaR

For the computation of the parametric, or variance-covariance VaR, we rely on a second order sensitivity-based P&L approximation

$$\pi_S = \sum_{i=1}^{n} D^i_{T_i} V \cdot Y_i + \frac{1}{2} \sum_{i,j=1}^{n} D^{i,j}_{T_i,T_j} V \cdot Y_i \cdot Y_j \tag{4.30}$$

with

- portfolio value V
- random variables Y_i representing risk factor returns; these are assumed to be multivariate normally distributed with zero mean and covariance matrix matrix $C = \{\rho_{i,k}\sigma_i\sigma_k\}_{i,k}$, where σ_i denotes the standard deviation of Y_i; covariance matrix C may be estimated using the Pearson estimator on historical return data $\{r_i(j)\}_{i,j}$. Since the raw estimate might not be positive semidefinite, we apply a salvaging algorithm to ensure this property, which basically replaces negative Eigenvalues by zero and renormalises the resulting matrix, see [20];
- first or second order derivative operators D, depending on the market factor specific shift type $T_i \in \{A, R, L\}$ (absolute shifts, relative shifts, absolute log-shifts), i.e.

$$\begin{aligned} D^i_A V(x) &= \frac{\partial V(x)}{\partial x_i} \\ D^i_R V(x) = D^i_L f(x) &= x_i \frac{\partial V(x)}{\partial x_i} \end{aligned}$$

  and using the short hand notation

$$D^{i,j}_{T_i,T_j} V(x) = D^i_{T_i} D^j_{T_j} V(x)$$

  In ORE, these first and second order sensitivities are computed as finite difference approximations (“bump and revalue”).

To approximate the p-quantile of π_S in (4.30) ORE offers the techniques outlined below.

4.5.4 Delta Gamma Normal Approximation

The distribution of (4.30) is non-normal due to the second order terms. The delta gamma normal approximation in ORE computes mean m and variance v of the portfolio value change π_S (discarding moments higher than two) following [21] and provides a simple VaR estimate

$$VaR = m + N^{-1}(q)\,\sqrt{v}$$

for the desired quantile q (N is the cumulative standard normal distribution). Omitting the second order terms in (4.30) yields the delta normal approximation.

4.5.5 Cornish-Fisher Expansion

The first four moments of the distribution of π_S in (4.30) can be computed in closed form using the covariance matrix C and the sensitivities of first and second order D_i and $D_{i,k}$, see e.g. [21]. Once these moments are known, an approximation to the true quantile of π_S can be computed using the Cornish-Fisher expansion, see also [7], which in practice often gives a decent approximation of the true value, but may also show bigger differences in certain configurations.

4.5.6 Saddlepoint Approximation

Another approximation of the true quantile of π_S can be computed using the Saddlepoint approximation using results from [22] and [23]. This method typically produces more accurate results than the Cornish-Fisher method, while still being fast to evaluate.

4.5.7 Monte Carlo Simulation

By simulating a large number of realisations of the return vector $Y = \{Y_i\}_i$ and computing the corresponding realisations of π_S in (4.30) we can estimate the desired quantile as the quantile of the empirical distribution generated by the Monte Carlo samples. Apart from the Monte Carlo Error no approximation is involved in this method, so that albeit slow it is well suited to produce values against which any other approximate approaches can be tested. Numerically, the simulation is implemented using a Cholesky Decomposition of the covariance matrix C in conjunction with a pseudo random number generator (Mersenne Twister) and an implementation of the inverse cumulative normal distribution to transform $U[0,1]$ variates to $N(0,1)$ variates.

Chapter 5

Capital

This chapter describes the various methods to calculate capital requirements in ORE.

5.1 Standardized Market Risk Capital (SMRC)

Calculating market risk capital requirements can be performed using the *Standardized Market Risk Capital (SMRC)* method. This method is defined by the regulator and based on the formula

$$\text{SMRC} = \text{Notional} \times \text{RiskWeight} \tag{5.1}$$

for every trade in scope.

SMRC is currently supported in ORE for the following trade types (unsupported types are ignored in calculations):

- *Bond*
- *ForwardBond*
- *BondOption*
- *CommodityForward*
- *CommodityOption*
- *CommoditySwap*
- *EquityOption*
- *EquityPosition*
- *EquityOptionPosition*
- *FXForward*
- *FXOption*
- *TotalReturnSwap*
- *ConvertibleBond*
- *ForwardRateAgreement*

- *CapFloor*
- *Swap*
- *Swaption*

5.1.1 Risk Weights

The risk weight depends on the type and the currencies involved in the trade. All trades supported in ORE and the corresponding risk weights are shown in Table 5.1. The distinction between *major* and *minor* currencies is given by the following list:

- `major`: USD, CAD, EUR, GBP, JPY, CHF
- `minor`: Any currency that is not major.

For trade types, where multiple currencies are involved, such as FxForward, the trade currencies are only classified as `major` if all currencies involved are major, and minor otherwise.

Trades which are based upon Swaps or Bonds depends upon the time until maturity of the underlying asset. As such trades with shorter time until maturity have smaller associated risk weights. Furthermore, in the case of trades dependent upon bonds, the rates are different for those which are based on US Government bonds.

5.1.2 Notional

The calculation of the notional of a trade can be involved as it depends on the trade type and the choice of the pricing engine. We refer to the documentation of those for technical details. For the trade types in listed in Table 5.1, the high-level methodology is as follows:

FxForward The FxForward trade type has a `BoughtAmount` and a `SoldAmount` in a `BoughtCurrency` and a `SoldCurrency`. The notional is calculated by converting both amounts into `BaseCcy` using the FX spot rate and then choosing the bigger of the two, i.e.

$$\mathrm{Notional} = \max(\mathrm{FX}_{\mathrm{base,bought}} \cdot \mathrm{BoughtAmount}, \mathrm{FX}_{\mathrm{base,sold}} \cdot \mathrm{SoldAmount})$$

FxOption The methodology is the same as for FxForward.

CommodityForward The CommodityForward trade type has a `Quantity` field and a `Strike` field. The notional is calculated as

$$\mathrm{Notional} = \mathrm{Strike} \cdot \mathrm{Quantity}\,.$$

CommoditySwap The `CommoditySwap` trade type contains a collection of legs. Each leg results in a sequence of flow amount between inception and maturity. The commodity swap notional is the sum of the (signed) notionals of all of its CommodityFloating legs. The notional of a CommodityFloating leg is then calculated by taking the earliest future flow amount of that leg. For variable-quantity swaps, we take the average quantity (taking into account spreads and gearing), while assuming the price of the earliest flow.

Trade Type	Currencies	Underlying	Maturity Time (Years)	Risk Weight
FxForward	major	-	-	6%
FxForward	minor	-	-	20%
FxOption	major	-	-	6%
FxOption	minor	-	-	20%
CommodityForward	all	-	-	20%
CommoditySwap	all	-	-	20%
CommodityOption	all	-	-	20%
EquityPosition	all	all	-	25%
EquityOption	all	all	-	25%
EquityOptionPosition	all	all	-	25%
	all	all	< 0.25	0%
	all	all	< 0.5	0.5%
	all	all	< 0.75	0.75%
	all	all	< 1	1%
Swap	all	all	< 2	1.5%
ForwardRateAgreement	all	all	< 3	2%
CapFloor	all	all	< 5	3%
Swaption	all	all	< 10	4%
	all	all	< 15	4.5%
	all	all	< 20	5%
	all	all	< 25	5.5%
	all	all	> 25	6%
ConvertibleBond	all	all	-	15%
	all	U.S. Govt. Bonds	< 5	1.5%
	all	U.S. Govt. Bonds	< 10	2.5%
	all	U.S. Govt. Bonds	< 15	2.75%
	all	U.S. Govt. Bonds	> 15	3%
	all	Other Bonds	< 1	2%
Bond	all	Other Bonds	< 2	3%
ForwardBond	all	Other Bonds	< 3	5%
BondOption	all	Other Bonds	< 5	6%
	all	Other Bonds	< 10	7%
	all	Other Bonds	< 15	7.5%
	all	Other Bonds	< 20	8%
	all	Other Bonds	< 25	8.5%
	all	Other Bonds	> 25	9%

Table 5.1: Risk Weights

CommodityOption For the `CommodityOption` trade type the notional is determined by the agreed `Strike` price times the corresponding `Quantity` value, both of which are provided in the trade data.

EquityPosition The `EquityPosition` trade type consists of an underlying asset (or basket of assets) with associated asset weight/s. The notional of the trade is given by the additional field `smrc_notional`. For each asset in the trade, SignedNotional = smrc_notional $*$ *weight*.

EquityOption In the case of `EquityOption` trades the notional is determined by the agreed `Strike` price times the corresponding `Quantity` value, both of which are provided in the trade data.

EquityOptionPosition The methodology is the same as for `EquityOption`.

ConvertibleBond The `ConvertibleBond` trade type has a notional field which may have an amortising structure. In this case the first of these notional values which occur after the provided `asof` date is used.

Bond The methodology is the same as for `ConvertibleBond`.

ForwardBond The methodology is the same as for `ConvertibleBond`.

BondOption The BondOption trade type contains information regarding the underlying option data including the `Strike` and `Quantity` values which are multiplied by one another to give the notional value.

Swap The `Swap` trade type has a `Notional` field for each leg present in the trade, here the value obtained from the first leg which appears in the input portfolio is used to represent the trade notional.

Swaption The `Swaption` trade type contains a collection of legs. Each leg results in a sequence of flow amount between inception and maturity. The notional is defined by taking the maximum over all legs and all current flow amounts after the `asof` date of the calculation.

ForwardRateAgreement The `ForwardRateAgreement` trade type has a `Notional` field which is used in determining the value.

CapFloor The methodology is the same as for `Swap`.

5.1.3 Aggregation & Offsetting of Positions

For each trade i in the portfolio, the SMRC charge SMRC_i is calculated using (5.1) and stored in a detailed report. The easiest aggregation of all the contributions of the trades into a single SMRC capital charge number is simply:

$$\text{SMRC} := \sum_i \text{SMRC}_i \tag{5.2}$$

Notice that this type of aggregation is very conservative as this formula does not take into account any offsetting effects between long and short positions of various trades in the portfolio.

Thus, we produce a second aggregated report, where offsetting between long and short positions of the same type of market risk is allowed. The precise definition of this depends on the trade type. We give a high-level overview of the methodology for some relevant trade types here:

FxForward An FxForward has a linear payoff, where one currency amount is bought and another is sold. We therefore think of an FxForward as having two legs - one that pays and one that receives. We compute the list of all currencies of all FxForwards in the portfolio and then for each currency j define a currency bucket CCY_j. In that bucket we sum up with a positive sign all the `boughtAmounts` of all FxForwards i with `boughtCurrency` equal to j and the same with the `soldAmounts`, but with a negative sign, thus calculating the total effective notional amount in that currency:

$$\mathrm{CCY}_{j,\mathrm{bought}} := \sum_{i,\mathrm{boughtCurrency}_i=j} \mathrm{boughtAmount}_i \cdot \mathrm{FX}_{\mathrm{base},\mathrm{bought}_i},$$
$$\mathrm{CCY}_{j,\mathrm{sold}} := \sum_{i,\mathrm{soldCurrency}_i=j} \mathrm{soldAmount}_i \cdot \mathrm{FX}_{\mathrm{base},\mathrm{sold}_i},$$
$$\mathrm{CCY}_j := \mathrm{CCY}_{j,\mathrm{bought}} - \mathrm{CCY}_{j,\mathrm{sold}}.$$

Finally, we aggregate the results of the currency buckets by weighing its absolute value with the $\mathrm{RiskWeight}_j$ of that currency, which is again 6%, if the currency is major and 20% otherwise:

$$\mathrm{SMRC}_{\mathrm{FxForward}} := \sum_j \mathrm{RiskWeight}_j \,|\, \mathrm{CCY}_j \,|.$$

FxOption An FxOption is not a linear trade and thus it cannot be decomposed as easily into legs like the FxForward. Therefore, for each FxOption i we compute the unordered set of the currency pair

$$\{\mathrm{BoughtCurrency}_i, \mathrm{SoldCurrency}_i\}$$

and then for each such currency pair $\mathrm{CCYPair}_j$ we sum up the signed notionals of the long and short put and call options i with that currency pair:

$$\mathrm{CCYPair}_j := \sum_{i,\mathrm{CCYPair}_i=j,} \mathrm{SignedNotional}_i,$$

where the $\mathrm{SignedNotional}_i$ of an option i has the same absolute value as the notional and the sign is given by Table 5.2. Finally, we compute the risk weight $\mathrm{RiskWeight}_j$ of each currency pair j, which is again 6% if both currencies are major and 20% otherwise. We then aggregate analogously

$$\mathrm{SMRC}_{\mathrm{FxOption}} := \sum_j \mathrm{RiskWeight}_j \,|\, \mathrm{CCYPair}_j \,|.$$

CommodityForward, CommoditySwap, CommodityOption These trades each have an underlying commodity, $\mathrm{Commodity}_\mathrm{i}$, which has an associated notional amount $\mathrm{SignedNotional}_\mathrm{i}$. For each unique commodity, $\mathrm{Commodity}_j$, we consider the portfolio netted total represented by $\mathrm{CommodityTotal}_j$ obtained by summing the signed notionals arising from trades associated with this commodity:

$$\mathrm{CommodityTotal}_j := \sum_{i,\mathrm{Commodity}_i=\mathrm{Commodity}_j,} \mathrm{SignedNotional}_i,$$

where the SignedNotional_i of a trade i has the same absolute value as the notional with sign given by Table 5.2. Finally, we compute the risk weight RiskWeight_j of each commodity j, which is 20%. We then aggregate analogously to obtain

$$\text{SMRC}_{\text{Commodity}} := \sum_j \text{RiskWeight}_j \,|\, \text{CommodityTotal}_j \,|.$$

EquityOption, EquityOptionPosition These trades depend upon an underlying equity, Equity_i, which has an associated notional amount $\text{SignedNotional}_\text{i}$. For each unique equity Equity_j we consider the portfolio netted total represented by EquityTotal_j obtained by summing up the signed notionals arising from trades associated with this equity given by:

$$\text{EquityTotal}_j := \sum_{i, \text{Equity}_i = \text{Equity}_j,} \text{SignedNotional}_i,$$

where the SignedNotional_i of a trade i has the same absolute value as the notional and the sign is given by Table 5.3 in the case of option based trades and simply positive (negative) for long (short) position trades. Finally, we compute the risk weight RiskWeight_j of each equity j, which is 25%. We then aggregate analogously

$$\text{SMRC}_{\text{Equity}} := \sum_j \text{RiskWeight}_j \,|\, \text{EquityTotal}_j \,|.$$

Swap, ForwardRateAgreement, CapFloor, Swaption In the case of these trades we consider only floating legs, and consider the time until maturity of the contract. As such for each trade i we consider the set of swap-maturity pairs $\{UnderlyingIndex_i, MaturityDate_i\}$ where the maturity times are considered on a discrete basis such that all trades with maturity within a certain window are grouped together, these windows are given in Table 5.1, e.g., all swaps for a certain index maturing in less than 0.25 years are grouped. Consequently for each swap-maturity pair SwapMaturity_j we sum up the signed notionals for each trade, which are determined by Table 5.4 in the case of swaption based trades and simply positive (negative) for long (short) position trades, to obtain the total for said pair given by

$$\text{SwapMaturityTotal}_j := \sum_{i, \text{SwapMaturity}_i = \text{SwapMaturity}_j} SignedNotional_i.$$

Finally for each swap-maturity pair j we compute the RiskWeight_j as given by Table 5.1 and again aggregate across all trades via

$$\text{SMRC}_{\text{SwapUnderlying}} := \sum_j \text{RiskWeight}_j \,|\, \text{SwapMaturityTotal}_j \,|.$$

ConvertibleBond Trades based upon convertible bonds have an underlying asset, BondUnderlying_i which determines the returns of the trade. Each corresponding notional is multiplied by the RiskWeight_j which is always given by 15% in this case and then aggregated analogously as

$$\text{SMRC}_{\text{BondUnderlying}} := \sum_j \text{RiskWeight}_j \,|\, \text{BondUnderlying}_j \,|.$$

Bond, ForwardBond, BondOption These trades are dependent upon an underlying bond asset, which itself has a given maturity date. As such for each trade i we consider the set of bond-maturity pairs $\{UnderlyingBond_i, MaturityDate_i\}$ where the maturity times are considered on a discrete basis such that all trades with maturity within a certain window are grouped together, these windows are given in Table 5.1, e.g., all unique U.S. government bonds maturing in less than 5 years are grouped. Consequently for each bond-maturity pair BondMaturity_j we sum up the signed notionals for each trade, which are determined by Table 5.3 in the case of option based trades and simply positive (negative) for long (short) position trades, to obtain the total for said pair given by

$$\text{BondMaturityTotal}_j := \sum_{i,\text{BondMaturity}_i=\text{BondMaturity}_j} SignedNotional_i.$$

The last distinction made is that those bonds issued by the U. S. Government are treated differently from all others as demonstrated in Table 5.1. Finally for each bond maturity pair j we compute the RiskWeight_j as given by Table 5.1 and again aggregate across all trades via

$$\text{SMRC}_{\text{BondUnderlying}} := \sum_j \text{RiskWeight}_j \,|\, \text{BondMaturityTotal}_j \,|.$$

BoughtCcy	SoldCcy	Type	LongShort	Sign
X	Y	call	long	+
X	Y	call	short	-
Y	X	call	long	-
Y	X	call	short	+
X	Y	put	long	-
X	Y	put	short	+
Y	X	put	long	+
Y	X	put	short	-

Table 5.2: FxOption Notional Signs

Type	LongShort	Sign
call	long	+
call	short	-
put	long	-
put	short	+

Table 5.3: Option Notional Signs

5.2 Counterparty Credit Risk Capital

Financial institutions either apply a *standardized* or an *advanced* approach for determining the regulatory capital amounts to be assigned to their derivative activity. The advanced approach is accessible to institutions with an internal model method

PayReceive	LongShort	Sign
pay	long	+
pay	short	-
receive	long	-
receive	short	+

Table 5.4: Swaption Notional Signs

(IMM) for credit risk capital which is approved by the regulator. This method involves sophisticated analysis of future exposures by Monte Carlo simulation methods using real-world measure risk factor evolutions [14]. Institutions without approved IMM have to apply a standardized approach instead, which is simplified in that it does not require Monte Carlo exposure simulation but resorts to formulas suggested by the Basel Commitee for Banking Supervision and enforced by the respective regulator. These formulas attempt to conservatively approximate the credit exposures which would have been obtained by more sophisticated IMM approaches.

The former standardized approach (Current Exposure Method) as published by the Basel Commitee for Banking Supervision (BCBS) in 2006 [2] is summarized in section 5.2.1. A revised standardized approach for counterparty credit risk from derivative acitvity (SA-CCR) as published in 2014 [4] is in effect from beginning of 2017.

5.2.1 Current Exposure Method (CEM)

The key quantity in the current standardized approach (or current exposure method, CEM) is the exposure at default (EaD) or *credit equivalent amount* which consists of two additive terms, current replacement cost and potential future exposure add-on,

$$\mathrm{EaD} = \mathrm{RC} + \mathrm{Notional} \times \mathrm{NettingFactor} \times \mathrm{AddOn}.$$

The replacement cost RC is simply given by the current exposure which is the current positive value of the netting set after subtracting collateral (C)

$$\mathrm{RC} = \max(0, \mathrm{PV} - \mathrm{C}).$$

Replacement cost is aggregated over all derivative contracts and netting sets.

The second term is supposed to approximately reflect the potential future exposure over the remaining life of the contract. It depends on the (fairly rough) product type classification and on time to maturity in three bands as shown in table 5.5 which is in use in this form since 1988.

The netting factor acknowledges netting also in the potential future exposure estimate of the netting set. If there is no netting as of today, i.e. net NPV equals gross NPV, the netting factor is equal to 1, otherwise it can be as low as 0.4. Additional netting benefit results from using a central clearing counterparty (CCP):

$$\mathrm{NettingFactor} = \begin{cases} 0.4 + 0.6 \times \dfrac{\max\left(\sum_i PV_i, 0\right)}{\sum_i \max(PV_i, 0)} & \text{bilateral netting} \\[2ex] 0.15 + 0.85 \times \dfrac{\max\left(\sum_i PV_i, 0\right)}{\sum_i \max(PV_i, 0)} & \text{central clearing} \end{cases}$$

Residual Maturity	Interest Rates	FX Gold	Equity	Precious Metals	Other Commodities
$\leq$ 1 Year	0.0%	1%	6%	7%	10%
1-5 Years	0.5%	5%	8%	7%	12%
$>$ 5 Years	1.5%	7.5%	10%	8%	15%

Table 5.5: Add-on factor by product and time to maturity. Single currency interest rate swaps are assigned a zero add-on, i.e. judged on replacement cost basis only, if their maturity is less than one year. Forwards, swaps, purchased options and derivative contracts not covered in the columns above shall be treated as "Other Commodities". Credit derivatives (total return swaps and credit default swaps) are treated separately with 5% and 10% add-ons depending on whether the reference obigation is regarded as "qualifying" (public sector entities (!), rated investment grade or approved by the regulator). N-th to default basket transactions are assigned an add-on based on the credit quality of n-th lowest credit quality in the basket.

Finally, the notional amounts that enter into the potential future exposure term are understood as effective notional amounts which e.g. take into account leverage which may be expressed through factors in structured product payoff formulas.

Note that CEM was valid until end of 2016.

5.2.2 Standardized Approach for Counterparty Credit Risk (SA-CCR)

With its 2014 publication [4], the Basel Committee for Banking Supervision has revised the standardized approach. The new method takes collateralization into account in a more detailed way than before which has the potential to reduce the credit equivalent amounts. On the other hand, the new method attempts to mimic a more conservative potential exposure, the *effective expected positive exposure* (EEPE) which tends to increase the resulting credit equivalent amounts. In summary, only a detailed impact analysis for specific portfolios will be able to tell whether the overall impact of the new method results in an increase or in a decrease of derivative capital charges. In the following we summarize the ingredients of the new methodology.

SA-CCR in ORE currently supports the following trade types per asset class (unsupported types are ignored in calculations with a structured warning raised):

- Foreign Exchange: *Swap* (cross currency), *CrossCurrencySwap*, *FxBarrierOption*, *FxForward*, *FxOption*, *FxTouchOption*
- Commodity: *CommodityForward*, *CommoditySwap*
- Interest Rate: *Swap* (Vanilla IR, basis and CPI swaps not yet supported), *Swaption* (European)
- Equity: *EquityOption*

Note for swap-type products that two legs are required.

5.2.2.1 Exposure at Default (EAD)

The EAD is still composed of a replacement cost and a potential future exposure add-on term but scaled up by a factor of 1.4 which is motivated by the committee's attempt to mimic a different (higher) exposure measure:

$$\mathrm{EAD} = 1.4 \times (\mathrm{RC} + \mathrm{PFE})$$

On the other hand, the replacement cost per netting set takes into account more details of the collateral agreement:

$$\mathrm{RC} = \max(\mathrm{PV} - \mathrm{C};\ \mathrm{TH} + \mathrm{MTA} - \mathrm{NICA};\ 0)$$

where

- PV is the netting set mark-to-market value,
- TH is the CSA's threshold amount,
- MTA the CSA's minimum transfer amount,
- NICA is the independent collateral amount (i.e. any received independent amount plus initial margin amount),
- C is the current collateral (i.e. variation margin plus NICA).

So even if the posted collateral C matches the PV so that the first term on the right-hand side vanishes, the various CSA slippage terms can cause a positive replacement cost contribution here. This supposedly imitates CVA behaviour.

For unmargined netting sets,

$$\mathrm{RC} = \max(\mathrm{PV} - \mathrm{C};\ 0).$$

5.2.2.2 Potential Future Exposure (PFE)

The PFE term is primarily driven by the aggregate add-on factor which is significantly more complex than in the current standardized approach's definition:

$$\mathrm{PFE} = \mathrm{Multiplier} \times \mathrm{AddOn}.$$

The PFE is obtained by scaling down the AddOn using a multiplier which recognises and rewards excess collateral:

$$\begin{aligned} \mathrm{Multiplier} &= \min\left(1; 0.05 + 0.95 \times \exp\left(\frac{\mathrm{PV} - \mathrm{C}}{1.9 \times \mathrm{AddOn}}\right)\right) \\ &= 1 \quad \text{if } PV \geq C, \\ &< 1 \quad \text{if } PV < C \text{ (excess collateral)}. \end{aligned}$$

The aggregate/netting set add-on is a composite

$$\mathrm{AddOn} = \sum_a \mathrm{AddOn}^{(a)}$$

where the sum is taken over the various asset classes in the netting set, and $\mathrm{AddOn}^{(a)}$ denotes the add-on factor for asset class a. The add-on factor within each asset class a

$$\mathrm{AddOn}^{(a)} = \sum_i \mathrm{AddOn}_i^{(a)} \tag{5.3}$$

is a sum of the hedging set add-ons (where $\mathrm{AddOn}_i^{(a)}$ is the add-on for hedging set i in asset class a). The list of possible hedging sets for each asset class is given in Table 5.6. The asset class assignment is based on a trade's "primary risk driver", but split assignment may be required for complex trades.

Asset Class	Hedging Set	Hedging Subset
Interest Rate	Currency	Maturity buckets (*1Y*, *1Y-5Y*, *5Y*)
Foreign Exchange	Currency pair	-
Equity	-	Qualifier
Credit	-	Qualifier
Commodity	*Energy, Metal, Agriculture, Other*	Qualifier/Group

Table 5.6: Hedging set/subset construction by asset class – See [4] paragraph 161.

Note:

- For Interest Rate, a trade is assigned to a hedging subset based on the end date of the period referenced by the underlying E_i.[1]
- For Commodity, similar underlyings can be grouped under the same hedging subset.[2] Currently, similar underlyings will be grouped together under the following categories: *Coal*, *Crude oil*, *Light Ends*, *Middle Distillates*, *Heavy Distillates*, *Natural Gas*, *Power*.
- For Equity and Credit, a single hedging set is used for the entire asset class. The hedging subset is then given by the underlying entity, where partial offsetting is applied across different entities, and full offset is applied within each entity.
- Within each asset class, a separate hedging set is reserved for basis trades and volatility/variance trades. Basis hedging sets are given in the format `QUALIFIER1/QUALIFIER2`. Volatility/variance trades are not yet supported.

As mentioned above, the potential future exposure term aims to mimic a particularly conservative exposure measure. This choice is built into the definition of the supervisory factors, quoting [4]: "A factor or factors specific to each asset class is used to **convert the effective notional amount into Effective EPE** based on the measured volatility of the asset class. Each factor has been calibrated to reflect the Effective EPE of a single at-the-money linear trade of unit notional and one-year maturity. This includes the estimate of realised volatilities assumed by supervisors for each underlying asset class." The supervisory factors are displayed in Table 5.7.

5.2.2.3 Trade-Specific Parameters

Before defining the hedging set add-on, $\mathrm{AddOn}_i^{(a)}$ (5.3), for each asset class we define the trade-specific parameters that will be used.

The trade specific parameters δ_i, $d_j^{(a)}$ and $MF_i^{(\mathrm{type})}$ are defined (for trade j and asset class a) as follows:

[1] See [4] paragraph 166.
[2] See [4] Example 3

1. $d_j^{(a)}$ **(trade-level adjusted notional)**
   - **Foreign Exchange**:
     - For trades where one of the legs is in the base currency: The adjusted notional is the foreign leg notional converted to the base currency.
     - For trades where both legs are denominated in a currency other than the base currency: Both leg notionals are converted to the base currency, and the larger of the 2 notionals is used as the adjusted notional.
   - **Interest Rate, Credit**: Notional $\times$ SD_j, where SD_j is the supervisory duration, with

     $$\mathrm{SD}_j = \frac{\exp(-0.05 \cdot S_j) - \exp(-0.05 \cdot E_j)}{0.05},$$

     where
     - S_j is the start date (in years) of the period referenced by the underlying,
     - E_j is the end date (in years) of the period referenced by the underlying.
   - **Equity, Commodity**: Price per unit $\times$ No. of units
   - Note: We index the adjusted notional by the asset class a is because complex trades can be assigned to multiple asset classes and hence have an adjusted notional in more than one asset class.
2. $MF_j^{(\mathbf{type})}$ **(maturity factor)**
   - For uncollateralized positions, this is computed from the time to maturity M_j (in years) of the trade: $MF_j^{(\text{unmargined})} = \sqrt{\min(\max(M_j, 2/52), 1)}$. Note the floor of 10 business days on M_j.
   - For collateralized positions, this is computed from the margin period of risk MPR (in years) used: $MF_j^{(\text{margined})} = 1.5 \cdot \sqrt{MPR}$
3. δ_j **(delta adjustment for direction and non-linearity)**
   - Options: In this case δ_j is an option delta (derived from the Black76 formula),

     $$\delta_j = \omega \cdot \Phi\left(\phi \cdot \frac{\ln(P_j/K_j) + 0.5\,\sigma_j^2\,T_j}{\sigma_j\,\sqrt{T_j}}\right),$$

     where
     - $\Phi(\cdot)$ is the cumulative normal distribution function,
     - P_j is the price of the underlying (typically the forward price),
     - K_j is the strike price of the option,
     - T_j is the latest option exercise date,
     - ω is $+1$ for long calls and short puts, -1 for short calls and long puts,
     - ϕ is $+1$ for calls, -1 for puts,
     - σ_j is the supervisory option volatility as defined in Table 5.7.

- CDO tranches: $\pm\frac{15}{(1+14\,A)(1+14\,D)}$ for purchased (sold) protection, where A and D denote the attachment and detachment point of the tranche, respectively
- Others: ± 1 depending on whether long or short in the primary risk factor

5.2.2.4 Hedging Set/Subset Add-On

We continue with sketching the hedging set level add-on from (5.3).

Interest Rate

Within currency hedging set i, each trade is assigned to 1 of 3 maturity buckets based on the end date of the period referenced by the trade's underlying:

- D_1: < 1 year
- D_2: 1-5 years
- D_3: > 5 years

The effective notional $D_{i,k}$ for maturity bucket k of currency hedging set i

$$D_{i,k} = \sum_{j=1}^{n} \delta_j \times d_j^{(\mathrm{IR})} \times \mathrm{MF}_j^{(\mathrm{type})}.$$

is calculated as the sum of all trades j in each maturity bucket.

Partial offsetting is applied when aggregating the contribution across the 3 maturity buckets, giving the effective notional for hedging set i:

$$\mathrm{EffectiveNotional}_i^{(\mathrm{IR})} = \sqrt{D_{i,1}^2 + D_{i,2}^2 + D_{i,3}^2 + 1.4 \cdot (D_{i,1} \cdot D_{i,2} + D_{i,2} \cdot D_{i,3}) + 0.6 \cdot D_{i,1} \cdot D_{i,3}}$$

Each contribution is a sum over all trades in each hedging set i and maturity bucket: One may choose not to apply any offset across maturity buckets, in which case

$$\mathrm{EffectiveNotional}_i^{(\mathrm{IR})} = |D_{i,1}| + |D_{i,2}| + |D_{i,3}|.$$

Finally, we multiply the effective notional by the supervisory factor (see Table 5.7) to obtain the add-on:

$$\mathrm{AddOn}_i^{(\mathrm{IR})} = SF_i^{(\mathrm{IR})} \times \mathrm{EffectiveNotional}_i^{(\mathrm{IR})}$$

Foreign Exchange

Unlike for IR instruments, FX does not use hedging subsets (i.e. the notional amounts are maturity independent) so trades within the same currency pair hedging set are allowed to fully offset each other. The effective notional calculation is similar to that of IR:

$$\mathrm{EffectiveNotional}_i^{(\mathrm{FX})} = \sum_{j=1}^{n} \delta_j \times d_j^{(\mathrm{FX})} \times MF_j^{(\mathrm{type})}$$

and

$$\mathrm{AddOn}_i^{(\mathrm{FX})} = SF_i^{(\mathrm{FX})} \times \left|\mathrm{EffectiveNotional}_i^{(\mathrm{FX})}\right|$$

Credit

All credit instruments are assigned to a single hedging set, and hedging subsets are defined by each underlying entity/name. Trades that reference the same entity are fully offset, giving the entity-level effective notional amount (for hedging subset k):

$$\text{EffectiveNotional}_k^{(\text{CR})} = \sum_{j=1}^{n} \delta_j \times d_j^{(\text{CR})} \times MF_j^{(\text{type})}$$

Multiplying by the supervisory factor, we get the entity-level add-on (for hedging subset k):

$$\text{AddOn}_k^{(\text{CR})} = SF_k^{(\text{CR})} \times \text{EffectiveNotional}_k^{(\text{CR})}$$

For single-name entities, the supervisory factor is determined by the credit rating, while for index entities this is determined based on whether the index is investment grade or speculative grade (see Table 5.7).

Finally, the asset class add-on is calculated by applying a partial offset across the entity-level add-ons:

$$\text{AddOn}^{(\text{CR})} = \sqrt{\underbrace{\left(\sum_k \rho_k^{(\text{CR})} \cdot \text{AddOn}_k^{(\text{CR})}\right)^2}_{\text{systematic component}} + \underbrace{\sum_k \left(1 - \left(\rho_k^{(\text{CR})}\right)^2\right) \cdot \left(\text{AddOn}_k^{(\text{CR})}\right)^2}_{\text{idiosyncratic component}}}$$

Equity

The hedging set/subset construction for equities is similar to that for credit, so the same calculation applies for effective notional where a hedging subset is formed for each equity underlying:

$$\text{EffectiveNotional}_k^{(\text{EQ})} = \sum_{j=1}^{n} \delta_j \times d_j^{(\text{EQ})} \times MF_j^{(\text{type})}$$

Likewise, for the entity-level add-on:

$$\text{AddOn}_k^{(\text{EQ})} = SF_k^{(\text{EQ})} \times \text{EffectiveNotional}_k^{(\text{EQ})}$$

There are only 2 supervisory factors for equities, based on whether the underlying is a single name or an index (see Table 5.7).

Finally, we apply partial offset once again across the entity-level add-ons to get the asset class add-on:

$$\text{AddOn}^{(\text{EQ})} = \sqrt{\underbrace{\left(\sum_k \rho_k^{(\text{EQ})} \cdot \text{AddOn}_k^{(\text{EQ})}\right)^2}_{\text{systematic component}} + \underbrace{\sum_k \left(1 - \left(\rho_k^{(\text{EQ})}\right)^2\right) \cdot \left(\text{AddOn}_k^{(\text{EQ})}\right)^2}_{\text{idiosyncratic component}}}$$

Commodity

The Commodity asset class uses 4 hedging sets (and no offsetting is allowed between hedging sets in any asset class), the Commodity add-on is more specifically defined as:

$$\text{AddOn}^{(\text{COM})} = \text{AddOn}^{(\text{COM})}_{\text{Energy}} + \text{AddOn}^{(\text{COM})}_{\text{Metal}} + \text{AddOn}^{(\text{COM})}_{\text{Agriculture}} + \text{AddOn}^{(\text{COM})}_{\text{Other}}$$

For Commodity, the calculation of hedging set level add-ons is the same as the calculation of asset class add-ons for Equity and Credit. As before, we start with calculating the effective notional at the entity level (i.e. hedging subset k) under hedging set i, applying full offset across trade contributions:

$$\text{EffectiveNotional}^{(\text{COM})}_{i,k} = \sum_{j=1}^{n} \delta_j \times d^{(\text{COM})}_j \times MF^{(\text{type})}_j$$

Then we calculate the add-on for hedging subset k:

$$\text{AddOn}^{(\text{COM})}_{i,k} = SF^{(\text{COM})}_i \times \text{EffectiveNotional}^{(\text{COM})}_{i,k}$$

Finally, we get the add-on for hedging set i (note the correlation terms are outside the sums as they apply to the hedging set, not to the hedging subset):

$$\text{AddOn}^{(\text{COM})}_i = \sqrt{\underbrace{\left(\rho^{(\text{COM})}_i \cdot \sum_k \text{AddOn}^{(\text{COM})}_{i,k}\right)^2}_{\text{systematic component}} + \underbrace{\left(1 - \left(\rho^{(\text{COM})}_i\right)^2\right) \cdot \sum_k \left(\text{AddOn}^{(\text{COM})}_{i,k}\right)^2}_{\text{idiosyncratic component}}}$$

Asset Class	Subclass	Supervisory Factor	Correlation	Supervisory Option Volatility
Interest rate		0.5 %	N/A	50%
Foreign exchange		4.0 %	N/A	15%
Credit, Single Name	AAA	0.38%	50%	100%
	AA	0.38%	50%	100%
	A	0.42%	50%	100%
	BBB	0.54%	50%	100%
	BB	1.06%	50%	100%
	B	1.6%	50%	100%
	CCC	6.0%	50%	100%
Credit, Index	IG	0.38%	80%	80%
	SG	1.06%	80%	80%
Equity, Single Name		32%	50%	120%
Equity, Index		20%	80%	75%
Commodity	Electricity	40%	40%	150%
	Oil/Gas	18%	40%	70%
	Metals	18%	40%	70%
	Agricultural	18%	40%	70%
	Other	18%	40%	70%

Table 5.7: Supervisory factors and option volatilities from [4] Table 2.

5.2.2.5 Capital Charge

The CCR capital charge is then computed as

$$K = EAD \times \underbrace{PD \times LGD}_{\text{Risk Weight}}$$

	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to BB-	Below BB-	Unrated
Sovereign	0%	20%	50%	100%	150%	100%
Financials	20%	30%	50%	100%	150%	100%
Corporate	20%	50%	75%	100%	150%	100%

Table 5.8: Example risk weights under the standardized approach for credit risk.

In the *Standardized Approach*, the risk weight is given by a fixed percentage depending on the counterparty type and its external rating [2, 13], see table 5.8. Under the *Internal Ratings-based Approach (IRB)*, banks can use their internal estimates of PD (*Foundation IRB*), or of PD and LGD (*Advanced IRB*).

5.2.2.6 Risk Weighted Assets (RWA)

RWA is calculated as a simple multiple of the capital charge

$$RWA = 12.5 \times K$$

5.2.2.7 Implementation Details

This section describes some of the requirements and implementation of the SA-CCR capital analytic in ORE.

Configuration Defaults

The SA-CCR capital analytic requires the following input files (with default values specific to the SA-CCR as outlined below; defaults that are not specific to any analytic are outlined in the corresponding subsection for that input file):

1. **Netting Set Definitions** (see the User Guide for a full description of the file format)
   - `ActiveCSAFlag`: Defaults to *False*
   - `MarginPeriodOfRisk`: Defaults to *2W*
   - `IndependentAmountHeld`: Defaults to zero
   - `ThresholdReceive`: Defaults to zero
   - `MinimumTransferAmountReceive`: Defaults to zero
   - `CalculateIMAmount`: Defaults to zero
   - `CalculateVMAmount`: Defaults to zero
2. **Counterparty Information** (see the User Guide for a full description of the file format)
   - `ClearingCounterparty`: Defaults to *False*
   - `SaCcrRiskWeight`: Defaults to *1.0*
3. **Collateral Balances** (see the User Guide for a full description of the file format)
   - `Currency`: Defaults to *USD*
   - `IndependentAmountHeld`: Defaults to zero

- `InitialMargin`: Defaults to zero
- `VariationMargin`: Defaults to zero

For each trade in the portfolio XML its netting set ID and counterparty ID (specified in the `Envelope` node) should have a corresponding entry in the netting set definitions and counterparty information files, respectively. If, for example, the corresponding netting set definition for the trade is missing, then an entry will be created internally with all the necessary inputs taking on the default values outlined above, with an appropriate warning provided in the log file.

Similarly for the collateral balances, an entry is required for all collateralised netting sets. If no entry is provided for the netting set of a given trade, a default collateral balance entry will be assumed, with an appropriate warning message.

If a configuration file is not provided at all (and hence all counterparties and netting sets are missing configurations), no warning messages will be given, and all entries in the configuration will assume the defaults.

Additional Requirements/Details

Since the SA-CCR aggregation step is done at the netting set level, there can only be one counterparty associated to each netting set at most, i.e. there can only be a one-to-one or many-to-one mapping from netting set ID to counterparty ID. This association is determined based on the portfolio XML. For example, for a two-trade portfolio, we would have the following mappings (from netting set to counterparty):

- One-to-one
  1. `NettingSetId` = "NS_A", `CounterpartyId` = "CPTY_A"
  2. `NettingSetId` = "NS_B", `CounterpartyId` = "CPTY_B"
- Many-to-one
  1. `NettingSetId` = "NS_A", `CounterpartyId` = "CPTY_A"
  2. `NettingSetId` = "NS_B", `CounterpartyId` = "CPTY_A"
- One-to-many (not allowed)
  1. `NettingSetId` = "NS_A", `CounterpartyId` = "CPTY_A"
  2. `NettingSetId` = "NS_A", `CounterpartyId` = "CPTY_B"

For a given netting set, if the initial margin amount is set to be calculated internally (see `CalculateIMAmount` in the User Guide's netting set definitions, an IM amount does not need to be provided in the collateral balances, and a SIMM amount will be calculated and used. If, however, an IM amount is still provided, then this provided balance will override the SIMM-generated amount.

This same procedure applies to the variation margin amount. In this case, the calculated VM will be equivalent to a perfect collateralisation of the netting set (i.e. VM = netting set NPV).

For clearing counterparties (see `ClearingCounterparty` in the counterparty information (see User Guide), the initial margin amount will be set to zero in the collateral balance, if provided.

Outputs and Reports

The SA-CCR capital analytic produces several output files:

- **saccr.csv**: Main SA-CCR results, broken down by netting set, asset class, and hedging set. Each row shows the calculated capital and its components at a specific aggregation level.
  - **NettingSet**: Identifier of the netting set. "All" means portfolio-level totals.
  - **AgreementType**, **CallType**, **InitialMarginType**, **LegalEntityId**: Collateral agreement and counterparty details.
  - **AssetClass**: Asset class (e.g., Commodity, FX, IR, Equity, All).
  - **HedgingSet**: Hedging set within the asset class (e.g., Metal, All).
  - **AddOn**: Add-on amount for the aggregation level.
  - **NPV**: Net present value of the trades in the aggregation.
  - **IndependentAmountHeld**, **InitialMargin**, **VariationMargin**: Collateral amounts.
  - **ThresholdAmount**, **MinimumTransferAmount**: CSA terms.
  - **RC**: Replacement cost.
  - **Multiplier**: Multiplier applied to the add-on.
  - **PFE**: Potential future exposure, $PFE = Multiplier \times AddOn$
  - **EAD**: Exposure at default, $\alpha(RC + PFE)$
  - **RW**: Risk weight.
  - **CC**: Capital charge.

  Rows with "All" in most columns show portfolio totals. Rows with specific netting sets, asset classes, or hedging sets show breakdowns at those levels. Empty fields indicate values not applicable at that level.

- **saccr_detail.csv** file provides a detailed, trade-level breakdown of the SA-CCR calculation. Each row corresponds to a single trade and shows all relevant parameters used in the exposure and add-on calculations. This report allows you to trace how each trade contributes to the overall capital charge. The columns are as follows:
  - **TradeId**: Unique identifier of the trade.
  - **TradeType**: Type of the trade (e.g., CommoditySwap, FxForward, Swaption).
  - **NettingSet**: Netting set identifier for the trade.

- **AgreementType**, **CallType**, **InitialMarginType**, **LegalEntityId**: Collateral agreement and counterparty details (may be empty if not applicable).
- **NPV**: Net present value of the trade.
- **AssetClass**: Asset class assigned to the trade (e.g., IR, FX, Commodity, Equity).
- **HedgingSet**: Hedging set within the asset class (e.g., currency pair for FX).
- **HedgingSubset**: Further subdivision within the hedging set (e.g., specific commodity group or maturity bucket).
- **Bucket**: Bucket or group used for add-on aggregation (e.g., maturity bucket for IR).
- **Qualifier**: Qualifier for the hedging set/subset (e.g., index name, underlying entity).
- **Currency**: Currency of the trade.
- **delta**: Supervisory delta
- **d**: Adjusted notional for the trade, as defined by SA-CCR rules. For IR and Credit derivatives it is the notional adjusted by the supervisory duration. For FX derivatives it is the notional value of the foreign currency. For Equity and Commodities its the current price times the quantity.
- **MF**: Maturity factor for the trade.
- **M**: Maturity of the trade (in years).
- **S**: Start date of the underlying period (in years, if applicable).
- **E**: End date of the underlying period (in years, if applicable).
- **T**: Latest option exercise date (in years, if applicable).
- **SD**: Supervisory duration (for IR and Credit).
- **CurrentPrice**: Current price of the underlying (if applicable).
- **NumNominalFlows**: Not used at the moment
- **Price**: Price used in calculating the supervisory delta for options (if applicable).
- **Strike**: Option strike price (if applicable).
- **Volatility**: Supervisory option volatility used for the trade (if applicable).

If there is a basis swap with two floating legs, there will be two entries in the report, one for each leg / main risk factor.

- **capital_crif.csv**: All data required to compute SA-CCR in the ISDA Regulatory Capital Model (CRIF) format.

5.3 CVA Capital

General provisions

- Regulatory CVA may differ from accounting CVA, e.g. excludes the effect of the bank's own default (DVA), there are several best practices constraints
- CVA risk is defined as the risk of losses arising from changing CVA values in response to changes in counterparty credit spreads and market risk factors
- Transactions with qualified central counterparties are excluded from CVA Capital calculations
- CVA Capital is calculated for the full portfolio (across all netting sets), including CVA hedges
- There are two approaches, basic (BA-CVA) and standardised (SA-CVA), the latter requires regulatory approval. SA-CVA banks can carve out netting sets and apply BA-CVA to these.
- There is a materiality threshold of 100 billion EUR aggregate notional of non-centrally cleared derivatives. When below, a bank can choose to set its CVA Capital to 100% of the CCR Capital. The regulator can remove this option.
- When calculating CCR Capital, the maturity adjustment factor may be capped at 1 for all netting sets contributing to CVA Capital.

5.3.1 Basic Approach, BA-CVA

There are two flavours of the basic approach

- a reduced version that does not recognise hedges
- a full version that does

Note: The implementation in ORE covers the reduced version so far.

5.3.1.1 Reduced Version

The total BA-CVA Capital charge according to the targeted revised framework [7] is

$$D_{BA-CVA} \times K_{reduced}$$

with discount scalar $D_{BA-CVA} = 0.65$ and

$$K_{reduced} = \sqrt{\underbrace{\left(\rho \sum_c SCVA_c\right)^2}_{\text{systematic component}} + \underbrace{(1-\rho^2)\sum_c SCVA_c^2}_{\text{idiosyncratic component}}}$$

where $\rho = 0.5$ and $SCVA_c$ is the stand-alone BA-CVA charge for counterparty c.

Stand-alone BA-CVA Capital charge, sum over netting sets NS:

$$SCVA_c = \frac{1}{\alpha} \cdot RW_c \cdot \sum_{NS} M_{NS} \cdot EAD_{NS} \cdot DF_{NS}$$

where

- $\alpha = 1.4$
- RW_c is the counterparty risk weight, see [6] page 111
- M_{NS} is the netting set's effective maturity, see paragraphs 38 and 39 of Annex 4 of the Basel II framework [1], page 216-217
- EAD_{NS} is the netting set's exposure at default, calculated in the same way as the bank calculates it for minimum capital requirements for CCR
- DF_{NS} is a supervisory discount factor, equal to 1 for banks that use IMM to calculate EAD, otherwise equal to $(1 - \exp(-0.05 \cdot M_{NS}))/(0.05 \cdot M_{NS})$

Note: The implementation in ORE

- uses the SA-CCR EAD amounts
- ignores the idiosyncratic component so that

$$K_{reduced} = \rho \sum_c SCVA_c$$

$$BA - CVA = D_{BA-CVA} \times \rho \times \sum_c SCVA_c$$

5.3.1.2 Full Version

Eligible hedges are single-name or index CDS, referencing the counterparty directly or a counterparty in the same sector and region.

$$K_{full} = \beta \cdot K_{reduced} + (1 - \beta) \cdot K_{hedged}$$

where $\beta = 0.25$ to floor the effect of hedging, and

$$K_{hedged} = \sqrt{\underbrace{\left(\rho \cdot \sum_c (SCVA_c - SNH_c) - IH\right)^2}_{\text{systematic component}} + \underbrace{(1 - \rho^2) \cdot \sum_c (SCVA_c - SNH_c)^2}_{\text{idiosyncratic component}} + \underbrace{\sum_c HMA_c}_{\text{indirect hedges}}}$$

where

- SNH_c is a sum across all single-name hedges that are taken out to hedge the CVA risk of counterparty c:

$$SNH_c = \sum_{h \in c} r_{rc} \cdot RW_h \cdot M_h^{SN} \cdot B_h^{SN} \cdot DF_h^{SN}$$

see [6] page 113 and [7].

- IH is a sum over index hedges the are taken out to hedge CVA risk:

$$IH = \sum_i RW_i \cdot M_i^{ind} \cdot B_i^{ind} \cdot DF_i^{ind}$$

see [6] page 113-114 and [7].

- HMA_c is a "hedging misalignment parameter" to avoid that single-name hedges can take the capital charge to zero:

$$HMA_c = \sum_{h \in c} (1 - r_{hc}^2) \cdot RW_h \cdot M_h^{SN} \cdot B_h^{SN} \cdot DF_h^{SN}$$

  with same parameters as in the calculation of SNH_c.

Note: The full version of BA-CVA is not implemented in ORE yet.

5.3.1.3 Implementation Details

Since the BA-CVA calculation is a relatively simple calculation that utilises the EAD amounts in a SA-CCR calculation, the same configuration for SA-CCR applies for BA-CVA. See Section 5.2.2.7.

5.3.2 Standard Approach, SA-CVA

SA-CVA uses as inputs the sensitivities of regulatory CVA (see below) to counterparty credit spreads and market risk factors driving covered transactions' values.

The SA-CVA calculation generally takes delta and vega risk into account for five risk types: interest rates (IR), foreign exchange (FX), reference credit spreads, equity and commodity. Note that vega risk includes sensitivity of option instruments and sensitivity of the CVA model calibration to input volatilities.

We denote s_k^{CVA} the sensitivity of the aggregate CVA to risk factor k and s_k^{Hdg} the sensitivity of all eligible CVA hedges to risk factor k. Eligible are hedges of both credit spreads and exposure components. Shift sizes are specified in section C.6 of [6].

Given the CVA sensitivities and regulatory risk weights and correlations, the calculation of SA-CVA is straightforward.

Bucket level capital charge:

$$K_b = \sqrt{\left[\sum_{k \in b} WS_k^2 + \sum_{k \neq l \in b} \rho_{kl} \cdot WS_k \cdot WS_l\right] + R \cdot \sum_{k \in b} \left(WS_k^{Hdg}\right)^2}$$

where $R = 0.01$ and

$$WS_k = WS_k^{CVA} + WS_k^{Hdg}, \qquad WS_k^{CVA} = RW_k \cdot s_k^{CVA}, \qquad WS_k^{Hdg} = RW_k \cdot s_k^{Hdg}$$

with risk weights RW_k and correlations ρ_{kl} as specified in Section C.6 of [6] and in [7].

Bucket-level capital charges must then be aggregated across buckets within each risk type :

$$K = m_{CVA} \cdot \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} \cdot K_b \cdot K_c}$$

with multiplier $m_{CVA} = 1.25$ and correlation parameters γ_{bc} as specified in Section C.6 of [6].

5.3.2.1 Regulatory CVA Calculation and CVA Sensitivity

Regulatory CVA is the basis for the calculation of the CVA risk capital requirement. Calculations of regulatory CVA must be performed for each counterparty with which a bank has at least one covered position.

Regulatory CVA at a counterparty level must be calculated according to the following principles [6] pages 115-117:

- Based on Monte Carlo simulation of exposure evolution, consistent with front office/accounting CVA
- Risk neutral probability measure
- Model calibration to market data where possible
- Use of PDs implied from credit spreads observed in the market, use of market-consensus LGDs
- Netting recognition applies as in accounting CVA
- Collateral (VM, IM): Exposure simulation must capture the effects of margining collateral that is recognised as a risk mitigant along each exposure path. All the relevant contractual features such as the nature of the margin agreement (unilateral vs bilateral), the frequency of margin calls, the type of collateral, thresholds, independent amounts, initial margins and minimum transfer amounts must be appropriately captured by the exposure model; the Margin Period of Risk has to be taken into account

The regulatory CVA calculation is based on ORE's XVA analytics. It takes CSA details for simulating VM balances into account (thresholds, minimum transfer amounts), as well as the Margin Period of Risk. Initial Margin is modelled as a stochastic Dynamic Delta VaR along all Monte Carlo paths.

The product scope for CVA sensitivity and SA-CVA is so far:

- FX Forwards
- FX Options
- Cross Currency Swaps

For this scope we assume independence of credit and other market factors. With this simplification, the calculation of CVA sensitivities w.r.t. credit factors does not require the recalculation of exposure profiles. IR/FX delta and vega calculation, however, does require the recalculation of exposures under each shift scenario. The number of scenarios is minimized and tailored to the portfolio. Moreover we make use of multithreading and further parallelization techniques to reduce calculation times. The shifts/recalculations required to cover the product scope above are listed below in table 5.9.

5.3.2.2 Implementation Details

This section describes some of the requirements and implementation of the SA-CVA capital analytic in ORE.

The SA-CVA capital analytic uses the following input files:

1. Netting Set Definitions (see User Guide)
2. Counterparty Information (see User Guide)

Optional Collateral Balances (see User Guide)

Optional Scenario Generator Data (see the `Parameters` block in the user guide's simulation parameterisation section)

Optional Cross Asset Model (see the `CrossAssetModel` block in the documentation above

Optional Instantaneous Correlations (see the `InstantaneousCorrelations` block in the documentation above

Optional Scenario Sim Market Parameters (see `Market` block in the documentation above

Risk Type	Risk Factor	Shift
IR Delta for currencies USD, EUR, GBP, AUD, CAD, SEK, JPY	Currency and tenor (1y, 2y, 5y, 10y, 30y)	Shift of each tenor point for all curves with the given currency curve, absolute shift size 1BP
IR Delta for any other currency	Currency	Parallel shift of all yield curves for given given currency by 1BP
FX Delta	Foreign currency (vs. a fixed domestic currency)	FX shift for any foreign1/foreign2 currency pair is obtained by triangulation from the "fundamental" foreign/domestic FX rates; relative shift by 1% for the base currency FX rate
FX Vega	Foreign currency (vs. a fixed domestic currency)	Volatility shift for any foreign1/foreign2 currency pair is implied from the "fundamental" foreign1/domestic and foreign2/domestic volatility and a fixed implied correlation; simultaneous 1% relative shift for all volatilities in the fundamental volatility surface
Counterparty Credit Delta	Entity and tenor point (0.5y, 1y, 3y, 5y, 10y)	Absolute shift of the relevant credit spread by 1BP, aggregation of sensitivities across entities within sector buckets 1-7
Reference Credit Delta	Reference credit sector (1-15)	Simultaneous absolute shift of the relevant credit spreads by 1BP, for all reference names in the bucket, across all tenor points

Table 5.9: Risk factors and shifts.

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