ORE Formula Based Coupon Module

Quaternion Risk Management

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Document History

ORE+ Release	Date	Author	Comment
na	29 August 2018	Peter Caspers	initial version
1.8.4.0	20 February 2019	Sarp Kaya Acar	add examples

Contents

1 Summary

This document describes the formula based coupon implemented in ORE+.

2 Methodology

2.1 Introduction

We consider a generalised structured coupon which at time T pays

$$\tau f(R_1(t), \ldots, R_n(t)) \tag{1}$$

where τ is the year fraction of the coupon, R_i's are IBOR and/or CMS rates fixed at time $t < T$ and f is a payoff function. Moreover we assume that the currency of the coupon can be different than the currency of the rates, i.e. the quanto-payoffs are allowed. For instance, let us consider the below example

$$f(R_1, R_2) = 1_{\{R_3 > 0.03\}} \max\{\min\{9 \cdot (R_2 - R_1) + 0.02, 0.08\}, 0.0\}, \tag{2}$$

with R_1 =GBP-CMS-2Y, R_2 =EUR-CMS-10Y, R_3 =USD-CMS-5Y. With the Formula-Based-Coupon module it is possible to price such kind of coupons by using the Monte-Carlo simulation technique. In the next section, we will present the underlying pricing model.

2.2 Pricing Model

For non-callable structured coupons, we generalise the model in [2], 13.16.2. We assume the rates to evolve with a shifted lognormal dynamics under the T-forward measure in the respective currency of R_i as

$$dR_i = \mu_i(R_i + d_i)dt + \sigma_i(R_i + d_i)dZ_i \tag{3}$$

with displacements $d_i > 0$, drifts μ_i and volatilities σ_i or alternatively with normal dynamics

$$dR_i = \mu_i dt + \sigma_i dZ_i \tag{4}$$

for $i = 1, \ldots, n$ where in both cases Z_i are correlated Brownian motions

$$dZ_i dZ_j = \rho_{i,j} dt \tag{5}$$

with a positive semi-definite correlation matrix $(\rho_{i,j})$. The drifts μ_i are determined using given convexity adjustments

$$c_i = E^T(R_i(t)) - R_i(0) \tag{6}$$

where the expectation is taken in the T-forward measure in the currency of the respective rate R_i. Slightly abusing notation c_i can be computed both using a model consistent with 3 resp. 4 (i.e. a Black76 style model) or also a different model like e.g. a full smile TSR model to compute CMS adjustments. While the latter choice formally introduces a model inconsistency it might still produce more market consistent prices at the end of the day, since it centers the distributions of the R_i around a mean that better captures

the market implied convexity adjustments of the rates R_i entering the structured coupon payoff.
Under shifted lognormal dynamics the average drift is then given by

$$\mu_i = \frac{\log((R_i(0) + d_i + c_i)/(R_i(0) + d_i))}{t} \tag{7}$$

and under normal dynamics

$$\mu_i = c_i/t \tag{8}$$

The NPV ν of the coupon is now given by

$$\nu = P(0,T)\tau E^T(f(R_1(t),\ldots,R_n(t))) \tag{9}$$

where $P(0,T)$ is the applicable discount factor for the payment time in the domestic currency and the expectation is taken in the T-forward measure in the domestic currency. To adjust 3 resp. 4 for the measure change between the currency of R_i and the domestic currency (if applicable), we apply the usual Black quanto adjustment

$$\mu_i \to \mu_i + \sigma_i \sigma_{i,X} \rho_{i,X} \tag{10}$$

where $\sigma_{i,X}$ is the volatility of the applicable FX rate and $\rho_{i,X}$ is the correlation between the Brownian motion driving the FX process and Z_i.
To evaluate the expectation in 9 a Monte Carlo simulation can be used to generate samples of the distribution of $(R_1,\ldots,R_n)$, evaluate f on these samples and average the results.

3 Parametrisation

3.1 Formula Based Leg Data

The formula based leg data allows to use complex formulas to describe coupon payoffs. Its `LegType` is `FormulaBased`, and it has the data section `FormulaBasedLegData`. It supports IBOR and CMS based payoffs with quanto and digital features. The following examples shows the definition of a coupon paying a capped / floored cross currency EUR-GBP CMS Spread contingent on a USD CMS barrier.
The `Index` field supports operations of the following kind:

- indices like IBOR and CMS indices, and constants as factors, spreads and/or cap/floor values;
- basic operations: $+$, $-$, $\cdot$, $/$;
- operators gtZero() (greater than zero) and geqZero() (greater than or equal zero) yielding 1 if the argument is > 0 (resp. ≥ 0) and zero otherwise
- functions: abs(), exp(), log(), min(), max(), pow()

In listing 1, we present the `FormulaBasedLegData` of the payoff in equation 2.

Listing 1: FormulaBasedLegData configuration.

```
<LegData>
  <LegType>FormulaBased</LegType>
  <Payer>true</Payer>
  <Currency>EUR</Currency>
   ...
  <FormulaBasedLegData>
    <Index>gtZero({USD-CMS-5Y}-0.03)*
              max(min(9.0*({EUR-CMS-10Y}-{GBP-CMS-2Y})+0.02,0.08),0.0)</Index>
    <IsInArrears>false</IsInArrears>
    <FixingDays>2</FixingDays>
  </FormulaBasedLegData>
   ...
</LegData>
```

Listing 2: Pricing engine configuration for formula based coupon pricer.

```
  <!-- Formula Based Coupons -->
  <Product type="FormulaBasedCoupon">
    <Model>BrigoMercurio</Model>
    <ModelParameters>
      <Parameter name="FXSource">ECB</Parameter>
    </ModelParameters>
    <Engine>MC</Engine>
    <EngineParameters>
      <Parameter name="Samples">1000</Parameter>
      <Parameter name="Seed">42</Parameter>
      <Parameter name="Sobol">Y</Parameter>
      <Parameter name="SalvagingAlgorithm">Spectral</Parameter>
    </EngineParameters>
  </Product>
```

4 Pricing Engine settings

The configuration of the pricing engine for the model introduced in section 2.2 is given in listing 2.

- The formula based coupon pricer uses the pricing engine configuration for `CapFlooredIborLeg` and `CMS`. For the configuration of these coupon pricers we refer to sections 7.3, 7.7.3 and 7.74 in [1].

- The parameter `FXSource` specifies the FX index tag to be used to look up FX-Ibor or FX-CMS correlations, see also 7.10.2 in [1].

5 Market Data

The relevant market data for the formula based coupon pricer encompasses

1. rate curves (for index projection and cashflow discounting)

2. cap / floor volatilities (for Ibor coupon pricers in the relevant currencies)

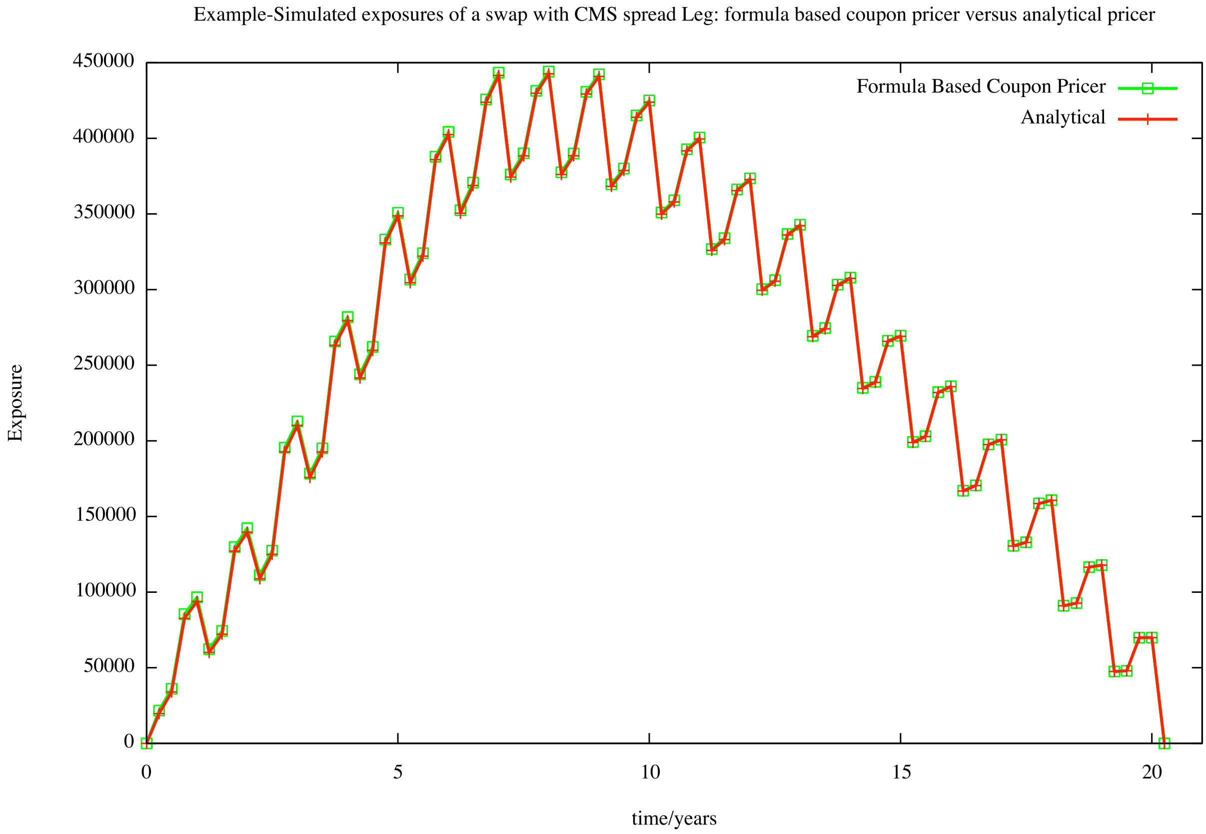

Figure 1: 20Y EUR interest rate swap with floored CMS spread leg, i.e. max(*EUR-CMS-10Y−EUR-CMS-1Y, 0.5%*)

3. swaption volatilities (for CMS coupon pricers in the relevant currencies)
4. correlation curves for the relevant Ibor-Ibor, Ibor-CMS, CMS-CMS, Ibor-FX, CMS-FX pairs

See [1] for details on the setup and configuration of this market data.

6 Examples

6.1 CMS Spread

We demonstrate a single currency Swap (currency EUR, maturity 20y, notional 10m, receive fixed 0.011244% annual, pay $\max\{\text{CMS-EUR-10Y} - \text{CMS-EUR-1Y}, 0.0\}$ semi-annual). We simulate the exposure with 1000 Monte-Carlo paths by using the formula based coupon pricer and the analytical CMS-Spread pricer [section 7.10.8, [1]]. The number of Monte-Carlo paths in formula based coupon pricer is 1000. EPE profiles with both runs are given in figure 1. We observed that the run with the formula based coupon pricer is approximately 2.5 times slower than the analytical one.

6.2 Digital CMS Spread

As a second example, we demonstrate a single currency Swap (currency EUR, maturity 20y, notional 10m, receive fixed 0.011244% annual, pay (CMS-EUR-10Y − CMS-EUR-1Y)+

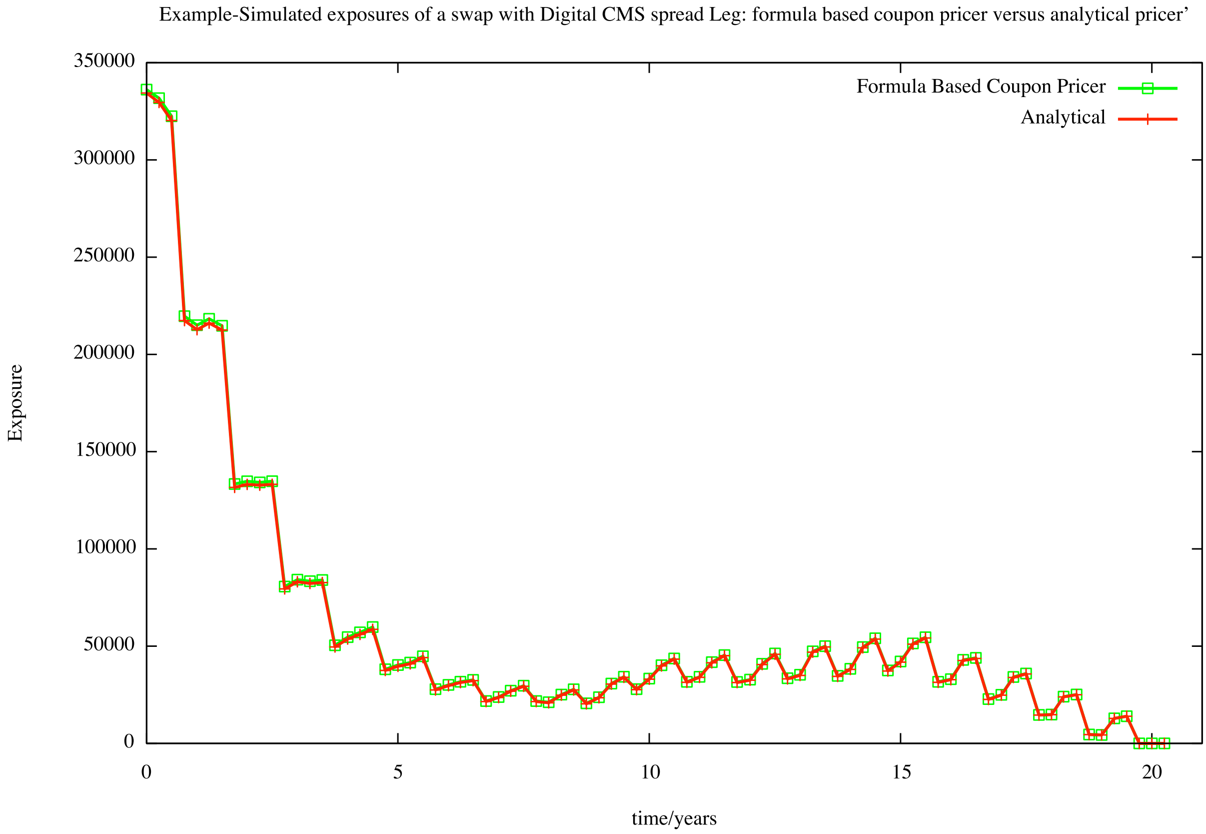

Figure 2: 20Y EUR interest rate swap with digital CMS spread leg, i.e. $(\textit{CMS-EUR-10Y} - \textit{CMS-EUR-1Y}) + 0.01 * 1_{\{\textit{CMS-EUR-10Y}-\textit{CMS-EUR-1Y}>0.0\}}$

$0.01 * 1_{\{\text{CMS-EUR-10Y}-\text{CMS-EUR-1Y}>0.0\}}$ semi-annual).

References

[1] ORE User Guide, http://www.opensourcerisk.org/documentation/

[2] Brigo, Damiano; Mercurio, Fabio: Interest Rate Models - Theory and Practice, 2nd Edition, Springer, 2006