Homework I

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Initialization

Run the following code to import the modules you'll need. After your finish the assignment, remember to run all cells and save the note book to your local machine as a PDF for gradescope submission.

```
In [1]: import os
   import math
   import random
   import glob
   import numpy as np
   from PIL import Image
   import matplotlib.pyplot as plt
   from skimage.util import montage

   np.random.seed(0)
```

1. Setup dataset

In this section we will download the dataset, unzip it and setup the paths to load images from.

This dataset is a tiny subset of ImageNet, a popular dataset for image classification.

This tiny dataset has **9538 training** images and **3856 test** images spanning **10 classes** {*fish*, *English-springer*, *cassette-player*, *chain-saw*, *church*, *French-horn*, *garbage-truck*, *gas-pump*, *golf-ball*, *parachute*} stored in the following directory structure:

```
dataset
---train
---class1
---class2
---test
---class1
---class2
```

The data has been cleaned and we have provided dataloading functions below so you can directly use the dataset.

```
In [2]: if not os.path.exists('content/imagenette.zip'):
    #!wget "https://drive.google.com/uc?export=download&id=1t3XtxcpVwZnKhsM95Q89MxYNlX5mj6aJ&confirm=
    !wget https://www.cs.cmu.edu/~deva/data/imagenette.zip -0 content/imagenette.zip
    !unzip -qq "content/imagenette/train'
    test_data_path = 'content/imagenette/test'
    train_image_paths = [] #to store image paths in list
    test_image_paths = []
    classes = []

#Get all the paths from train_data_path and append image paths and class to to respective lists

for data_path in glob.glob(train_data_path + '/*'):
        classes.append(data_path.split(')')[-1])
        train_image_paths.append(glob.glob(data_path + '/*'))
```

```
for data path in glob.glob(test data path + '/*'):
            test_image_paths.append(glob.glob(data_path + '/*'))
        train image paths = list(sum(train image paths,[]))
        random.shuffle(train_image_paths)
        test_image_paths = list(sum(test_image_paths,[]))
        random.shuffle(test_image_paths)
        idx_to_class = {i:j for i, j in enumerate(classes)}
        class_to_idx = {value:key for key,value in idx_to_class.items()}
In [4]: def LoadData(img_paths,img_size,class_to_idx):
         n = len(img_paths)
         Images = np.zeros((n,img_size,img_size,3),dtype='uint8')
         Labels = np.zeros(n)
          for i in range(n):
            path = img_paths[i]
            Images[i,:,:,:] = np.asarray(Image.open(path).resize((img_size,img_size)));
            Labels[i] = class to idx[path.split('/')[-2]]
          return Images, Labels
        # Load images as size 32x32; you can try with img_size = 64 to check if it improves the accuracy
        img size = 32
        Train_Images, Train_Labels = LoadData(train_image_paths, img_size, class_to_idx)
        Test_Images, Test_Labels = LoadData( test_image_paths, img_size, class_to_idx)
In [5]: # Visualize the first 5 images of the 10 classes
        plt.figure(figsize=(15,15))
        for i in range(10):
          plt.subplot(10,1,i+1)
          ind = np.nonzero(Train_Labels == i)[0]
          plt.imshow(montage(Train_Images[ind[:5],:],grid_shape=(1,5),channel_axis=3))
          plt.axis('off');
          plt.title(idx to class[i])
```



Debug Flag

Set the debug flag to true when testing. Setting the debug flag to true will let the dataloader use only 20% of the training dataset, which makes everything run faster. This will make testing the code easier.

Once you finish the coding part please make sure to change the flag to False and rerun all the cells. This will make the colab ready for submission.

```
In [6]: DEBUG = False
        # Take a smaller subset of the training set for efficient execution of kNN
        # We also create a small validation set
        if DEBUG:
          num train = 1900
          num\_test = 700
        else:
          num train = 9000
          num_test = 3856
        X_train = Train_Images[:num_train].reshape(num_train,-1).astype('float64')
        y_train = Train_Labels[:num_train]
        X_test = Test_Images[:num_test].reshape(num_test,-1).astype('float64')
        y_test = Test_Labels[:num_test]
        print('Train data shape: ' , X_train.shape)
        print('Train labels shape: ', y_train.shape)
        print('Test data shape: ' , X_test.shape)
print('Test labels shape: ' , y_test.shape)
       Train data shape: (9000, 3072)
       Train labels shape: (9000,)
       Test data shape: (3856, 3072)
       Test labels shape: (3856,)
```

Problem 3.1

(a) Define the KNearestNeighbor class

```
In [7]: from collections import Counter
        class KNearestNeighbor(object):
          """ a kNN classifier with L2 distance """
          def __init__(self):
            pass
          def train(self, X, y):
            Train the classifier. For k-nearest neighbors this is just
            memorizing the training data.
            - X: A numpy array of shape (num_train, D) containing the training data
             consisting of num_train samples each of dimension D.
            - y: A numpy array of shape (N,) containing the training labels, where
                 y[i] is the label for X[i].
            self.X_train = X
            self.y\_train = y
          def predict(self, X, k=1, num_loops=0):
            Predict labels for test data using this classifier.
            - X: A numpy array of shape (num test, D) containing test data consisting
                 of num_test samples each of dimension D.
```

```
- k: The number of nearest neighbors that vote for the predicted labels.
  - num loops: Determines which implementation to use to compute distances
   between training points and testing points.
 Returns:
  - y: A numpy array of shape (num test,) containing predicted labels for the
   test data, where y[i] is the predicted label for the test point X[i].
 if num_loops == 0:
    dists = self.compute_distances_no_loops(X)
 elif num_loops == 1:
    dists = self.compute_distances_one_loop(X)
 elif num_loops == 2:
    dists = self.compute_distances_two_loops(X)
  else:
    raise ValueError('Invalid value %d for num loops' % num loops)
 return self.predict_labels(dists, k=k)
def compute distances two loops(self, X):
 Compute the 12 distance between each test point in X and each training point
 in self.X train using a nested loop over both the training data and the
 Inputs:
 - X: A numpy array of shape (num_test, D) containing test data.
 Returns:
  dists: A numpy array of shape (num test, num train) where dists[i, j]
   is the Euclidean distance between the ith test point and the jth training
    point.
 num_test = X.shape[0]
 num_train = self.X_train.shape[0]
 dists = np.zeros((num_test, num_train))
 for i in range(num test):
    for j in range(num_train):
      # Compute the l2 distance between the ith test image and the jth
     dists[i,j] = np.linalg.norm(X[i] - self.X_train[j])
 return dists
def compute_distances_one_loop(self, X):
  Compute the 12 distance between each test point in X and each training point
  in self.X_train using a single loop over the test data.
 Input / Output: Same as compute_distances_two_loops
 num_test = X.shape[0]
 num_train = self.X_train.shape[0]
 dists = np.zeros((num_test, num_train))
 for i in range(num test):
   # Compute the 12 distance between the ith test point and all training
   # points, and store the result in dists[i, :].
   dists[i, :] = np.linalg.norm(self.X_train - X[i], axis=1)
  return dists
def compute_distances_no_loops(self, X):
  Compute the l2 distance between each test point in X and each training point
 in self.X_train using no explicit loops.
 Input / Output: Same as compute_distances_two_loops
 num test = X.shape[0]
 num train = self.X train.shape[0]
 dists = np.zeros((num test, num train))
 train_norm = np.sum(self.X_train**2, axis=1)
 test_norm = np.sum(X**2, axis=1)[:, np.newaxis]
 cross_term = np.dot(X, self.X_train.T)
 dists = np.sqrt(test_norm + train_norm - 2 * cross_term)
```

```
return dists
def predict_labels(self, dists, k=1):
 Given a matrix of distances between test points and training points,
 predict a label for each test point.
 Inputs:
  - dists: A numpy array of shape (num_test, num_train) where dists[i, j]
    gives the distance betwen the ith test point and the jth training point.
  - y: A numpy array of shape (num test,) containing predicted labels for the
    test data, where y[i] is the predicted label for the test point X[i].
  - knn_idxs: List of arrays, containing Indexes of the k nearest neighbors
    for the test data. So, for num_tests, it will be a list of length
    num_tests with each element of the list, an array of size 'k'. This will
   be used for visualization purposes later.
 num test = dists.shape[0]
 y_pred = np.zeros(num test)
  knn_idxs = []
 for i in range(num_test):
   closest_y = []
   # Use the distance matrix to find the k nearest neighbors of the ith
   # testing element, and use self.y_train to find the labels of these
    # neighbors. Store these labels in closest_y.
   top_k_indx = np.argsort(dists[i])[:k]
    closest_y = self.y_train[top_k_indx]
    knn_idxs.append(top_k_indx)
    vote = Counter(closest_y)
    count = vote.most_common()
    y_pred[i] = count[0][0]
  return y pred, knn idxs
```

(b) Check L2 distance implementation

Now, let's do some checks to see if you have implemented the functions correctly. We will first calculate distances using *compute_distance_two_loops* and check accuracy for k=1 and k=3. Then, we will compare the *compute_distance_one_loop* and *compute_distance_no_loop* with *compute_distance_two_loops* to ensure all results are consistent.

Initialize the KNN Classifier

```
In [8]: classifier = KNearestNeighbor()
  classifier.train(X_train,y_train)
```

Compute the distance between the training and test set. This might take some time to run since we are running the two loops function which is not efficient.

6 to 8 mins for full dataset | 2 to 3 mins for debug dataset

```
In [9]: dists_two = classifier.compute_distances_two_loops(X_test)
```

Now, let's do some checks to see if you have implemented the functions correctly. We will first calculate the distances using compute_distance_two_loops function and check the accuracies for k=1 and k=3. Then, we will compare the compute_distance_one_loop and compute_distance_no_loop functions with it to check their correctness.

Predict labels and check accuracy for k = 1. You should expect to see approximately 28% accuracy for full dataset. (Accuracy below 24% on full dataset (Debug = False) will not be given full grades)

```
In [10]: y_test_pred, k_idxs = classifier.predict_labels(dists_two, k=1)
# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 1096 / 3856 correct => accuracy: 0.284232

Now lets check the one loop implementation. This should also take some time to run.

4 to 6 mins for full dataset | 1 to 2 mins for debug dataset

Note: This function can possibly take a little more time that two loop implementation because of some quirks in python, numpy and cpu processing. It is fine as long as the final output shows no difference below.

Difference was: 0.000000 Good! The distance matrices are the same

Now lets check the vectorized implementation. This should take less than 30 secs to run for full dataset.

```
In [12]: # Now implement the fully vectorized version inside compute_distances_no_loops
# and run the code
dists_no = classifier.compute_distances_no_loops(X_test)
# check that the dist ance matrix agrees with the one we computed before:
difference = np.linalg.norm(dists_two - dists_no, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')</pre>
```

Difference was: 0.000000 Good! The distance matrices are the same

Let's compare how fast the implementations are You should see significantly faster performance with the fully vectorized implementation

```
In [13]:
    def time_function(f, *args):
        """
        Call a function f with args and return the time (in seconds) that it took to execute.
        """
        import time
        tic = time.time()
        f(*args)
        toc = time.time()
        return toc - tic

two_loop_time = time_function(classifier.compute_distances_two_loops,X_test)
print('Two loop version took %f seconds' % two_loop_time)

one_loop_time = time_function(classifier.compute_distances_one_loop,X_test)
print('One loop version took %f seconds' % one_loop_time)
```

```
no_loop_time = time_function(classifier.compute_distances_no_loops,X_test)
print('No loop version took %f seconds' % no_loop_time)
# you should see significantly faster performance with the fully vectorized implementation
```

Two loop version took 138.603475 seconds One loop version took 241.449752 seconds No loop version took 1.107140 seconds

From this point on, we will use the efficient no loop implementation

The given accuracy of 29% is much better than chance accuracy of

A: Naive chance is 10% considering each of the 10 classes in the dataset has equal amounts of samples.

Though the no-loop impermentation is far faster, there maybe situations where one_loop or two_loop implementations are useful, such as [HINT: Imagine really large training set and or testset]

A: Vectorization comess at the cost of memory usage as all data needs to be loaded to perform the operation in one step. Thus, when handling large datasets, or for that matter large batches of a dataset, if the system memory is contrained vectorization may not be posisble. On the other extreme, when the data is very small, the upfront cost of copying everything to memory may actually be larger than performing the iterative operation.

```
In [14]: y_test_pred, k_idxs = classifier.predict_labels(dists_no, k=3)
# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 1131 / 3856 correct => accuracy: 0.293309

Visualize KNN results

Let's visualize the K nearest images for some randomly selected examples from the test set using the k_idxs list you returned in predict_labels.

Here the leftmost column is the input image from the test set and rest of the columns are the K nearest neighbors from the training set

```
In [15]:

def visualize_knn(classifier,X_test,N=5, K=7):
    # This visualization routine makes use of GLOBAL Train_Images and Test_Images variables
    # to visualize the K nearest neighbors of the first N Test Images

dist = classifier.compute_distances_no_loops(X_test[:N,:])
    _, k_idxs = classifier.predict_labels(dist,k=K)
    k_idxs = np.vstack(k_idxs)
    testim = montage(Test_Images[:N,:],grid_shape=(N,1),channel_axis=3)
    trainim = montage(Train_Images[k_idxs.ravel(),:],grid_shape=(N,K),channel_axis=3)
    plt.imshow(np.concatenate((testim,trainim),axis=1))
    plt.axis('off');
    plt.title('Test [leftmost column], K_neighbors [right columns]');

visualize_knn(classifier,X_test)
```

Test [leftmost column], K neighbors [right columns]



Normalizing image descriptors:

Let us try normalizing each image here by subtracting by its mean and scaling to have unit norm.

```
In [16]: # Normalize each image descriptor to have zero-mean and unit-length

X_train_norm = X_train
X_test_norm = X_test

train_mean = np.mean(X_train_norm, axis=1, keepdims=True)

train_std = np.std(X_train_norm, axis=1, keepdims=True)

test_mean = np.mean(X_test_norm, axis=1, keepdims=True)

test_std = np.std(X_test_norm, axis=1, keepdims=True)

# ===== your code here! =====

# Normalize each image descriptor to have zero-mean and unit-length
# If X is the descriptor vector for a given image, then sum_i X[i] = 0 and sum_i X[i]**2 = 1

X_train_norm = (X_train_norm - train_mean)/ train_std
X_test_norm = (X_test_norm - test_mean) / test_std
# ==== end of code =====

print('Train data shape: ', X_train_norm.shape)
print('Test data shape: ', X_test_norm.shape)
Train data shape: (9000, 3072)
```

We calculate the accuracies again using k = 1 and k = 3 and see that the accuracies are much better compared to those we obtained without any preprocessing on the images!

```
In [17]: classifier = KNearestNeighbor()
    classifier.train(X_train_norm,y_train)

# Classify using the efficient no_loops implementation
    dists = classifier.compute_distances_no_loops(X_test_norm)
    y_test_pred, k_labels = classifier.predict_labels(dists, k=3)

# Compute and print the fraction of correctly predicted examples
    num_correct = np.sum(y_test_pred == y_test)
    accuracy = float(num_correct) / num_test
    print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))

Got 1310 / 3856 correct => accuracy: 0.339730
```

Test data shape: (3856, 3072)

Written question: Normalization produces image descriptors that have unit length. Prove that minimizing the euclidean distance of such descriptors is equivalent to maximizing the cosine similarity. Here is an example of latex in markdown that might be helpful: $||x-y||^2 = x^Tx - 2x^Ty + y^Ty$

A:

Cosine similarity gives an idea of the angular distance of two vectors, regardless of their magnitude. And is defined as

$$\cos heta = rac{x^T y}{||x||||y||}$$

Given that we have normalized our vectors, the magnitude of our vectors is one, such that,

$$\cos \theta = x^T y$$

Similarly, we can simplify the euclidean distance,

$$\left|\left|x - y\right|\right|^2 = 1 - 2(x^T y) + 1$$

 $\left|\left|x - y\right|\right|^2 = 2(1 - x^T y)$

Subtituting the cosine in this expression,

$$\left|\left|x-y\right|\right|^2 = 2(1-\cos\theta)$$

Here we can see that by maximizing the $\cos \theta$ we minimize the euclidean distance.

KNN with HOG

The previous parts all directly used raw pixels from input images to compute distances with k-NN. In this part, we will first use the Histogram of Oriented Gradients (HOG) as features for each image. We will use these features with our kNN implementation to find the nearest neighbours. Please read the descriptions and fill in the functions below.

```
In [18]: def compute_gradient(image):
           is_color = len(image.shape) == 3
           if is_color:
             image = image.mean(axis=2)
           angles = np.zeros like(image)
           delta_x = np.zeros_like(image)
           delta_y = np.zeros_like(image)
           height, width = image.shape
           # Compute the gradients along the rows and columns as two arrays.
           for i in range(height):
             for j in range(width):
               # Gradient along the rows
               if j == 0:
                 delta_x[i, j] = image[i, j + 1] - image[i, j]
               elif j == width - 1:
                 delta_x[i, j] = image[i, j] - image[i, j - 1]
               else:
                 delta_x[i,j] = (image[i, j+1] - image[i,j-1]) /2.0
               # Gradient along the columns
               if i == 0:
                 delta_y[i, j] = image[i + 1, j] - image[i, j]
               elif i == height - 1:
                 delta_y[i, j] = image[i, j] - image[i - 1, j]
```

```
else:
    delta_y[i,j] = (image[i+1, j] - image[i-1,j]) /2.0

# Compute the magnitude as the square root of the sum of the squares of both gradients
magnitudes = np.sqrt(delta_x ** 2 + delta_y ** 2)
# Sanity check with numpy's gradient
gradient_error = np.linalg.norm(magnitudes - np.linalg.norm(np.gradient(image), axis=0))
assert np.allclose(magnitudes, np.linalg.norm(np.gradient(image), axis=0)), f"Error, gradient inc
# Compute the angles as the inverse tangent of the gradients along the rows and the gradients alc
for row in range(image.shape[0]):
    for col in range(image.shape[1]):
        angles[row, col] = math.degrees(math.atan2(delta_y[row, col], delta_x[row, col]))
return magnitudes, angles
```

```
In [19]: def bin_gradient(angles, magnitudes, n_orient, pixels_per_cell):
           Given the gradient orientations and magnitudes of an image, creates
           a histogram of orientations weighted by gradient magnitudes
           Inputs:
           - angles: A numpy array of shape (32, 32) where angles[i,j]
               is the angle of the gradient at the (i,j) pixel in the input image.
           - magnitudes: A numpy array of shape (32, 32) where magnitudes[i,j]
               is the magnitude of the gradient at the (i,j) pixel in the input image.
           - n_orient: An int representing the number of orientations to bin in histogram
           - pixels_per_cell: An int representing the number of rows/columns of pixels
               in each spatial cell
           - oriented_histogram: A numpy array of shape (32/4=8, 32/4=8,9)
               for pixels_per_cell=4 and n_orient=9
           n_y, n_x = angles.shape
           oriented_histogram = np.zeros((int(n_y//pixels_per_cell),int(n_x//pixels_per_cell),n_orient))
           # Iterate through each pixel in every cell
           for py in range(0, n_y, pixels_per_cell):
             for px in range(0, n_x, pixels_per_cell):
               for i in range(pixels_per_cell):
                 for k in range(pixels_per_cell):
                   val = magnitudes[py + i, px + k]
                   bin_idx = int(angles[py +i, px + k] // (180 / n_orient))
                   if bin_idx == n_orient:
                     bin_idx = n_orient - 1 # wrap around the bin idx
                   oriented_histogram[py // pixels_per_cell, px // pixels_per_cell, bin_idx] += val
           return oriented_histogram
```

NOTE: Once we create a histogram based on the gradient of the image we need to normalize it. Gradients of an image are sensitive to overall lighting. If you make the image darker by dividing all pixel values by 2, the gradient magnitude will change by half, and therefore the histogram values will change by half.

Ideally, we want our image features to be independent of lighting variations. In other words, we would like to "normalize" the histogram so they are not affected by lighting variations.

We have provided the normalization code below.

```
Returns:
normalized blocks: A numpy array of k,num orient) where normalized blocks[i,j] is a normalized
n blocks y = oriented histogram.shape[0]-cells per block+1
n_blocks_x = oriented_histogram.shape[1]-cells_per_block+1
normalized_blocks = np.zeros((n_blocks_y,n_blocks_x,cells_per_block,cells_per_block,oriented_hist
# ===== your code here! =====
for y in range(n blocks y):
  for x in range(n blocks x):
    block = oriented_histogram[y:y+cells_per_block,x:x+cells_per_block] # Get corresponding secti
    block = block - np.mean(block)
    block = block / np.sqrt(np.sum(block**2) + epsilon) # Normalize block
    if clip:
      block = np.minimum(block, 0.2)
      block = block - np.mean(block)
      block = block / np.sqrt(np.sum(block**2) + epsilon)
    normalized\_blocks[y,x] = block
return normalized blocks
```

After implementing your HOG functions, please run the cells below to test the results. You should expect to get an accuracy slightly higher than that with unnormalized raw pixels.

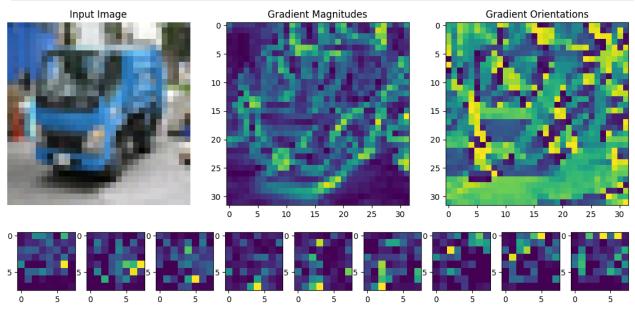
```
In [21]: def compute hog(image, n orient=9, pixels per cell=4, cells per block=4):
           Builds a Histogram of Oriented Gradients (HOG) weighted by gradient magnitudes
           from an input image
           Inputs:
           - image: A numpy array of shape (32, 32) containing one grayscaled image.
           Outputs:
           - histogram: A 1D numpy array that represents the HOG descriptor for the image.
           assert(image.dtype == 'float64')
           # Read in image and convert to grayscale
           if len(image.shape) > 2:
            image = np.mean(image,2)
           # Compute gradient
           magnitudes, angles = compute gradient(image)
           # Bin gradients into cells
           oriented histogram = bin gradient(angles, magnitudes, n orient, pixels per cell)
           # Block normalize the cells
           normalized_blocks = block_normalize(oriented_histogram, cells_per_block)
           # Return flattened descriptor (without making an additional copy)
           return normalized blocks.ravel()
```

```
In [22]: # Check out HOG descriptor for a single image
         \#image = X_{train[0].mean(2)} \# Initially, build representation for grayscale image
         image = X_train[0].reshape(img_size,img_size,3);
         plt.figure(figsize=(14,8))
         plt.subplot(1,3,1)
         plt.imshow(image.astype('uint8'));
         plt.axis('off')
         plt.title('Input Image')
         pixels_per_cell=4
         cells_per_block=4
         n_orient=9
         angle step = 180 // n orient
         # Step 1: compute gradients
         magnitudes, angles = compute_gradient(image)
         plt.subplot(1,3,2)
         plt.imshow(magnitudes)
         plt.title('Gradient Magnitudes')
```

```
plt.subplot(1,3,3)
plt.imshow(angles)
plt.title('Gradient Orientations')

# Step 2: Bin gradients into cells
oriented_histogram = bin_gradient(angles, magnitudes, n_orient, pixels_per_cell)
plt.figure(figsize=(14,8))
#plt.suptitle('Oriented Histograms')
for i in range(n_orient):
   plt.subplot(1,n_orient,i+1)
   plt.imshow(oriented_histogram[:,:,i])

# Step 3: Block normalize the cells
normalized_blocks = block_normalize(oriented_histogram, cells_per_block)
```



This part will take some time to run for the full dataset. Approx 1 to 2mins.

Got 1425 / 3856 correct => accuracy: 0.369554

You can also visualize the K nearest images for some randomly selected examples from the test set using the k_idxs list you returned in predict_labels trained with HOG descriptors.

```
In [26]: visualize_knn(classifier,X_test_hog)
```

print('Got %d / %d correct => accuracy: %f' % (num correct, num test, accuracy))

Test [leftmost column], K_neighbors [right columns]

