Analytical

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1 (a) $x(n) = x(n-1) + 5$

$x(1) = 0$

Sub $n = 2$

$x(2) = x(2-1)+5$

$= x(1)+5$

$= 0+5 = 5$

$x(3) = x(3-1)+5$

$= x(2)+5$

$= 5+5 = 10$

$x(4) = x(4-1)+5$

$= x(3)+5$

$= 10+5 = 15$

Sequence : 0, 5, 10, 15 . . . . .

By generalising,

$x(n) = 5n-5$ for $n>1$

c) $x(n) = 3x(n-1)$ for $n>1$

$x(1) = 4$

Sub $n = 2$

$x(2) = 3x(2-1)$

$= 3x(1)$

$= 3 \cdot 4 = 12$

Sub $n = 3$

$x(3) = 3x(3-1)$

$= 3x(2)$

$= 3 \cdot 12 = 36$

Sub $n = 4$

$x(4) = 3x(4-1)$

$= 3x(3)$

$= 3 \cdot 36 = 108$

Sub $n = 5$

$x(5) = 3x(5-1)$

$= 3x(4)$

$= 3 \cdot 108 = 324$

Sequence: $4, 12 + 36 + 324$.

Generalize:

$f(n) = 4 * 3^{n-1}$

(c) $x(n) = x(n/2)+n$

$x(1) = 1$ $\quad$ $n = 2^k$

Sub $n=2$

$x(2) = x(2/2)+2$

$= x(1)+2$

$= 1+2 = 3$

Sub $n=3$

$x(4) = x(4/2)+4$

$= x(2)+4$

$= 3+4 = 7$

Sub $n=8$

$x(6) = x(8/2)+8$

$= x(4)+8$

$= 7+8 = 15$

Sequence: $1+3+7+15 \ldots$

Generalize: $2^n - 1 = f(n)$

$n = 2k$

$f(n) = 2^{2k} - 1$

(d) $x(n) = x(n/3) + 1 \qquad n = 3^k$

$x(1) = 1$

sub $n = 3$

$x(3) = x(3/3) + 1$

$= x(1) + 1$

$= 1 + 1 = 2$

$n = 9$

$x(9) = x(9/3) + 1$

$= x(3) + 1$

$= 2 + 1 = 3$

$n = 27$

$x(27) = x(27/3) + 1$

$= x(9) + 1$

$= \cancel{2+1+1}$

$= 3 + 1 = 4$

sequence: $1, 2, 3, 4 \ldots\ldots n$

generalize: $x(n) = \log_3 n$

$n = 3^k$

$x(k) = \log_3 3^k$

i) $T(n) = T(n/2) + 1$

Let's use backward substitution.

$n = n/2$

$$T(n/2) = T\left(\frac{n}{2^2}\right) + n$$

$$T(n) = T\left[\frac{n}{2^2}\right] + n + 1$$

$n = n/2^2$

$$T\left(\frac{n}{2^2}\right) = T\left(\frac{n}{2^3}\right) + n$$

$$T(n) = T\left[\frac{n}{2^3}\right] + n + n + 1$$

$$T(n) = T\left[\frac{n}{2^3}\right] + 2n + 1$$

The Pattern:

$$T(n) = T\left[\frac{n}{2^k}\right] + k$$

$$\frac{n}{2^k} = 1 \qquad n = 2^k \qquad k = \frac{n}{2}$$

Taking log on both sides

$$\log n^k = \log_2 n$$

$$T(n) = T(1) + \log_2 n$$

$$T(n) = c + \log_2 n$$

$\therefore$ $T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$

Let's use Master Theorem

$T(n) = aT(n/b) + f(n)$

$a = 1+1 = 2$

$b = 3$

$f(n) = c(n)$

Step I:

$\log_b a = \log_3 2$ $\qquad$ $f(n) = n^k \log n^p$

$= 0.630$ $\qquad$ $K = 1 \quad P = 0$

Step II:

$\log_b a \ \Delta \ K$

$0.630 < 1$

It comes under case - III

Step III:

$P \ \Delta \ K$

It comes under $O(n^2)$

$f(n) = O(n^1)$

$f(n) = O(n)$

(a) What does the algorithm compute?

The above given algorithm finds the minimum element of an array, by breaking the problems into sub problems.

(b) setup a recurrence relation:

$T(n) = T(n-1)+1$

$T(1) = 0$

$T(n) = T(1) + T(n-1)$

$= 0 + n-1$

$T(n) = n-1$

Time complexity: $O(n)$

4) Analayse the order of growth.

$F(n) = 2n^2+5 \quad g(n) = 7n$

$n=1$

$f(n) = 2(1)^2+5 \quad g(n) = 7 \cdot 1$

$= 2+5 \quad = 7$

$= 7$

$n=2$

$f(2) = 2(2)^2+5 \quad g(2) = 7 \cdot 2$

$= 8+5 = 13 \quad = 14$

$n=3 \quad f(3) = 2(3)^2+5 \quad g(3) = 7 \cdot 3$

$= 18+5 = 23 \quad = 21$

$n=4 \quad f(4) = 2(4)^2+5 \quad g(4) = 7 \cdot 4$

$= 37 \quad = 28$

By analysing $f(n) \geq g(n) \cdot c$ $\therefore$ It is ~~Best~~ Best case.

$f(n) = \omega(g(n))$