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E102 Spring 2024

Advanced Systems Engineering

Digital Controller Project

Due Date:

Teams with a senior: 5pm Friday May 3
Teams with juniors: 9am Thursday May 9

Please complete and submit this project workbook detailing the design, simulation, and experimental testing of digital controllers for an overdamped second order LTI system to satisfy the following specifications for the response to a step reference input of 2.5V:

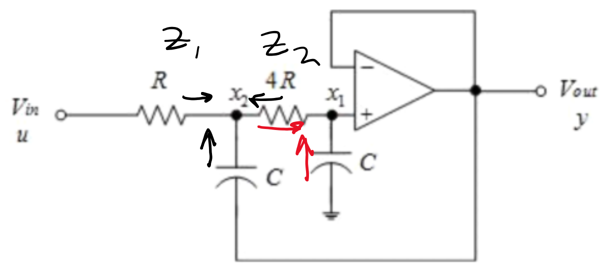
- **less than 1% peak overshoot**
- **zero steady state error**
- **minimize the control input keeping 1% settling time under 4 s**
- **sample at 10 Hz**
- **control input saturates at 5V**

Timeline:

1. **Team Project posted on Canvas on April 18th.**
2. **Pick up experimental kit for your team from the engineering stockroom starting April 18th. Complete the "Using the Arduino Uno as a digital controller" tutorial from the class directories.**
3. **Return kit (disassembled) to the engineering stockroom and submit this Project Workbook on Gradescope for your team by <u>Friday May 3 (Seniors) or Thursday May 9 (Juniors).</u>**

Project Tasks

1. Consider the overdamped second order circuit shown below:



$C = 10\mu F; R = 50k\Omega$

Determine the transfer function $H(s) = \dfrac{Y(s)}{U(s)}$ [1]

op-amp in negative feedback.

$V_+ = V_- \quad i_{in} = 0$

$$\frac{V_1 - V_{out}}{4R} + \frac{-V_{out}}{1/Cs} = 0$$

$$V_1 = V_{out}(1 + 4RCs)$$

$$4R\left(\frac{V_{in} - V_1}{R} + \frac{V_{out} - V_1}{4R} + \frac{V_{out} - V_1}{1/Cs}\right) = 0$$

$$4(V_{in} - V_1) + V_{out} - V_1 + 4RCs(V_{out} - V_1) = 0$$

$$4V_{in} + V_{out}(1 + 4RCs) + V_1(-5 - 4RCs) = 0$$

$$4V_{in} + V_{out}(1 + 4RCs) + V_{out}(1 + 4RCs)(-5 - 4RCs) = 0$$

$$4V_{in} + V_{out}(1 + 4RCs + (1 + 4RCs)(-5 - 4RCs)) = 0$$

$$4V_{in} + V_{out}(1 + 4RCs)(1 - 5 - 4RCs) = 0$$

$$4V_{in} + V_{out}(1 + 4RCs)(-4 - 4RCs) = 0$$

$$V_{out}(1 + 4RCs)(-4 - 4RCs) = -4V_{in}$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{-4}{(1 + 4RCs)(-4 - 4RCs)} = \frac{1}{(1 + 4RCs)(1 + RCs)}$$

$R = 50k\Omega$
$C = 10\mu F$

$$\boxed{H(s) = \frac{1}{s^2 + 2.5s + 1}}$$

[1] All derivations and design calculations should be inserted in the text boxes provided. Handwritten equations are acceptable

Determine the state space representation
$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}\mathbf{x}(t)$$

$$H(s) = \frac{1}{s^2 + 2.5s + 1}$$

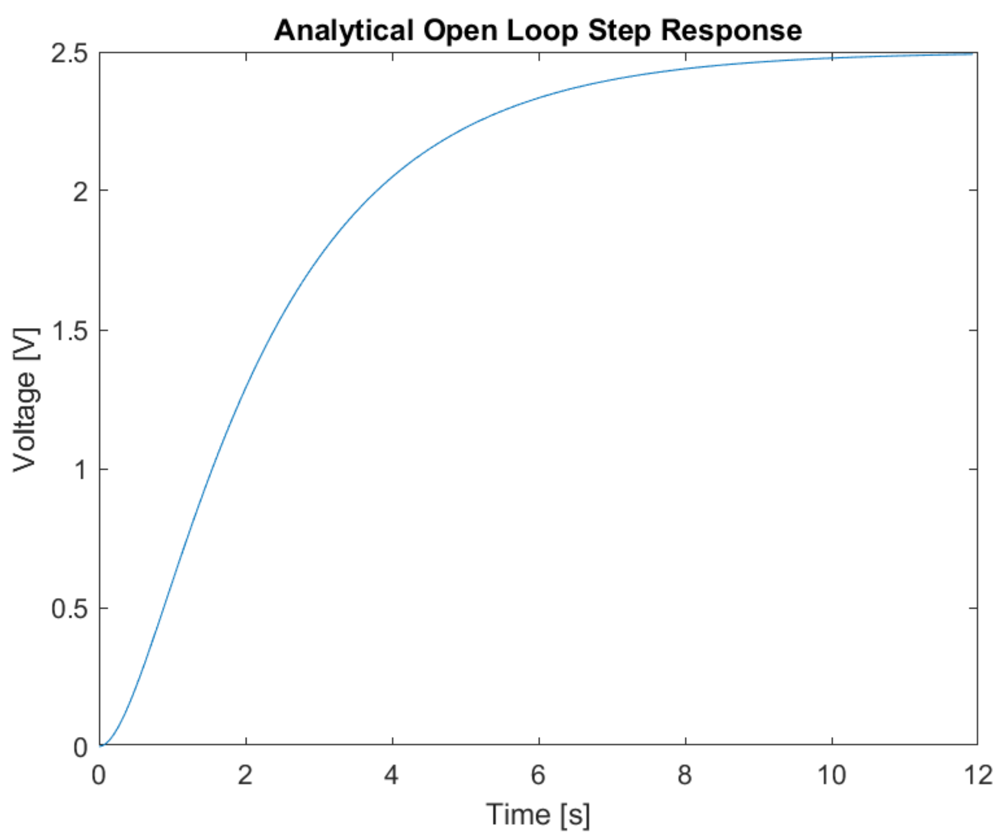
CCF form:

$$\vec{x} = \begin{bmatrix} \dot{x}_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} \dot{V}_{out} \\ V_{out} \end{bmatrix} \qquad A_c = \begin{bmatrix} -2.5 & -1 \\ 1 & 0 \end{bmatrix} \qquad B_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = V_{out} \quad u = V_{in} \qquad C_c = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D_c = 0$$

$$\boxed{\begin{aligned} \begin{bmatrix} \ddot{V}_{out} \\ \dot{V}_{out} \end{bmatrix} &= \begin{bmatrix} -2.5 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{V}_{out} \\ V_{out} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_{in} \\ V_{out} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{V}_{out} \\ V_{out} \end{bmatrix} + 0\,V_{in} \end{aligned}}$$

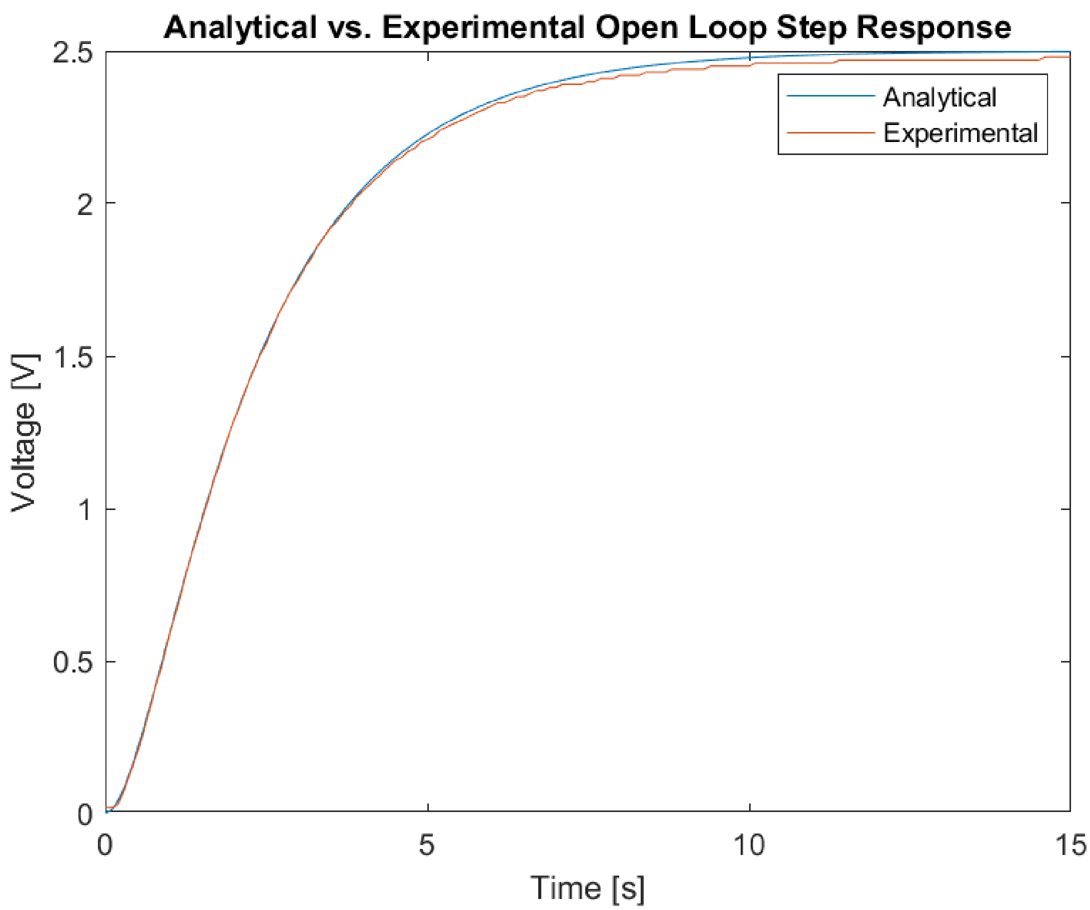
Determine the step response $y_{step}(t)$



2. Build the circuit with the LMC6484 op amp (data sheet in class directory) and measure the open loop step response experimentally by coding the Arduino Uno microprocessor to interface with the circuit.

Insert a fully labeled experimental open loop step response plot for a 2.5V step input[2] with an overlay of the analytical step response from part 1

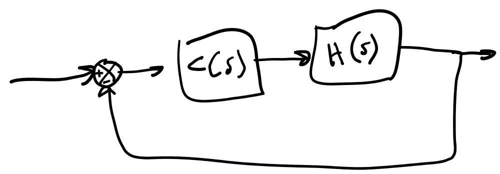
(insert figure here)



[2] All plot figures submitted in this workbook must be fully labeled with title, axis labels, and legends for multiple plots. The quality of the figures will be considered a part of the assessment. See the plot examples in the "Using the Arduino Uno as a digital controller" tutorial document

3. Design a unity feedback continuous-time closed loop system for the circuit (plant) with a PI compensator $C(s) = K_p + \frac{K_i}{s}$ for 70º phase margin and crossover frequency $\omega_{co} = 1\, rad/s$.

$$H(s) = \frac{1}{s^2 + 2.5s + 1}$$



$$T_{OL}(s) = C(s)H(s) = \frac{K_p + \frac{K_i}{s}}{s^2 + 2.5s + 1}$$

crossover frequency: $|T_{OL}(jw_{co})| = 1.$ ①

Phase margin: $\phi_m = \measuredangle T_{OL}(jw_{co}) + 180^\circ$ ②

Plug in $w_{co} = 1$ and $\phi_m = 70^\circ$ into ① and ②.

① $$|T_{OL}(jw_{co})| = \left| \frac{K_p + \frac{K_i}{j}}{-1 + 2.5j + 1} \right| = \frac{\sqrt{K_p^2 + K_i^2}}{2.5} = 1$$

$$K_p^2 + K_i^2 = 6.25$$

② $$70^\circ = \tan^{-1}\left(\frac{-K_i}{K_p}\right) - 90^\circ + 180^\circ$$

$$-20^\circ = \tan^{-1}\left(\frac{-K_i}{K_p}\right)$$

$$-0.364 = \frac{-K_i}{K_p} \qquad K_i = 0.364 K_p$$

Plug into ①.

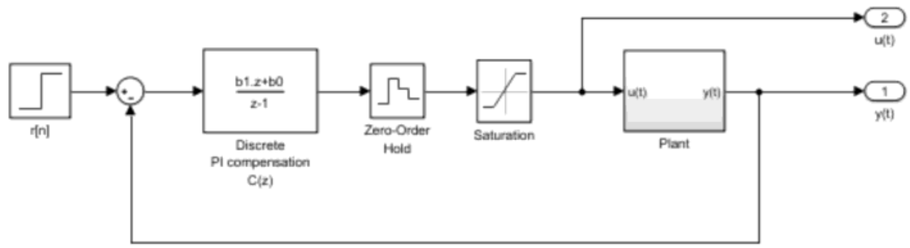
7.5625

$$K_p^2 + 0.1325 K_p^2 = 6.25$$

$$\therefore \boxed{\begin{aligned} K_p &= 2.349 \\ K_i &= 0.855 \end{aligned}}$$

[2.4345 -2.349]

4. Build a SIMULINK model for digital PI control of the circuit with a saturated control $0 \le u \le 5V$ and sample time $T = 0.1s$



Simulink =>Discrete =>Discrete Transfer Fcn.   Sample time=> T
Simulink =>Discrete =>Zero-Order Hold.   Sample time=> T

Emulate (discretize) the PI compensator from part 3 using the backward differencing method.

$$C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s}$$

Turn into differential equation.

$$U(s) = E(s)\left(K_p + \frac{K_i}{s}\right)$$

$$\mathcal{L}^{-1}\left[\, U(s)s = E(s)sK_p + E(s)K_i \,\right]$$

$$\downarrow$$

$$u'(t) = K_p e'(t) + K_i e(t)$$

Discretize by using $\left.\frac{df}{dt}\right|_{nT} \approx \frac{f_n - f_{n-1}}{T}$ where $T = 0.1$ sec.

$$\frac{u_n - u_{n-1}}{T} = K_p \frac{e_n - e_{n-1}}{T} + K_i e_n$$

$$u_n - u_{n-1} = K_p(e_n - e_{n-1}) + K_i T e_n$$

$$u_n = u_{n-1} + e_n\underbrace{(K_p + K_i T)}_{b_0} + e_{n-1}\underbrace{(-K_p)}_{b_1}$$

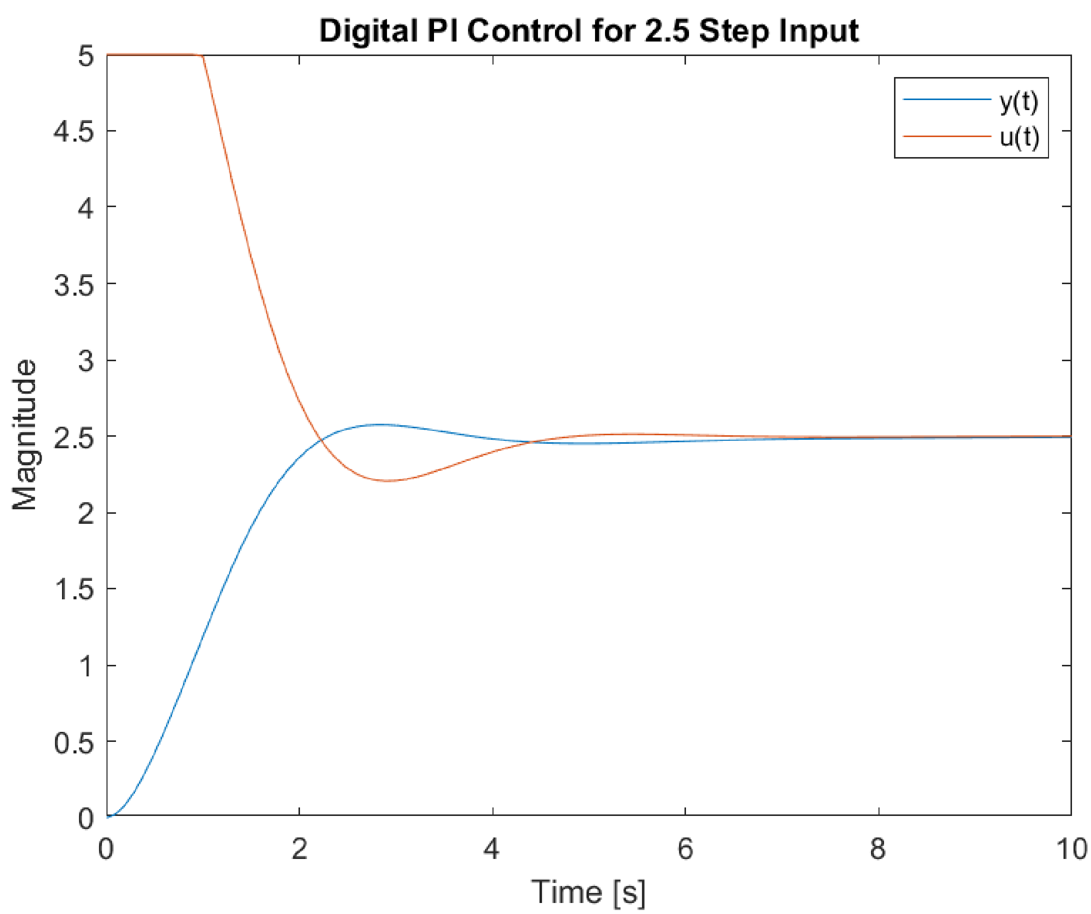
$$b_0 = 2.4345 \qquad b_1 = -2.349$$

$$C(z) = \frac{b_0 + b_1 z^{-1}}{1 - z^{-1}}$$

$$\boxed{C(z) = \frac{2.4345 - 2.349 z^{-1}}{1 - z^{-1}}}$$

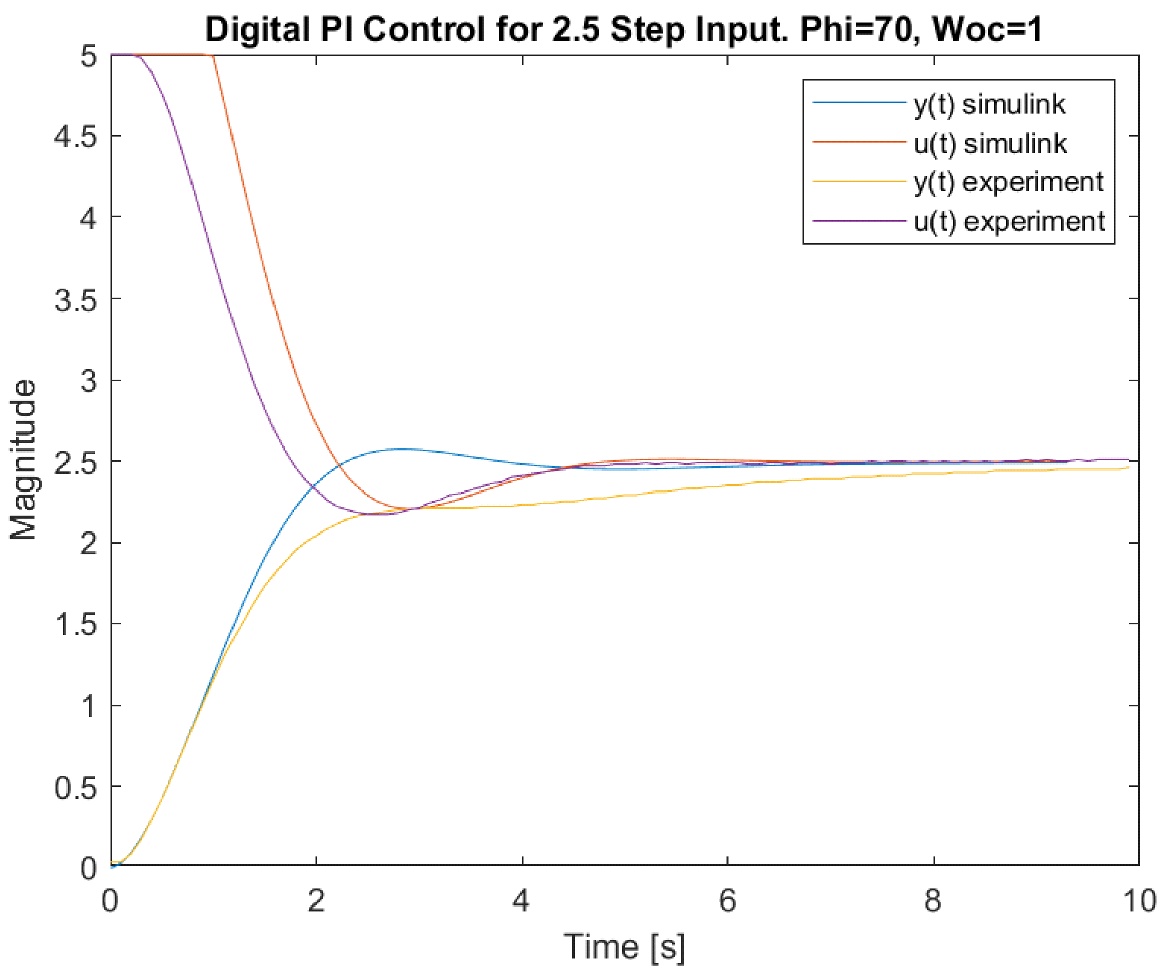
Insert a fully labeled simulated closed loop step response plot of the digital control system for a 2.5V step reference input $r[n]$. Design for a phase margin $\phi_m = 70^o$ at crossover frequency $\omega_{co} = 1\, rad/s$ Plot both the response $y(t)$ and the control input $u(t)$ on the same figure.

(insert figure here)



5. Program the discrete PI closed loop control for the LMC6484 op amp circuit Insert a fully labeled experimental closed loop step response plot for a 2.5V step reference input, with an overlay of the simulated step response from part 4. Plot both the response $y[n]$ and the control input $u[n]$ on the same figure.

(insert figure here)



Complete a table of trials of values of design parameters ϕ_m, ω_{co} with the resulting experimental values of 1% settling time $t_{settling}$; peak overshoot M_p; steady state error e_{ss} and the summation of the absolute deviations of the control input from the final steady state control input $U = \sum_{n=0}^{\infty} |u[n] - u_{ss}|$ where $u_{ss} = \lim_{n \to \infty} u[n]$.

ϕ_m	ω_{co}	$t_{settling}$	M_p	e_{ss}	$U = \sum_{n=0}^{\infty} \lvert u[n] - u_{ss} \rvert$
70	1	11.6 sec	N.A.	0	34.01
70	2	12.8 sec	N.A.	0	35.75
60	1	3.8 sec	0.4%	0	38.39

6. Design an observer-based discrete-time state feedback control of the circuit (plant)

- Determine the equivalent discrete-time state space description of the plant for a sample period $T = 0.1s$ Use MATLAB **c2d** command.
  $\mathbf{x}[n+1] = \mathbf{A_d}\mathbf{x}[n] + \mathbf{B_d}u[n]$
  $y[n] = \mathbf{C_d}\mathbf{x}[n] + D_d u[n]$ Record $\mathbf{A_d}, \mathbf{B_d}, \mathbf{C_d}, D_d$

$$A_d = \begin{bmatrix} 0.7746 & -0.0883 \\ 0.0883 & 0.9954 \end{bmatrix} \quad C_d = [0, \; 1]$$

$$B_d = \begin{bmatrix} 0.0883 \\ 0.0046 \end{bmatrix} \quad D_d = 0$$

- Design a full state feedback controller to minimize the objective function:
  $$J = \frac{1}{2}\sum_{n=0}^{\infty}\left(Q_{11}x_1[n]^2 + Q_{22}x_2[n]^2 + u[n]^2\right) \qquad \text{with } Q_{11} = 100; Q_{22} = 1;$$
  Use MATLAB **dlqr** command . Record the full state feedback gain vector ***K*** and the discrete-time closed loop poles

$$K = \begin{bmatrix} 4.6334 \\ -0.0260 \end{bmatrix}$$

$$\text{poles} = \{0.9862, \; 0.3746\}$$

- Select the reference gain K_r for zero steady state error to a constant reference input. Record K_r

$$K_r = \frac{1}{(-C_d - D_d K)(I - A_d + B_d K)^{-1} + D_d}$$

$$K_r = -0.974$$

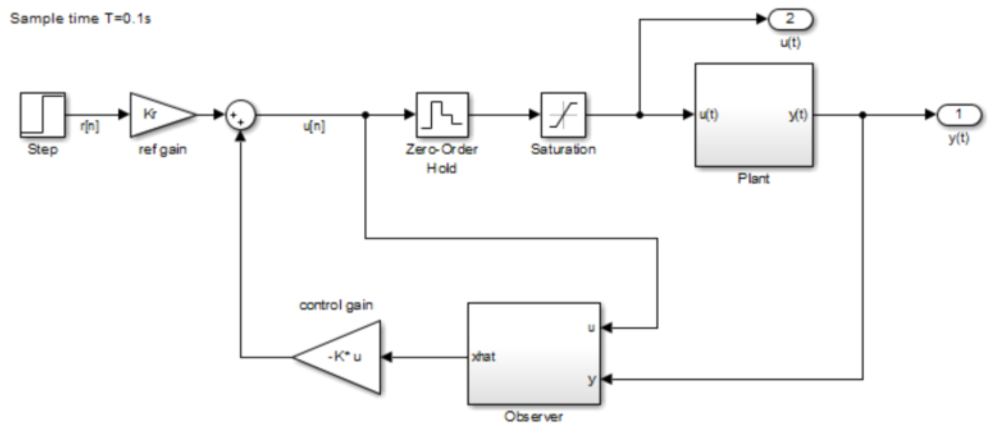
- Design an observer with pole locations at twice the speed of the closed loop poles Record **L** and the observer poles

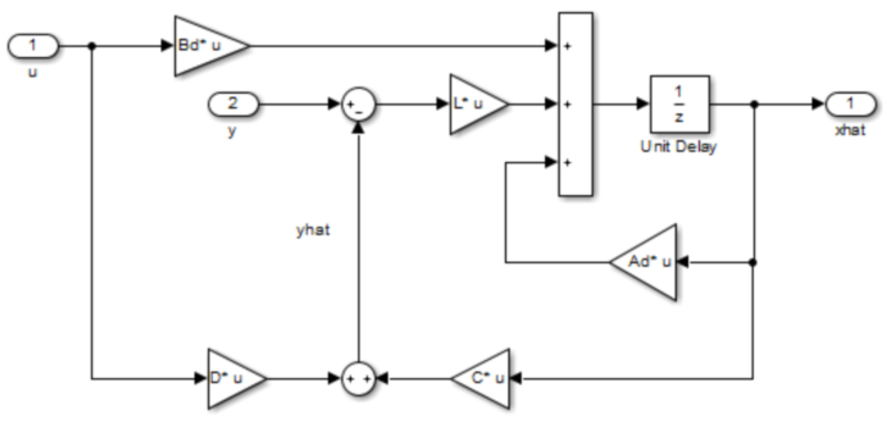
Using place on $\frac{1}{2} \cdot P_{controller}$

for $P_{observer} = \{0.1873, 0.4931\}$

$$L = \begin{bmatrix} -0.4257 \\ 0.6804 \end{bmatrix}$$

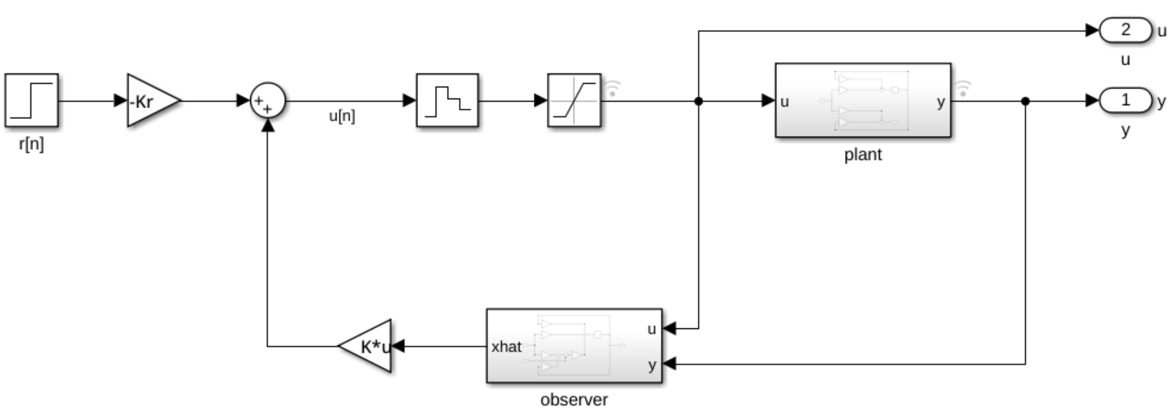
7. Build a SIMULINK model for observer-based state feedback control of the circuit with a saturated control $0 \le u \le 5V$





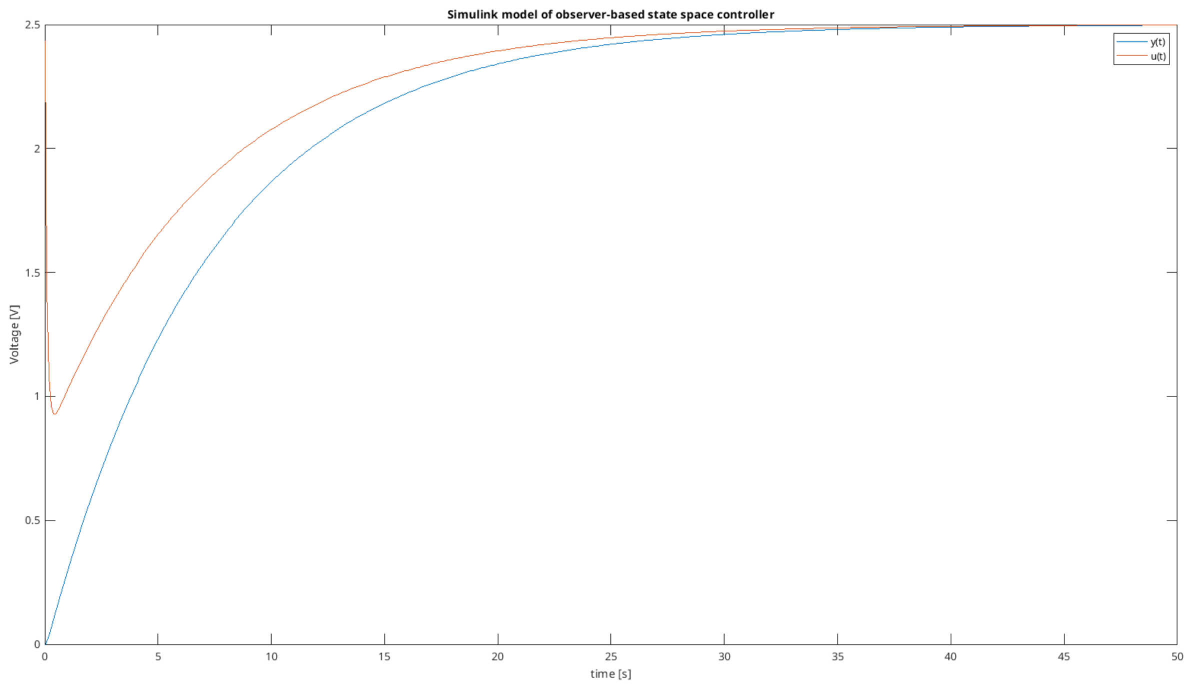
Discrete Observer Subsystem

*Simulink =>Gain =>*Multiplication=>Matrix(K*u)
Simulink =>Discrete =>Unit Delay.   Sample time=> T

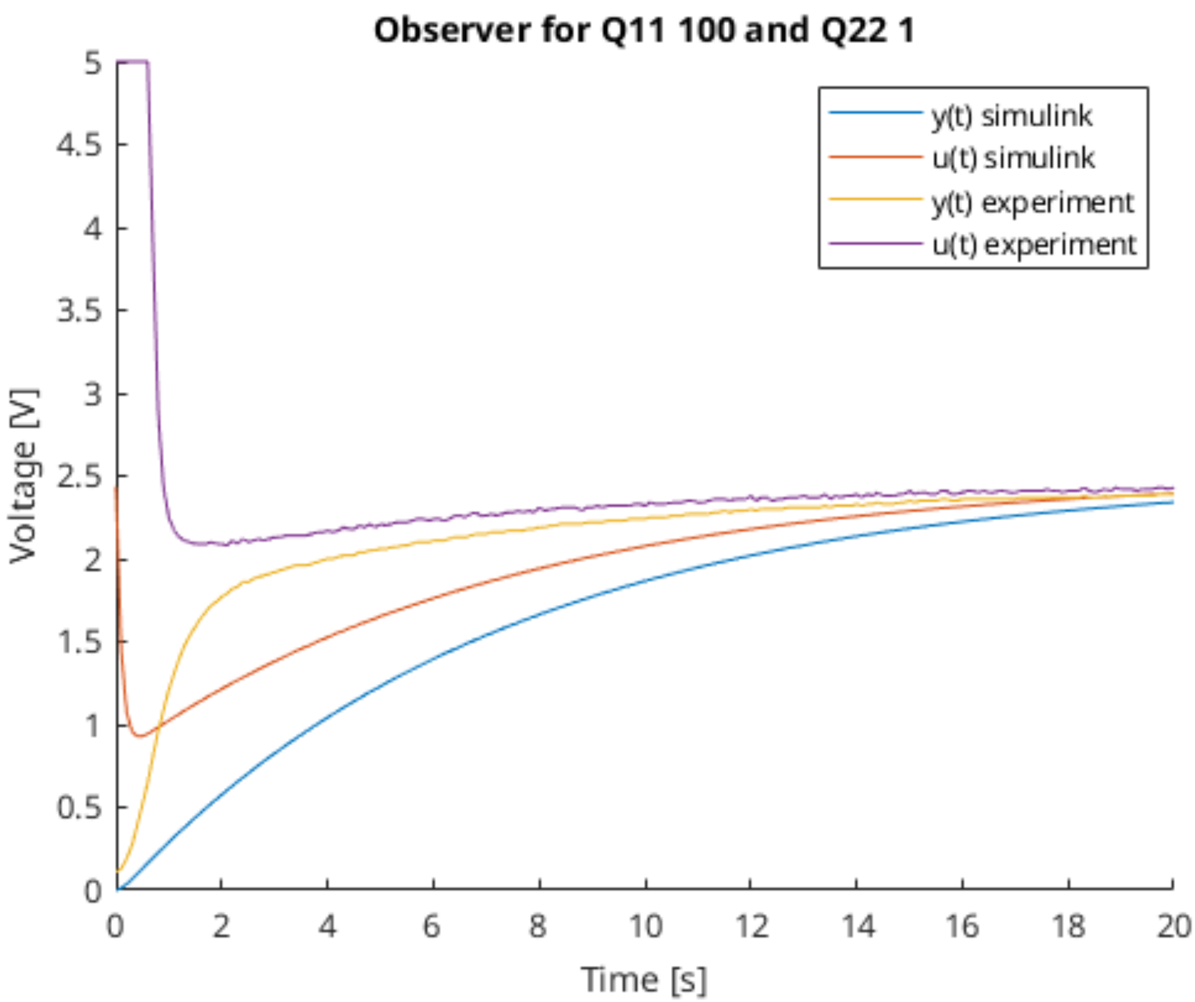


Insert a fully labeled simulated closed loop step response plot of the digital control system for a 2.5V step reference input $r[n]$. Plot both the response $y(t)$ and the control input $u(t)$ on the same figure.

(insert figure here)



8. Program the observer-based state feedback control for the LMC6484 op amp circuit
Insert a fully labeled experimental closed loop step response plot for a 2.5V step reference input, with an overlay of the simulated step response from part 7
Plot both the response $y[n]$ and the control input $u[n]$ on the same figure.



Complete a table of trials of values of design control parameters Q_{11}, Q_{22} with the resulting experimental values of 1% settling time $t_{settling}$; peak overshoot M_p; steady state error e_{ss} and the summation of the absolute deviations of the control input from the final steady state control input $U = \sum_{n=0}^{\infty} |u[n] - u_{ss}|$ where $u_{ss} = \lim_{n \to \infty} u[n]$.

Q_{11}	Q_{22}	$t_{settling}$ (s)	M_p(%)	e_{ss}	$U = \sum_{n=0}^{\infty} \lvert u[n] - u_{ss} \rvert$
100	1	25.8285	0.4115	2.4%	74.87
1	1	7.7520	0.4032	0.4%	55.12
1	100	2.2833	0.4	0.4%	29.53

9. Insert a short (1 page) discussion of your rationale for the choice of the control parameters and the effect of your choices on the experimental results

For PI control, both the phase margin and the crossover frequency were tuned for the system to satisfy the design parameters. As the phase margin was increased from 70 degrees, marginal changes were visible in the step response. However, when the phase margin was decreased, the system became more unstable which yielded a faster settling time and a higher overshoot. When the crossover frequency was increased or decreased, the system became drastically more unstable, yielding far too high overshoot. We obtained the best-performing system by keeping the crossover frequency at its nominal value of 1 rad/s and decreasing the phase margin to 60. This allowed the system to be just fast enough without overshooting beyond 1%.

For the observer, emphasizing the optimization of Vout over the derivative of Vout results in a very fast settling time that is focused on maximizing the correctness of Vout. Thus a Q11 of 1 and a Q22 of 100 results. There was almost no discrepancy between the simulation and the actual circuit in this case, but other values of Q11 and Q22 which emphasized the derivative of Vout over Vout or emphasized neither resulted in terrible performance as seen in the chart on the previous page.

10. Insert a short (1 page) discussion of the sources of error between the simulated and experimental step response results

For PI control, when the phase margin and crossover frequencies were tuned, we saw varying degrees of discrepancy between the experimental and simulated results. Interestingly, there was the least discrepancy when the phase margin was at 50. When the phase margin was increased or decreased from 50, the results diverged from each other. We saw the greatest discrepancy between experiment and simulation when the crossover frequency was greatly altered, such as doubling it from 1 rad/s to 2 rad/s. The simulated response behaved as expected, but the experimental results behaved quite abnormally with no overshoot while having oscillcations and an extremely long settling time. These discrepancies are not due to high variations in control inputs applying strenuous demands to the op-amp since the U factor does not correlate to smaller discrepancies. Rather, they could be due to non-idealities of our experimental setup at very specific rates of change in input voltage. Perhaps a crossover frequency of 1 rad/s is the most ideal for this discrete feedback system at this specific sampling frequency, or perhaps a phase margin of 50 applies strikes the perfect balance between speed and stability to allow simulink to accurately track the response.

For the observer controlled system, the only thing that could have been a source of error is the op-amp approximation that the gain is infinite. This assumption was made when calculating the initial transfer function of the circuit. When the gain is not infinite, building an op-amp circuit in feedback results in a small amount of error. Since op amps have a very large open-loop gain, this error is miniscule, which matches what we actually see when taking measurements.