PCV Exercise 2

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1 Theory

If x is an ideal point and H is some homography, is x'=Hx also always an ideal point? What about an ideal line l? How do you interpret the result geometrically?

Neither ideal points nor ideal lines necessarily stay ideal in a homography. Many simple transformations, like translation or rotation, are homographies, but they do not affect w and thus cannot influence whether a point/line is ideal or not. However, it is possible to end up with non-ideal points/lines in a homography, if it is or contains a perspective transformation. In this case, geometrically speaking, a vanishing point for ideal points or the horizon for the ideal line is now part of the visible scene.

How many distinct ideal points exist? How many distinct ideal lines exist?

For any given Homography, there are an infinite amount of ideal points, which all are part of a single ideal line.

Let l=(a, b, c) be a line. Give a formula for the following line representations

Axis intercept form

$$\frac{-xa}{c} - \frac{yb}{c} - 1 = 0$$

Hessian normal form

$$\Phi = atan\left(\frac{-b}{a}\right)$$
$$d = \sqrt{\frac{c^2}{a^2 + b^2}}$$

$$x\ cos(\Phi) + y\ sin(\Phi) - d = 0$$

What is a Homography?

A Homography is any projective transformation which preserves straight lines and is invertible.

Which conditions on the image acquisition have to be fulfilled in order to model the image transformation successfully as a 2D homography ?

There must be no rotation that is not around the projection axis. FUrthermore, the objects in the image should be far away from the projection plane.

How many degrees of freedom does a 2D homography have?

A 2D Homography is a 3×3 Matrix, affecting homogeneous coordinates, leading to 9 variables. However, we can reduce this to 8, as H and λH affect homogeneous coordinates the same way. So if we divide H $h_{3,3}$, we end up with a Matrix with 8 Degrees of freedom, without changing the transformation itself:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix}$$

Why should the homogeneous points be normalized before creating the design matrix?

Making sure the points all have the same scale, improves numerical stability. Addition and Multiplication of very large and very small numbers leads to significant errors in floating point arithmetic.

What are possible reasons the images don't align perfectly?

The image might not align perfectly due to imperfectly chosen point-pairs or bad image acquisition. Whenever there are disparities between images, it might lead to bad alignment.