Exercise 01

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1. You would like to compute the connecting line between two 2D points. What happens, if the two points are identical?

2. Where does the general line $x \cos \varphi + y \sin \varphi = d$ intersect the line $(0, 0, 1)^T$ given in homogeneous coordinates? How can this point be interpreted?

3. Show that the horizon is a straight line by showing that three points on the horizon are always collinear.

1) if two points are identical then:

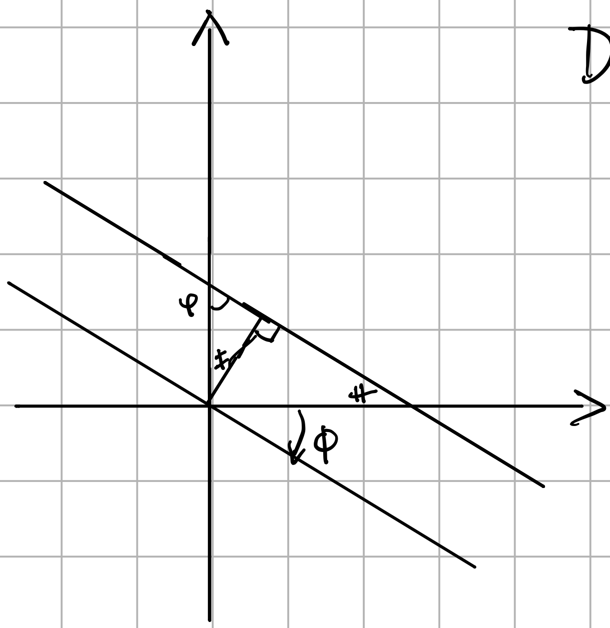
$$l = P_1 \times P_2 = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

is not defined as $\|l\| = 0$

2) $l_{inf} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$: ideal line at infinity

$$l_g = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ -d \end{pmatrix} \rightarrow P_{inf} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ -d \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{pmatrix} :$$

ideal point at infinity



Direction $\tan\phi = \frac{-\cos\varphi}{\sin\varphi} = \frac{-1}{\tan\varphi}$

$\phi = -(90° - \varphi) = \varphi - 90°$

3) point on the horizon: $\begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$

$P_i = \begin{pmatrix} u_i \\ v_i \\ 0 \end{pmatrix}$ $\quad \det \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 0 & 0 & 0 \end{bmatrix} \overset{!}{=} 0$ [redacted]

1. The two points **x** = $(2, 3)^T$ and **y** = $(-4, 5)^T$ are given.
   a) Determine the connecting line **l** between the two points.

   b) *Move* **x** and **y** in the direction **t** = $(6, -7)^T$,
   rotate afterwards using the angle φ = 15° and finally
   scale with factor λ = 8.

   c) Accomplish the same operations with the line **l**.

2. Check whether the transformed points **x**' and **y**' are on the transformed line **l**'.

1) a) $l = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \times \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 22 \end{bmatrix}$

b) <u>Move in direction $(6,-7)^T$</u>

$$x' = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}$$

$$y' = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

<u>Rotate $\varphi = 15^\circ$</u>

$$x'' = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}}_{x'} = \begin{bmatrix} 8c+4s \\ 8s-4c \\ 1 \end{bmatrix}$$

$$y'' = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}}_{y'} = \begin{bmatrix} 2c+2s \\ 2s-2c \\ 1 \end{bmatrix}$$

<u>Scale with factor $\lambda = 8$</u>

$$x''' = 8 \cdot x'' = 32 \begin{bmatrix} 2c+s \\ 2s-c \\ 1/32 \end{bmatrix}$$

$$y''' = 8 \cdot y'' = 16 \begin{bmatrix} c+s \\ s-c \\ 1/16 \end{bmatrix}$$

c) $H = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$

$= 8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/8 \end{bmatrix} \begin{bmatrix} c & -s & 6c+7s \\ s & c & 6s-7c \\ 0 & 0 & 1 \end{bmatrix}$

$= 8 \begin{bmatrix} c & -s & 6c+7s \\ s & c & 6s-7c \\ 0 & 0 & 1/8 \end{bmatrix}$

$\Rightarrow H^{-1} = \frac{1}{8} \begin{bmatrix} c & s & -48 \\ -s & c & 56 \\ 0 & 0 & 8 \end{bmatrix}$

$\Rightarrow H^{-T} = \frac{1}{8} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ -48 & 56 & 8 \end{bmatrix}$

$\Rightarrow$ Transformed line: $l' = H^{-T} l$

$= \frac{1}{8} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ -48 & 56 & 8 \end{bmatrix} \begin{bmatrix} -2 \\ 6 \\ 22 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -2c+6s \\ -2s-6c \\ -64 \end{bmatrix}$

2) $x'''^T \cdot l' = 32 \,[2c+s \quad 2s-c \quad 1/32] \cdot \frac{1}{8} \begin{bmatrix} -2c+6s \\ -2c-6c \\ -64 \end{bmatrix}$

$$= 4\left[(2c+s)(-2c+6s)+(2s-c)(-2s-6c)-2\right]$$

$$= 4\left(-4c^2 + \cancel{12cs} - \cancel{2cs} + 6s^2 - 4s^2 - \cancel{12cs} + \cancel{2cs} + 6c^2 - 2\right)$$

$$= 4\left(2c^2 + 2s^2 - 2\right) = 0 \qquad \blacksquare$$

$$y'''^{T} \cdot l' = 16 \begin{bmatrix} c+s & s-c & 1/16 \end{bmatrix} \frac{1}{8} \begin{bmatrix} -2c+6s \\ -2s-6c \\ -64 \end{bmatrix}$$

$$= 2\left[(c+s)(-2c+6s)+(s-c)(-2s-6c)-4\right]$$

$$= 2\left(-2c^2 + \cancel{6cs} - \cancel{2cs} + 6s^2 - 2s^2 - \cancel{6sc} + \cancel{2cs} + 6c^2 - 4\right)$$

$$= 2\left(4c^2 + 4s^2 - 4\right) = 0 \qquad \blacksquare$$