Final Exam
CSCE 590 - Section 002: Optimization   Instructor - Vignesh Narayanan
Spring 2024
Due Date - April 30, 11.59 PM

Instructions:

- Submit a single PDF with solutions. Use the following format to name the PDF file - "CSCE590-FinalExam-Lastname."
- Do not use any optimization package for implementing the optimization algorithm
- Wherever applicable, include detailed steps/calculations in your answers
- If the answers are not readable, they will not be graded
- Points will be deducted if a problem is not solved as well as (solution) presented in its entirety.
- If you refer to any resource to get your solutions, add an acknowledgement with your solution (details of the source, e.g., book, website, etc.). Points will not be deducted for the same.
- Include codes to the problems as links and make them accessible; and add figures if appropriate
- For the coding problems, if the step size, initial guess, and terminating conditions are not provided, use reasonable choices for them in your code.

Part A: (40 points)

1. Prepare a (precise and detailed) summary of the following :

   (a) Definitions of atleast four optimization problems and four useful theorems that were covered in the course.

   (b) Main results pertaining to each of the optimization problems listed in your solution of item (a); exclude the theorems included in your solution of item (a). (do not just copy the statements given in the lecture slides as is, provide their implications)

   (c) Pseudo-code or algorithm for atleast four algorithms covered in the class

Part B: (60 points)

1. Given the inconsistent linear system

$$\begin{array}{rcrcrcl} x_1 & + & x_2 & + & x_3 & = & 3 \\ & & & & x_3 & = & 1 \\ x_1 & & & + & x_3 & = & 2 \\ 2x_1 & & & + & 5x_3 & = & 8 \\ -7x_1 & + & 8x_2 & & & = & 0 \\ x_1 & + & 2x_2 & - & x_3 & = & 1. \end{array}$$

- Formulate the least squares optimization problem and find the least squares solution.
- Solve the optimization problem $\|Ax - b\|_2^2 + \lambda \|x\|_2^2$ for some $\lambda \in (0, 1)$.
- Formulate a linear programming problem to solve this linear system and solve the linear program using any method discussed in our class.

- Starting with the solution from each of the three problems above, find the best approximate solution in any two dimensional subspace.

2. Consider the following problem $\min(2x - x^2) \quad s.t. \quad 0 \leq x \leq 4$.

- Is this problem a convex optimization problem? Provide supporting arguments to your answer.
- What are all the local and global optimal solutions to the problem.