Psychological Review
1982, Vol. 89, No. 4, 334–368

0033-295X/82/8904-0334$00.75

Tracing the Dynamic Changes in Perceived Tonal Organization in a Spatial Representation of Musical Keys

Carol L. Krumhansl
Cornell University

Edward J. Kessler
Stanford University

The cognitive representation of harmonic and tonal structure in Western music is investigated using a tone-profile technique. In this method listeners rate how well single tones (any one of the 12 tones of the chromatic scale) follow a musical element such as a scale, chord, or cadence. Very stable rating profiles reflecting the tonal hierarchies in major and minor keys are obtained, which, when intercorrelated and analyzed using multidimensional scaling, produce a four-dimensional spatial map of the distances between keys. The keys are located on the surface of a torus, in which the circle of fifths and the parallel and relative relations between major and minor keys are represented. In addition, single chords (major, minor, diminished, and dominant seventh) are found to be closely associated with the major and minor keys in which they play harmonic functions. The developing and changing sense of key during sequences of chords is traced by obtaining probe tone ratings following each chord in 10 different sequences, 8 of which contain modulations (changes) between keys. Modulations between closely related keys are found to be effected more immediately than are modulations between relatively distant keys. In all cases beyond the initial chord, the sense of the prevailing key is stronger than that produced by the last heard chord in isolation. Thus, listeners integrate harmonic functions over multiple chords, developing a sense of key that may need to be reevaluated as additional chords are sounded. It is suggested that the perceived relations between chords and keys and between different keys are mediated through an internal representation of the hierarchy of tonal functions of single tones in music.

Music consists of tones varying in pitch, duration, loudness, and timbre, but the perception of music extends well beyond the registration of these physical attributes of the musical stimulus. Indeed, music contains considerable structure even in the relations that obtain among the individual tones. For example, the durations are such that metrical and rhythmic patterns emerge from the succession of tones, and these patterns in combination with other features define natural boundaries, or phrases, within the musical composition. Variations in loudness serve to highlight rhythmic patterns, further emphasize phrase structure, and distinguish between tones constituting the primary melodic line and tones serving more ornamental or harmonic functions. Timbral characteristics may additionally provide important cues for the overall structure of the musical composition.

In Western music pitch relationships are probably the most developed and essential to defining organization. Despite the tremendous variation found in the selection and ordering of pitches in the musical literature, certain underlying regularities can be observed and described. The perception of musical structure depends on the processing of pitch information with reference to a system of knowledge about the conventional uses of pitches within the musical tradition. Thus, cognitive processes are significantly involved in music perception, and empirical work in this area is directed at the description of these processes and the internal represen-

This research was supported in part by a grant from the National Science Foundation (BNS-81-03570) to the first author. The authors are grateful to David M. Green for the use of the facilities in the Psychophysics Laboratory at Harvard University, to Murray Spiegel and David Wilson for technical advice and assistance, and to Roger N. Shepard and an anonymous reviewer for their extensive comments on an earlier draft.

Requests for reprints should be sent to Carol L. Krumhansl, Department of Psychology, Uris Hall, Cornell University, Ithaca, New York 14853.

tation of pitch relations essential for the assimilation of musical organization.

In our work we rely heavily on the characterization of pitch structure provided by music theorists whose analyses of musical scores and introspective probings have yielded a large body of theoretical writings. Whereas much of this work, such as Piston's (1962) *Harmony*, is limited to the description of compositional conventions in existing music, other treatments (e.g., Berry, 1976; Schenker, 1906/1954, 1935/1979; Schoenberg, 1911/1978, 1969) point to the form that a psychological model of music processing might take. Whether these intuitions will in fact prove to elucidate the psychological study of music perception is a matter yet to be decided, although preliminary evidence does suggest that listeners familiar with Western music possess internal cognitive structures and processing strategies simliar to those presumed by music theorists. At a minimum, music theory provides a convenient terminology for describing the structure of music itself and is a rich source of predictions and hypotheses for empirical investigation.

Music theorists suggest that pitch structure in traditional Western music can best be understood in terms of the concept of tonality, or musical key, which will be described more fully later. In a tonal system one single pitch, called the tonic, functions as a central reference point for the system as a whole. Every other tone, and each chord, plays a unique function with respect to that abstract tonal center and these functions are summarized in a hierarchy of structural stability. Moreover, different musical keys have special associations to each other and are described as more or less distantly related. This investigation provides a quantitative, empirical measure of the perceived relatedness between the individual pitches and an instantiated tonal center, replicating and extending an earlier study by Krumhansl and Shepard (1979). These quantitative results are then used to obtain a spatial map of musical keys, reflecting the distances between the different keys. Next, we characterize the harmonic roles of individual chords with respect to different tonal centers in this system. Finally, we trace in the obtained spatial map how the sense of key develops and changes over time when the listener hears well-structured sequences of chords.

Music-Theoretic Descriptions of Tonality

Music-theoretic analyses are directed at specifying the tonal functions of the musical elements—single tones and tones sounded simultaneously as chords—in the prevailing tonality or key. Every musical key is associated with a hierarchy that applies to the set of musical pitches, 12 in each octave range. This hierarchy distinguishes between tones on the basis of their relative prominence or stability. The tonic tone is described as the single most central and stable pitch within the set of musical pitches. This tone gives the name to the key; for example, the note C is the tonic of the key of C. The tonic is also the first tone of the scale (sometimes referred to as the first scale degree), is typically sounded near the beginning of a piece of music, and is the tone that most frequently occurs at the end of major phrase boundaries. Next in the tonal hierarchy are the other scale tones, usually conforming to the interval patterns of the major or minor diatonic scale. Among these tones the fifth and third scale degrees are relatively stable; these tones together with the tonic form the major triad chord, which like the scale is a distinctive marker of the tonal center or key. Finally, the tones that are not contained within the scale, called nondiatonic, are the least stable and appear infrequently and with less rhythmic stress. This tonal hierarchy is at the core of music-theoretic accounts of structure in tonal music.

In addition, tones are frequently sounded simultaneously in chords, and a similar hierarchy is presumed to apply to the set of chords. The most frequently used chords are those consisting of three different pitches, which are therefore called triads. Triads are constructed from a root tone, which is a note of the scale, the tone a major or minor third above the root, and a third tone a major or minor third above the second tone. The octave equivalent of one or more of these pitches is typically sounded simultaneously. The position of the root of the triad in the scale is used to designate the chord. For example, the chord built on the first scale degree is indicated by I, the chord built on the second by II, and so on. The chords built on each of the seven scale degrees constitute the basic set of harmonies of the key. The tonic, I, triad is described as the most stable chord within the system, followed by the V, or dominant chord, the IV, VI, and II chords, and finally the III and VII chords. Chords outside this set are called nondiatonic and are infrequently sounded; when present they demand a return to the more stable elements. These hierarchies among tones and chords are used to provide varying degrees of tension and resolution at different points in time throughout a composition, which is the fundamental organizing principle of tonal music.

Tonal structure is even more complex than just outlined, for a hierarchy of levels is believed to exist in the instantiation of tonal regions. Chomsky (1965) has postulated a surface and deep structure for language, and Schenker (1906/1954, 1935/1979) has similarly found tonal music to be analyzable on foreground, middle ground, and background levels. Three levels of key instantiation may be distinguished: the primary key, a temporary alternative key, and the key of a single chord. The primary key is the major or minor key that is given emphasis and usually begins and ends the piece. In between, modulations (changes to different keys) may establish new though usually related keys. Moreover, each individual chord may take on some of the properties of the tonic of a key, a process called "tonicization," which differs from modulation by having a much briefer influence. We will return to these ideas later, as they become relevant for the interpretation of certain experimental results.

Psychological Models of Pitch Structure

Music theory suggests that musical structure in tonal music can be accounted for by the hierarchies that apply to the musical elements and that each key is associated with a particular ordering of stability within the set of single tones and chords. A number of psychological investigations have obtained quantitative empirical support for the internalization of these hierarchies by listeners familiar with tonal music. One study by Krumhansl and Shepard (1979) asked listeners to rate how well each different tone of the chromatic scale in an octave range completed a seven-tone C major scale. The ratings given by listeners having the greatest familiarity with tonal music recovered the tonal hierarchy predicted by music theorists: The tonic tone received the highest ratings, followed by the third and fifth scale degrees, the remaining scale degrees, and finally the nondiatonic tones. This hierarchy was also obtained in a scaling study of tones presented in a well-defined tonal context (Krumhansl, 1979). In that study the tones most stable within the tonal system were perceived as most closely interrelated, and those less stable as more distantly related. Moreover, this latter study revealed a pattern of temporal asymmetries in the judgments of pairs of tones that depended systematically on their relative stability in the instantiated key. Thus, in both these studies the pattern of results was highly specific to the hierarchy that applies to the tonality of the context.

One feature of these results should be emphasized. In both studies it was found that intervals of equal size were not perceived as equivalent but depended instead on the tonal functions of the individual tones within the context key. In the Krumhansl and Shepard (1979) scale-completion study, the tone a certain number of half steps above the implied tonic received a different rating than did the tone an equal number of half steps below the tonic. For instance, the note C♯, which is one half step above the implied tonic, C, was given a much lower rating than was the note B, which is a half step below the implied tonic. This may be understood by the fact that the C♯ is nondiatonic within the context key, but the B plays the function of the melodically significant leading tone within the key. Similarly, the scaling study (Krumhansl, 1979) found that the judged musical relatedness between two tones forming a fixed interval depended not simply on interval size per se but on the tonal functions of the individual tones. Again, for example, different ratings were given to intervals of equal size, such as that formed by C and C♯ and that formed by C and B. These findings indicate that some systematic transformation must intervene between geometrically regular representations of pitch structure, such as those proposed by Shepard (1982a; see also Shepard, 1981, 1982b), and the pitch relations as they are perceived in a tonal context. Moreover, these modifications may be accounted for by the unique function of each pitch within the instantiated key, as described by music theorists.

There is also empirical support for the music-theoretic description of a hierarchy of stability that applies to chords. Two studies (Bharucha & Krumhansl, in press; Krumhansl, Bharucha, & Kessler, 1982) have recovered the predicted hierarchy in which the tonic, I, chord is the most stable, followed by the V chord, the IV, VI, and II chords, and finally the III and VII chords. This hierarchy has been obtained even in the absence of a well-defined tonal context (Bharucha & Krumhansl, in press). However, the judged relations between chords were affected when a key context was provided (Bharucha & Krumhansl, in press; Krumhansl, Bharucha, & Castellano, in press). These alterations in judgments, which are summarized in three context-dependent principles, are a function of the distance between the instantiated context key and the key in which the chords play harmonic roles. As with single tones, then, the perception of chords depends significantly on their functions within the established key. This suggests that if invariant relationships are found in music, they are neither at the level of single tones nor at the level of chords but instead may be found at the more abstract level of musical keys or tonal centers. The focus of the present investigation is the characterization of the psychological representation of structure at the level of musical keys.

A number of psychological studies using memory performance provide further evidence for the importance of tonal organization in music perception. Well-structured tonal sequences have been found to be remembered better than those not conforming to key structure. This result has been obtained using both melodic sequences (Cohen, 1975; Dewar, 1974; Dewar, Cuddy, & Mewhort, 1977; Francès, 1972) and harmonic sequences, that is, chord sequences (Bharucha & Krumhansl, in press; Krumhansl et al., in press). A number of results reflect the differential stability of diatonic and nondiatonic elements. Tones and chords that are nondiatonic within an implied key have been found to be recognized correctly less often than are diatonic tones and chords (Bharucha & Krumhansl, in press; Krumhansl, 1979). Moreover, diatonic elements are more frequently confused with other diatonic elements than they are with nondiatonic elements (Bharucha & Krumhansl, in press; Dewar, 1974; Dowling, 1978). Finally, asymmetries have been found such that nondiatonic tones and chords are more frequently confused with diatonic elements than diatonic elements are confused with those that do not belong to the key (Bharucha & Krumhansl, in press; Dowling & Bartlett, Note 1). The magnitude of each of these effects has been shown to depend systematically on the choice of the context key (Krumhansl et al., in press). These memory results provide convergent evidence for the structural relations obtained in the empirical studies described earlier.

Overview

In this article we investigate three closely related aspects of the representation and processing of tonal structure, each of which is presented in a separate section. In the first we obtain a quantitative measure of the degree to which each individual tone in an octave range is related to an abstract tonal center. These quantitative measurements are then used to derive a spatial map of the major and minor keys, representing the distances between different tonal centers. In the second section we determine the relations between chords of different types and these tonal centers, representing this information by points in the spatial representation of tonal regions. In the final section we assess how the sense of key develops and changes over time during sequences of chords, some of which contain modulations (shifts) between different keys. Our focus in the last two sections is on the manner in which chords and chord sequences relate to tonal regions as they are laid out in the spatial representation of keys described in the first section. Each one of these sections begins with a short description of the relevant music theory, followed where possible by a brief summary of previous empirical investigations related to the issue considered in that section.

Distances Between Keys

Determining the psychological distance between keys is important to an understanding of the perception of music in the Western tonal tradition. In music the juxtaposition of keys can be used to highlight structural elements, provide contrasts of varying degrees, and add expressive qualities to a piece (Berry, 1976, p. 15; Rosen, 1972, pp. 26, 27; Salzer, 1962, p. 20). Some keys are so closely related that a change from one to another can hardly be detected, whereas others are so distantly related that shifts between them seem abrupt and unpleasant. The measurement of the psychological distances between tonal centers may elucidate the nature of the internal representation of musical structure that underlies these perceptual phenomena. In addition, as suggested earlier, invariant relations may be contained in the structure at the level of abstract tonal centers.

Music-Theoretic Descriptions of Key Distance

Music theory presumes that several features of keys may determine the accessibility or ease of modulation between them, which is considered to be a rough measure of distance between the keys. For major keys three different features all consistently predict the same pattern of interkey relatedness: the number of tones shared by their scales, the difference in the number of sharps or flats in the key signature, and the distance around the circle of fifths. For the major keys the difference in the key signatures is inversely related to the number of shared scale tones. For example, two major keys that differ only in terms of one sharp or flat have six shared scale tones. Thus, G major (one sharp) and F major (one flat) both differ from C major (no sharps or flats) in terms of one scale tone, and the tonic of G is a fifth above and the tonic of F a fifth below that of C major. D major (two sharps) and B^b major (two flats) each have two scale degrees not shared by C major, and their tonics are separated by two fifths above and below the tonic of C major. This fifth connection continues to indicate distance between major keys until after six fifths above or below any given key the key of the tritone, the most distantly related major key, is reached in both directions. Because this fifth relationship folds back on itself, it is conveniently pictured as a circle, called the circle of fifths, first proposed by Heinichen (Forte, 1979, p. 27). Thus, the distances between major keys are fairly easy to specify unambiguously.

Major and minor keys are often used within the same piece, making the distance between keys of different modes an important issue. However, introduction of the minor keys complicates the picture considerably. To every major key there corresponds a minor key that has the same key signature, called the *relative minor*, whose tonic is a minor third below (or a major sixth above) that of the major key. For example, A minor is the relative minor of C major. Despite the shared key signatures, the relative major and minor keys do not overlap completely in their scale tones. The scale of the relative minor key has in fact three different forms: the harmonic minor form in which the seventh scale tone is raised by a half step, the ascending melodic form in which the sixth and seventh scale tones are each raised by half steps, and the descending melodic form (or natural form) in which the scale tones are exactly those of the relative major key. These different forms make impossible the precise definition of major–minor key distance as the number of shared scale tones. Nonetheless, a major key and its relative minor key are considered closely related by virtue of their shared key signatures.

Additionally, there is a close tie between a major key and the minor key built on the same tonic, which is called the parallel minor. This is true even though their key signatures differ in terms of three sharps or flats. For example, C major (with no sharps or flats) has C minor (with three flats) as its parallel minor. Despite the differences in key signatures, these keys share most of the structurally stable tones—the tonic, fourth, and fifth scale degrees—as well as the seventh, leading tone in the harmonic and ascending melodic forms. Possibly even more important is the fact that they also share the essential dominant, V, chord, which prepares the tonic of either of the parallel keys (Schoenberg, 1969, p. 51).

Thus, every major key has two closely related minor keys—the relative and parallel minors. Or, looked at another way, any single minor key is closely tied to two different major keys that are relatively distant on the circle of fifths. This fact makes it impossible to accommodate the minor keys within the scheme of the circle of fifths and has led Schoenberg (1969, pp. 20, 30) to abandon the circle of fifths model and to propose an alternative formulation, shown in Figure 1. In this model there is a single central reference key, with other keys seen as related regions. The figure shows the key regions surrounding the C major and C minor keys. For both, the representation consists of alternating columns of major and minor keys such that each major key is flanked by its relative minor key on one side and its parallel minor key on the other. The keys within each column are separated by fifths. The minor-key model is clearly a subset of the one for major keys. According to Schoenberg (1969, p. 30) a minor tonic does not maintain as direct control over its regions as does a major tonic, and consequently, the number of directly related regions is smaller for minor than for major keys.

To anticipate our results somewhat, we obtain a spatial representation of keys that is strikingly similar to the one proposed by Schoenberg (1969, pp. 20, 30). There are two important differences, however. First, Schoenberg's key regions are centered around a particular major or minor key, and thus, this scheme does not simultaneously accommodate all major and minor key relations. Instead, a different representation is required for each key. In contrast, we obtain a map of key regions that depicts all interkey relations as distances between points in a metric space. Second, Schoenberg does not attempt to make fine discriminations in tonal distance as, for instance, he equates the distance between a major key and its parallel and relative minors. Despite his statements

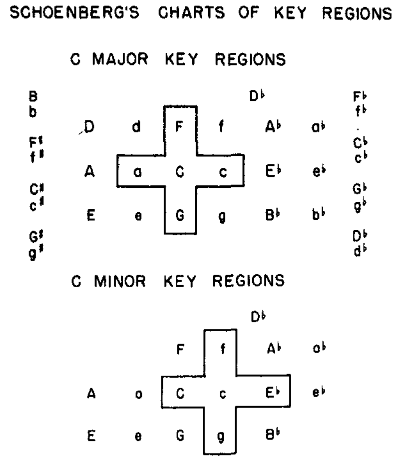

Figure 1. Schoenberg's spatial map of the key regions surrounding C major and C minor keys (adapted from Schoenberg, 1969). (The circle of fifths relation is represented as the vertical dimension, and the parallel and relative relations between major keys—indicated by capital letters—and minor keys—indicated by lowercase letters—as the horizontal dimension.)

that in common practice a major key is closer to its relative than its parallel minor (Schoenberg, 1911/1978, p. 153; 1969, p. 68), this difference is not structurally evident in the model. (There is some debate on this point. Piston [1962] finds that parallel majors and minors are practically identical and that classical composition in the 18th and 19th centuries tended "to regard the two modes as simply two aspects of one tonality" [p. 145].) In any case, our method produces interkey distances that permit precise comparisons.

Empirical Studies of Key Distance

Although the issue of key distance has not been extensively investigated in psychology, there are a number of studies demonstrating key distance effects. Memory performance for transposed sequences has been found to be better when the sequence is shifted to a closely related key than to a distantly related key (Bartlett & Dowling, 1980; Cohen, 1975; Cuddy, Cohen, & Miller, 1979). In these studies listeners are required to judge whether two melodic sequences are the same, except for transposition to a different key (which maintains the relative sizes of all the intervals). Accuracy in this task is higher when the melody is transposed to a closely related key. More recently, Krumhansl et al. (in press) have identified three principles governing harmonic relations that depend on the distance of the context key from the key in which the chords have basic harmonic functions. The probability that a chord is correctly recognized was found to be a decreasing function of this distance. Moreover, the psychological proximity of two different chords in the same key, measured both in terms of direct relatedness judgments and memory confusions, decreased as interkey distance increased. Finally, this distance was found to affect the magnitude and direction of temporal asymmetries in relatedness judgments and memory confusions. Although these studies have not provided a measure of distance between musical keys, they do suggest that the music-theoretic description of structure at the level of tonal centers is applicable to the process of music perception.

The Present Approach to Determining Key Distance

In our empirical determination of key distances, we chose not to employ recognition memory for transposed sequences, which has yielded key distance effects in previous studies (Bartlett & Dowling, 1980; Cohen, 1975; Cuddy et al., 1979). Three reasons for this were, first, in order to get very precise results, we would need sequences that are uniformly tonally unambiguous, which would be difficult if not impossible to construct. Second, we were concerned that transpositions between closely related keys might occasionally be rejected owing to confusions with tonal answers (Dowling, 1978). Finally, if both major and minor sequences were employed, transpositions between them would result in changed intervals in melodic sequences or changed chord types in harmonic sequences, making the results difficult to interpret solely in terms of key distance. Estimating key distances from changes in the patterns of confusion errors or relatedness judgments as a function of the context key (such as those observed in Bharucha & Krumhansl, in press; Krumhansl et al., in press) would be quite indirect as well as necessitate a very large number of observations.

For these reasons we chose to employ the method introduced in the earlier scale-completion study (Krumhansl & Shepard, 1979), which we will refer to here as the tone-profile technique. The results of that study, in which listeners rated how well each musical pitch in an octave range (the pitches of the chromatic scale) completed a seven-note major scale, were described earlier. The rating profiles for the listeners with a moderate to high level of musical training revealed considerable structure, which accorded well with music-theoretic descriptions of the tonal hierarchy. This result suggests that rating profiles of this sort may serve as a distinctive marker of tonal organization that could serve as a basis for determining distances between keys.

In the first experiment we obtained rating profiles for the 12 different musical elements listed in Table 1. Each of these musical elements was presented with each of the 12 tones of the chromatic scale, which will be called probe tones. In other words, every possible element–probe combination was presented and for each the listeners rated how well the probe tone fit, in a musical sense, with the element just heard. This procedure yielded a rating profile for each of the 12 elements studied. The tones and chords employed were constructed of pitches sounded in five octaves using an amplitude envelope that tapered off at both low and high ends of the frequency range (after Shepard, 1964).

Four precautions were taken to maximize the stability of the rating profiles. First, the same musical element appeared throughout a block of trials. Second, to minimize carryover effects between blocks, each new element was presented in a different key from that of the previous block, and each block began with a few practice trials to acclimate the listener to the new element in the new key. Third, the experiment contained exact replications and also replications in which the element was transposed to a different reference pitch. Fourth, listeners selected for the experiment formed a relatively homogeneous group in terms of musical background, with a moderate to high degree of musical sophistication.

Table 1
Musical Elements Used in Experiment 1

Element	Example
Ascending major scale	D major scale: D E F# G A B C# D
Ascending harmonic minor scale	A minor scale: A B C D E F G# A
Major chord	E major chord: E G# B
Minor chord	D minor chord: D F A
Diminished chord	F# diminished chord: F# A C
Dominant seventh chord	Dominant seventh chord on G: G B D F
IV-V-I cadence in major	Cadence in C major: F major chord, G major chord, C major chord
II-V-I cadence in major	Cadence in F major: G minor chord, C major chord, F major chord
VI-V-I cadence in major	Cadence in D major: B minor chord, A major chord, D major chord
IV-V-I cadence in minor	Cadence in E minor: A minor chord, B major chord, E minor chord
II-V-I cadence in minor	Cadence in F# minor: G# diminished chord, C# major chord, F# minor chord
VI-V-I cadence in minor	Cadence in B minor: G major chord, F# major chord, B minor chord

Experiment 1

Method

Subjects. Ten Harvard University undergraduates served as subjects and were paid at the rate of $3 per hour. Each subject participated in two experimental sessions lasting a total of approximately 2.5 hours. All subjects had a minimum of 5 years of formal musical instruction. Responses to a questionnaire on musical background indicated that the subjects had received instruction on musical instruments and voice for an average of 10.9 years. In terms of performing experience, the participants in the experiment had performed for an average of 12.0 years in instrumental groups and 2.1 years in choral groups. Currently, subjects were playing music an average of 3.1 hours per week and listening to music for 23.3 hours per week. One subject reported having taken a "very basic" introductory course in music theory, but the remaining subjects had no formal music theory background. All subjects reported having normal hearing and no one reported having absolute pitch.

Apparatus. The stimulus tapes (Maxell UD 35-90) were recorded at 7.5 inches per second (19 cm per second) on a Sony TC-540 tape recorder. During the experimental sessions the tapes were played back using the same tape recorder and Telephonics TDH-50 headphones. All the stimuli were prepared digitally on a PDP-15 computer in the Psychophysics Laboratory at Harvard University and were played through a digital-to-analog converter under computer control. The digital representations were sampled at a rate of 5,000 points per second. A temporal amplitude envelope with a 10 msec rise and decay reduced onset and offset clicks introduced by the hardware. Finally, a low-pass filter with cutoff set at 2450 Hz eliminated high-frequency noise.

Two kinds of musical units were produced: chords and single tones. The major, minor, and diminished chords each consisted of 15 sine wave components (three in each of five octaves), the dominant seventh chords consisted of 20 sine waves (four in each octave), and the single tones consisted of 5 sine waves (one in each octave). The components were selected from the 12 notes of the chromatic scale, whose frequencies were spaced equally in log frequency (equal tempered tuning) and were based on a 440 Hz A. The amplitudes of the selected components were determined by a loudness envelope over the five-octave range (from 77.8 Hz to 2349 Hz), which consisted of three parts: a gradually increasing section over the first octave and a half, a constant section over the middle two octaves, and a symmetrically decreasing section over the last octave and a half. The corresponding amplitude of the sine wave components at each frequency was determined using the amplitude-loudness curves of Fletcher and Munson (1933). This method, originally introduced by Shepard (1964) and described in detail elsewhere (Krumhansl et al., 1982), produces tones and chords with an organlike sound, without any clearly defined lowest or highest component frequencies. The composite chords and single tones were played at 71 db (A) that, because of the difference in number of sine wave components, necessitated a slight upward shift of the loudness envelope for the single tones and a slight downward shift for the dominant seventh chords.

Stimulus materials. The 12 elements used in the experiment, illustrated in Table 1, are ascending major scale, ascending harmonic minor scale, major chord, minor chord, diminished chord, dominant seventh chord, and three cadences (IV-V-I, II-V-I, and VI-V-I) in both major and minor keys. Each trial consisted of an element followed by a probe tone. The same element was used in 14 consecutive trials to form a block of trials. The first two trials in each block were for practice.

On the other 12 trials of the block, each possible probe tone from the chromatic scale was paired with the element in a random order. A stimulus tape contained 12 element blocks, one for each element. A replication tape contained the identical elements as the first tape but in a different random order and with a different order of probe tones. A transposition tape used the same element types but each was sounded in a different pitch range than on the original tapes. Again, there was a replication tape with the transposed elements randomized as above. The subjects heard the four different tapes in different orders such that during each of the two experimental sessions, a subject heard one complete replication—either both original or both transposition tapes.

On each trial the element was heard first, followed by a 1-sec pause, and then the probe tone sounded for .5 sec. The timing of the different elements was as follows: The tonic tones of the two scales (major and minor) were sounded for .5 sec, and the remaining scale tones for .25 sec, with a .19-sec pause between scale tones. The single-chord elements were played for .5 sec, as were the chords in the three-chord cadences. In the cadences there was approximately .25 sec between chords. A 4-sec interval between trials allowed subjects to record their responses.

Procedure. At the start of the first session, subjects were instructed to rate on a 7-point scale how well, in a musical sense, each probe tone fit into or went with the musical element just heard. On this scale, 1 was designated "fits poorly" and 7 was designated "fits well," and subjects were encouraged to use the full range of the response scale. Subjects responded to three complete blocks of practice trials before beginning the experimental tapes. After all experimental tapes had been presented, the subjects completed the short questionnaire about their musical backgrounds.

Results and Discussion

The 12 types of musical elements listed in Table 1 were each probed by the 12 tones of the chromatic scale. The set of judgments for each of the musical elements will be referred to as the profile for that element. These profiles will be analyzed in a number of ways. First, the stabilities of the profiles are evaluated using intersubject correlations and correlations between the profiles obtained for transposed elements. Second, the profiles themselves are intercorrelated and separate composite profiles are constructed for major and minor keys. Finally, correlations between these appropriately shifted major and minor profiles, providing a measure of interkey relatedness, are analyzed using multidimensional scaling, yielding a spatial map of the 24 major and minor keys. The discussion of chord-key relations that derive from the obtained profiles will be deferred until the next section.

Intersubject differences. No large individual differences were expected because the subjects were selected to be comparable in terms of their musical backgrounds. In order to evaluate consistency across listeners, intersubject correlations on the response profiles (averaged over the two exact replications) were computed. These correlations averaged .585, ranging from .412 to .740, and were all significant ($p < .01$). (The correlations between the two replications for each subject ranged from .241 to .658, averaging .492, suggesting that error variability may be substantially attenuating the intersubject correlations that were computed on the responses averaged over the two replications.) Multidimensional scaling (Kruskal, 1964; Shepard, 1962a, 1962b; the computer program used was KYST—Kruskal, Young, & Seery, Note 2) and hierarchical clustering (Johnson, 1967; the program used was AGCLUS—Olivier, Note 3) techniques were applied to the matrix of intersubject correlations. Neither of these analyses produced solutions that grouped listeners together in a way that related to any aspect of their musical backgrounds. Consequently, individual differences were not explored further, and the remaining analyses are based on the rating profiles averaged across the 10 subjects.

Transpositions. Each element was represented in two different pitch ranges in the experiment; these are called transposition replications. Although subjects, particularly those with less musical training, have been found to have some difficulty producing transpositions of tone sequences (Attneave & Olson, 1971), it is commonly believed that transpositions are perceived as equivalent, except perhaps by listeners with absolute pitch. Because not one of our subjects had absolute pitch, high correlations were expected between the ratings when the profiles were shifted appropriately to compensate for the interval of transposition. These transposition correlations were high, averaging .839, so the profiles used in the remaining analyses were those obtained by averaging the transposed ratings.

Major and minor key profiles. One central purpose of this experiment was to establish reliable profiles for major and minor keys so that these could be used in later analyses. In addition, the major key profile from this experiment can be compared with that of Krumhansl and Shepard (1979). Correlations between the 12 different profiles showed a very similar pattern of probe tone judgments for the major chord element and the three cadences in major, IV–V–I, II–V–I, and VI–V–I. The average correlation between these (.896, $p < .01$) indicates a consistent pattern of ratings for these elements. The profile for the major scale was somewhat less similar, having an average correlation of .796 with the other major profiles. Consequently, we will take as the major key profile the average ratings given the 12 probe tones for the major chord and the three cadences in major. This profile is shown at the top of Figure 2, where the average probe tone judgments are plotted with respect to C major. Of course, the profile for any other major key would be identical, except shifted the appropriate number of semitones up or down. Similarly, the profiles for the minor chord and the cadences in minor were very much alike, with an average correlation of .910 ($p < .01$). Again, the minor scale profile was less similar, with an average correlation of .727 with the others. Thus, the minor key profile, also shown in Figure 2, is the average of the profiles for the minor chord and the minor cadences.

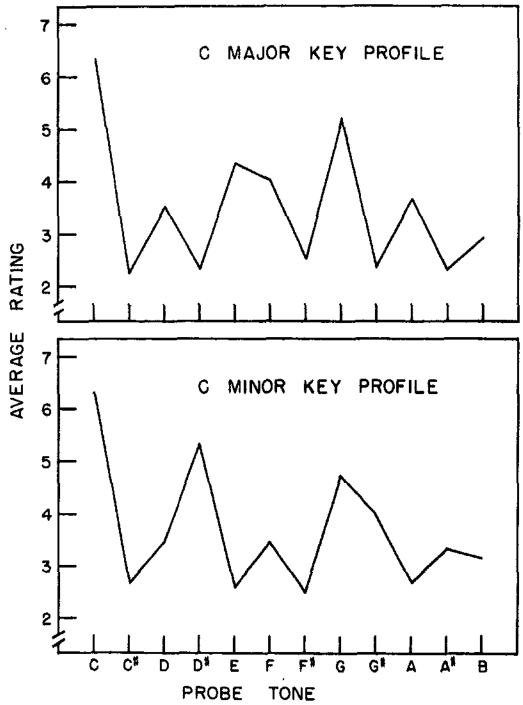

Figure 2. The obtained major and minor key profiles from the first experiment. (The profile for the major key [upper graph] is the average rating given each of the 12 tones of the chromatic scale following a major chord and the three cadences [IV–V–I, II–V–I, and VI–V–I] in a major key. The minor key profile [lower graph] is averaged over the minor chord and the three cadences in minor. The profiles are shown with respect to C major and minor, respectively.)

The major and minor key profiles share a number of features with each other and also with the scale-completion judgments of Krumhansl and Shepard (1979). In both key profiles the tonic, C, received higher ratings than all the other tones: $t(9) = 16.84$ for major, and $t(9) = 13.42$ for minor, $p < .001$ for both. In addition, all nontonic scale tones (using the harmonic form for minor) had higher average ratings than did nondiatonic tones: $t(9) = 6.05$ and 9.23, $p < .001$, for major and minor, respectively. Within the set of diatonic tones, the components of the tonic chord, C, E, and G in major and C, E^b ($D^\#$), and G in minor, were judged as fitting more closely with the major and minor elements than were the other diatonic tones: $t(9) = 16.28$ and 9.77, $p < .001$, for major and minor, respectively. Thus, the ratings in this study strongly confirmed the hierarchy obtained by Krumhansl and Shepard (1979). This hierarchy was also evident in the multidimensional scaling solution of tones judged in a major scale context (Krumhansl, 1979) and is entirely consistent with the qualitative predictions of music theory.

Interkey distance. We next used the major and minor key profiles, which have been shown to be extremely reliable and interpretable, as an indirect measure of interkey distance. The correlations between the profiles, shifted to the appropriate tonics, are taken as a measure of this distance. To illustrate this process, Figure 3 shows the profile for C major superimposed on the profile for A minor (the C minor profile shifted down three half steps or equivalently up nine half steps). The ratings of the two profiles were then correlated. These two particular profiles were quite similar, giving a high correlation, as would be expected for a major key and its relative minor. This same procedure was applied to all major–major, ma-

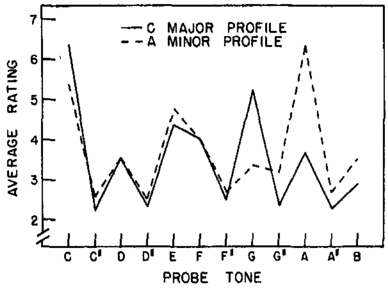

Figure 3. The C major key profile and the A minor key profile are superimposed to illustrate the derivation of the interkey distances. (The A minor profile was obtained by shifting the C minor profile down three steps of the chromatic scale. The values of the two profiles were then correlated to give a measure of the distance between C major and A minor keys.)

jor–minor, and minor–minor key pairs. The obtained pattern of correlations was extremely regular and reflected to a striking degree the several features discussed earlier that are supposed by music theorists to determine key distance. (Other measures of key distance, such as the summed squared distances between the rating profiles, were also explored and produced almost identical results). The regularities in these measures of key distance are brought out more clearly in the following analyses, however, so a full discussion will be postponed until later.

Multidimensional scaling of major and minor profile correlations. The interkey correlation matrix was then analyzed using the multidimensional scaling program MDSCAL (Kruskal, Note 4). A satisfactory solution was obtained only in four dimensions, with a stress value (Formula 1) of .017. The standard, principal-axis orientation option was employed. This solution is shown in its 2 two-dimensional projections in Figure 4. In the first two dimensions, the major and minor keys are arranged around a circle so that any major key is flanked on either side by its dominant and subdominant major keys (whose tonics are a fifth above and below that of the first major key). The same holds true for the minor keys. Thus, in the first two dimensions we obtain the circle of fifths for major and minor keys.

However, the placement of the minor key circle with respect to the major key circle poses problems for interpretation. In the solution the closest minor key to any major key is neither its relative nor its parallel minor but instead the minor key built on the second scale degree (the relative minor of the subdominant). This placement undoubtedly reflects a compromise between the close tie of a major key to both its relative and parallel minor keys, which are separated by three steps around the circle of fifths. (For example, the C major profile correlates strongly with both the A minor and C minor profiles but less strongly with that of D minor). Because of this tension produced by the relative and parallel relations, the two-dimensional solution (which was almost identical to the left panel of Figure 4) was rejected in favor of the four-dimensional solution pictured here.

The projection in the third and fourth dimensions represents the parallel and relative relations between major and minor keys directly. The keys again form a circular configuration, but with a slightly smaller radius. On this circle any given major key is flanked by its parallel minor on one side and by its relative minor on the other, and likewise, any minor key is located next to its parallel and relative major keys. In order to accomplish this, the solution identified as a single point all major keys whose tonics differ by a major third and also all minor keys differing by a major third. The musical interpretation of this identification is not immediately obvious; however, viewing this four-dimensional solution in a slightly different but equivalent way reveals much more clearly the structure of the interkey relations.

Toroidal representation of key distance. The low stress value of the four-dimensional solution is evidence that this representation precisely accounts for the intercorrelations between profiles. However, the 2 two-dimensional projections posed problems for interpretation. In each of the two-dimensional projections, circular configurations were obtained. Thus, the points fall on a surface that can be described as a circle in two dimensions crossed with a circle in two more dimensions. But this is exactly the topological definition of a torus ($S^1 \times S^1$). Each point in four dimensions can be uniquely described by the angular distance, θ, around one circle and the angular distance, ϕ, around the second circle. The (θ, ϕ) values for each key can then be plotted in two di-

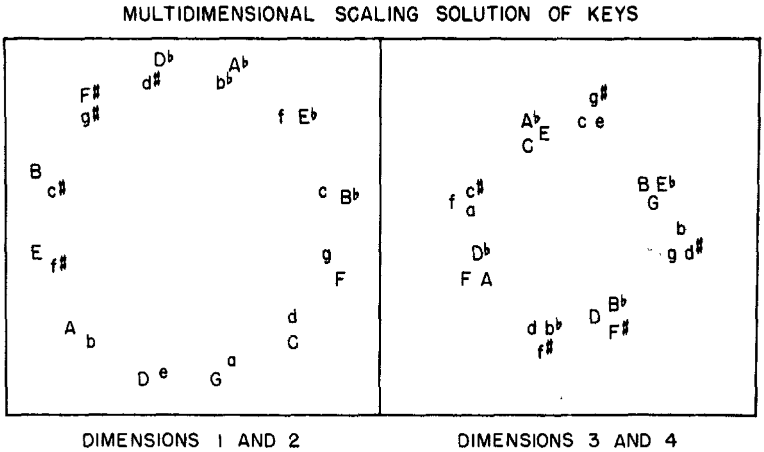

Figure 4. The four-dimensional multidimensional scaling solution of the intercorrelations between the 24 major and minor key profiles (stress = .017). (The projection of the solution onto the first two dimensions is shown on the left. In this projection the circle of fifths for major and minor keys was obtained. The projection onto the last two dimensions is shown on the right. Major and minor keys separated by an interval of a major third were represented as single points in the solution. Each major key was located next to its parallel minor key on one side and its relative minor key on the other. Similarly, each minor key was flanked by its parallel and relative major keys.)

mensions as shown in Figure 5, where it is understood that the opposite edges of the rectangle are identified.

To see the similarity between this representation and the more familiar (but misleading) description of a torus as an inner-tube, first visualize folding the rectangle over to identify the two horizontal edges, making a hollow tube, followed by wrapping it around to line up the two open ends. Not only does this introduce distortions of distances (the inner radius is smaller than the outer radius) but it also leads to false intuitions about the interdependence of coordinates in the different dimensions. For these reasons the flattened out representation shown in Figure 5 is preferable.

The (θ, ϕ) coordinates of the 24 keys were computed from the four-dimensional scaling solution and are shown in the torus representation in Figure 5. Because of small deviations from perfect circularity in the obtained scaling solution, ideal coordinates were computed for each key such that the points were constrained to fall on the surface of the torus. The program CMH2 (Cliff, 1966), which shifts, rotates, and expands or contracts one configuration to best match a second configuration, was used to compare the ideal configuration with that actually obtained in the scaling solution. This program gave a correlation of .998, indicating that the torus is in fact a very accurate representation of the four-dimensional scaling solution of the profile correlations.

More importantly, when viewed as (θ, ϕ) coordinates on a torus, the pattern of interkey distances becomes entirely interpretable. First of all, the keys separated by fifths fall on a path that wraps three times around the torus before joining up with itself again; the major keys fall on one such path, and the minor keys on another, parallel path. These are lined up so that any major key is flanked by its relative minor on one side and its parallel minor on the other. These parallel, relative, and fifth relations are made explicit for C major in Figure 5.

There is a striking similarity between the map of tonal regions given by Schoenberg (1969) for major and minor keys (Figure 1) and local regions of the toroidal representation obtained here (Figure 5). The torus, however, has the advantage of simultaneously depicting all interkey relations, not just those immediately surrounding a single major or minor key. Moreover, precise quantitative comparisons between interkey distances can be made from the torus representation. For example, a major key is in fact found to be more closely related to its relative than to its parallel minor as seen,

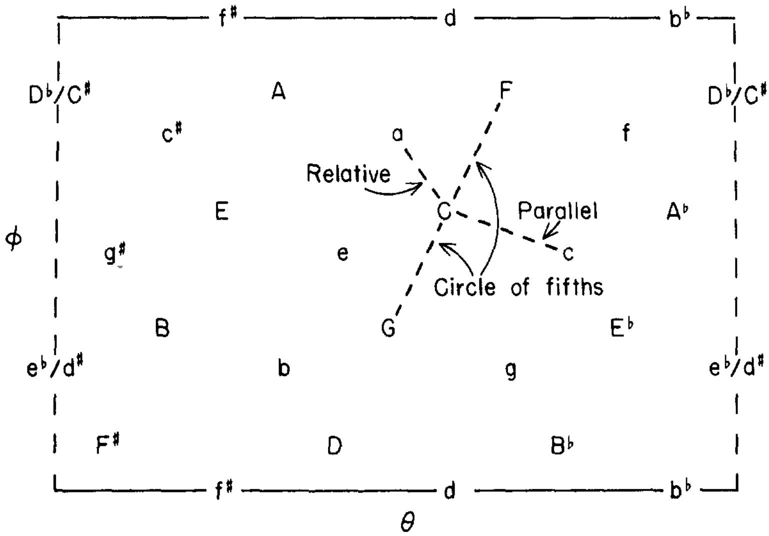

Figure 5. An equivalent two-dimensional map of the multidimensional scaling solution of the 24 major and minor keys. (The vertical [dashed] edges are identified and the horizontal [solid] edges are identified, giving a torus. The circle of fifths and parallel and relative relations for the C major key are noted.)

for instance, as the closer distance between C major and A minor than between C major and C minor. Another finding is that a major key is closer to the minor key built on its third scale degree (the relative minor of the dominant key) than it is to the minor key built on its second scale degree (the relative minor of the subdominant key). For instance, C major and E minor are closer than are C major and D minor. In fact, this was anticipated by Schoenberg (1969, p. 68) and may reflect the greater number of chords shared by the major key and the relative minor of the dominant key. Other comparisons of this sort can easily be made using this spatial map of key regions. Moreover, this representation provides a framework for approaching the problem taken up later of how chords relate to different tonal centers and how the sense of key develops as the listener hears sequences of chords.

Other spatial representations. Other previously proposed spatial representations have been described in detail by Shepard (1982a; see also Shepard, 1981, 1982b) and will be mentioned only briefly here. The first such representation in the literature is the helical structure of single tones (Drobisch, 1846, cited in Ruckmick, 1929; Pickler, 1966; Revesz, 1954; Shepard, 1964). In this three-dimensional configuration, the single tones are spaced along a helical path in order of increasing pitch height such that tones separated by an octave interval are relatively close as the helix winds back over itself on successive turns. The projection of the points on the plane perpendicular to the vertical axis of the helix is often referred to as "tone chroma," and the projection on the axis as "tone height." This representation, then, simultaneously specifies the close relation between tones separated by small intervals and that between tones at octave intervals. This helical representation is preferable for musical pitches to the unidimensional psychophysical scale of pitch that combines both frequency and log frequency as proposed by Stevens (Stevens & Volkmann, 1940; Stevens, Volkmann, & Newman, 1937), because there is a strong identification between tones differing by octaves. This octave effect is seen in their interchangeable use in music theory and composition and in judgments of intertone relatedness (Allen, 1967; Boring, 1942, p. 376; Krumhansl, 1979; Licklider, 1951, pp. 1003–1004; and numerous other treatments). Shepard (1982a) argues that the tones should be equally spaced in terms of log frequency around the helix because both the selection of musical tones and performance in transposition tasks (Attneave & Olson, 1971) indicate that the log frequency scale applies to musical pitches.

Shepard (1981, 1982a, 1982b) has recently proposed more complex representations of single tones. In the representation referred to as the "melodic map" (Shepard, 1982a, Figure 4), the set of 12 tones in an octave range are placed on the surface of the torus in four dimensions. As in the configuration obtained here, the first two dimensions correspond to the circle of fifths. The last two dimensions constitute the chroma circle of the helical structure proposed earlier. However, introducing the circle of fifths splits apart the chroma circle into two whole-tone scales forming a double helix that wraps around the surface of the torus. Shepard provided evidence that an additional fifth dimension might be needed to account for tone height.

Empirical support for Shepard's melodic map comes from the earlier scale-completion study (Krumhansl & Shepard, 1979). There are a number of reasons why the present analysis, which is based on similar data, produces different results. The first reason is that the method of obtaining the matrix of interelement distances was different in the two cases. In the Shepard analysis the value for tones separated by each number of half steps was the average of the rating given to the tone that number of half steps above the implied tonic and the rating given to the tone that number of half steps below the tonic. As noted earlier, however, intervals of equal size often received significantly different ratings, and these differences could be understood in terms of their functional interpretations within the tonal hierarchy of the instantiated key. Consequently, Shepard's analysis loses considerable structure contained in the rating profiles. The present analysis used instead the correlation between the shifted profiles as the distance measure. This latter measure is most properly interpreted as a measure of interkey distance because it is based on the similarity of tonal functions of the individual musical tones in the different keys. A second, related reason for differences between the two representations is that tone height does not apply to an abstract tonal center, because it is independent of the octave in which the tonic is sounded. Moreover, the probe tones and context elements used in this study were constructed of frequencies sounded in five octaves simultaneously, thus minimizing tone height effects. Nonetheless, it is significant that the chroma circle did not emerge from the present analysis because Shepard (1964) demonstrated that this circular dimension is independent of tone height.

Shepard's (1982a, Figure 8) "harmonic map," which is a transformation of the melodic map, is identical to the spatial representation obtained here except for scale factors and the inclusion of points for major and minor keys in the present results. Here, however, we give this configuration a different interpretation. Shepard's proposed representation depicts the relations between single tones, with certain chord, scale, and key relations reflected as patterned subsets within the spatial map. Thus, this repesentation brings out structures at all three levels: single pitches, chords, and keys. In contrast, we interpret the spatial configuration obtained here as depicting structure at only the level of abstract tonal centers and presume that additional or other structural features apply to single tones and chords, particularly those that reflect the context-specific hierarchy of stability described by music theorists. Empirical evidence supporting this view has been obtained in a number of studies (Bharucha & Krumhansl, in press; Krumhansl, 1979; Krumhansl et al., 1982, in press; Krumhansl & Shepard, 1979) and was summarized earlier in this article.

That Shepard's (1982a) harmonic map appears as a subset of the spatial representation of musical keys obtained here, however, may be accounted for by the central function of the tonic tone in the structural hierarchy that applies to each major and minor key. In other words, the convergence between Shepard's harmonic map and the toroidal map of musical keys may be understood in terms of the close association between an abstact tonal center and the tone that is its tonic. To test this hypothesis an additional analysis of the profiles obtained in the present experiment was performed. In this analysis a measure of the psychological proximity between single tones was estimated by correlating the ratings given to one tone in the 24 major and minor keys with the ratings given to the other tone in the 24 keys. This measure reflects how similar the two tones are in their tonal functions across all possible musical keys. Multidimensional scaling applied to these correlations recovered the spatial configuration of Shepard's harmonic map (stress = .007). The first two dimensions contained the circle of fifths, and the second two dimensions contained a circular configuration in which tones differing by a major third were identified as a single point; the ratio of the radii was 4:3. Thus, Shepard's harmonic map may be obtained from the probe tone ratings and may be understood as being derivative of structure at the level of abstract tonal centers and mediated through the central function of the tonic tone in the hierarchy of stability.

Relations Between Chords and Tones

In much of tonal music, groups of tones are sounded simultaneously in chords. Although it is a straightforward matter to specify the chords that constitute the basic set of harmonies of a particular key, as described earlier, the converse problem of determining the prevailing key from a sequence of chords is much more difficult. The key of a piece of music is not explicit in the music, except as the key signature of the written score, and even this does not distinguish between major and minor modes. The listener must extract key information from the sequence of chords itself, using knowledge about the harmonic functions of chords in the different musical keys. In this section we turn to the problem of characterizing the listener's knowledge of these harmonic functions.

Music-Theoretic Descriptions of Chord Functions

Seven chords form the basic set of harmonies of a particular key and are often denoted by roman numerals to designate the position of the root of the chord in the scale of the key. The tones of the chords are usually selected from the major scale for major keys and from the harmonic minor scale for minor keys. Because of differences between the intervals of the major and minor scales, different chord types appear in major and minor keys. In a major key, I, IV, and V are major chords; II, III, and VI are minor; and the VII chord is diminished. A minor key has major chords as V and VI, minor chords as I and IV, diminished chords as II and VII, and an augmented chord as III, which, however, is often used in its major form. (For this reason we do not consider the augmented chord further here.) Despite these differences in chord types, the roman numeral notation used in music theory implies there is a similarity in the effect of chords built on the same scale degrees in major and minor keys. In practice, usually a fourth tone repeating one of the other three in a different octave is also simultaneously sounded. In addition, the other chord tones may be placed in any different octave, yielding different inversions. These inversions differ in terms of the lowest bass tone and in the specific intervals contained in the chords. Whether these inversions produce significantly distinctive effects or simply serve to introduce greater variety is a matter of some debate (see, for example, Rameau, 1722/1971; Schoenberg, 1969, p. 6). The issue of chord inversions will not be considered further here because our experiments employed chords with each component sounded in five octaves, using an amplitude envelope that tapered off at both low and high ends of the frequency range.

Not only are there restrictions on the particular chords used in a key but there are also restrictions on the orders in which they can appear. Moreover, certain chords within the set are described as playing more structurally significant roles than others, and these are typically sounded more frequently and in more prominent temporal positions. In Western music the key sense is primarily a product of chords used in this highly constrained fashion. Despite these conventions there usually remains some residual ambiguity of key. At the most basic level, this ambiguity results because any given chord (and sometimes even longer sequences of chords) can simultaneously function in more than one key. In fact, major and minor chords each play roles in at least five different keys, and a diminished chord in three keys. For instance, a C major chord can function as I in C major, V in F major, IV in G major, V in F minor, or as VI in E minor. For Berry (1976) the dual or multiple meanings of harmonies are fundamental to tonal music:

> The designation by roman numeral symbols of a harmony in two or three lights . . . is not an inconsistency; it is a realistic representation of the range of functional

significances attending a harmony of both primary and secondary contexts of tonal reference. The indication of such multiple functions is a reflection of the depth of tonal-harmonic meaning and of the way harmony is heard in tonal music. (p. 67)

Thus, any single chord may be interpreted as simultaneously playing various different harmonic functions, but there may be a tendency to assign the chord one single most likely role. Schenker (1906/1954) notes the tendency to "ascribe to any major or minor triad, first of all, the meaning of the tonic" (p. 252). This interpretation may, however, need to be reevaluated as subsequent chords are heard that are incompatible with the previously assigned harmonic function. In Western music, then, there is a complicated set of circumstances that leads to the designation of the key of a piece of music.

Empirical Studies of Chord Functions

Previous investigations of chord functions have focused on the relations between different chords rather than those between chords and abstract tonal centers. Nonetheless, the results of these studies on chords have shown clear influences of organization at the level of tonal centers. One scaling study (Krumhansl et al., 1982) used the set of chords from three closely related keys: C major, G major, and A minor. In that study the chords of the three keys were shown to be organized in an overlapping fashion that preserved the internal chord hierarchy within each key, despite the multiple functioning of chords in several keys. The chords of G major and A minor that are shared with C major were centrally located, whereas those unique to G major were separated from those unique to A minor, which is relatively remote from G major. In a complementary study Bharucha and Krumhansl (in press) investigated the chords of maximally distant major keys, finding a clear separation between the chords of distant keys. This separation was also reflected in memory confusions. Finally, Krumhansl et al. (in press) observed systematic alterations of perceived interrelations between chords as the context key was varied. These effects were a function of the distance between the key in which the chords function and the key of the context, as measured by the number of steps around the circle of fifths. (Only major key contexts, whose distances can be described by the circle of fifths, were used in that study.) Thus, there is considerable evidence from these empirical investigations that chords and their interrelations are perceived with respect to a system of related tonal centers.

The Present Approach to Representing Chord Functions

The following analysis addresses the problem of how the perceived relation between a chord and its abstract tonal center, that is, its harmonic function, may be represented using the spatial map of musical keys described in the last section. We test the specific hypothesis that a chord is perceived as closely related to the abstract tonal centers in which it plays its most harmonically significant roles. If this is the case, then interkey distances in the spatial map of the major and minor keys must reflect the multiple interpretations that are possible for each single chord. In other words, keys sharing chords must be located in proximal positions on the surface of the torus. The profiles obtained for major, minor, diminished, and dominant seventh chords using the method described earlier are used to test these predictions.

Results and Discussion

Chord–key profile correlations. Each of the four chord profiles (for major, minor, diminished, and dominant seventh chords) was correlated with each of the composite major and minor key profiles (Figure 2). The key profiles were shifted to all possible tonics so that the relation between each of the 4 chords and each of the 24 major and minor keys could be determined. Not surprisingly, the major chord profile correlated highly with the key in which it is the I chord, as did the minor chord with the corresponding minor key. This would be expected because the chord profiles and the cadence profiles were averaged to determine the key profiles, and the tonic interpretation is assumed most likely for major or minor chords. Other patterns in the correlations are brought out more clearly in the next analysis.

Chord positions on the torus. Earlier, the argument was made that keys sharing chords should be perceived as closely related because this would enable listeners to entertain multiple functional roles in different keys. The torus representation provides a convenient framework for testing this prediction as well as for representing the relative strengths with which a single chord is interpreted in its different possible roles.

The correlations between the chord and key profiles were analyzed using the program PREFMAP (Chang & Carroll, Note 5; see Carroll, 1972, for a more complete description of the method). This program determines a point or vector in multidimensional space so as to best reflect the psychological distances (or preferences) between an object and a set of other objects. The set of other objects is also represented as points in the space in an a priori configuration that is often a previously obtained multidimensional scaling solution. The a priori configuration used here was the idealized coordinates of the musical keys on the torus as described earlier. We elected to use the Phase 4 option to find a vector to represent the correlations between each obtained profile and that of each of the 24 major and minor keys. This vector was determined in the first two dimensions separately from the last two dimensions because this ensures that the combined vector in four-dimensional space would in fact intersect the torus. (Fitting the correlations simultaneously in all four dimensions, using the Phase 4 option and using the ideal point option of Phase 3, was also tried in various cases, yielding very similar results.)

The results for the C major, A minor, and B diminished chords are shown in Figure 6. These chords were selected for visual presentation because they all function in the C major key, which is a convenient reference key. Analogous results would hold for any other chords. The point shown for the C major chord at the top of Figure 6 is the intersection with the torus of the vector obtained in the first two dimensions and the vector obtained in the last two dimensions. That this is an accurate representation of the correlations between the chord and the keys is indicated by the measure of fit for the first two and last two dimensions (r = .975 and .585, respectively). As expected, the C major chord is very close to the C major key in which it functions as the I chord. It is, however, drawn slightly toward the F major and F minor keys in which it plays the strong dominant, V, function. The C major chord is also the IV of G major and VI of E minor, which are its slightly weaker harmonic roles, and these keys are more distant from the C major chord. Thus, we see that the position of the C major chord is a compromise of its roles in the different keys, but it is located relatively close to all keys in which it functions.

The center of Figure 6 shows the position of the A minor chord in the map of key regions. The goodness-of-fit measure was slightly lower in the first two dimensions, r = .923, and slightly higher in the last two dimensions, r = .615. Again, the strongest interpretation of the chord is as the tonic, I, of A minor. In addition, its roles in F major (as III), C major (as VI), G major (as II), and E minor (as IV) are also reflected by the close distance between the A minor chord and these keys. That these functions do not draw the chord point closer to these keys may be a result of the strong tendency for the parallel major key, A, to dominate its parallel minor. Schenker (1906/1954) notes the tendency for "every passage in minor to be resolved into major" (p. 54). Given this, and the strong parallel major–minor relation, the position of the minor chord can be understood as reflecting the tension caused by the opposite forces pulling it toward its relative and parallel major keys.

The position of the B diminished chord is shown at the bottom of Figure 6. For this chord the measure of fit was .963 in the first two dimensions and .636 in the last two. This diminished chord is located near C major, in which it is the VII chord; C minor, in which it is also the VII chord; and A minor, in which it is the II chord. That it is drawn slightly toward G major may reflect the overlap between the B diminished chord and the tonic, I, chord of G major. The position on the torus of the dominant seventh chord was also determined. The vectors obtained for the dominant seventh chord built on G (which is the V^7 of C major) intersected the torus at a point between C and G major, almost identical to the position of the B diminished chord. This confirms the interpretation of the leading tone triad, VII, as the dominant seventh chord with a missing root.

Thus, this result supports the music-theoretic description of these two chords as functionally similar.

In sum, this analysis of chord functions showed that major, minor, diminished, and dominant seventh chords are perceived as closely related to the tonal centers in which they play significant harmonic roles. The precise position obtained for chords that function in multiple keys represents a compromise of the combined influences of their various roles in these keys. Moreover, this analysis made clear the fact that keys sharing chords are located in proximal positions on the surface of the torus. In this is seen the intimate connection between chords and keys in tonal music. That an extremely regular representation of interkey distances was obtained may be understood as resulting in part from the complex interlocking pattern of multiple chord functions. The relative positions of the keys on the torus, then, depend to some extent on the chords that simultaneously function in different tonal centers. Also, the perceptual coherence achieved during well-structured chord sequences would seem to result because each of the chords in the key and in closely related keys is located proximally in this representation, a property also obtained in the scaling studies of chords (Bharucha & Krumhansl, in press; Krumhansl et al., 1982, in press). The pattern of chord-key relations, then, substantiates earlier characterizations of harmonic relations between chords, partially explains the structure found at the level of abstract tonal centers, and indicates a means through which a sense of key might evolve during the perception of harmonic sequences. We turn now to an investigation that traces how the key sense develops and changes as a well-structured sequence of chords unfolds in time.

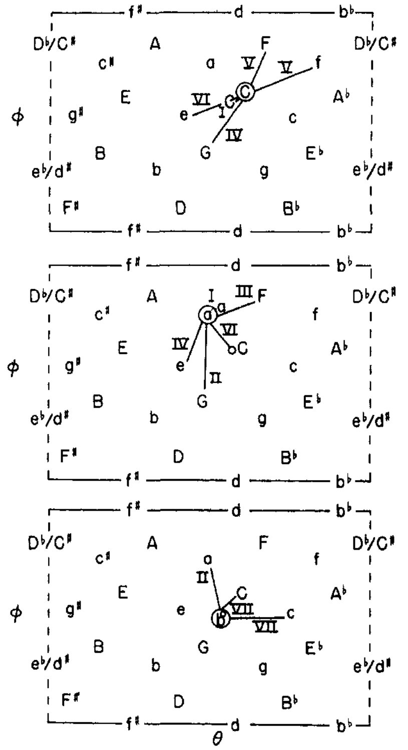

Figure 6. The relations between the C major, A minor, and B diminished chords and the 24 keys are illustrated in the top, middle, and bottom panels, respectively. (The point for each chord is the intersection of the vectors obtained by PREFMAP and the torus representation of the keys. For each chord its harmonic functions in the keys are indicated by roman numerals. The dominant seventh chord on G [not shown] was located in approximately the same position as the B diminished chord.)

Chord Sequences: The Developing and Changing Sense of Key

Neisser (1976) has described perceiving as an ongoing process in which the internal schema is continuously modified as external stimulus information becomes available:

> The listener develops more or less specific readinesses (anticipations) for what will come next, based on information he has already picked up. These anticipations—which themselves must be formulated in terms of temporal patterns, not of isolated moments—govern what he will pick up next, and in turn are modified by it. (p. 27)

In music perception this continually interactive process between the internal schema and external stimulus may perhaps be seen most clearly as the developing and changing key sense when a harmonic sequence is being heard over time. We have already described the process of perceiving tonal organization as one in which chords are interpreted in their multiple functions, giving rise to certain hypotheses about a likely key region and inducing expectations that its closely associated tones and chords will be sounded. However, the hypothesized tonal center may need to be reevaluated as the sequence progresses. Given the spatial map of key regions, and the inherently temporal structure of music, we are in a unique position to investigate the dynamic changes that occur in the internal representation during the perception of harmonic sequences. In particular, we are concerned with the question of how the sense of key is initially extracted from the sequence and how modulations (shifts) between tonal regions are assimilated.

Music-Theoretic Descriptions of Tonal Organization

In Western music a composition is typically organized around a primary key, and the chords employed are for the most part contained within the basic set of harmonies of that key. However, shifts to other temporary keys are often employed and are used to provide organization to the piece as a whole. Generally, this shift, or modulation, is to a closely related key and is effected through the use of a chord that is shared by the two keys. A chord that functions in this way is called a pivot chord and is usually followed fairly immediately by a cadence, that is, a strong key-defining sequence of chords that most frequently contains the V and I chords of the new key. Schenker (1906/1954) notes, "In general, a cadence in the new key has proved to be the most suitable means to fortify the new key and thus to make the modulations real and complete" (p. 322).

In addition, Schenker (1906/1954, 1935/1979) has suggested a hierarchy of instantiated tonal regions, which was mentioned earlier in this article. Schenker's influence is clearly evident in recent theoretical writings by Lerdahl and Jackendoff (1977, in press) and Deutsch and Feroe (1981). His work also has direct relevance to the present results. Schenker proposed a system of musical analysis that identifies the surface elements of the music in terms of their relations to fixed underlying musical structures. That is, at the very deepest level a few structural elements are identified, and the musical tones are described as they relate to these structural elements in a hierarchy of increasing abstraction. This system characterizes the way in which the momentary influences of the surface level are transformed into a coherent whole.

According to Schenker any piece of music can be analyzed at its most basic level, called background or Ursatz, in terms of a single tonal center. This key is most often explicitly instantiated at the beginning and end and continues to exert its influence throughout the piece, despite possible modulations. Indeed, Schenker (1906/1954, p. xxii) rejects the concept that a complete modulation ever takes place on the background level; a new key may achieve some independence but will function only on the middle ground or foreground levels, giving preeminence to the primary key. A surprising number of music theorists agree on the importance of the primary key. For example, Rosen (1972) states, "A passage in a tonal work that is outside the tonic is dissonant in relation to the whole piece, and demands resolution if the form is to be completely closed" (p. 26). Essentially the same position is taken by Schoenberg (1969):

> Every digression from the tonic is considered to be still within the tonality, whether directly or indirectly, closely or remotely related. In other words, there is only *one tonality* in a piece, and every segment formerly considered as another tonality is only a region, a harmonic contrast within that tonality. (p. 19)

However, the position that there is a unique underlying tonality has been criticized as being too inflexible, and to date there is no evidence that most listeners actually maintain a fixed sense of key throughout an extended musical passage.

The middle ground in Schenker's hierarchy represents the piece of music in terms of the more structurally significant events (chords and tones). Thus, at this level the music is specified in a more simplified form than in the complete score. Modulations between keys are reflected at this level of analysis as well as the foreground level. The foreground level is the actual music itself. Each individual chord has a tendency to be interpreted as the tonic of a key, a process called tonicization. Schenker (1906/1954) says, "Not only at the beginning of a composition but also in the midst of it, each [chord] manifests an irresistible urge to attain the value of the tonic for itself" (p. 256). Tonicization is expected to be most pronounced when the chord is preceded by its own dominant (V) and during sections of greatest key ambiguity and instability. Thus, Schenker

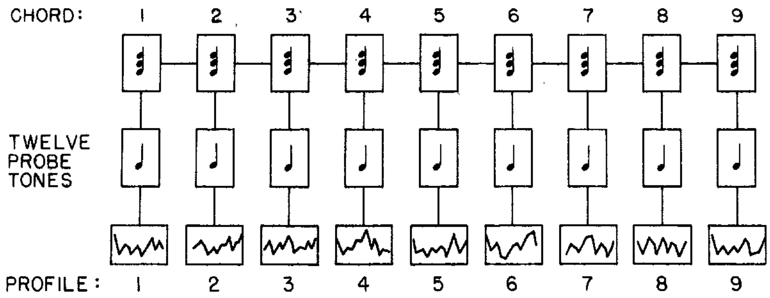

Figure 7. The design of Experiment 2, using chord sequences. (The first chord of each sequence was followed by each of the 12 probe tones within the chromatic scale, giving the first rating profile. Next, the first two chords of each sequence were paired with the 12 probe tones, giving the second profile. This process was continued until profiles were obtained for sequences of Lengths 1 through 9.)

proposes that pitch structure may be analyzed in terms of a hierarchy of increasing abstraction, with the least abstract level being the actual tone, intermediate levels corresponding to the more structurally significant elements within the key prevailing in that section of the piece, and the deepest level specifying a single key that is presumed to underlie the entire composition.

The Present Approach to Assessing Perceived Tonal Organization

Various aspects of Schenker's (1906/1954, 1935/1979) description of a tonal hierarchy will be investigated in the next experiment, which again used the tone-profile technique employed earlier. The design of the experiment is shown in Figure 7. Each of the sequences consisted of nine chords. Ten different sequences were used; their structure will be described later. In the experiment the listener first heard the very first chord of each sequence, followed by each of the 12 possible probe tones of the chromatic scale, as in the first experiment. This gave a profile of ratings for each of the first chords of the 10 sequences. Following this, the listener hears the first two chords of each sequence, again paired with all possible probe tones, giving a profile for two-chord sequences. This procedure was continued until profiles were obtained for sequences of Lengths 1 through 9. It is important to note that at no time had the listener heard the sequence beyond the point at which it was being probed. That is, the rating profiles were based only on the chords presented up to that point of the sequence. These profiles were then correlated with the major and minor key profiles obtained earlier to trace the unfolding process through which a sense of key is achieved.

The 10 harmonic sequences are shown in Table 2. The first two sequences were constructed from the chords of a single key, those of C major for the first sequence and those of C minor for the second; these will be referred to as the intended keys. The chord functions within each key are shown in the table as roman numerals, as in a standard harmonic analysis. These sequences were constructed so as to be relatively tonally unambiguous, although as noted earlier the multiple harmonic functions of individual chords necessarily entails a certain residual ambiguity. Nonetheless, the selection of chords from a single key tends to reduce this ambiguity. In addition, the temporal order of the chords was guided by music-theoretic descriptions of permissible (or probable) chord sequences in music. For this we relied on the table of chord progressions contained in Piston (1962, p. 18). Furthermore, these two sequences began with the harmonically significant IV–V progression and ended with the relatively strong II–VI–I cadence, both of which would be expected to be effective instantiators of key.

The other eight sequences contained modulations (changes) between keys; these are shown as Sequences 3 through 10 in Table 2. Both the chords themselves and the roman numeral analysis in the two keys are shown in the table. These modulating sequences all had the same general structure. Each of them began with a cadence, IV–V–I or II–V–I, in the first key, which was either major

Table 2
Chord Sequences

Sequence number	Modulation	First key	Second key	Chord 1	Chord 2	Chord 3	Chord 4	Chord 5	Chord 6	Chord 7	Chord 8	Chord 9
1	No	C major	C major	F major C: IV	G major V	A minor VI	F major IV	C major I	A minor VI	D minor II	G major V	C major I
2	No	C minor	C minor	F minor c: IV	G major V	A^b major VI	F minor IV	C minor I	A^b major VI	D diminished II	G major V	C minor I
3	Yes: Direct and close	C major	G major	F major C: IV G:	G major V (I)	C major I (IV)	A minor VI (II)	E minor III VI	B minor III	E minor (III) VI	D major V	G major (V) I
4	Yes: Indirect and remote	C major	B^b major	F major C: IV B^b: (V)	G major V	C major I	A minor VI	F major IV V	G minor VI	E^b major IV	F major (IV) V	B^b major I
5	Yes: Direct and close	C major	A minor	F major C: IV a: (VI)	G major V	C major I	F major IV (VI)	D minor II IV	E major V	B diminished (VII) II	E major V	A minor (VI) I
6	Yes: Indirect and remote	C major	D minor	F major C: IV d:	G major V	C major I	F major IV	D minor II I	B^b major VI	E diminished II	A major V	D minor (II) I
7	Yes: Close	C minor	F minor	D diminished c: II f:	G major V	C minor I	A^b major VI	F minor IV I	D^b major VI	B^b minor IV	C major V	F minor (IV) I
8	Yes: Distant	C minor	$C^{\#}$ minor	D diminished c: II $c^{\#}$:	G major V	C minor I	G major V	A^b major VI V	A major VI	$F^{\#}$ minor IV	$G^{\#}$ major (VI) V	$C^{\#}$ minor I
9	Yes: Close	C minor	C major	D diminished c: II C:	G major V (V)	C minor I	A^b major VI	G major V V	A minor VI	F major IV	G major (V) V	C major I
10	Yes: Indirect but close	C minor	A^b major	D diminished c: II A^b:	G major V	C minor I (III)	A^b major VI (I)	F minor IV VI	E^b major V	B^b minor II	E^b major V	A^b major (VI) I

or minor, and ended with one of these two cadences in the second key, which again was either major or minor. That is, the first three and last three chords constituted a cadence in the first and second keys, respectively. In each sequence the fourth chord belonged to the first key. The modulation was effected through the use of a pivot chord—a chord that functions harmonically in both keys—in the fifth position. Again we were guided in this choice by music theory, which describes the use of pivot chords as the most common means of modulation. The sixth chord belonged to the new key but not the initial key, thus reinforcing the sense of the new key, which was expected to become solidified by the cadence in the last three positions.

As noted earlier, keys sharing chords are located in proximal regions on the torus, and thus, all modulations employing pivot chords are necessarily between relatively close regions. However, some of the pairs of keys represented in the modulating sequences are more closely related than others. Of the modulating sequences beginning in a major key, the relations between C major and G major (Sequence 3) and between C major and A minor (Sequence 5) are classified by Schoenberg (1969, p. 68) as direct and close, that between C major and B♭ major (Sequence 4) and C major and D minor (Sequence 6) as indirect and remote. These are also more distant on the toroidal representation of the key regions. Of the sequences beginning in C minor, the relation to F minor (Sequence 7) and to C major (Sequence 9) are described by Schoenberg (1969, p. 75) as close, the relation to A♭ major (Sequence 10) as indirect but close, and the relation to C♯ minor (Sequence 8) as distant. Again, these classifications are substantiated by the obtained interkey distances. Consequently, of the modulating sequences, five (Sequences 3, 5, 7, 9, and 10) will be considered as moving between closely related key regions, and three (Sequences 4, 6, and 8) between relatively distant regions.

Certain limitations of the roman numeral analyses of the chord sequences provided in Table 2, which are those yielded by the standard theory of harmonic analysis, should be emphasized. Such analyses do not make explicit the full range of tonal ambiguity of the individual chords, their varying degrees of stability or the possibility that the tonal functions of the chords may need to be reevaluated as the sequence progresses. Less conservative approaches might interpret the chord functions in terms of a wider variety of keys and indicate how earlier chord functions may be reinterpreted in light of harmonic information following later in the sequence. However, the tone-profile technique employed here enables us to assess directly how the listener's sense of key develops and changes, providing a quantitative measure of the relative strengths of tonal interpretations at each point in time. In this way we are able to monitor the perceptual processes giving rise to the sense of key.

In addition, a number of specific issues raised by Schenker's (1906/1954, 1935/1979) description of tonal hierarchies will be addressed. His claim that at the deepest background level the listener maintains a fixed underlying sense of key, will be evaluated by determining the extent to which the profile ratings reflect the tonality of the first key after a modulation has occurred. Changes at the middle ground in Schenker's hierarchy are investigated by describing the time course of the development of these alternative, contrasting tonal interpretations in the modulating sequences. In this connection differences between modulations to more closely and more distantly related tonal regions are examined. Finally, the momentary influence of the individual chords, which may be considered to be the psychological correlate of Schenker's foreground level, will be assessed by determining the influence of the last heard chord on the rating profiles.

Experiment 2

Method

Subjects. Fourteen adult subjects from the Harvard University and Cornell University communities, who were recruited through posted notices in the psychology departments, participated in the experiment. They were paid at the rate of $12 for two experimental sessions lasting a total of approximately 3½ hours. A musical-background questionnaire showed that subjects had an average of 8.6 years instruction on musical instruments and voice and 6.4 years experience performing in instrumental groups and 2.3 years in choral groups. Currently, the subjects were playing an instrument or singing an average of 7.7 hours per week and were listening to music for 14.6 hours per week. Four of the subjects had taken one or more courses in music theory; the remaining subjects had no background in music theory. Again, all reported normal hearing, and no one reported having absolute pitch.

Apparatus and stimulus materials. The apparatus was identical to that used in Experiment 1, and the chords of the sequences and probe tones were produced in the same way. The 10 sequences shown in Table 2 were described in detail earlier. The durations of the chords and probe tones were as follows: Each chord had a duration of .5 sec, with .125 sec between chords. There was a pause of .5 sec between the last chord and the probe tone, which sounded for .5 sec, and a 4-sec pause between trials.

Procedure. Before beginning, the design of the experiment was explained to each subject. That is, they were told that they would hear chord sequences that varied from one to nine chords, each followed by a single probe tone. For each trial they were to rate on a scale from 1 to 7 how well the final probe tone fit in with the sequence of chords on the trial. The first session began with a practice tape containing trials of Sequence Lengths 1, 3, 5, 7, and 9, with a modulating sequence that was different from any actually heard in the experiment. They then heard the first chords of each of the 10 sequences (duplications eliminated) with all possible probe tones. These were followed by all of the different two-chord sequences, and the sequence length was increased until finally the complete nine-chord sequences were presented. Although the sequences in Table 2 are described with respect to C major or minor, in the experiment the reference key was shifted randomly from trial to trial, and all chord sequences of a given length were randomly intermixed. Lastly, a questionnaire about musical background was completed by each subject.

Results and Discussion

Individual differences. Intersubject correlations of the responses were generally high, with an average correlation of .669 between any individual subject's responses and the average responses for the group. Again, no systematic differences between subjects appeared that related to musical background. All further analyses are based on the average responses across subjects.

Correlations between sequence and key profiles: Nonmodulating sequences. The first main results are pictured in Figure 8. The solid line in the graph on the top left shows the average correlation between the obtained profiles for the nonmodulating Sequences 1 and 2 and the profile for the intended C major and C minor keys. In other words, the profile found after each chord in the sequence was correlated with the profile for the key from which the chords were drawn. These correlations were all quite high and tended to increase as the sequences progressed. Thus, it appears that the chord sequences do in fact instantiate, at least to some degree, the intended key.

There are peaks in the correlations at the fifth and ninth chord positions, where the tonic, C major or C minor chord, was sounded. These can be interpreted as the local effects of the chords in their tonic functions within the prevailing keys. In addition, there is a somewhat lower peak after the second chord, where the harmonically significant IV–V progression has occurred. Thus, even before the tonic has been heard, the obtained profile correlated strongly with that of the intended key. At each of the nine serial positions, it is possible to determine the extent of the local key effect of the last heard chord on the prevailing tonality of the sequence. To do this the chord–key correlations described earlier were compared with the correlations obtained in this experiment. These correlations between the individual chords and the key predict how strongly each chord in isolation would be expected to relate to the intended key of the sequence and, thus, serve as a baseline against which the strength of the intended key can be compared. To the extent that the obtained correlations exceed the local chord effects, the profiles reflect integration over multiple chords, leading to a sense of the overall key. These chord–key correlations averaged over Sequences 1 and 2 are plotted as the dotted line in Figure 8.

If complete tonicization is occurring, the obtained correlations will be expected to approximate these chord–key correlations. Complete tonicization is seen when the tonic, I, chord of the intended key is sounded in Positions 5 and 9. At all the nontonic positions, however, we see that the correlations with the prevailing key were stronger than would be predicted on the basis of the single chords alone. The difference between the graphs, indicated by the shaded area, is a measure of the extent to which the listener perceives the prevailing tonal organization as opposed to the tonal organization of the last heard chord. (However, this difference in the first position undoubtedly simply reflects some instability of the ratings that was characteristic of the early positions of the sequences.) These results indicate that during a harmonic sequence, listeners do develop a sense of key that is at least partially independent of the individual chords. That the two functions tend to move in parallel does suggest, however, that local effects of tonicization are occurring to some degree. Moreover, the fact that the correlations between the obtained profile and the prevailing key profile are less than unity means that the intended key is less than completely instantiated.

To summarize the results for the nonmodulating sequences, a sense of the intended key is evidenced by the relatively high correlations with that key's profile. The correlations tended to increase as the sequence progressed. Some effects of tonicization occurred, as indicated by the covariation between the obtained correlations and the correlations between the last chord profile and that of the prevailing key. For each nontonic chord in the sequence, however, the correlation with the prevailing key was stronger than would be predicted on the basis of the last heard chord. This suggests that listeners do in fact process the multiple harmonic functions of the chords and integrate this information as additional chords are heard, giving rise to a key sense that is partially independent of the individual chords. The correlations with the intended key, though, were less than perfect, reflecting some ambiguity and some local effects of the chords themselves. Thus, the profile measure is simultaneously reflecting both the underlying prevailing key (at the background and middle ground levels) and the effect of the single chords (at the foreground level).

Sequences modulating between closely related keys. The modulating sequences can be used to separate effects at background and middle ground levels. The results for the five modulations between closely related keys are shown at the center of Figure 8. On the left the correlations between the profile obtained at each position of the sequence and the profile for the intended first key are graphed as the solid line; the correlations from the first experiment between the last heard chord and the first key are indicated by the dashed line. On the right are the analogous correlations with the profile of the intended second key.

The graph on the left shows that the strength of the first key grows during the first three chords, which constitute a cadence in the first key, and then gradually declines throughout the remainder of the sequence. A dip occurs at the sixth chord, which is a member of the basic set of harmonies of the second, but not of the first, key. At each nontonic position along the sequence after the initial chord, the correlation with the profile of the first key is stronger than would be predicted on the basis of the last heard chord alone, as indicated by the shaded region. This demonstrates the continued influence of the first key even after the modu-

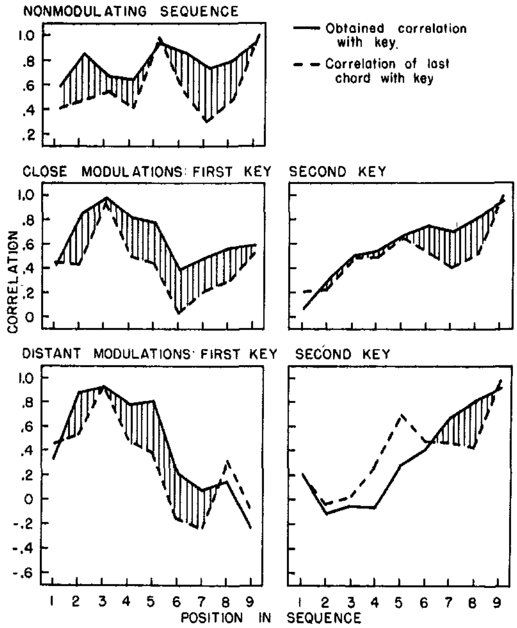

Figure 8. The correlations with the intended key or keys in Experiment 2. (The solid line on the top left figure shows the correlation between the obtained profile at each point along the sequence and the profile of the intended key averaged over the two nonmodulating sequences. The dashed line indicates the correlation between the last heard chord and the intended key derived from the first experiment. The extent to which the prevailing key was stronger than the local chord effects is indicated by the shaded area. The graphs in the center of the figure show the average results for the five sequences that modulated between closely related keys. The correlations between the profile for the first key and both the obtained and single chord profiles are plotted as the solid and dashed lines, respectively. On the right are shown the analogous correlations with the second key at each point along the sequence. The graphs at the bottom of the figure show the results for the three sequences that modulated between relatively distant keys.)

lation has occurred. Again the effect of tonicization of each chord is apparent as the covariation between the obtained correlation with the first key profiles and the correlation between the last heard chord and first key profiles. The difference between these functions, however, tends to decrease toward the end of the sequence. Although listeners do achieve a sense of the initial key and tend to maintain it throughout the sequence, there is some evidence here that the strength of the first key, beyond the local chord effects, declines as the sequence moves into the new key.

On the right, the strength of the second, closely related key is seen to increase throughout the sequence. Initially, the obtained correlations matched almost perfectly those that would be expected on the basis of the last heard chord alone. As soon as a chord belonging to the second but not the first key was heard in the sixth position, however, the obtained and single-chord correlations deviate, with a stronger second-key effect than would be expected on the basis of the remaining chords in isolation. Thus, although local chord effects account for the responses throughout the first five chords, the listeners were apparently prepared to shift to the new key as soon as a chord unique to it was heard.

These results for modulations between closely related keys showed that listeners gradually shifted their key sense from the region of the first key toward the region of the second key. Some residual effect of the first key, however, was maintained throughout the entire sequence. This supports the notion that, at least for modulations between closely related keys, once a key has been established its influence continues to be exerted despite the modulation to a different key. Thus, effects of the predominant key at the background level are separated from the events at middle ground and foreground levels. However, the predominant key appears to prepare the listener for modulations to closely related regions, for as soon as a chord unique to the new key is heard, that key is perceived more strongly than would be expected on the basis of that single chord in isolation. In this we see that even when the initial key sense has been achieved, listeners entertain alternative, closely related key regions and are prepared to shift toward them as soon as chords unique to them are sounded. The strength of the new key continues to increase and achieves a relatively high level even before the cadence in the new key has been heard.

Sequences modulating between distantly related keys. A somewhat different picture emerges, however, for the sequences that contain modulations between distantly related keys. These results are shown at the bottom of Figure 8. On the left, the correlations with the first key are seen to rise steadily throughout the cadence in the first three positions and continue to maintain a relatively high level until they drop dramatically as soon as chords of the new key are introduced in Positions 6 through 9. Despite this rapid decline the correlations with the first key continue to be stronger than those predicted by the most recently heard chord until the last two positions, in which the single-chord correlations with the first key become stronger. That is, the effect of the first key is maintained for some time after the modulation but is greatly weakened, and eventually the association between the individual chords and the first key appears to be suppressed by the introduction of the new key. The extent of chord tonicization, as before, is evident in the parallel movement of the two functions.

The suppression effect between distantly related keys is even more apparent in the obtained correlations with the second key, shown at the bottom right of Figure 8. After the first chord, where local chord effects account for the results, there is an increasing tendency for the strength of the second key to be lower than predicted by its association with each individual chord. This difference continues to increase through the fifth position, despite that fact that the fifth chord—the pivot chord—is contained in the second key. Even after the sixth chord of the sequence, which is unique to the new key, the strength of the new key is somewhat weaker than the local chord effects would predict. Only following the seventh chord does the strength of the new key exceed that which would be predicted solely on the basis of the most recently heard chord.

These results contrast with those obtained for the modulations between closely related keys. For those sequences the sense of the first key was maintained throughout the entire sequence despite the modulation to the new key. For the distant modulations, although the first key continued to exert its influence after the modulation, its effect was very much weakened, and the relations between the individual chords and the first key were suppressed even before the completion of a cadence in the new key. These listeners appeared unable to maintain the sense of the primary key at the background level when the sequence modulated to a relatively distant key. Moreover, whereas there was a readiness to move to new, closely related keys, there was a certain measure of resistance to shift to distantly related keys, as seen in the later development of the sense of the new key in the sequences modulating between distant keys.

Although three of the sequences are described here as distant modulations, this is only relative to the close modulations. In fact, only one of these three modulations (that between C minor and C# minor) was described by Schoenberg (1969, pp. 68 and 75) as truly distant; the others were classified as indirect and remote. By electing to structure the modulating sequences using a pivot chord shared by the two keys, modulations between more distantly related regions could not be included. This is a consequence of the fact, discussed earlier, that keys sharing chords are at least relatively close. Perhaps even more striking suppression effects might have been obtained for modulations that moved between less closely related keys constructed in some way other than through a single pivot chord. However, the pivot-chord method is most representative of traditional harmonic practice, and even within the range of key distances employed, regular and interpretable differences were found between the close and relatively distant modulations.

Traces of the developing and changing key sense on the torus. The last analysis pointed to a sense of key that developed and changed as the chord sequences unfolded in time. In that analysis we grouped together the two nonmodulating sequences, the five close modulations, and the three relatively distant modulations. In order to explore in more detail the perceived tonal structure of each of these sequences, we again used the program PREFMAP (Chang & Carroll, Note 5). The input to this program was the matrix of correlations between the rating profile after each successive chord of each of the sequences and the 24 major and minor key profiles of the first experiment. Again, the a priori configuration was the set of idealized coordinates of the keys on the torus, and the Phase 4 option was used to find vectors in the first two and last two dimensions separately, as described earlier. The results of these analyses can be represented as the intersection with the torus of the two resulting vectors for each of the nine positions along the sequence. The measure of fit between the resulting vectors and the correlation data will not be given for each case separately. However, in all cases the fit in the first two dimensions was better (average $r = .940$) than in the last two dimensions (average $r = .589$). There was a tendency, however, for the first two dimensions to account for more of the variance for the sections analyzed in major ($r = .970$) than for the sections in minor ($r = .906$). The last two dimensions, in contrast, accounted for more variance for sections in minor ($r = .632$) than in major ($r = .553$).

Figure 9 shows the results for the sequence in C major (Sequence 1) on the left and for the sequence in C minor (Sequence 2) on the right. The points are labeled by the position in the sequence of the last heard chord. On the left the first point shows the tonicization of the first chord, F major. When the G major chord follows, the point moves down toward C major. Following this the points tend to cluster in the region around the C major key, although the positions vary somewhat. This movement reflects local influences of the individual chords, as can be seen by comparing the positions of the points with the individual chords of the sequence in Table 2. For example, movement of the third point toward A minor is accounted for by the A minor chord as the third chord in the sequence, the movement of the fourth point toward F major by the F major chord in position four, and so on. Thus, in these traces we again see the joint influences of the prevailing C major region and the local influences of chords as they function in their multiple harmonic roles.

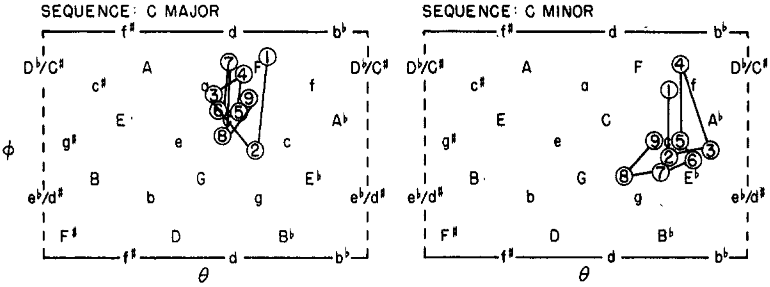

Figure 9. The developing sense of key is traced on the torus for the nonmodulating sequences in the C major key (on the left) and the C minor key (on the right). (The points, which are the intersection of the vectors obtained by PREFMAP and the torus, are labeled by the position of the last heard chord in the sequence.)

On the right the position of the first point indicates tonicization of the F minor chord in the first position of the sequence in C minor. After the second chord, G major, is heard, the C minor key appears to be fully established. However, following this, strong local chord effects are evident, as the third point moves toward A^b major, and the fourth point toward F minor. After this, the tonic, C minor chord, is sounded, and the remaining points cluster more closely around the C minor key. For this sequence, then, it appears that the prevailing key of C minor becomes established later in the sequence than C major did in the first sequence, perhaps due to the greater instability of the minor mode in its attraction to both relative and parallel major keys. In spite of the use of chords with the same roman numerals (chord functions) in these two sequences, this analysis points to significant differences in the functioning of the analogous chords across modes and in the ways in which minor and major keys are processed and organized.

Figure 10 shows the results of the analyses on the modulating sequences that began in C major. On the left are the two close modulations, to G major and A minor, and on

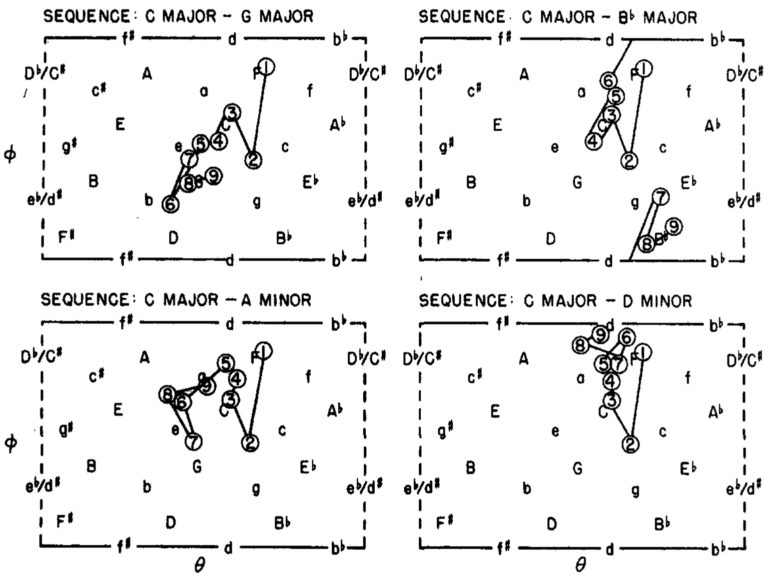

Figure 10. The key sense is traced on the torus for the four modulating sequences that began in the C major key. (The two plots on the left show the results for those sequences that modulated between closely related keys, and the two plots on the right show the results for the more distant modulations.)

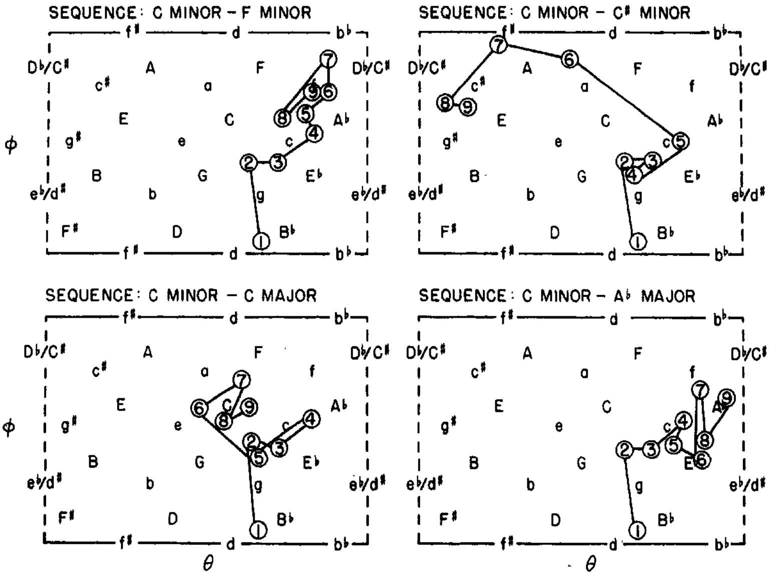

Figure 11. The results for the four sequences that begin in the C minor key are shown in the four plots. (The two on the left and the one on the lower right are for sequences that modulated between closely related keys; the one on the upper right shows the trace obtained for a modulation to a distantly related key.)

the right are the more distant modulations, to B^b major and D minor. In all of these cases, the key of C major is firmly established by the time the third chord has been heard, that is, after the IV–V–I cadence in C major. Following this, the points tend to move toward the new key center, showing some local effects of the individual chords, until by the eighth and ninth chords the new key has been quite firmly established. For both close modulations the points have all moved to the vicinity of the second key from the sixth chord on. For the more distant modulations, the continued pull of the first key is evident later in the sequence.

Similar results were found for the modulating sequences that began in C minor (shown in Figure 11). The two modulations on the left were close modulations, to F minor and C major. The one on the lower right to A^b major was called by Schoenberg (1969, p. 75) indirect but close. For these sequences the points have all shifted toward the new key from the sixth chord on. The figure in the upper right is the only sequence included in this experiment that was classified by Schoenberg as truly distant. In that plot we see a dramatic shift occurring between the first and second keys. One special feature of this last result should be noted. As the torus wraps around, the C minor and C$^\#$ minor keys are not in fact maximally distantly related, as is necessitated by the shared A^b (G$^\#$) major chord. However, the trajectory that is followed between the two keys does not follow the shortest path between the keys but rather takes the longer path around the torus in the other direction.

This last result, that the path taken between two keys need not be the shortest path on the torus, indicates that the perceived distance between keys during modulations depends on more than just absolute key distance. Instead, it depends on how the modulation is achieved, that is, on the specific chords that appear in the sequence. In particular, the number and nature of pivot chords, the duration of the transitional region, whatever intermediate regions are emphasized, and also the direction of the modulation (i.e., which key is first and which is second) are all factors that may influence whether the modulation is heard as smooth or abrupt and whether the keys are perceived as closely or distantly related. We suggest, then, that trajectories of the sort presented here, which trace the key sense as it develops and changes over time, may more fully capture the perceptual phenomena of shifting tonal regions than simple schemes that classify pairs of keys in terms of their relative distances.

General Discussion

The experiments reported here were directed at describing the internal representation of pitch relations in listeners familiar with Western music. Music theorists characterize pitch structure in tonal music as the functioning of tones and chords within an abstract tonal center, or musical key. Different tone and chord functions are associated with distinctive properties, certain elements taking more structurally significant roles than others. Additional structure is presumed to exist in the varying degrees of relatedness between different musical keys. Here we investigated the question of whether listeners do, as suggested, process individual tones and chords with reference to a system of abstract tonal centers, and if so, how the perceived tonal functions of the tones and chords and the relations between keys might be represented quantitatively.

We first obtained a measure of the strength of association between each musical pitch of the chromatic scale and an abstract tonal center. The abstract tonal center was instantiated by a musical element, such as a scale, chord, or cadence, that would be expected to be a relatively unambiguous indicator of the major or minor key. Each of the 12 notes of the chromatic scale was presented following each of these musical elements. The listeners' ratings of how well the individual tones fit with these musical elements contained considerable structure, the highest rating being given to the tonic tone, the first degree of the scale. This was followed by the third and fifth scale degrees, which together with the first scale degree form the tonic, I, chord. For major key elements the rating for the fifth scale degree was higher than that for the third, whereas the opposite order was found for minor keys. The fourth scale degree for major and the sixth scale degree for minor followed next. The remaining scale degrees had intermediate ratings and, finally, the nondiatonic tones had the lowest ratings. These rating profiles agree with the qualitative account of tonal functions provided by music theorists, and the profile for the elements defining major keys replicated the results of an earlier study (Krumhansl & Shepard, 1979), which introduced the basic technique used here.

After reliable rating profiles were obtained for major and minor keys, which serve as distinctive markers of the instantiated key, the profiles were shifted and intercorrelated. This provided a measure of distances between any pair of keys (major or minor) based on the similarity of the tonal hierarchy in the two keys. These correlations showed that each major key was closely related to its dominant and subdominant major keys and to its relative and parallel minor keys. Similarly, each minor key was closely tied to its dominant and subdominant minor keys and to its relative and parallel major keys. The multidimensional scaling technique, which transforms a set of measured interobject proximities into a spatial configuration of points so as to best account for the proximity data, was applied to the matrix of correlations. That analysis produced a very regular configuration of points in four dimensions that accounted almost perfectly for the correlations computed between the rating profiles. The 24 major and minor keys were situated on the surface of a torus so as to directly reflect the circle of fifths relations as well as the parallel and relative relations between major and minor keys.

When the obtained torus representation was viewed as a rectangle in two dimensions with opposite edges identified, the results were strikingly similar to the map of key regions provided by Schoenberg (1969, pp. 20, 30), arrived at through analysis of structure in musical compositions. Thus, this empirically derived spatial representation of the 24 keys strongly confirms the music-theoretic description of structure at the level of abstract tonal centers. However, measures of key distance proposed by music theorists, such as overlapping scale tones, differences in key signature, or distances around the circle of fifths, do not provide the kind of precise quantitative measure obtained here through comparing the relative strengths of the individual tones within the different keys. Although all of these factors are reflected, directly or indirectly, in the profiles, neither singly nor in combination do they yield the kind of unambiguous measure of key distance produced by the present method.

What is more interesting from the psychological point of view is that listeners, at least those with fairly extensive experience with tonal music, have apparently internalized musical structure at the more abstract level of tonal centers. In the present context this representation was derived in a somewhat indirect way from the profile ratings. However, other studies have demonstrated key distance effects on recognition of transposed melodic sequences (Bartlett & Dowling, 1980; Cohen, 1975; Cuddy et al., 1979). Confusions between chords and judgments of harmonic relations have similarly been found to depend on key distance (Krumhansl et al., in press). In combination with these studies, the findings obtained here provide quite strong evidence for an internal representation of interkey structure. Moreover, the present approach suggests a mechanism through which the relations between keys might become internalized.

The method used here to recover the distances between keys was based on the relative stability of the single tones within the key instantiated by the context element. This raises the possibility that the internalization of interkey structure might likewise be derivative of the perceptual assignment of tones to positions within the hierarchy of tonal functions. The tonal hierarchy of single pitches is, according to this view, psychologically primary, and perceived key distances depend on the similarity of the hierarchies associated with the keys. The primacy of the level of tones makes logical sense because, after all, music is constructed from the simultaneous and successive combination of individual pitches.

However, the suggestion that the tonal hierarchy is fundamental raises the question of how the tonal hierarchy itself may be acquired. In music certain tones are emphasized by their more frequent occurrence, particularly at phrase beginnings and endings, and these tones typically have longer durations and are given greater rhythmic stress. Moreover, the selection of these pitches maintains certain fixed intervals, conforming in Western music to the interval patterns of the major and minor diatonic scales. The diatonic scales are constructed of a distinctive pattern of whole and half steps between successive scale degrees, and this interval structure, which is maintained in all tonal music, may further facilitate the internalization of pitch relations (Balzano, 1980). Thus, music itself contains numerous features that would promote the assignment of pitches to various positions within a hierarchy. In this connection it should be noted that despite the fact that other musical cultures employ different sets of pitches than those employed in Western music, there are a number of other traditions in which a roughly analogous hierarchy exists within the set of musical tones. For example, Indian music is typically constructed around a few single pitches that are used more frequently and appear in more prominent positions within the sequence; other tones appear less frequently, are given less rhythmic emphasis, and tend to be followed by the more focal tones (Jairazbhoy, 1971; Meyer, 1956).

Both the construction and perception of pitch sequences around one or a few tones may reflect a general cognitive phenomenon that has been shown to apply to a wide variety of perceptual and semantic domains (Rosch, 1975; see Smith & Medin, 1981, for a recent summary of related work). According to this view there are certain members of natural categories that function as cognitive reference points, or prototypes, for the category as a whole. These elements are described as the most representative of the category, in relation to which all other category members are seen. Krumhansl (1979) noted the applicability of this description to the most structurally stable pitches in the tonal system in music.

Finally, psychophysical accounts (Helmholtz, 1863/1954) of musical structure have focused on the coincidence between the intervals formed by the tones most stable in the tonal hierarchy, such as fifths and major thirds, and the overtone structure of naturally produced tones. The frequency ratios of tones forming these intervals can be represented as ratios of small integers, which may be related to the phenomenal description of these intervals as sounding more consonant than others. Despite Krumhansl's (1979) demonstration that judgments of the relations between tones forming fixed intervals depend on the broader context in which they are embedded, and the cross-cultural diversity found in the selection of intervals, the role played by the factors of overtone structure and frequency ratios in the initial construction of the tonal hierarchy, at least that found in Western music, cannot be ruled out.

In any case, we suppose that the hierarchy of tonal stability is acquired through experience with the structural relations that obtain in the music itself. In support of this hypothesis, Krumhansl and Shepard (1979) found that listeners with more extensive musical backgrounds produced much more finely differentiated judgments of how well single tones completed a scale sequence. Whereas the responses of these listeners reflected the tonal hierarchy, listeners with less training emphasized tone-height differences. In a recent developmental study, Krumhansl and Keil (in press) found a developmental trend in judgments of pairs of tones embedded within short tonal melodies. The youngest children, who were in the first two grades of elementary school, distinguished only between scale and nonscale tones. The additional distinction between the tonic, third, and fifth scale degrees and the other scale tones emerged only later in the developmental sequence, although it was firmly established by Grades 5 and 6. Only the responses of adult listeners in that study reflected the functional equivalence of the tonic tones separated by an octave. A tone-height effect was found for all groups of listeners, and the magnitude of this effect did not change with the age of the listeners. The developmental trend observed may be characterized as a pattern of increasing differentiation between the tonal roles of the individual pitches within the context key. This description also applies to differences observed as a function of training in the Krumhansl and Shepard (1979) study, although the ordering of the components differed in the two cases.

Dowling (1978) has described the diatonic scale structure as a well-learned perceptual schema and has stressed its importance for the perceptual organization of tone sequences. That this schema appears quite early developmentally was suggested to account for Bartlett and Dowling's (1980) finding that recognition of transposed sequences was affected by the distance of the transposition for children of elementary school age. Arbitrary pitches cannot be accommodated within the interval schema of musical pitches. In support of this, Krumhansl and Shepard (1979) found that pitches halfway between chromatic scale steps (quarter tones) did not appear to be discriminated from their neighboring musical tones, and Siegel and Siegel (1977a, 1977b) have shown categorical perception of pitch among musicians. Moreover, the tones must conform to diatonic scale structure as evidenced by poorer memory for random sequences (Cohen, 1975; Dewar, 1974; Dewar et al., 1977; Francès, 1972). Finally, even if the pitches conform to some scale, the sequence will be most easily assimilated and remembered if it is constructed so as to emphasize those pitches most important within the diatonic heirarchy (Cohen, 1975). Thus, there is considerable support for the position that the tonal hierarchy described by music theorists is internalized by listeners and significantly affects the perceptual processes through which muscial tones are encoded and remembered. In the present article we have suggested that this tonal hierarchy may be psychologically fundamental and have shown that the perceived relationships between different abstract tonal centers may be derived from it.

Although we have emphasized the tonal interpretation of single tones here, music theorists have focused on the description of the harmonic functions of chords rather than single tones. Indeed, much of traditional Western music is constructed from tones sounded simultaneously in chords. The question addressed in the second section of this article, then, was how chord functions might be represented within the obtained spatial representation of tonal regions. In order to assess the relations between chords and keys, the ratings profiles for chords were compared with those for the major and minor keys. Each of the four chords—major, minor, diminished, and dominant seventh—was found to be closely associated with the keys in which it appears, and the degree of association reflected the relative strengths of the different functional roles in the hierarchy described by music theorists and obtained empirically in the studies of Krumhansl et al. (1982) and Bharucha and Krumhansl (in press). In the present investigation both the major and minor chords were found to be closest to the key in which they function as the tonic, I, chord. In the case of the major chord, it was next closest to the major key in which it is the dominant, V, chord. For the minor chord, after the tonic interpretation, the VI function in a major key was the next strongest. The diminished chord was closely tied to the major chord in which it plays the VII function, and the dominant seventh chord was close to the major key in which it is the V^7.

A subsequent analysis of these chord–key relations represented the chords as points in the map of musical keys. This analysis brought out more clearly the feature that each chord is closely associated with the musical keys in which it functions. Thus, the obtained chord positions reflected the inherent ambiguity of chord functions that comes about because each chord may simultaneously function within multiple tonal centers. As noted earlier, it is a simple matter to specify the chords of any single key but impossible to assign a single key to any individual chord or, indeed, a set of chords. If a sense of key is ever to be achieved, the listener must be able to entertain the possible harmonic functions of each chord and integrate this information as the musical sequence progresses. This process will be facilitated if the internal representation is such that each chord is closely tied to the tonal centers in which it functions, and this property was found in the present results. Thus, the chord–key distances indicate that the internal representation supports the simultaneous interpretation of chords in their various harmonic roles.

Given the finding that chords are positioned close to the keys in which they function, it must necessarily be the case that keys sharing chords are located close to one another in the map of key regions. This result might suggest that chord overlap can be used as a measure of key distance. There are problems, however, with this approach. Most notably, although parallel major and minor keys were found to be closely related, they in fact share only two chords. Thus, although shared chords belong to closely associated tonal centers, not all closely positioned keys have a substantial number of chords in common. The measure of key distance derived from the tonal hierarchies of single tones provided a more satisfactory measure of key distance, which nonetheless gave rise to a spatial map that was able to accommodate the complex interlocking pattern of chord functions.

In the final section we investigated the way in which the sense of key develops and changes during extended chord sequences. In discussing the results it was useful to employ the notion of different levels of tonal organization at foreground, middle ground, and background levels (Schenker, 1906/1954, 1935/1979). The first level corresponds to the local effects of the individual chords, particularly as major and minor chords tend to suggest the key in which they are the tonic chord. For all of the sequences, this effect of tonicization was apparent as the covariation between the observed strength of the prevailing key and what would be predicted if each chord of the sequences had been heard in isolation. In addition, when vectors representing the relative strengths of the different keys at each point in time were projected onto the torus, the resulting traces clearly exhibited variations attributable to the effect of the individual chords.

In all cases, however, a sense of the prevailing key above and beyond these single-chord effects was clearly evident at all steps beyond the initial position in the sequences. To separate middle ground from background levels, some of the sequences contained modulations between keys. Schenker (1906/1954, 1935/1979) holds that at the very deepest background level, a single key once established remains invariant despite modulations that may affect the other levels. For the modulations between closely related keys, the sense of the first key was maintained throughout the sequence despite modulations to the new key. The strength of the first key, however, tended to decrease compared with the new key as the sequence progressed and to become more dependent on local chord effects. In contrast, for the modulations between relatively distant keys, the sense of the first key was entirely absent by the end of the sequences. In fact, the second key appeared to suppress the first key below the level of local chord effects, indicating that listeners may not maintain the key at the background level when relatively distant modulations occur. This suppression effect between distantly related keys was also evident at the beginning of the sequences containing distant modulations. For these the strength of the second key (which was to be established later) was again lower than would be expected based on the individual chords early in the sequence, and this difference tended to increase as the first key became more firmly established.

One other difference was found between close and relatively distant modulations. For the close modulations, as soon as a chord unique to the new key was heard, a sense of the new key was apparent. In contrast, when the modulation was to a less closely related key, it took longer to achieve the new key sense. This effect for distant modulations was seen in both the correlations with the new key and in the traces on the torus, where there was a greater tendency for the points to remain in the vicinity of the first key after the modulation had occurred. Thus, it appears that an established sense of key tends to prepare the listener for modulations to new closely related keys but induces a certain measure of resistance to move to remote tonal centers.

The obtained differences between close and distant modulations indicate that key distance does indeed affect the perception of modulations between keys. However, key distance alone does not entirely account for the ease or difficulty of modulation. Instead, difficulty depends to a great degree on the way in which the modulating sequence is constructed, that is, on the particular chords of the sequence. In some cases the path taken between keys may be relatively direct, in other cases less so. In one of the modulating sequences studied, the path taken between the first and second keys followed the longer, rather than the shorter, path between the two keys on the surface of the torus. Had the sequence used different intermediary chords, a shorter path might have been traversed instead and the modulation perceived as being more direct. Thus, key distance per se is only one factor that affects the ease with which a modulation is assimilated. Tracing the changing key sense in a spatial map of the sort obtained here provides more information about the shifting tonal sense than does a simple classification of key relations.

In these results we find that listeners integrate information from successive chords, extracting a sense of key that grows over time, which in turn induces expectancies about possible alternative tonal centers that may later be introduced. The key sense, then, comes about through a process of perceiving the individual chords in their relations to one another and in their multiple functional roles within different tonal centers. This process is facilitated by an internal representation of interkey distances that supports these multiple interpretations, one in which keys sharing chords are closely positioned and the chords are closely tied to the keys in which they function.

At the most basic level, however, we believe that relations between keys and between individual chords and keys are all mediated through the internalized hierarchy of tonal functions at the level of individual tones. Indeed, the present results are all based on the analysis of tonal hierarchies, as reflected in the rating profiles of single tones. In Western music this hierarchy gives rise to harmonic properties of simultaneously sounded tones in chords that function within different, interrelated tonal centers in a way that appears to be unparalleled in other musical cultures. The present study provides empirical evidence that listeners familiar with Western music do in fact perceive musical organization at these more abstract levels and provides a quantitative description of the internal representation of these musical structures.

Reference Notes

1. Dowling, W. J., & Bartlett, J. C. *Assimilation of brief atonal melodies to tonal prototypes: Asymmetrical effects of judgment.* Paper presented at the meeting of the Psychonomic Society, Philadelphia, November 1981.
2. Kruskal, J. B., Young, F. W., & Seery, J. B. *How to use KYST, a very flexible program to do multidimensional scaling and unfolding.* Murray Hill, N.J.: Bell Laboratories, 1973.

3. Olivier, D. C. *Aggregative hierarchical clustering program: Program write up* (Computation Center Documentation). Cambridge, Mass.: Harvard University, 1973.
4. Kruskal, J. B. *How to use* MDSCAL, *a program to do multidimensional scaling and multidimensional unfolding.* Murray Hill, N.J.: Bell Laboratories.
5. Chang, J.-J., & Carroll, J. D. *How to use* PREFMAP and PREFMAP2: *Programs which relate preference data to multidimensional scaling solutions.* Murray Hill, N.J.: Bell Laboratories.

References

Allen, D. Octave discriminability of musical and nonmusical subjects. *Psychonomic Science*, 1967, *7*, 421–422.

Attneave, F., & Olson, R. K. Pitch as medium: A new approach to psychological scaling. *American Journal of Psychology*, 1971, *84*, 147–166.

Balzano, G. J. The group-theoretic description of 12-fold and microtonal pitch systems. *Computer Music Journal*, 1980, *4*, 66–84.

Bartlett, J. C., & Dowling, W. J. Recognition of transposed melodies: A key-distance effect in developmental perspective. *Journal of Experimental Psychology: Human Perception and Performance*, 1980, *6*, 501–515.

Berry, W. *Structural functions in music.* Englewood Cliffs, N.J.: Prentice-Hall, 1976.

Bharucha, J. J., & Krumhansl, C. L. The representation of harmonic structure in music: Hierarchies of stability as a function of context. *Cognition*, in press.

Boring, E. G. *Sensation and perception in the history of experimental psychology.* New York: Appleton-Century, 1942.

Carroll, J. D. Individual differences in multidimensional scaling. In R. N. Shepard, A. K. Romney, & S. Nerlove (Eds.), *Multidimensional scaling: Theory and application in the behavioral sciences* (Vol. 1). New York: Academic Press, 1972.

Chomsky, N. *Aspects of the theory of syntax.* Cambridge, Mass.: M.I.T. Press, 1965.

Cliff, N. Orthogonal rotation to congruence. *Psychometrika*, 1966, *31*, 33–42.

Cohen, A. *Perception of tone sequences from the Western-European chromatic scale: Tonality, transposition and the pitch set.* Unpublished doctoral dissertation, Queen's University at Kingston, Ontario, Canada, 1975.

Cuddy, L. L., Cohen, A. J., & Miller, J. Melody recognition: The experimental application of musical rules. *Canadian Journal of Psychology*, 1979, *33*, 148–157.

Deutsch, D., & Feroe, J. The internal representation of pitch sequences in tonal music. *Psychological Review*, 1981, *88*, 503–522.

Dewar, K. M. *Context effects in recognition memory for tones.* Unpublished doctoral dissertation, Queen's University at Kingston, Ontario, Canada, 1974.

Dewar, K. M., Cuddy, L. L., & Mewhort, D. J. K. Recognition memory for single tones with and without context. *Journal of Experimental Psychology: Human Learning and Memory*, 1977, *3*, 60–67.

Dowling, W. J. Scale and contour: Two components of a theory of memory for melodies. *Psychological Review*, 1978, *85*, 341–354.

Fletcher, H., & Munson, W. A. Loudness, its definition, measurement and calculation. *Journal of the Acoustical Society of America*, 1933, *5*, 82–108.

Forte, A. *Tonal harmony in concept and practice* (3rd ed.). New York: Holt, Rinehart & Winston, 1979.

Francès, R. *La perception de la musique* (2nd ed.). Paris: Vrin, 1972.

Helmholtz, H. von. *On the sensations of tone as a physiological basis for the theory of music* (A. J. Ellis, Ed. and trans.). New York: Dover, 1954. (Originally published, 1863).

Jairazbhoy, N. A. *The ragas of North Indian music: Their structure and evolution.* London: Faber & Faber (Middletown, Conn.: Wesleyan University Press), 1971.

Johnson, S. C. Hierarchical clustering schemes. *Psychometrika*, 1967, *32*, 241–254.

Krumhansl, C. L. The psychological representation of musical pitch in a tonal context. *Cognitive Psychology*, 1979, *11*, 346–374.

Krumhansl, C. L., Bharucha, J., & Castellano, M. A. Key distance effects on perceived harmonic structure in music. *Perception & Psychophysics*, in press.

Krumhansl, C. L., Bharucha, J. J., & Kessler, E. J. Perceived harmonic structure of chords in three related musical keys. *Journal of Experimental Psychology: Human Perception and Performance*, 1982, *8*, 24–36.

Krumhansl, C. L., & Keil, F. C. Acquisition of the hierarchy of tonal functions in music. *Memory & Cognition*, in press.

Krumhansl, C. L., & Shepard, R. N. Quantification of the hierarchy of tonal functions within a diatonic context. *Journal of Experimental Psychology: Human Perception and Performance*, 1979, *5*, 579–594.

Kruskal, J. B. Nonmetric multidimensional scaling: A numerical method. *Psychometrika*, 1964, *29*, 28–42.

Lerdahl, F., & Jackendoff, R. Toward a formal theory of tonal music. *Journal of Music Theory*, 1977, *21*, 111–171.

Lerdahl, F., & Jackendoff, R. *A generative theory of tonal music.* Cambridge, Mass.: MIT Press, in press.

Licklider, J. C. R. Basic correlates of the auditory stimulus. In S. S. Stevens (Ed.), *Handbook of experimental psychology.* New York: Wiley, 1951.

Meyer, L. *Emotion and meaning in music.* Chicago: University of Chicago Press, 1956.

Neisser, U. *Cognition and reality.* San Francisco: Freeman, 1976.

Pickler, A. G. Logarithmic frequency systems. *Journal of the Acoustical Society of America*, 1966, *39*, 1102–1110.

Piston, W. *Harmony* (3rd ed.). New York: Norton, 1962.

Rameau, J.-P. *Treatise on harmony* (P. Gossett, trans.). New York: Dover, 1971. (Originally published, 1722.)

Revesz, G. *Introduction to the psychology of music.* Norman: University of Oklahoma Press, 1954.

Rosch, E. Cognitive reference points. *Cognitive Psychology*, 1975, *7*, 532–547.

Rosen, C. *The classical style.* New York: Norton, 1972.

Ruckmick, C. A. A new classification of tonal qualities. *Psychological Review*, 1929, *36*, 172–180.

Salzer, F. *Structural hearing: Tonal coherence in music* (Vol. 1). New York: Dover, 1962.

Schenker, H. *Harmony* (O. Jones, Ed., and E. M. Borgese, trans.). Cambridge, Mass.: M.I.T. Press, 1954. (Originally published, 1906.)

Schenker, H. *Free composition* (E. Oster, Ed. and trans.). New York: Longman, 1979. (Originally published, 1935.)

Schoenberg, A. *Structural functions of harmony* (Rev. ed.). New York: Norton, 1969.

Schoenberg, A. *Theory of harmony* (R. Carter, trans.). London: Faber & Faber, 1978. (Originally published, 1911.)

Shepard, R. N. The analysis of proximities: Multidimensional scaling with an unknown distance function. I. *Psychometrika*, 1962, *27*, 125–140. (a)

Shepard, R. N. The analysis of proximities: Multidimensional scaling with an unknown distance function. II. *Psychometrika*, 1962, *27*, 219–246. (b)

Shepard, R. N. Circularity in judgments of relative pitch. *Journal of the Acoustical Society of America*, 1964, *36*, 2346–2353.

Shepard, R. N. Individual differences in the perception of musical pitch. In *Documentary report of the Ann Arbor symposium: Applications of psychology to the teaching and learning of music*. Reston, Va.: Music Educators National Conference, 1981.

Shepard, R. N. Geometrical approximations to the structure of pitch. *Psychological Review*, 1982, *89*, 305–333. (a)

Shepard, R. N. Structural representations of musical pitch. In D. Deutsh (Ed.), *Psychology of music*. New York: Academic Press, 1982. (b)

Siegel, J. A., & Siegel, W. Absolute identification of notes and intervals by musicians. *Perception & Psychophysics*, 1977, *21*, 143–152. (a)

Siegel, J. A., & Siegel, W. Categorical perception of tonal intervals: Musicians can't tell sharp from flat. *Perception & Psychophysics*, 1977, *21*, 399–407. (b)

Smith, E. E., & Medin, D. L. *Concepts and categories*. Cambridge, Mass.: Harvard University Press, 1981.

Stevens, S. S., & Volkmann, J. The relation of pitch to frequency: A revised scale. *American Journal of Psychology*, 1940, *53*, 329–353.

Stevens, S. S., Volkmann, J., & Newman, E. B. A scale for the measurement of the psychological magnitude of pitch. *Journal of the Acoustical Society of America*, 1937, *8*, 185–190.

Received July 2, 1981
Revision received February 8, 1982 ■