Roadmap for
Effective Reduced-Body Lattice EFT Models using Parametric Matrix Models

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1 Introduction

The central idea in this project is that many nuclei may be able to be effectively described by models not of the individual nucleons, but instead of composite "particles" that are themselves made up of nucleons. This is not a wholly new idea, as we already describe nuclei themselves by the interactions of nucleons, composite particles made up of quarks and gluons. Additionally we have seen that the structure of some nuclei are well described by the interactions of alpha particles [3].

Just as the pairwise interaction of quarks and gluons becomes a many-body interaction when we instead consider nucleons, we can expect that the structure—specifically the locality—of the interaction between nucleons will change when we instead consider composite particles.

Parametric matrix models [1] (PMMs) are an obvious choice for this task.

2 Project Goals

This project will be considered successful if we can demonstrate the ability for PMMs to

- Learn an effective fewer-body model of a nucleus from high-fidelity lattice EFT data.
- Make predictions or evaluations of the underlying structure of a nucleus in an unsupervised manner.

2.1 Stretch Goals

1. Using the learned PMM can we say that the structure of ^{16}O is well described by a system of just a ^{12}C and an α particle, four individual alpha particles, some other configuration, or is it irreducible beyond the nucleon level?
2. Can we use the learned PMM to emulate the lattice EFT calculations over model parameters?

3 Project Plan

3.1 Understand and Reimplement Original PMM [Complete]

Understand and reimplement the original PMM algorithm [1].

3.2 Implement the Eigenvector PMM for Eigenstsates of a 1D Toy Model [Complete]

Using the extended PMM algorithm for state vectors, implement the algorithm for a simple toy model (harmonic oscillator) such that it is able to effectively emulate the ground state or several low lying states for varying Hamiltonian parameters.

PMM algorithm research outside this project has produced multiple internal notes and codes for this task.

3.2.1 Extend to Density Data

As data from lattice EFT simulations will only be 1-body densities, we will need to modify the above vector PMM implementation to learn from density data.

A central task will be to determine an appropriate loss function for comparing the density data as distributions. Some potential loss functions include

- Kullback-Leibler divergence
- Jensen-Shannon divergence
- Relative von Neumann entropy
- Mean squared error and its variants

PMM algorithm research outside this project has produced multiple internal notes and codes for this task.

3.3 Focus in on Two-Body Toy 3D Lattice System

Instead of using the POD-based PMM algorithm, we instead will impose that the latent space for the PMM is also a 3D lattice. Additionally we will focus on the two-body case, which can be written as a one-body problem in relative coordinates. In doing so, the form of the effective kinetic energy operator will be unchanged. The implementation of the effective potential energy operator should allow for the distance-dependence to be specified and therefore the short-ranged-ness of the interaction to be controlled and imposed. For instance,

$$V(r) = \begin{cases} V_0/r^2 & r < r_0 \\ 0 & r \geq r_0 \end{cases}.$$

The PMM implementation will need to be able to use a different number of lattice points for the latent space and the true data space. This will require suitable modifications to the loss function. One simple approach would be to interpolate the smaller space to the larger space, though this may prove to be too computationally expensive or noise-sensitive.

Additionally, complications may arise from the inherent translational and rotational symmetries of both the data and the latent space. This problem has already been encountered and addressed in the "registration" of pinhole algorithm data from lattice EFT and it is therefore expected that similar techniques and insights will be applicable here.

3.4 Extension to Two-Body PMM Emulation of an A-Body Toy 3D Lattice System

Extending the previous PMM to a toy A-body system should be straightforward. A suitable toy system will need to be identified. The comparison of 1-body densities between the PMM and the true data will be unchanged from the two-body case.

The PMM will be informed by the expected mass of each of the two bodies, which in this case would allow the reduced mass of the two-body system (and therefore the mass of the one-body model) to be determined.

3.5 N-Body PMM Emulation of an A-Body Toy System for $2 \leq N < A$

The final step before applying the PMM to real lattice EFT data will be to extend the PMM to N-body systems. This will require implementing short- and finite-ranged many-body interactions in the PMM. How exactly these interactions should be parametrized is an open question.

3.6 Application to Lattice EFT Data

The conclusion of this project will be to acquire lattice EFT data for systems which are expected to be well-described by a reduced-body model, such as ^{9}Be $\left(^8\text{Be} + n\right)$ [2] and ^{12}C $(\alpha + \alpha + \alpha)$ [3]. This data is originally in the form of hundreds of thousands of samples from the pinhole algorithm, but will be pre-processed into 3D 1-body densities. A PMM will be trained on each of these systems. From this, we will analyze the properties of the effective interactions learned by the PMM and the accuracy of the reduced-body model will be evaluated.

3.7 Unsupervised Identification of Nuclear Structure

Either as future work following the demonstration of the PMM's ability to learn effective reduced-body models or as powerful application of this technology, we will attempt to use the PMM to identify the structure of a nucleus for which it is not certain to be well-described by relatively few clusters. A clear candidate nucleus in this case is ^{16}O, which may be well-described by

- $^{12}\text{C} + \alpha$
- $\alpha + \alpha + \alpha + \alpha$
- $^8\text{Be} + ^8\text{Be}$
- $^8\text{Be} + \alpha + \alpha$
- ...

The central idea behind identifying the structure in an unsupervised fashion would be to fit several reduced-body PMMs, each assuming one of the above structures, to the data. The structure that best fits the data, if any, can be said to be a likely candidate for the structure of the nucleus.

References

[1] Patrick Cook, Danny Jammooa, Morten Hjorth-Jensen, Daniel D. Lee, and Dean Lee. Parametric matrix models. https://arxiv.org/abs/2401.11694, 2024.

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[3] Shihang Shen, Serdar Elhatisari, Timo A. Lähde, Dean Lee, Bing-Nan Lu, and Ulf-G. Meißner. Emergent geometry and duality in the carbon nucleus. *Nature Communications*, 14(1), May 2023.