Definitions

Adiabatic Process A process in which no heat transfer occurs
Isothermal All points on a surface are the same temperature
Diffuse No prefered direction for outgoing rays
Specular Prefered direction for outgoing rays
Black Body Perfect Emitter and Absorber ($\varepsilon = 1, \alpha = 1$)
Gray Body Properties are independent of wavelength
Irradiance (G) Flux of energy that irradiates a surface
Radiosity (J) total flux of radiative energy away from a surface
Reflectivity (ρ) How much of the irradiance is reflected
Absorptibity (α) How much of the irradiance is absorbed
Transmissivity (τ) How much of the irradiance passes through the material

When Assumptions Can Be Made

- Steady State:
- 1D Conduction: When conduction is significant in only a single direction
- $\tau = 0$: material is opaque or thick (>1 wavelength of light)

Constants

Boltzmann Constant

$$k_B = 1.38 \times 10^{-23} \frac{J}{K}$$

Stefan-Boltzmann Constant

$$\sigma_{\text{SB}} = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

Governing Equation

$$E_{\text{in}} - E_{\text{out}} + E_{\text{gen}} = E_{\text{st}}$$

Conduction

Fourier Law of Heat Conduction

$$q'' = -k\frac{\partial T}{\partial x}$$

Heat Diffusion Equation

Cartesian Coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

Cylindrical Coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \phi}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

Boundary Conditions

1. Constant Surface Temperature: $T(0,t) = T_s$
2. Constant Surface Heat Flux
   a. Finite Heat Flux: $-k\frac{\partial T}{\partial x}\mid_{x=0} = q_s''$
   b. Adiabatic or Insulated Surface: $-k\frac{\partial T}{\partial x}\mid_{x=0} = 0$
3. Convection Surface Condtion: $-k\frac{\partial T}{\partial x}\mid_{x=0} = h[T_\infty - T(0,t)]$

Thermal Diffusivity

The ratio of how a material conducts thermal energy to how well it stores thermal energy.

$$\alpha = \frac{k}{\rho c_p}$$

Circuit Analogy

Cartesian Coordinates:

$$R_{\text{cond}} = \frac{L}{kA_s}$$

Cylindrical Coordinates:

$$R_{\text{cond}} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

Spherical Coordinates:

$$R_{\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Overall Heat Transfer Coefficient

Used for composite materials such as walls with different materials.

$$U = \frac{1}{R_{\text{tot}}A_s}$$

Contact Resistance

To account for gaps due to surface roughness between mating surfaces.

$$R''_{\text{tc}} = \frac{T_A - T_B}{q_x''}$$

Porous Materials

These materials have pockets of liquid which greatly affects the thermal conductivity, so the average thermal conductivity is used to ease the calculates called k_{eff}.

Porosity (void fraction): ε Fluid Thermal Conductivity: k_f Solid Thermal Conductivity: k_s

$$k_{\text{eff,min}} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{\text{eff,max}} = \varepsilon k_f + (1-\varepsilon)k_s$$

$$k_{\text{eff}} = \left[\frac{k_f + 2k_s - 2\varepsilon(k_s - k_f)}{k_f + 2k_s + \varepsilon(k_s - k_f)}\right]k_s$$

Fins (Extended Surfaces)

(Page 118 has table of fin efficiencies for various shapes Basic Heat and Mass Transfer)

$$m = \left(\frac{hP}{kA_c}\right)^{\frac{1}{2}} = \left(\frac{2h}{kt}\right)^{\frac{1}{2}} = \left(\frac{4h}{kD}\right)^{\frac{1}{2}}$$

Fin Effectiveness:

$$\varepsilon_f = \left(\frac{kP}{hA_c}\right)^{\frac{1}{2}}$$

Fin Efficiency:

$$\eta_f = \frac{\tanh(mL)}{mL}$$

Total Surface Efficiency:

$$\eta_t = 1 - \frac{A_f}{A}(1 - \eta_f)$$

Resistance of a Fin: The resistance for a single fin

$$R_{t,f} = \frac{1}{hA_f\eta_f}$$

Thermal Resistance of Finned Surface: The resistance for an entire finned surface

$$R = \frac{1}{hA\eta_t}$$

Transient Conduction

Biot Number

Ratio of thermal resistances between conduction and convection. If Bi << 1 then the conduction resistance is much less than the convective resistance and it is safe to assume uniform temperature distribution.

$$\text{Bi} \equiv \frac{\bar{h}L}{k}$$

Fourier Number

Dimensionless Time

$$\text{Fo} \equiv \frac{\alpha t}{L_c^2}$$

Lumped Thermal Capacitance

Condition: $\text{Bi} < 0.1$ (error associated with method is negligible)

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-\text{Bi} \cdot \text{Fo})$$

Problem-Solving Strategy

(Page 209 Basic Heat and Mass Transfer)

1. Calculate Bi and if Bi < 0.1 then use Lumped Thermal Capacitance
2. Calculate Fourier Number. If Fo < 0.05 then Eqn 3.61 (pg. 194)
3. Calculate Fourier Number. If 0.05 < Fo < 0.2 then Eqn 3.72 and 3.73 (pg. 205)
4. Calculate Fourier Number. If Fo > 0.2 then Eqn 3.75 - 3.77 (pg. 206)

Convection

$$q_{\text{local}} = hA(T_s - T_\infty)$$

$$q_{\text{global}} = UA(T_s - T_\infty)$$

Thermal Resistance

$$R = \frac{1}{hA}$$

Reynold's Number

$$\text{Re}_x = \frac{\rho u_\infty x}{\mu} = \frac{u_\infty x}{\nu}$$

Nusselt Number

$$\text{Nu} \equiv \frac{hL}{k_f}$$

Flat plate in Laminar Flow:

$$\text{Nu} = 0.664\text{Re}_L^{\frac{1}{2}}\text{Pr}^{\frac{1}{3}}$$

Prandtl Number

$$\text{Pr} \equiv \frac{\nu}{k}$$

Convective Heat Transfer Coefficient

$$h = -\frac{k_f}{T_s - T_\infty}\frac{\partial T}{\partial y}\ |_{y=0}$$

Radiation

Stefan-Boltzmann Law

$$dq = dU + Pd\mathbb{V}$$

Energy Density (ϵ)
Internal Energy (U) = $\epsilon\mathbb{V}$
Pressure (P) = $\frac{1}{3}\epsilon$

Spectral Energy Density

$$E(\lambda) = \frac{k_B T}{\lambda^4}$$

$$E(f) = \frac{k_B T}{c^3} f^3$$

Boltzmann Const (k_B) = $1.38 \times 10^{-23} \frac{J}{K}$

Total Radiative Energy

$$E = \int_0^\infty E(\lambda) d\lambda = \int_0^\infty E(f) df$$

Emissivity (Emittance)

The fraction of emissive power a real body ($E(T)$) emits compared to a black body ($E_b(T)$).

$$\varepsilon \equiv \frac{E(T)}{E_b(T)}$$

Emissive Power

$$E(T) = \varepsilon \sigma_{\text{SB}} T^4$$

Kirchoff's Law

A body in thermodynamic equilibrium must emit as much energy as it absorbs in each direction at each wavelength. This is to avoid violating the 2nd Law of Thermodynamics.

$$\varepsilon_\lambda(T, \theta, \phi) = \alpha_\lambda(T, \theta, \phi)$$

Diffuse Form:

$$\varepsilon_\lambda(T) = \alpha_\lambda(T)$$

Diffuse and Gray Form:

$$\varepsilon(T) = \alpha(T)$$

Wiens Displacement Relation

$$\lambda_{\text{peak}} T = \text{const} = 2898 \mu m K$$

View Factors

The view factor is a correction to the Stefan-Boltzmann constant for black bodies. The view factor is a function of the surface area of two materials, the angle between the normals of each surface and the ray of radiation (β_1, β_2), and the distance between the two surfaces. (See page 561 in AHTT for table of view factors)

$$F_{1-2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos(\beta_1)\cos(\beta_2)}{\pi s^2} dA_2 dA_1$$

$$Q_{\text{net}(1-2)} = A_1 F_{1-2} \sigma_{\text{SB}} (T_1^4 - T_2^4)$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

Irradiance

The flux of energy that irradiates a surface.

$$\rho + \alpha + \tau = 1$$

Radiosity

The total flux of radiative energy away from a surface.

$$J = E + \rho G = \varepsilon E_b + \rho G$$

Circuit Analogy

Surface Resistance for a diffuse, gray surface:

$$R_{\text{surf}} = \frac{1 - \varepsilon}{\varepsilon A}$$

Geometrical Resistance for a diffuse, gray surface:

$$R_{\text{geo}} = \frac{1}{A_1 F_{1-2}}$$

Total Heat Flux between two diffuse, gray surfaces:

$$Q_{\text{net}_{1-2}} = \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{R_{\text{surf}_1} + R_{\text{geo}_{1-2}} + R_{\text{surf}_2}}$$

Heat Pipes

Saturated liquids evaporate by absorbing heat from a higher temperature and saturated vapors condense by releasing heat to a lower temperature.

$$\Delta P_{\text{capillary}} - \Delta P_{\text{gravitational}} = \Delta P_L + \Delta P_V$$

Effective Length of a Heat Pipe

$$L_{\text{eff}} = L_A + \frac{1}{2}(L_E + L_C)$$

Capillary Pressure

Pore Radius (r_p) Surface Tension (γ)

$$\Delta P_{\text{capillary}} = 2\gamma \left[\frac{\cos(\theta_E)}{r_p} - \frac{\cos(\theta_C)}{r_p} \right]$$

Gravitational Pressures

Angle of Heat Pipe (ϕ)

$$\Delta P_{\text{gravitational}} = \rho g (L_{\text{eff}} \sin(\phi))$$

Darcy Relation

Flow Rate (Q_f) Permeability (K_p) Effective Wick Area (A_w)

$$Q_f = -\frac{\mathrm{K}_p}{\mu} A_w \left[\frac{\Delta P_L}{L_{\text{eff}}} \right]$$

Liquid Phase Change Pressure Drop

$$\Delta P_L = \frac{\mu L_{\text{eff}}}{\mathrm{K}_p A_w} (\dot{m}, \rho_L)$$

Vapor Related Pressure Drop

$$\Delta P_V = \left(\frac{1}{2} \rho_v v^2 \right) \left(\frac{64}{\text{Re}} \right) \left[\frac{L_{\text{eff}}}{4 \frac{A_w}{\rho}} \right]$$

Heat Pipe Equation

$$q \cdot L_{\text{eff}} = \left(\frac{\rho_L \gamma h_{\text{fg}}}{\mu_L} \right) [\mathrm{K}_p A_w] \left[\frac{2}{r_p} \right]$$

Weber Number

$$\text{We} = \frac{\rho v^2}{\gamma} l$$

Issues with Heat Pipes

1. Sonic limit (liquid metal heat pipes)
2. Entrainment -> dry out
3. Boiling limitation (bubbles in wick)
4. Chokig of vapor flux

Heat Exchangers

$$\mathbb{C}_h = \dot{m}_h C_h$$

$$\mathbb{C}_c = \dot{m}_c C_c$$

Power Lost

$$dq = -\dot{m}_h C_h dT_h$$

Power Gained

$$dq = \dot{m}_c C_c dT_c$$

Log-Mean Temperature Difference

$$\Delta T_{\text{LMTD}} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

Heat Flux

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left[\frac{1}{\dot{m}_c C_c} + \frac{1}{\dot{m}_h C_h}\right]$$

$$q = UA\Delta T_{\text{LMTD}}$$

Max Heat Flux

$$q_{\text{max}} = \mathbb{C}_{\text{min}}(T_{\text{hi}} - T_{\text{ci}})$$

Heat Exchanger Effectiveness

$$\varepsilon = \frac{q}{q_{\text{max}}}$$

Number of Transfer Units

$$\text{NTU} = \frac{UA}{\mathbb{C}_{\text{min}}}$$

Parallel Flow

$$\Delta T_1 = T_{\text{hi}} - T_{\text{ci}}$$

$$\Delta T_2 = T_{\text{ho}} - T_{\text{co}}$$

Counter Flow

$$\Delta T_1 = T_{\text{hi}} - T_{\text{co}}$$

$$\Delta T_2 = T_{\text{ho}} - T_{\text{ci}}$$

Thermal Electrics

$$m = \frac{R}{\mathbb{R}}$$

$$Z = \frac{S^2}{K\mathbb{R}}$$

Seebeck Effect

$$S = \frac{\Delta V}{\Delta T}$$

Heat Engine

Carnot Efficiency

$$\eta_{\text{carnot}} = \frac{T_H - T_C}{T_H}$$

Carzon-Ahlborn Efficiency

$$\eta_{c-a} = 1 - \sqrt{\frac{T_C}{T_H}}$$

Thermal Conductance

$$K = \frac{k_1 A_1}{l_1} + \frac{k_2 A_2}{l_2}$$

Resistance of Materials

$$\mathbb{R} = \frac{\rho_1 l_1}{A_1} + \frac{\rho_2 l_2}{A_2}$$

Current

$$I = \frac{S(T_H - T_C)}{R + \mathbb{R}}$$

Thermal Electric Efficiency

$$\eta_{\text{TE}} = \eta_{\text{carnot}} \left[\frac{\left(\frac{m}{m+1}\right)}{1 + \frac{K\mathbb{R}}{S^2}\left(\frac{m+1}{T_H}\right) - \frac{1}{2}\eta_{\text{carnot}}\left(\frac{1}{m+1}\right)} \right]$$

Geometric Constraint

$$\sqrt{\frac{k_2 \rho_1}{k_1 \rho_2}} = \frac{A_1}{A_2}$$

Optimal m

$$m_{\text{optimal}} = \sqrt{1 + \frac{1}{2} Z (T_H + T_C)}$$

Peltier Cooler

$$q_\Pi = q_o + q_T = \Pi I$$

$$q_T = \frac{1}{2} I^2 \mathbb{R} + K(T_H - T_C)$$

Coefficient of Perfomance

$$\text{COP} = \frac{T_C}{T_H - T_C}$$

Peltier Coefficient

$$\Pi = S T_C$$

Critical Current

$$I_C = \frac{S T_C}{\mathbb{R}}$$

General Equations of Usefulness

$$L_c = \frac{V}{A_s}$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Surface Area of a Sphere

$$A_s = 4\pi r^2$$

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

Surface Area of a Cylinder

$$A_s = 2\pi r h$$

Volume of a Cylinder

$$V = \pi r^2 h$$

Error Function

$$\mathrm{erf}(\eta) = \left(\frac{2}{\pi^{\frac{1}{2}}}\right) \int_0^{\eta} e^{-u^2}\, du$$

Complimentary Error Function:

$$\mathrm{erfc}(\eta) = 1 - \mathrm{erf}(\eta)$$

Mass Flow Rate

$$\dot{m} = Q_f \cdot \rho_f$$

Differential Equations

First Order ODE

$$\frac{\partial f}{\partial t} + p(t)f(t) = g(t)$$

1. Find $\mu(t)$

$$\mu(t) = \exp\left(\int p(t)dt\right)$$

2. Multiply by $\mu(t)$

$$\frac{d}{dt}(\mu(t)f(t)) = \mu(t)g(t)$$

3. Integrate Both Sides

$$\int \frac{d}{\mathrm{dt}}(\mu(t)f(t))\ \mathrm{dt} = \int \mu(t)g(t)\ \mathrm{dt}$$

$$\mu(t)f(t) = \int \mu(t)g(t)\ \mathrm{dt}$$

$$f(t) = \frac{1}{\mu(t)} \int \mu(t)g(t)\ \mathrm{dt}$$

Second Order ODE

$$\frac{\partial^2 y(t)}{\partial t^2} + p(t)\frac{\partial y(t)}{\partial t} + q(t)y(t) = g(t)$$

1. Find the two roots (assume $y = \exp(rt)$)

$$\frac{\partial y(t)}{\partial t} = r\exp(rt)$$

$$\frac{\partial^2 y(t)}{\partial t^2} = r^2\exp(rt)$$

$$r_1, r_2 = \frac{-p(t) \pm \sqrt{p(t)^2 - 4 \cdot q(t)}}{2}$$

2. Subsitute roots into general solution

$$y(t) = c_1\exp(r_1 t) + c_2\exp(r_2 t)$$

3. Use Initial Conditions to find c_1 and c_2