Definitions

Adiabatic Process A process in which no heat transfer occurs
Isothermal All points on a surface are the same temperature
Diffuse No prefered direction for outgoing rays
Specular Prefered direction for outgoing rays
Black Body Perfect Emitter and Absorber ($\varepsilon = 1, \alpha = 1$)
Gray Body Properties are independent of wavelength
Irradiance (G) Flux of energy that irradiates a surface
Radiosity (J) total flux of radiative energy away from a surface
Reflectivity (ρ) How much of the irradiance is reflected
Absorptibity (α) How much of the irradiance is absorbed
Transmissivity (τ) How much of the irradiance passes through the material

When Assumptions Can Be Made

- Steady State:
- 1D Conduction: When conduction is significant in only a single direction
- $\tau = 0$: material is opaque or thick (>1 wavelength of light)

Governing Equation

$$E_{\text{in}} - E_{\text{out}} + E_{\text{gen}} = E_{\text{st}}$$

Conduction

Heat Diffusion Equation

Cartesian Coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

Cylindrical Coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2(\theta)}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin(\theta)}\frac{\partial}{\partial \phi}\left(k \sin\theta \frac{\partial T}{\partial \theta}\right) + \dot{q}''' = \rho c_p \frac{\partial T}{\partial t}$$

Boundary Conditions

1. Constant Surface Temperature: $T(0, t) = T_s$
2. Constant Surface Heat Flux
   a. Finite Heat Flux: $-k\frac{\partial T}{\partial x} \mid_{x=0} = q''_s$
   b. Adiabatic or Insulated Surface: $-k\frac{\partial T}{\partial x} \mid_{x=0} = 0$
3. Convection Surface Condtion: $-k\frac{\partial T}{\partial x} \mid_{x=0} = h[T_\infty - T(0, t)]$

Thermal Diffusivity

The ratio of how a material conducts thermal energy to how well it stores thermal energy.

$$\alpha = \frac{k}{\rho c_p}$$

Circuit Analogy

Cartesian Coordinates:

$$R_{\text{cond}} = \frac{L}{kA_s}$$

Cylindrical Coordinates:

$$R_{\text{cond}} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

Spherical Coordinates:

$$R_{\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Overall Heat Transfer Coefficient

Used for composite materials such as walls with different materials.

$$U = \frac{1}{R_{\text{tot}} A_s}$$

Contact Resistance

To account for gaps due to surface roughness between mating surfaces.

$$R''_{\text{tc}} = \frac{T_A - T_B}{q''_x}$$

Porous Materials

These materials have pockets of liquid which greatly affects the thermal conductivity, so the average thermal conductivity is used to ease the calculates called k_{eff}.

Porosity (void fraction): ε Fluid Thermal Conductivity: k_f Solid Thermal Conductivity: k_s

$$k_{\text{eff,min}} = \frac{1}{\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}}$$

$$k_{\text{eff,max}} = \varepsilon k_f + (1 - \varepsilon)k_s$$

$$k_{\text{eff}} = \left[\frac{k_f + 2k_s - 2\varepsilon(k_s - k_f)}{k_f + 2k_s + \varepsilon(k_s - k_f)}\right] k_s$$

Fins (Extended Surfaces)

(Page 118 has table of fin efficiencies for various shapes Basic Heat and Mass Transfer)

$$m = \left(\frac{hP}{kA_c}\right)^{\frac{1}{2}} = \left(\frac{2h}{kt}\right)^{\frac{1}{2}} = \left(\frac{4h}{kD}\right)^{\frac{1}{2}}$$

Fin Effectiveness:

$$\varepsilon_f = \left(\frac{kP}{hA_c}\right)^{\frac{1}{2}}$$

Fin Efficiency:

$$\eta_f = \frac{\tanh(mL)}{mL}$$

Resistance:

$$R_{t,f} = \frac{1}{hA_f \eta_f}$$

Transient Conduction

Biot Number

Ratio of thermal resistances between conduction and convection. If Bi << 1 then the conduction resistance is much less than the convective resistance and it is safe to assume uniform temperature distribution.

$$\text{Bi} \equiv \frac{\bar{h}L}{k}$$

Fourier Number

Dimensionless Time

$$\text{Fo} \equiv \frac{\alpha t}{L_c^2}$$

Lumped Thermal Capacitance

Condition: $\text{Bi} < 0.1$ (error associated with method is negligible)

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-\text{Bi} \cdot \text{Fo})$$

Problem-Solving Strategy

(Page 209 Basic Heat and Mass Transfer)

1. Calculate Bi and if Bi < 0.1 then use Lumped Thermal Capacitance
2. Calculate Fourier Number. If Fo < 0.05 then Eqn 3.61 (pg. 194)
3. Calculate Fourier Number. If 0.05 < Fo < 0.2 then Eqn 3.72 and 3.73 (pg. 205)
4. Calculate Fourier Number. If Fo > 0.2 then Eqn 3.75 - 3.77 (pg. 206)

Convection

Reynold's Number

$$\text{Re}_x = \frac{\rho u_\infty x}{\mu} = \frac{u_\infty x}{\nu}$$

Nusselt Number

$$\text{Nu} \equiv \frac{hL}{k_f}$$

Flat plate in Laminar Flow:

$$\text{Nu} = 0.664 \text{Re}_L^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}}$$

Prandtl Number

$$\text{Pr} \equiv \frac{\nu}{k}$$

Convective Heat Transfer Coefficient

$$h = -\frac{k}{T_w - T_\infty}\frac{\partial T}{\partial y} \mid_{y=0}$$

Radiation

Stefan-Boltzmann Law

$$dq = dU + Pd\mathbb{V}$$

Energy Density (ϵ)
Internal Energy (U) = $\epsilon\mathbb{V}$
Pressure (P) = $\frac{1}{3}\epsilon$

Spectral Energy Density

$$E(\lambda) = \frac{k_B T}{\lambda^4}$$

$$E(f) = \frac{k_B T}{c^3} f^3$$

Boltzmann Const (k_B) = $1.38 \times 10^{-23} \frac{J}{K}$

Total Radiative Energy

$$E = \int_0^\infty E(\lambda) d\lambda = \int_0^\infty E(f) df$$

Emissivity (Emittance)

The fraction of emissive power a real body ($E(T)$) emits compared to a black body ($E_b(T)$).

$$\varepsilon \equiv \frac{E(T)}{E_b(T)}$$

Emissive Power

$$E(T) = \varepsilon \sigma_{\text{SB}} T^4$$

Kirchoff's Law

A body in thermodynamic equilibrium must emit as much energy as it absorbs in each direction at each wavelength. This is to avoid violating the 2nd Law of Thermodynamics.

$$\varepsilon_\lambda(T, \theta, \phi) = \alpha_\lambda(T, \theta, \phi)$$

Diffuse Form:

$$\varepsilon_\lambda(T) = \alpha_\lambda(T)$$

Diffuse and Gray Form:

$$\varepsilon(T) = \alpha(T)$$

Wiens Displacement Relation

$$\lambda_{\text{peak}} T = \text{const} = 2898 \mu m K$$

View Factors

The view factor is a correction to the Stefan-Boltzmann constant for black bodies. The view factor is a function of the surface area of two materials, the angle between the normals of each surface and the ray of radiation (β_1, β_2), and the distance between the two surfaces. (See page 561 in AHTT for table of view factors)

$$F_{1-2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos(\beta_1)\cos(\beta_2)}{\pi s^2} dA_2 dA_1$$

$$Q_{\text{net}(1-2)} = A_1 F_{1-2} \sigma_{\text{SB}} (T_1^4 - T_2^4)$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

Irradiance

The flux of energy that irradiates a surface.

$$\rho + \alpha + \tau = 1$$

Radiosity

The total flux of radiative energy away from a surface.

$$J = E + \rho G = \varepsilon E_b + \rho G$$

Circuit Analogy

Surface Resistance for a diffuse, gray surface:

$$R_{\text{surf}} = \frac{1 - \varepsilon}{\varepsilon A}$$

Geometrical Resistance for a diffuse, gray surface:

$$R_{\text{geo}} = \frac{1}{A_1 F_{1-2}}$$

Total Heat Flux between two diffuse, gray surfaces:

$$Q_{\text{net}_{1-2}} = \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{R_{\text{surf}_1} + R_{\text{geo}_{1-2}} + R_{\text{surf}_2}}$$

Heat Pipes

$$L_c = \frac{V}{A_s}$$

General Equations of Usefulness

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Surface Area of a Sphere

$$A_s = 4\pi r^2$$

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

Surface Area of a Cylinder

$$A_s = 2\pi r h$$

Volume of a Cylinder

$$V = \pi r^2 h$$

Error Function

$$\text{erf}(\eta) = \left(\frac{2}{\pi^{\frac{1}{2}}}\right)\int_0^\eta e^{-u^2} du$$

Complimentary Error Function:

$$\text{erfc}(\eta) = 1 - \text{erf}(\eta)$$