

Problem 3

Tuesday, 13 August 2024

- (a) Give, with brief justification, an expression for the number of clauses of φ_G in terms of m (the number of edges of G) and t (the number of triangles in G). For full marks, the number of clauses produced by your **awk** program (**prob2.awk**) must be correct as well as equalling the expression you give here.

In the algorithm implemented in the awk file, it first creates a clause $(v_i|v_k)$ for each line which represents 1 edge (ignoring the first line which tells our program how many lines there are). Therefore during this, it generates m number of edges (1 clause for each edge). Once it finishes looping through all the lines, it goes through an algorithm that finds all sets of 3 edges that share 3 vertices in the text file and for each triangle found, generates a clause $(\sim v_i, \sim v_j, \sim v_k)$ to emit the triangle. Therefore it generates 1 clause for each triangle.

In total the number of clauses generated will be $m + t$.

The output for the awk function in prob2 aligns with this rule:

$(v_1|v_2) \& (v_1|v_3) \& (v_1|v_4) \& (v_2|v_3) \& (v_2|v_6) \& (v_3|v_5) \& (v_4|v_7) \& (v_5|v_7) \& (v_6|v_7) \& (\sim v_1|\sim v_2|\sim v_3)$

This contains 10 clauses, 9 edges + 1 triangle.

- (b) Give, with brief justification, an expression for an upper bound on the number of triangles t in G in terms of m . (You don't need to find the tightest bound possible, but it needs to be tight enough to support your argument in part (c).)

The number of triangles cannot exceed the number of edges on the tree. Since 3 edges are required to form a triangle, we cannot form n count of triangles with n edges.

$t < m$

- (c) Based on your answers to (a) and (b), give with brief justification an upper bound on the number of clauses of φ_G , expressed *only* in terms of m .

Question A we determined that total clauses = $m + t$

Question B we determined that bound for t is m

Therefore total clauses $< 2 * m$