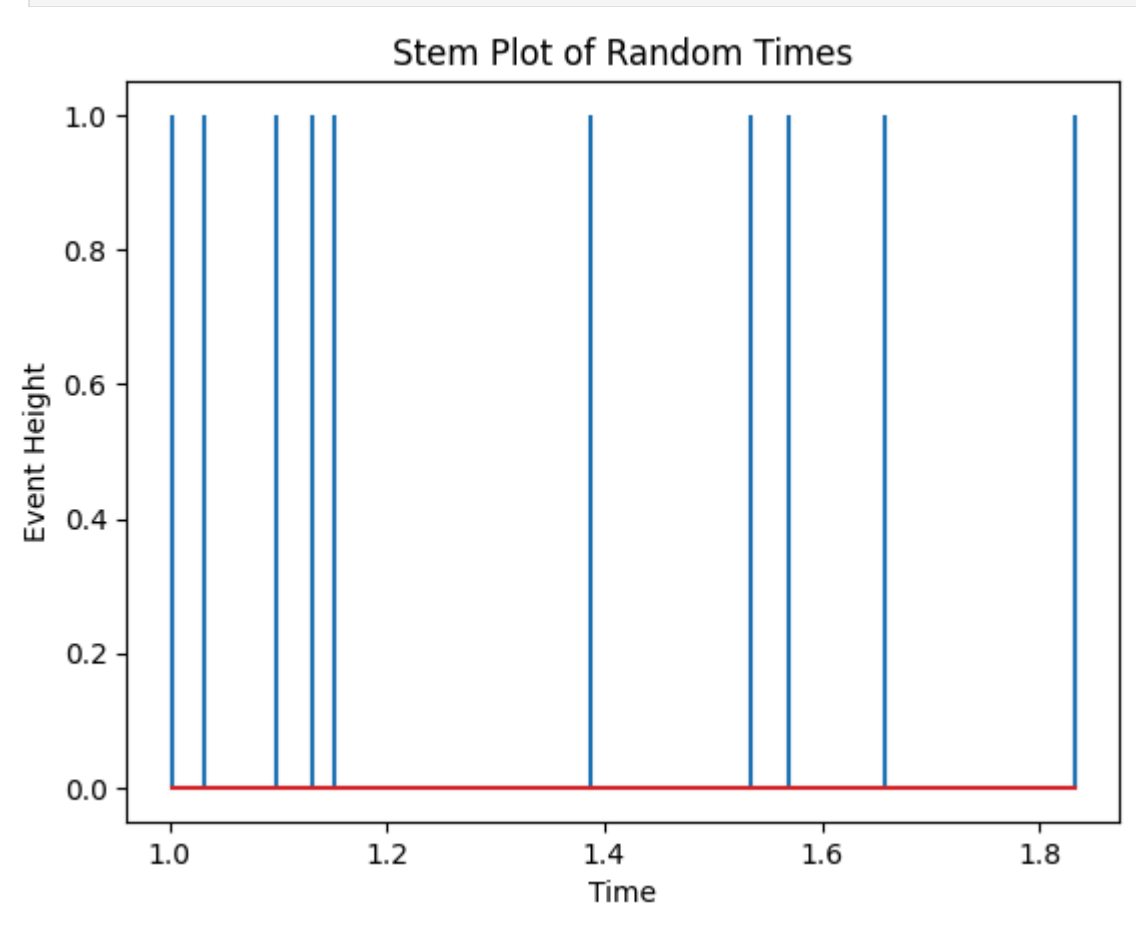
```python
from A2b_rhl72 import *
import numpy as np
import matplotlib.pyplot as plt
```

1a

```python
def randtimes(N, t1, t2):
    """
    Simulate a Poisson process

    N - number of random times

    [ t1 , t2 ] - time interval
    """
    random_times = [random.uniform(t1,t2) for _ in range(N)]
    return random_times

def plotflash(times):
    """
    Plot the given times as a stem plot with unit heights.
    """
    plt.stem(times, [1] * len(times), markerfmt='')
    plt.xlabel('Time')
    plt.ylabel('Event Height')
    plt.title('Stem Plot of Random Times')
    plt.show()
```

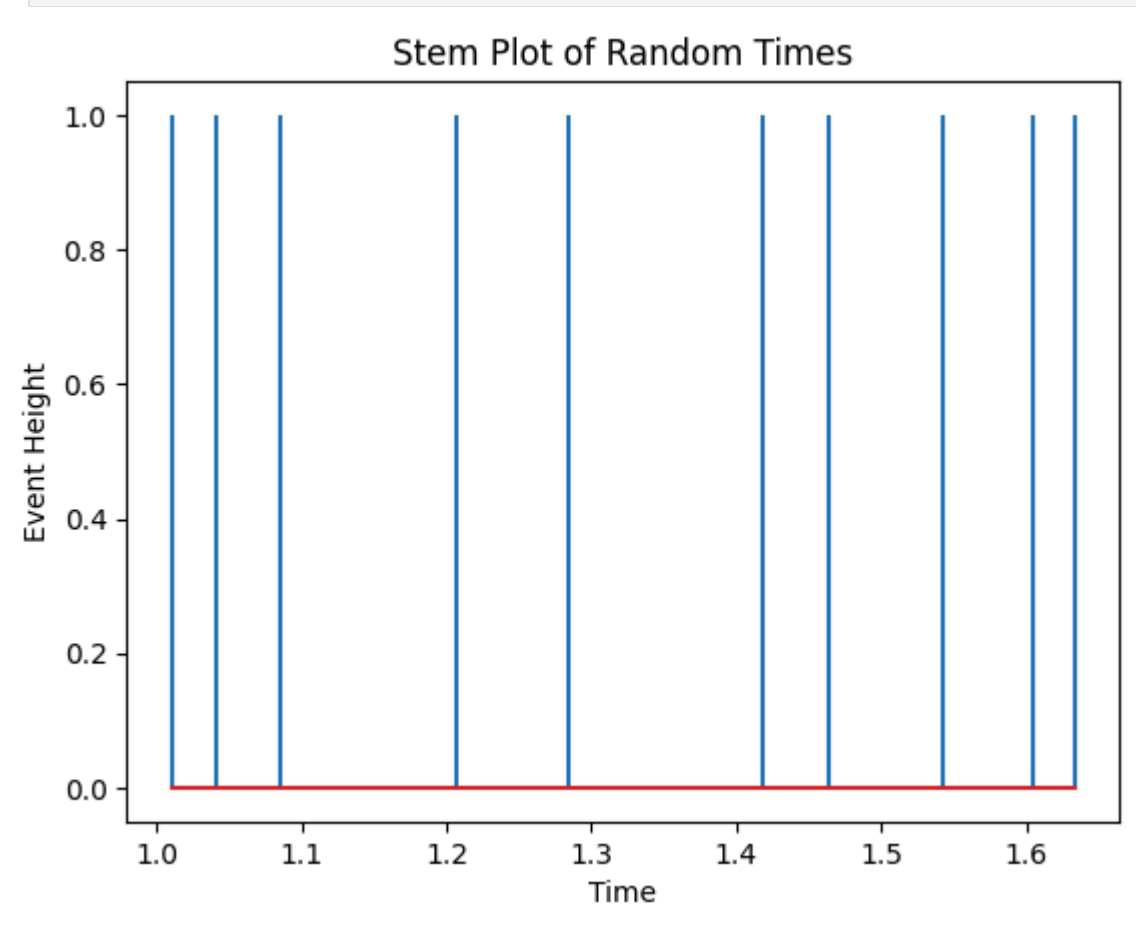
```python
t = randtimes(10, t1=1, t2=2)
plotflash(t)
```



1b

```python
def randintervals(N, lambd, t1):
    """
    Generate N random time intervals from t1 with lambda rate.
    """
    intervals = np.random.exponential(scale=1.0/lambd, size = N)

    event_times = t1 + np.cumsum(intervals)

    return event_times
```

```python
intervals = randintervals(10,10,1)
plotflash(intervals)
```



1c

The two methods produce statistically equivalent photon arrivals overall but differ in appearance. The randtimes method scatters N events uniformly over the interval. The randintervals method has time differences betwen events drawn from a exponential distribution. This results in periods with many closely spaced events and other periods with long gaps.

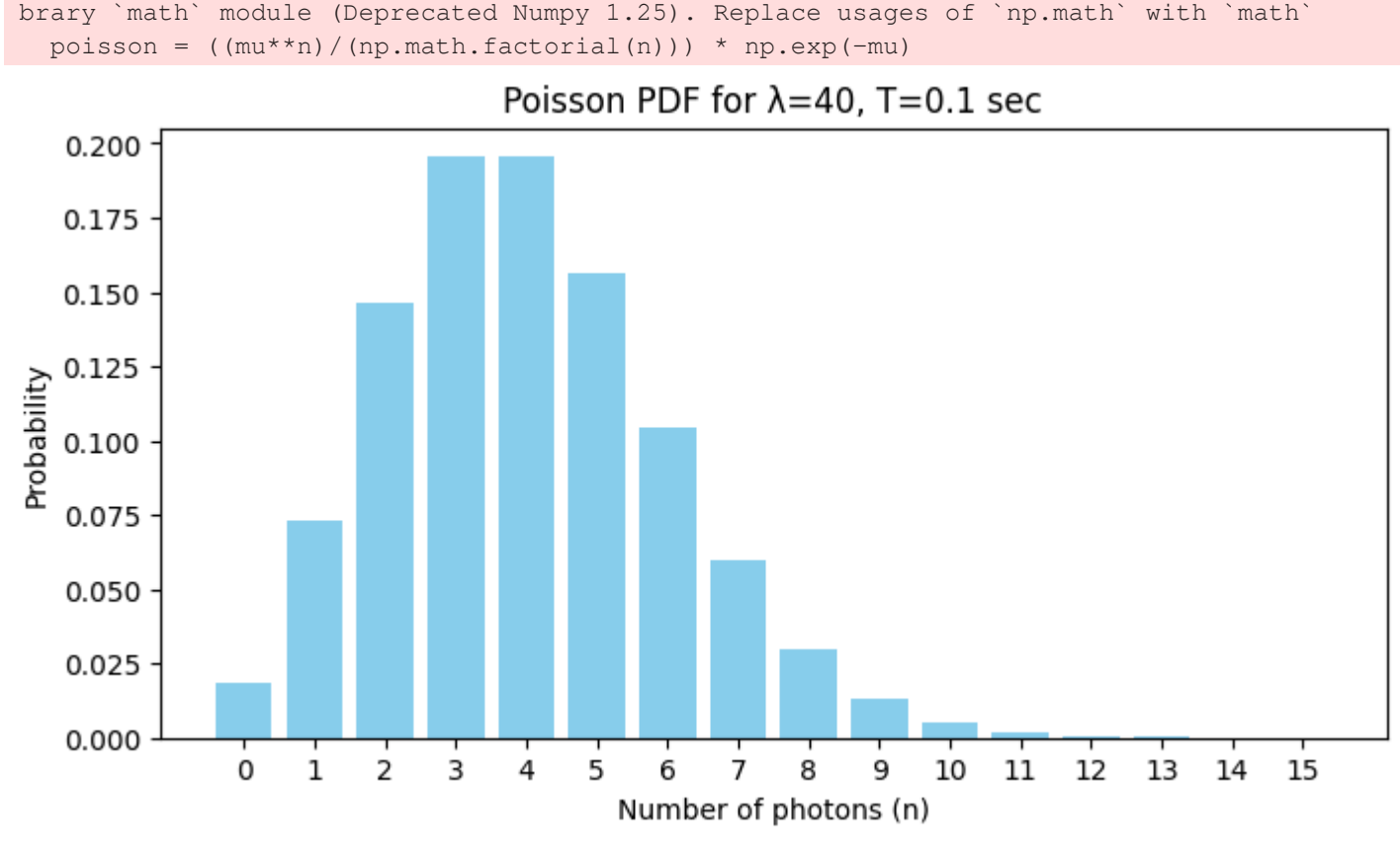
Noting that to actually see the flash of light, a certain threshold number of photons must be seen in a time interval, but if the arrivals of photons are probabalistic, so is the percieving of the photons. Seeing the flash is "inherently" probabalistic.

2a

```python
def poisson_pdf(n, lambd, T):
    """
    Compute Poisson probability for n events in time T
    """
    mu = lambd * T

    poisson = ((mu**n)/(np.math.factorial(n))) * np.exp(-mu)

    return poisson
```

```python
# Let's plot the pdf for n = 0 to 15 for a given rate.
lam_rate = 40    # events per second, say
T = 0.1          # 100 msec
n_values = np.arange(0, 16)
pdf_vals = [poisson_pdf(n, lambd=lam_rate, T=T) for n in n_values]


plt.figure(figsize=(8,4))
plt.bar(n_values, pdf_vals, color='skyblue')
plt.xlabel('Number of photons (n)')
plt.ylabel('Probability')
plt.title('Poisson PDF for λ=40, T=0.1 sec')
plt.xticks(n_values)
plt.show()
```

```
C:\Users\Ryan\AppData\Local\Temp\ipykernel_5600\1394053765.py:7: DeprecationWarning: `np.math` is a deprecated alias for the standard li
brary `math` module (Deprecated Numpy 1.25). Replace usages of `np.math` with `math`
  poisson = ((mu**n)/(np.math.factorial(n))) * np.exp(-mu)
```




```python
# And for fun lets just double the photon rate to see if the probabilities go up twice as much.

lam_rate = 80    # events per second, say
T = 0.1          # 100 msec
n_values = np.arange(0, 16)
pdf_vals = [poisson_pdf(n, lambd=lam_rate, T=T) for n in n_values]


plt.figure(figsize=(8,4))
plt.bar(n_values, pdf_vals, color='skyblue')
plt.xlabel('Number of photons (n)')
plt.ylabel('Probability')
plt.title('Poisson PDF for λ=40, T=0.1 sec')
plt.xticks(n_values)
plt.show()
```

```
C:\Users\Ryan\AppData\Local\Temp\ipykernel_5600\1394053765.py:7: DeprecationWarning: `np.math` is a deprecated alias for the standard li
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  poisson = ((mu**n)/(np.math.factorial(n))) * np.exp(-mu)
```
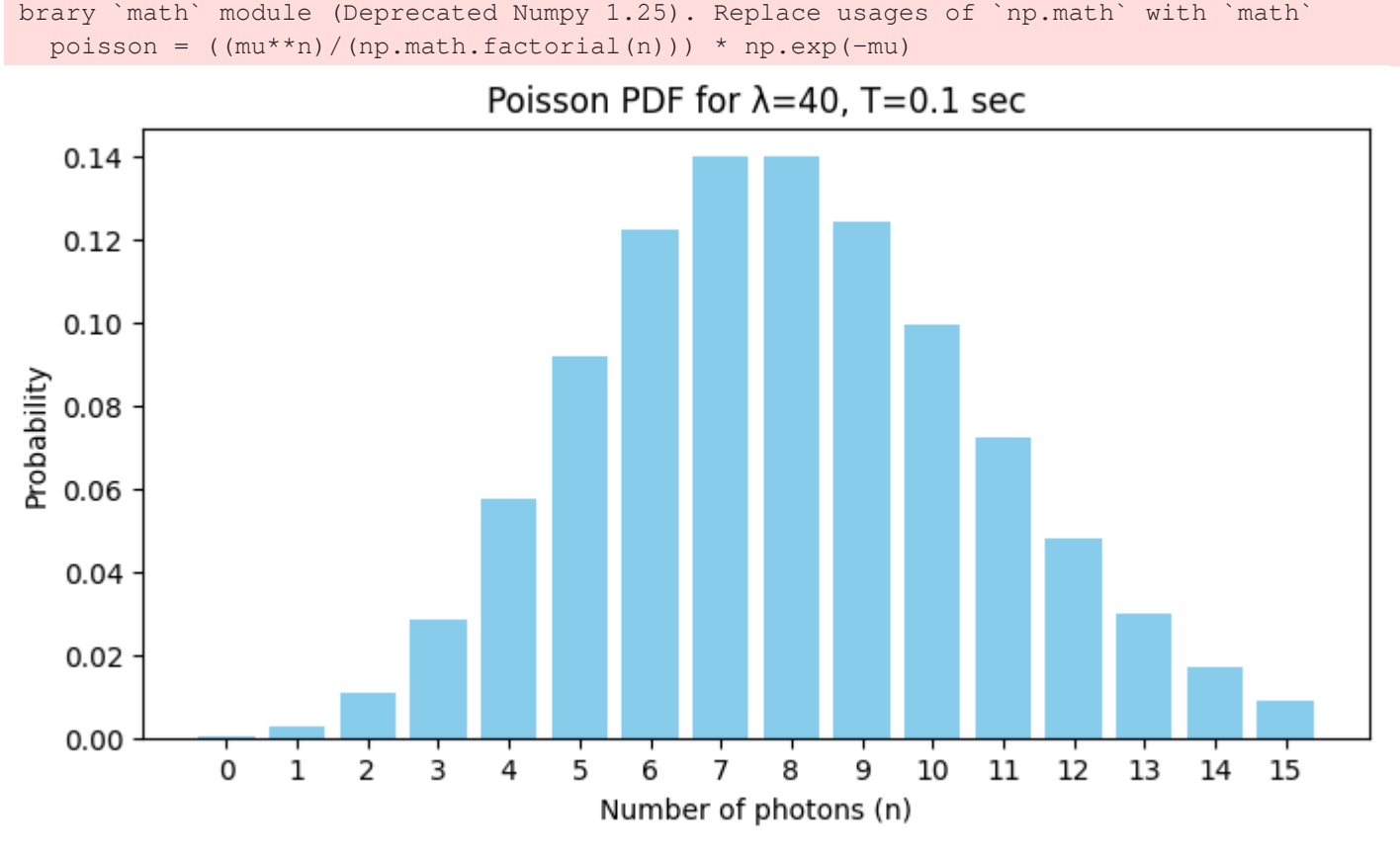



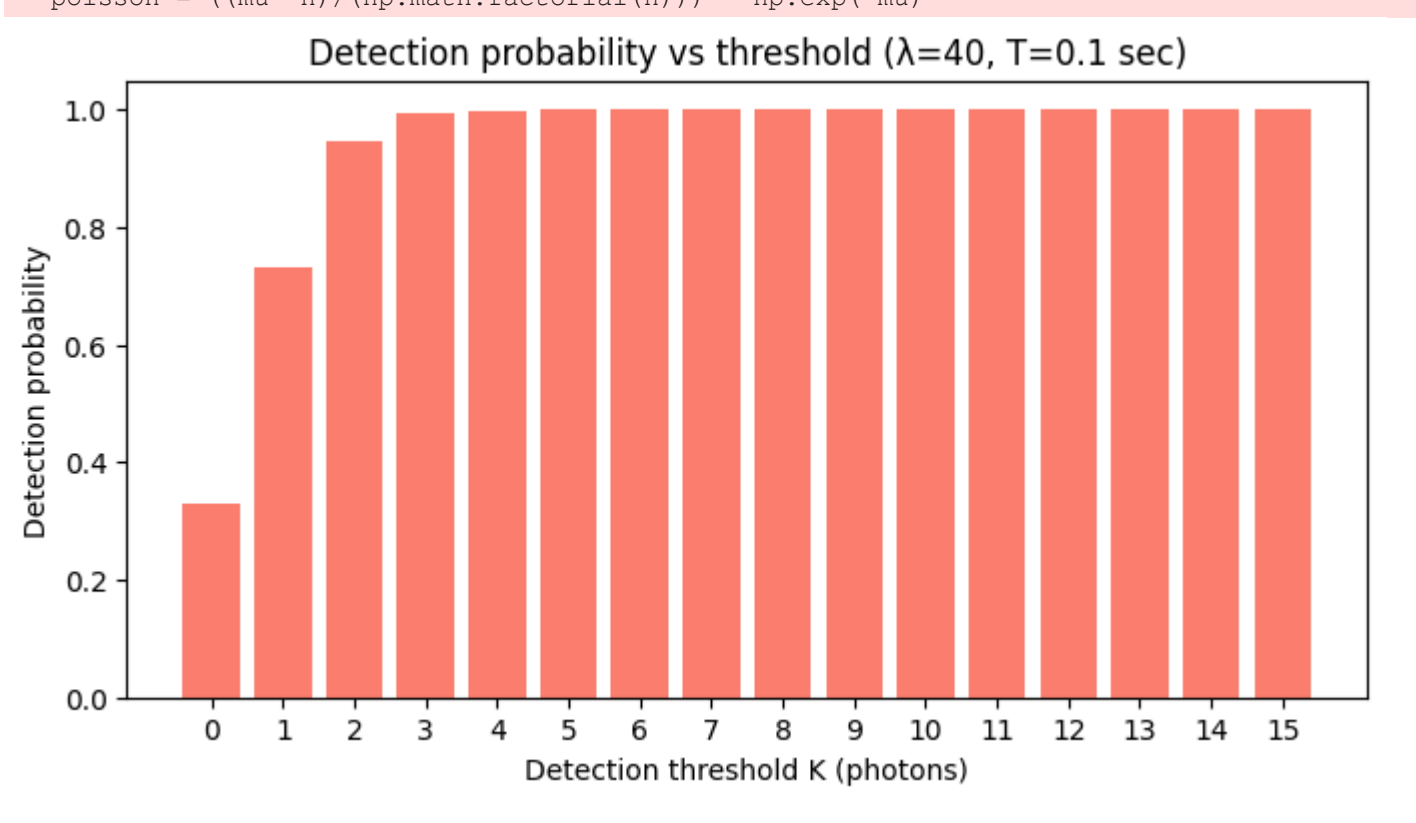
As you can observe, the mean of the distribution shifted almost to twice as much photons.

2b

```python
def detectionprob(K, lambd=40, T=0.1):
    """
    Probability of detecting the flash
    """

    mu = lambd * T
    return 1-poisson_pdf(K, mu, T)
```

```python
# Let's plot the detection probability as a function of the threshold K.
K_values = np.arange(0, 16)
det_prob_values = [detectionprob(K, lambd=40, T=0.1) for K in K_values]

plt.figure(figsize=(8,4))
plt.bar(K_values, det_prob_values, color='salmon')
plt.xlabel('Detection threshold K (photons)')
plt.ylabel('Detection probability')
plt.title('Detection probability vs threshold (λ=40, T=0.1 sec)')
plt.xticks(K_values)
plt.show()
```

```
C:\Users\Ryan\AppData\Local\Temp\ipykernel_5600\1394053765.py:7: DeprecationWarning: `np.math` is a deprecated alias for the standard li
brary `math` module (Deprecated Numpy 1.25). Replace usages of `np.math` with `math`
  poisson = ((mu**n)/(np.math.factorial(n))) * np.exp(-mu)
```



3a

```python
def lightflash(lambd, t1=0.8, t2=2.2):
    """
    Simulate a Poisson process with rate `lambd` within the interval [t1, t2].

    lambd - Poisson rate (events per unit time)
    t1, t2 - Start and end time of the process
    """
    T = t2 - t1  # Duration of process

    # Sample the number of events from a Poisson distribution
    N = np.random.poisson(lambd * T)  # Number of events

    # Generate random event times
    event_times = randtimes(N, t1, t2)

    # Return sorted event times
    return np.sort(event_times)
```

```python
def plotflash(photon_times, ax):
    """
    Plot photon arrival times as vertical lines.

    photon_times: array of arrival times
    ax: matplotlib axis object
    """
    ax.eventplot(photon_times, orientation='horizontal', colors='black', linewidths=0.8)
    ax.set_ylabel("Photon Events")

def simulate_flash():
    """
    Simulate a flash of light with three stages:
    1. Full photon stream.
    2. Photons passing through a shutter that is open from s1 to s2.
    3. Photons detected by rods (each photon is detected with probability α).

    The simulation is done in milliseconds.
    """
    # Parameters
    lam = 100         # photon rate: 100 photons per ms
    full_t1, full_t2 = 0.8, 2.2  # flash interval in ms
    s1, s2 = 1.0, 2.0            # shutter open from 1 to 2 ms
    alpha = 0.06               # fraction absorbed/detected by rods

    # Generate full photon stream
    full_stream = lightflash(lam, t1=full_t1, t2=full_t2)

    # Photons passing through shutter: those events between s1 and s2.
    shutter_stream = full_stream[(full_stream >= s1) & (full_stream <= s2)]

    # Detected photons: each photon from the shutter stream is detected with probability α.
    detected_stream = shutter_stream[np.random.rand(len(shutter_stream)) < alpha]

    # Create subplots with aligned x-axes (in ms)
    fig, axes = plt.subplots(3, 1, sharex=True, figsize=(10, 6))

    # Plot full stream
    plotflash(full_stream, ax=axes[0])
    axes[0].set_title("Full Photon Stream (λ=100 photons/ms)")
    axes[0].set_xlim(full_t1, full_t2)

    # Plot after shutter
    plotflash(shutter_stream, ax=axes[1])
    axes[1].set_title("After Shutter (open from %.1f to %.1f ms)" % (s1, s2))

    # Plot detected photons
    plotflash(detected_stream, ax=axes[2])
    axes[2].set_title("Detected by rods (α = %.2f)" % alpha)
    axes[2].set_xlabel("Time (ms)")

    plt.tight_layout()


simulate_flash()
```

I is the number of photons. $Photons$

lambda is the number of photons per period. $\frac{Photons}{msec}$

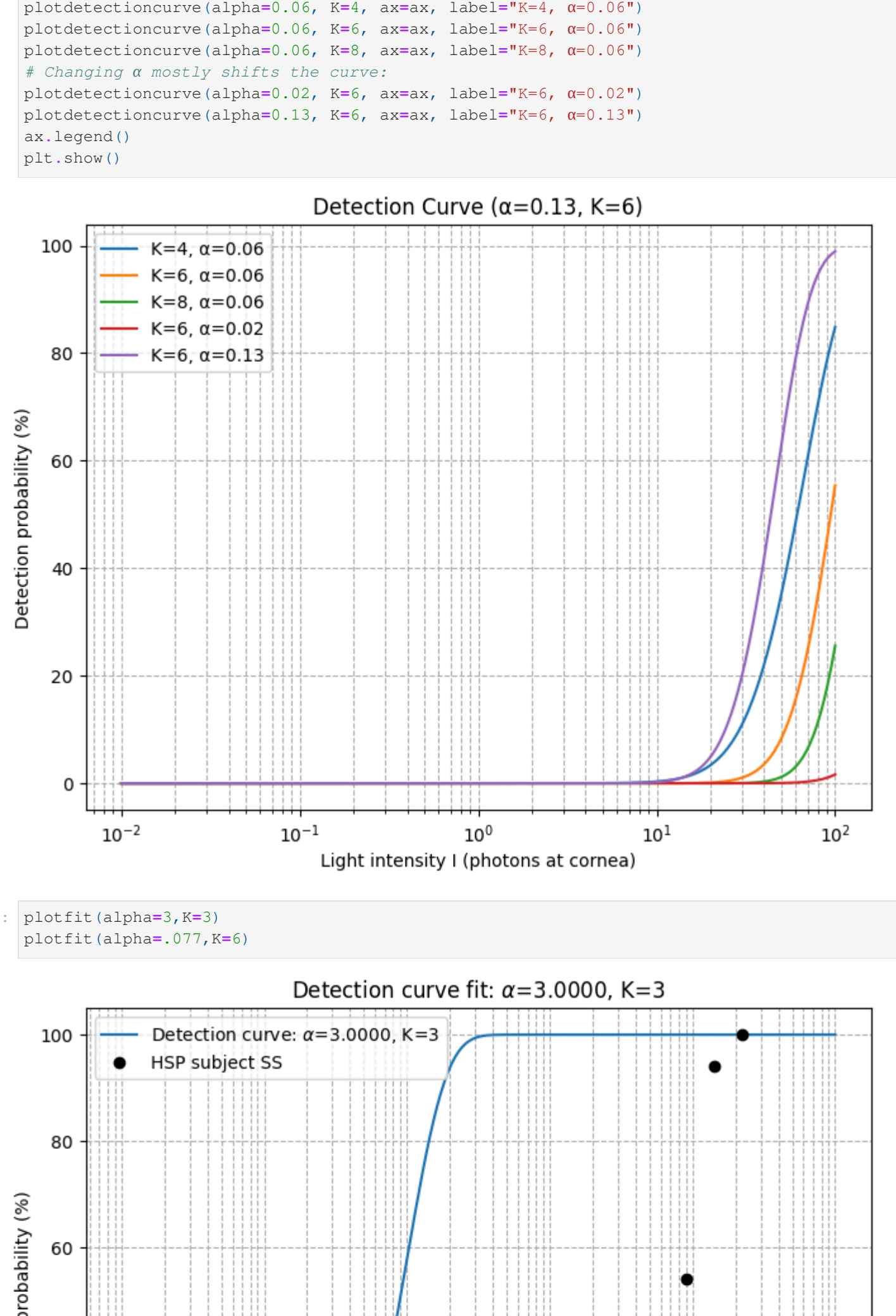
T is time interval $msec$

alpha is the absorbtion rate. $\frac{Photons}{msec}$

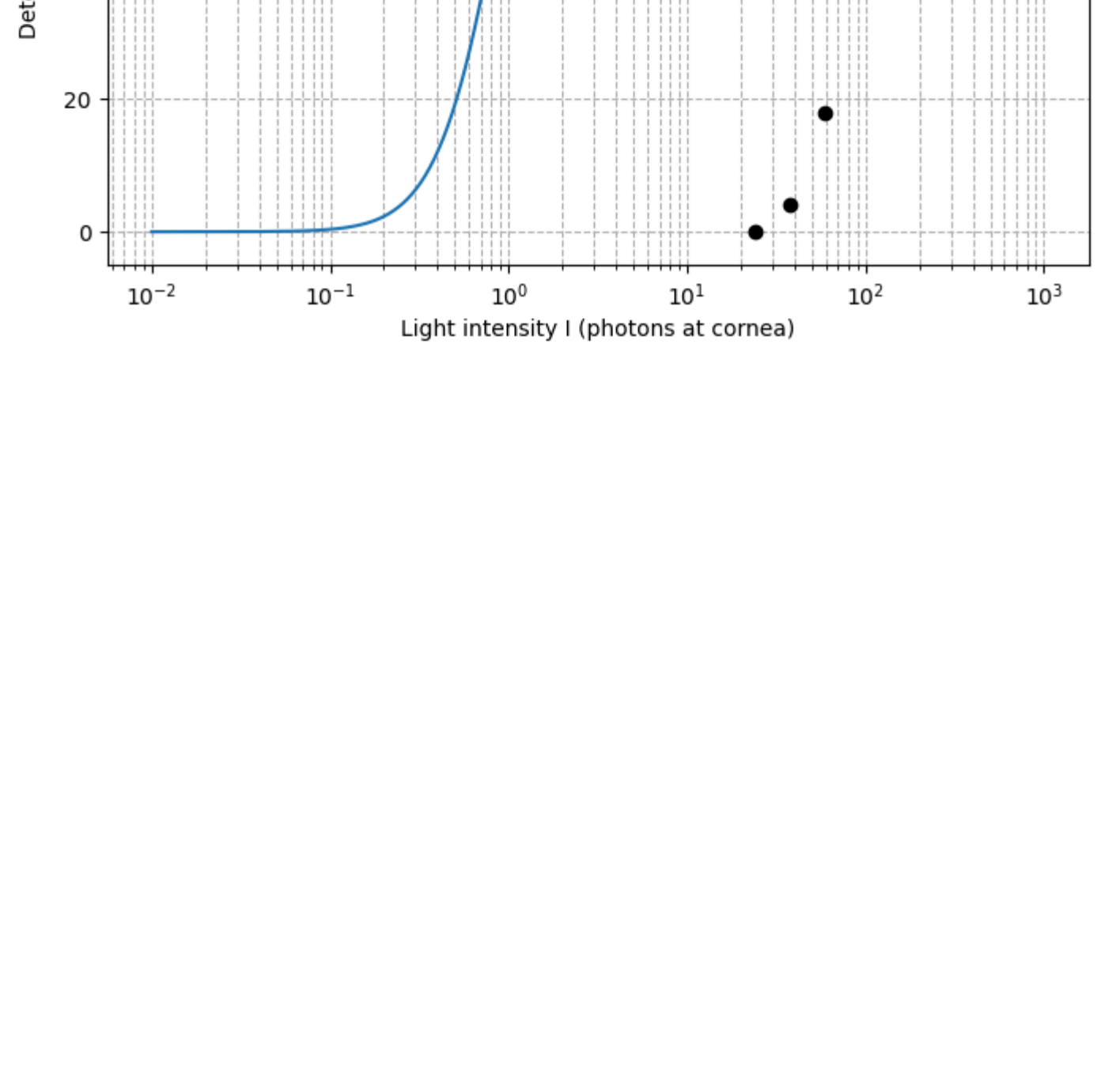
K is the threshhold for detecting the light per period. $Photons$

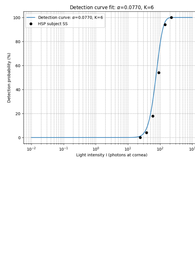
```python
# 3c. Plot the percent of flashes detected as a function of light intensity I.
def plotdetectioncurve(alpha=0.5, K=6, ax=None, label=None):
    """
    Plot the detection probability as a function of intensity I.
    I is the total number of photons arriving at the cornea.
    The x-axis is on a logarithmic scale and ranges from 0.01 to 100.
    """
    I_values = np.logspace(-2, 2, 200)
    prob_values = [probseeing(I, alpha=alpha, K=K) for I in I_values]
    if ax is None:
        fig, ax = plt.subplots(figsize=(8,6))
    ax.plot(I_values, np.array(prob_values)*100, label=label)
    ax.set_xscale('log')
    ax.set_xlabel("Light intensity I (photons at cornea)")
    ax.set_ylabel("Detection probability (%)")
    ax.set_title("Detection Curve (α=%.2f, K=%d)" % (alpha, K))
    ax.grid(True, which="both", ls="--")
    return ax
```

```python
# Let's overlay a few curves to see the effects of varying α and K.
fig, ax = plt.subplots(figsize=(8,6))
# Changing K changes the curve's shape:
plotdetectioncurve(alpha=0.06, K=4, ax=ax, label="K=4, α=0.06")
plotdetectioncurve(alpha=0.06, K=6, ax=ax, label="K=6, α=0.06")
plotdetectioncurve(alpha=0.06, K=8, ax=ax, label="K=8, α=0.06")
# Changing α mostly shifts the curve:
plotdetectioncurve(alpha=0.02, K=6, ax=ax, label="K=6, α=0.02")
plotdetectioncurve(alpha=0.13, K=6, ax=ax, label="K=6, α=0.13")
ax.legend()
plt.show()
```




```python
plotfit(alpha=3,K=3)
plotfit(alpha=.077,K=6)
```




Detection curve fit: σ=0.0770, K=6
Detection curve: σ=0.0770, K=6
HSP subject 53
Detection probability (%)
100
80
60
40
20
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Light intensity I (photons at cornea)