► BEN DE BONDT, *Some remarks on Namba-type forcings.*
Institut de Mathématiques de Jussieu (IMJ-PRG), Université Paris Cité, Bâtiment Sophie Germain, 8 Place Aurélie Nemours, 75013 Paris, France.
E-mail: ben.de-bondt@imj-prg.fr.

For the purpose of this abstract, let "a Namba-type forcing" be any forcing that forces ω_2 to get cofinality ω and doesn't collapse ω_1. It is well known that the existence of a semiproper Namba-type forcing is equivalent to a Strong Chang's Conjecture, but that instead the existence of a stationary set preserving Namba-type forcing is provable in plain ZFC. However, in the context of questions on iterated-forcing-using-side-conditions, it is natural to ask whether one can demand more than mere stationary set preservation and get provably in ZFC a Namba-type forcing that allows many (but not necessarily club many) models for which there exist sufficient semi-generic conditions. In this talk I will discuss a "side-condition version" $\mathbb{P}$ of Namba forcing and explain that there exists a very natural projective stationary family of countable elementary submodels of H_θ such that $\mathbb{P}$ is semiproper *with respect to these models.* In fact, we can consider a notion of strong semiproperness, in analogy to the notion of strong properness and verify that $\mathbb{P}$ satisfies it, again with respect to these distinguished models.

As an application of this approach towards Namba forcing, we discuss a particularly natural presentation of an "ersatz iterated Namba forcing" which, given an increasing sequence $(\kappa_\alpha : \alpha < \gamma)$ of regular cardinals $\geq \omega_2$, adds for every $\alpha < \gamma$ a countable cofinal subset of κ_α, while at the same time preserving stationarity of stationary subsets of ω_1. In the proof, we will make strong use of a technique involving labelled trees and games played on such trees that appears in [1].

Finally, we will mention closely related ongoing work and remaining questions.
This talk is based on joint work with my thesis supervisor Boban Veličković.

[1] Matthew Foreman and Menachem Magidor, *Mutually stationary sequences of sets and the non-saturation of the non-stationary ideal on $P_\kappa(\lambda)$*, ***Acta Mathematica***, vol. 186 (2001), no. 2, pp. 271–300.

[2] Saharon Shelah, ***Proper and Improper Forcing***, Perspectives in Logic, vol. 5, Cambridge University Press, 1998.