Reachability and Safety Games under TSO Semantics

SCooL / GandALF 2024 in Reykjavik

Stephan Spengler

Uppsala University, Sweden

20 June 2024

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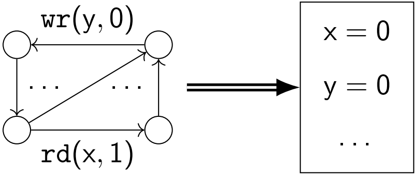
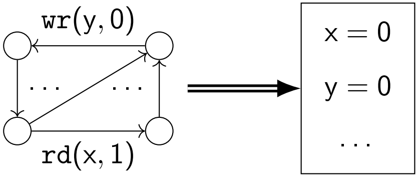


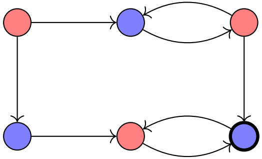
$x = 0$

$y = 0$

$\dots$

Reachability and Safety Games under TSO semantics





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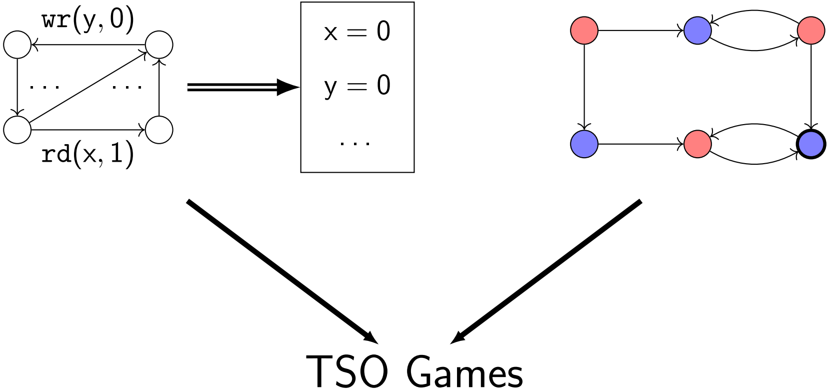

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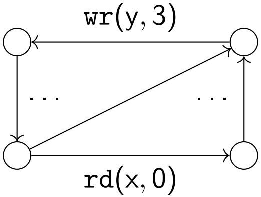
. . .

TSO Games

Total Store Order

Total Store Order

Proc^1:



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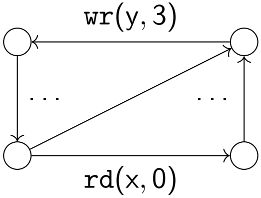
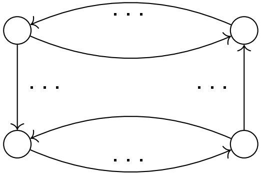


Proc^2:



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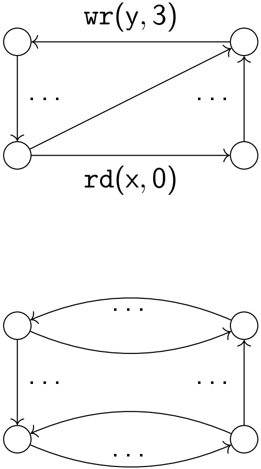

Memory:

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. . .

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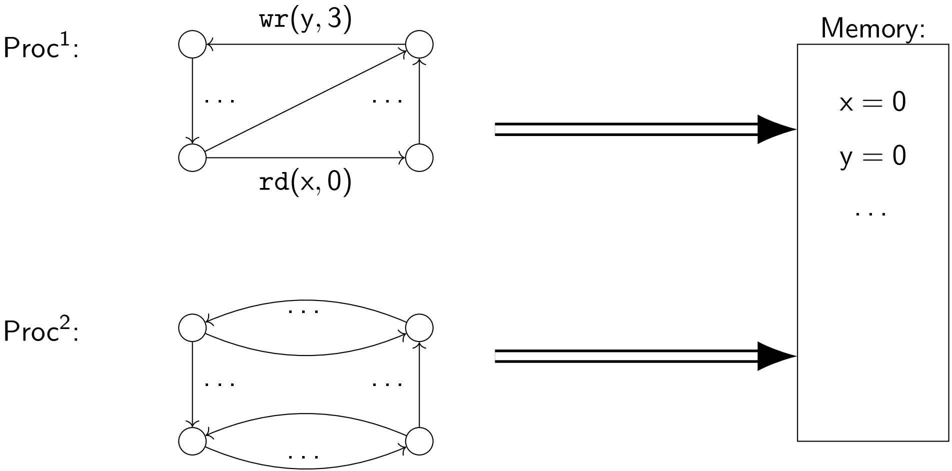

Proc^1:

$\texttt{wr}(y, 3)$

$\texttt{rd}(x, 0)$

Proc^2:

Memory:

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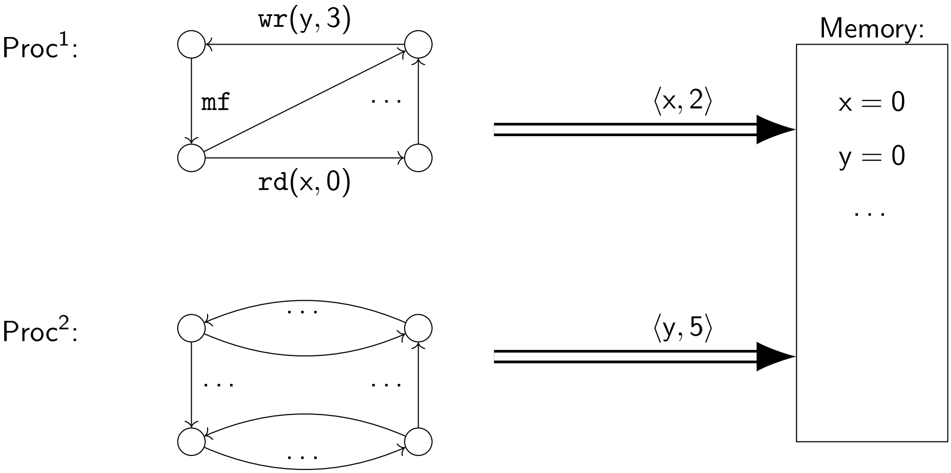
$\dots$

Total Store Order



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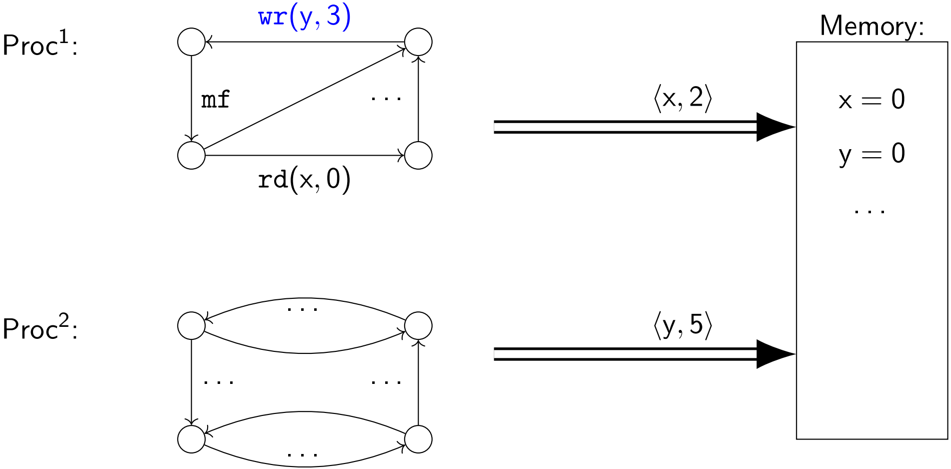

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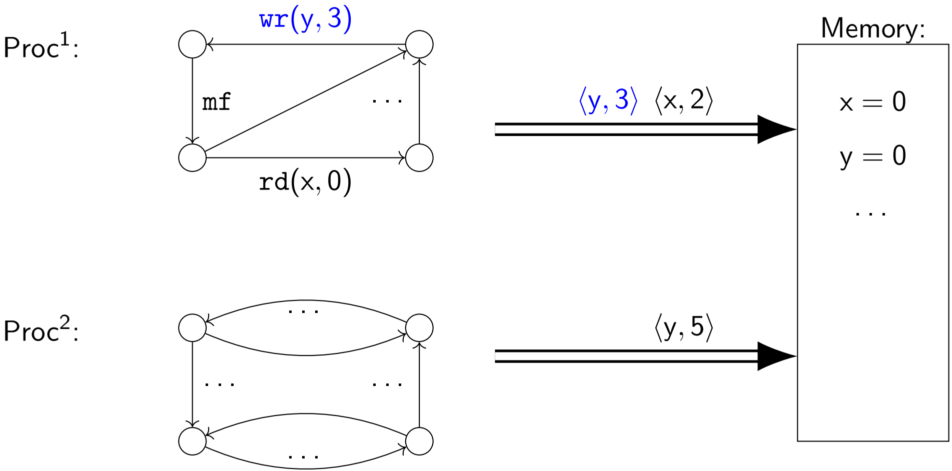


Proc^1:

$\mathtt{wr}(y, 3)$

mf

. . .

$\mathtt{rd}(x, 0)$

$\langle y, 3\rangle$ $\langle x, 2\rangle$

Memory:

$x = 0$

$y = 0$

. . .

Proc^2:

. . .

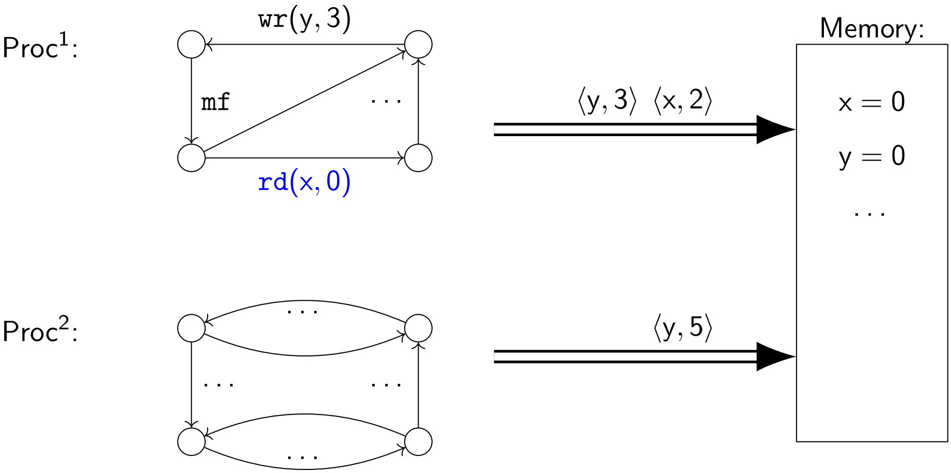
. . .

. . .

. . .

$\langle y, 5\rangle$

Total Store Order



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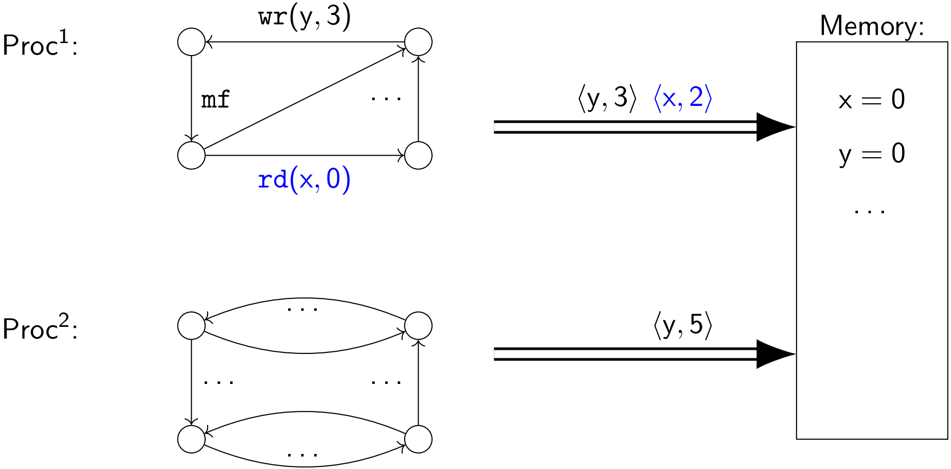

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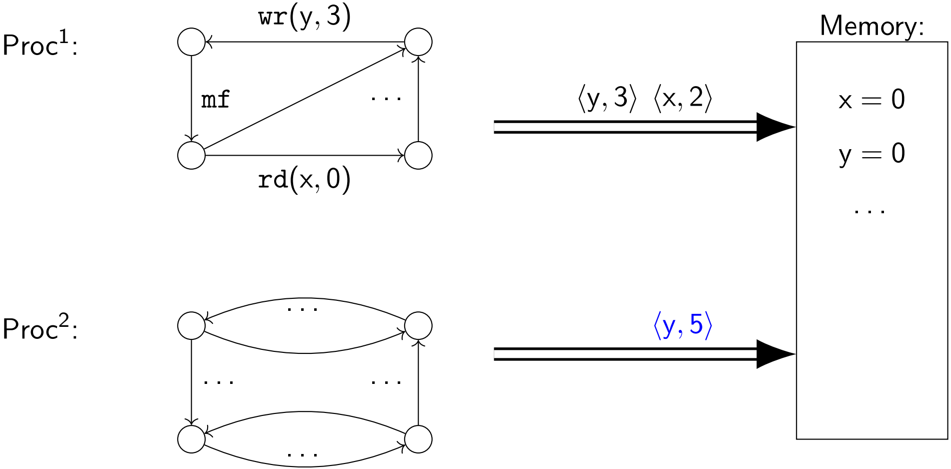

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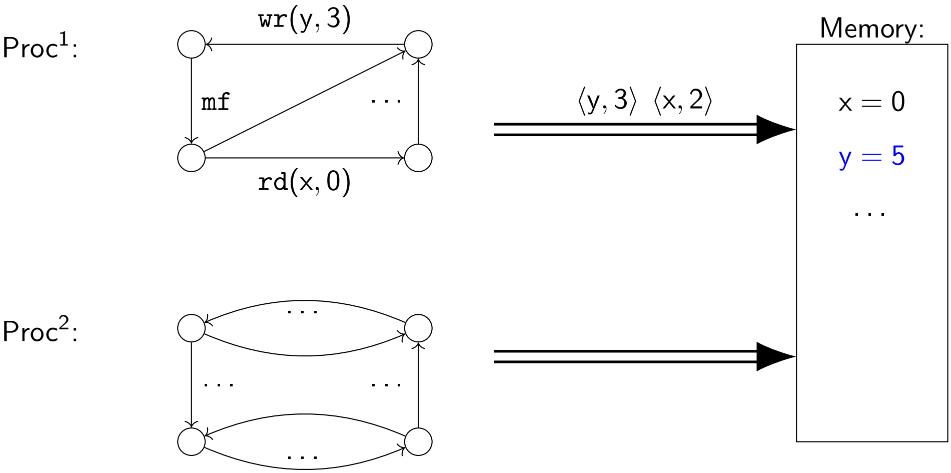

Games

Games

- players A and B

Games

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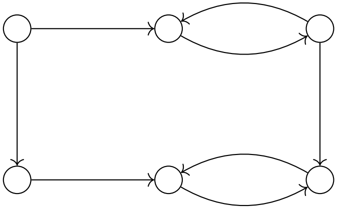
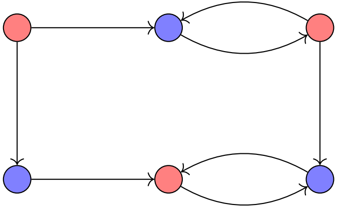
- configurations C
- transition relation $\longrightarrow$



Games

- players A and B
- configurations $C = C_A \cup C_B$
- transition relation $\longrightarrow$
  - $\longrightarrow \subseteq (C_A \times C_B) \cup (C_B \times C_A)$



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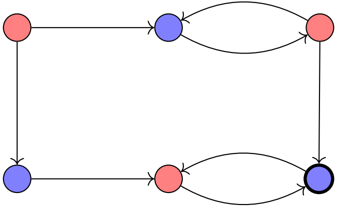

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  - A tries to reach C_F
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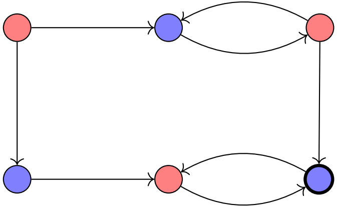

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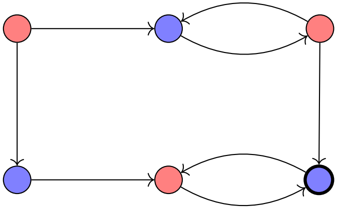
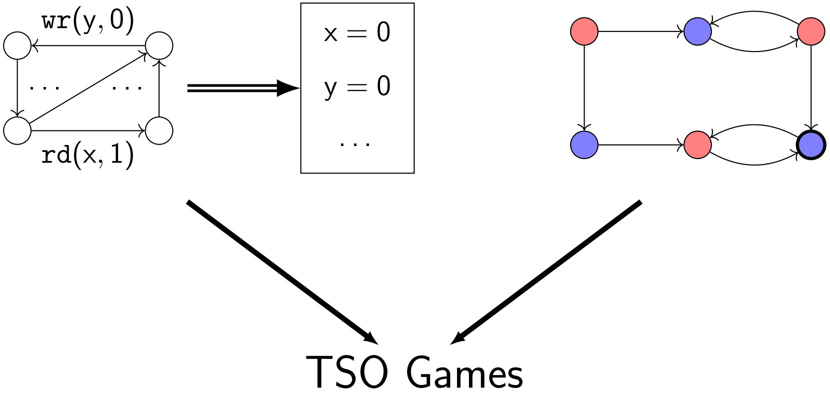
- reachability game:
  - A tries to reach C_F
  - B tries to avoid C_F
- safety game: reversed roles



TSO Games



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TSO Games

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TSO (configuration graph): $c_1 \xrightarrow{\text{instr}} c_2 \xrightarrow{\mathtt{up}^*} c_3$

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TSO Game: $c_1 \xrightarrow{\hspace{3em}} c_2 \xrightarrow{\hspace{3em}} c_3$

TSO Games

TSO (configuration graph):

$$\begin{tikzcd}
c_1 \arrow[r, "\text{instr}"] & c_2 \arrow[r, "\mathrm{up}^*"] \arrow[loop below, "\mathrm{up}^0"] & c_3
\end{tikzcd}$$

TSO Game:

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TSO Games

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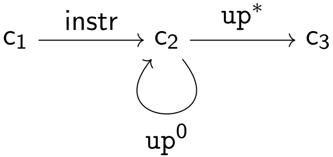

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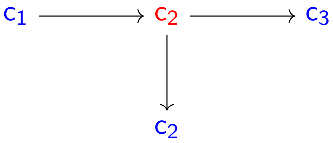

TSO Games

TSO (configuration graph):

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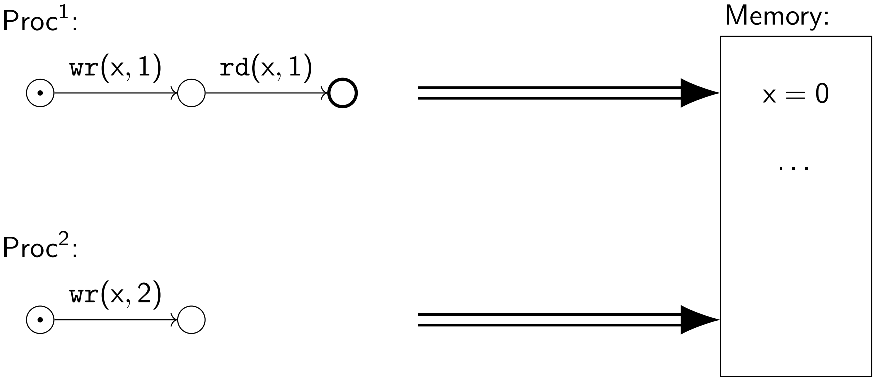
TSO Game:

$$\begin{tikzcd} c_1 \arrow[r] & c_2 \arrow[r] \arrow[d] & c_3 \\ & c_2 & \end{tikzcd}$$

process player / *update player*

TSO Games - Reachability Problem



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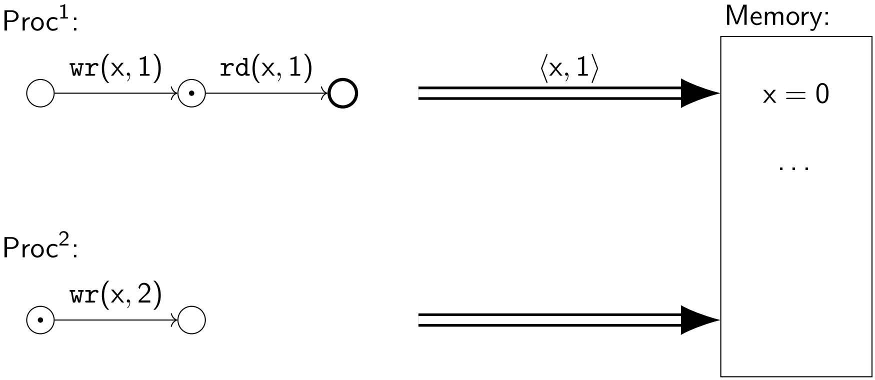

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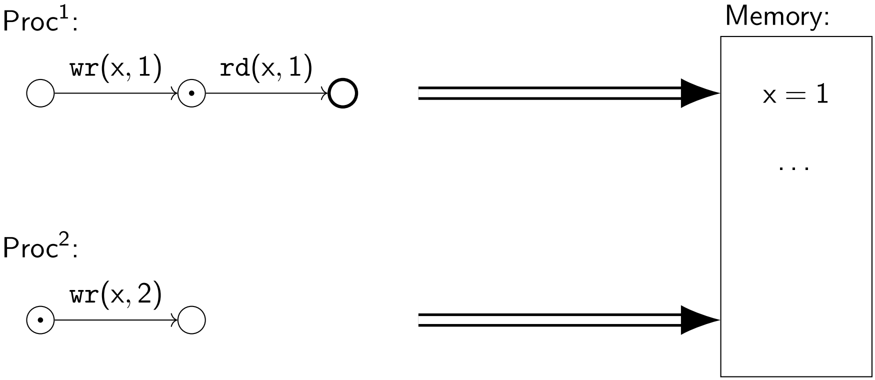

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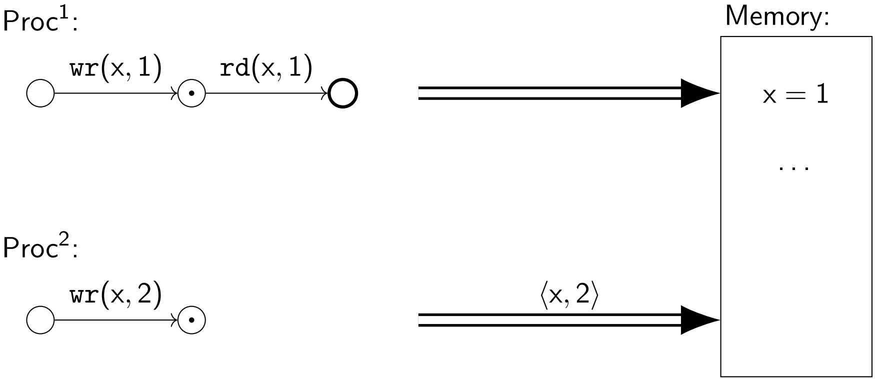

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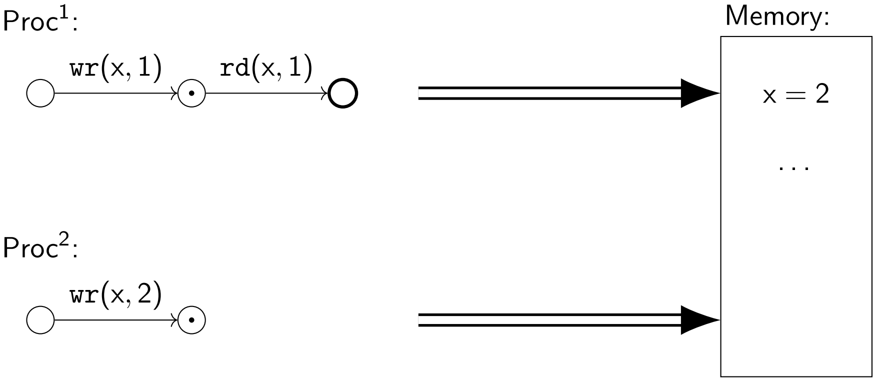

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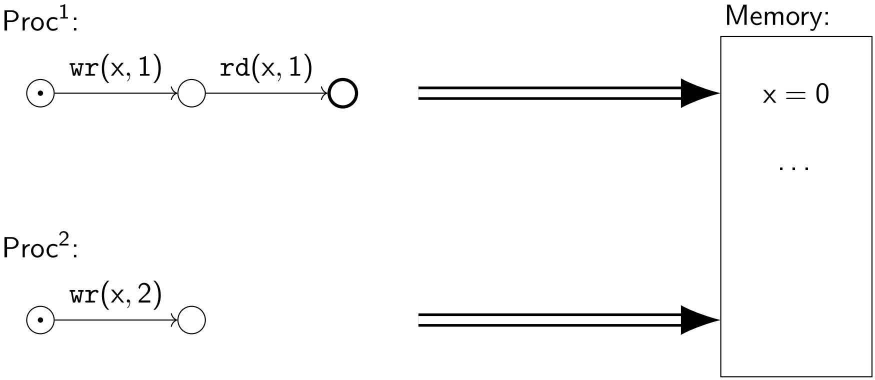

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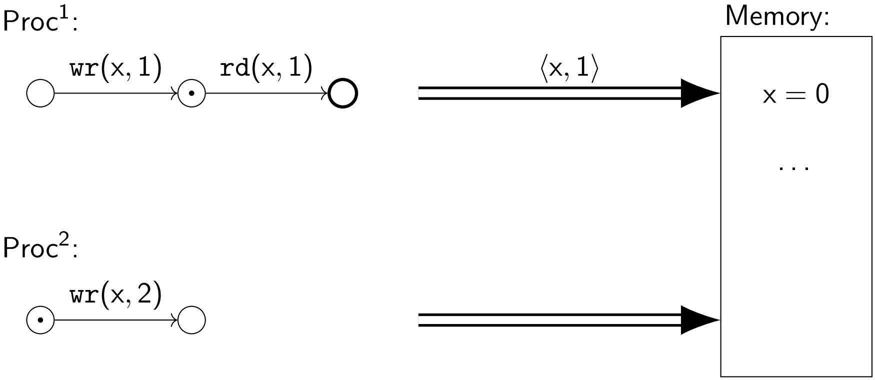

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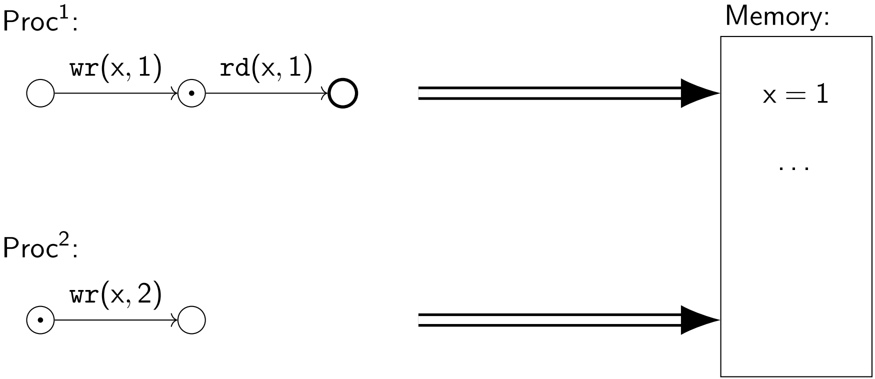

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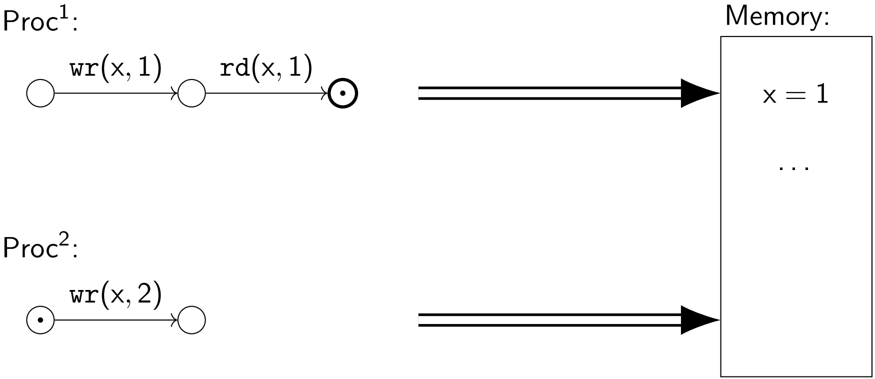

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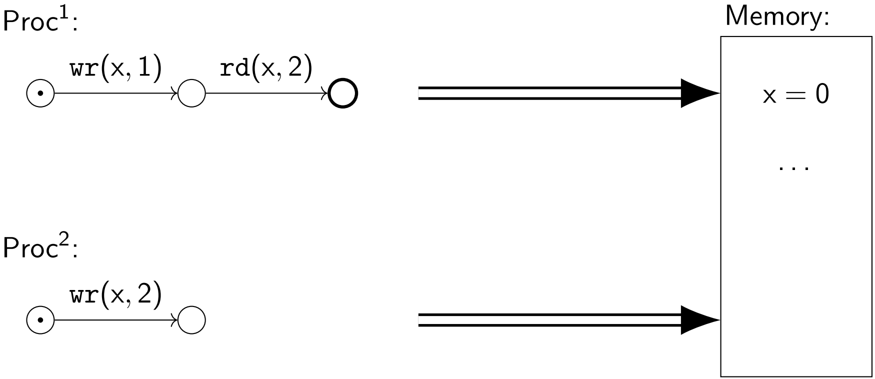

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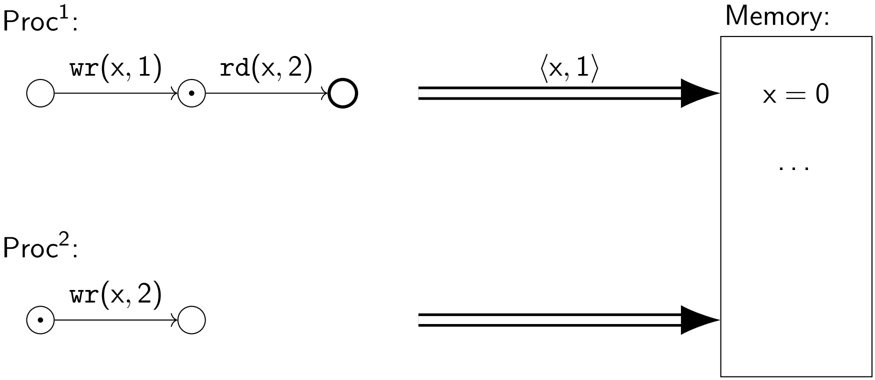

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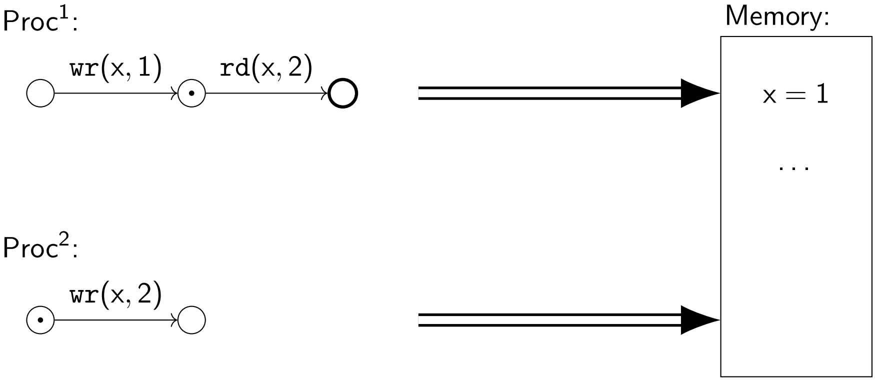

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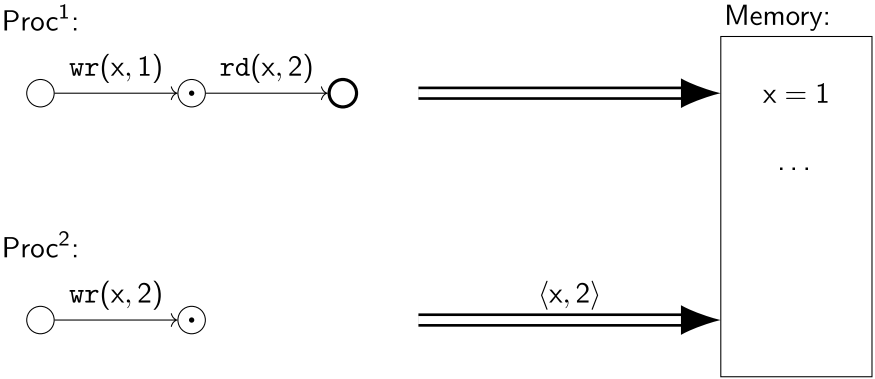

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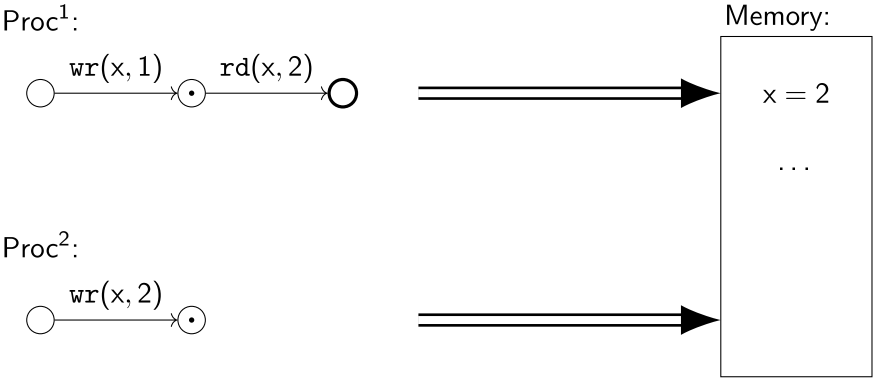

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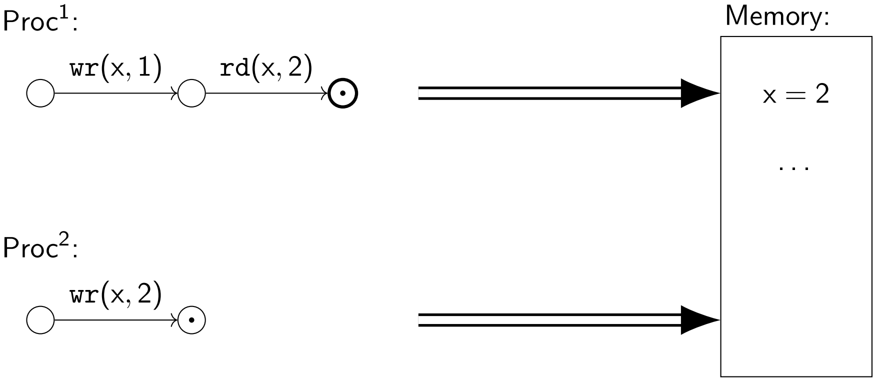

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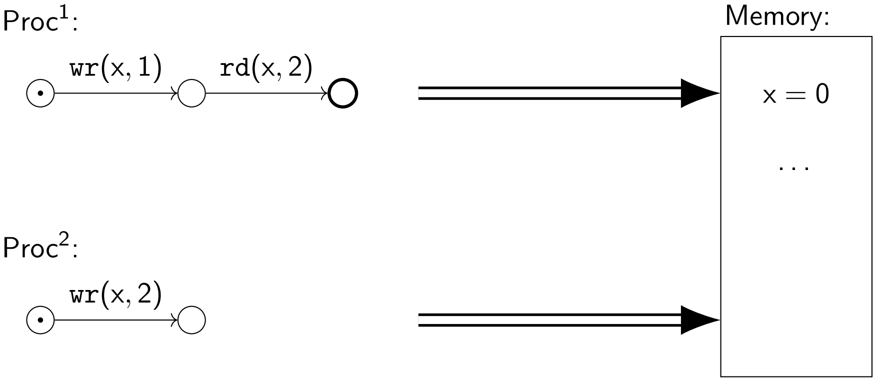

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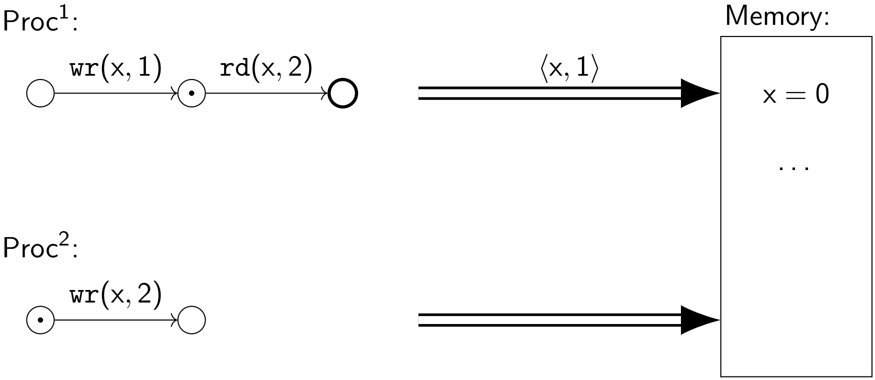

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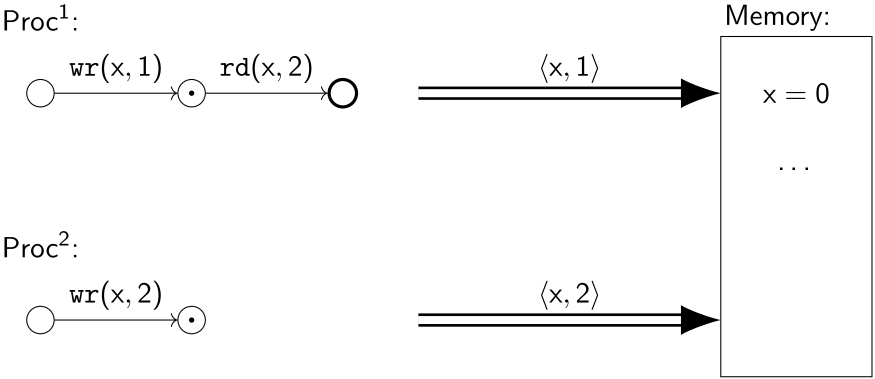

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- analysis reduces to single-process programs (finite behaviour)

- complexity: PSPACE-complete

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  Eventually, every buffer message must be updated to the memory.

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safety games?

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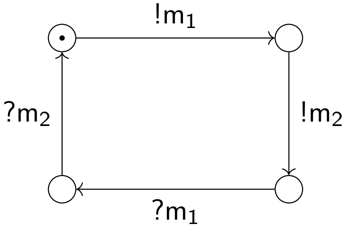
Idea: Reduction from *Perfect Channel Systems*

- nondeterministic finite state automata augmented by FIFO *channel*



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Idea: Reduction from *Perfect Channel Systems*

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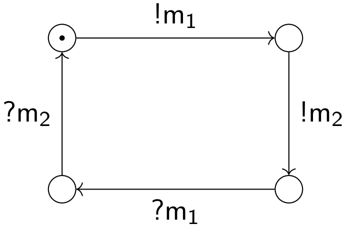

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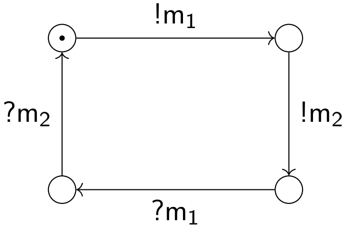
Idea: Reduction from *Perfect Channel Systems*

- nondeterministic finite state automata augmented by FIFO *channel*
- use TSO buffer to simulate channel
- reduce PCS reachability (undecidable) to TSO reachability game



Update Fairness - PCS Reduction

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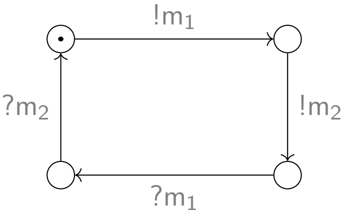

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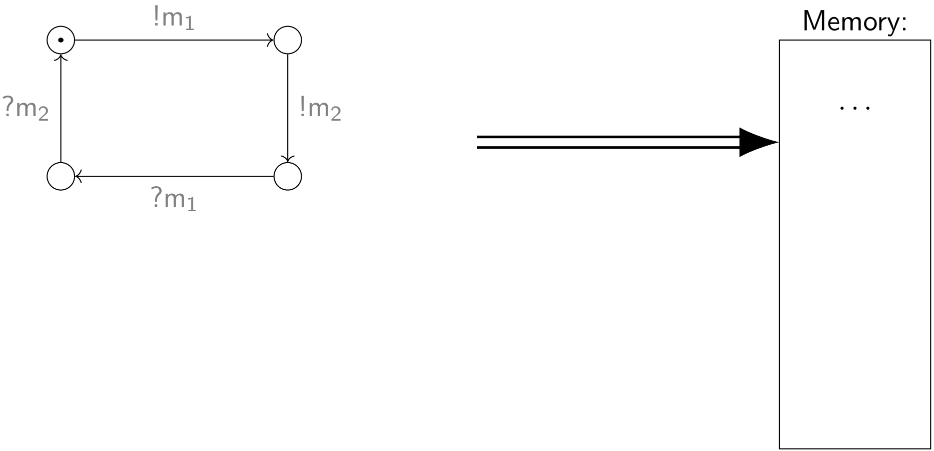

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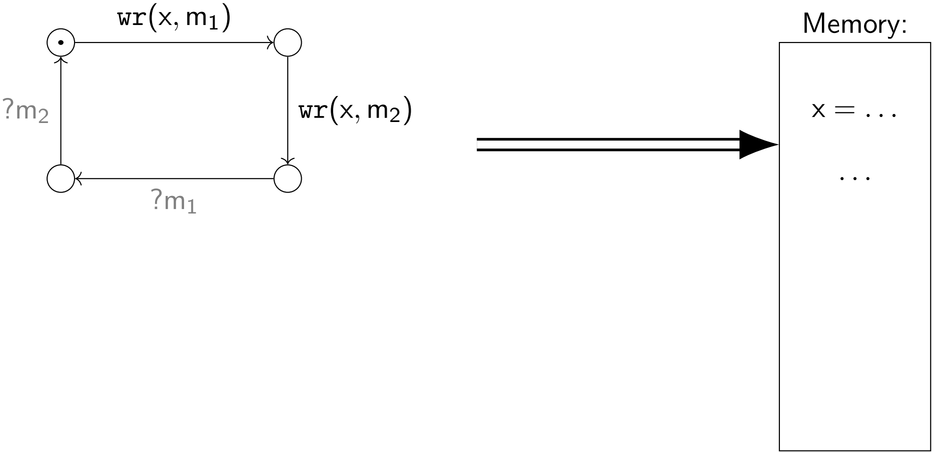

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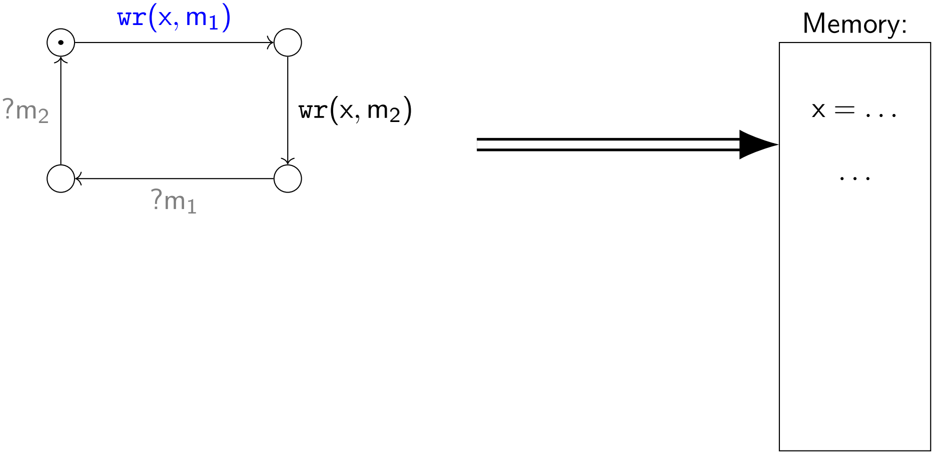

Memory:

$x = \ldots$

$\ldots$

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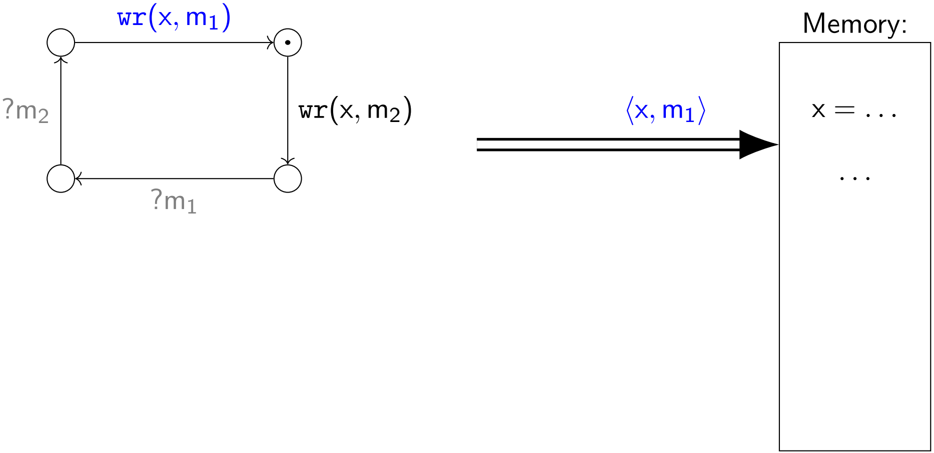

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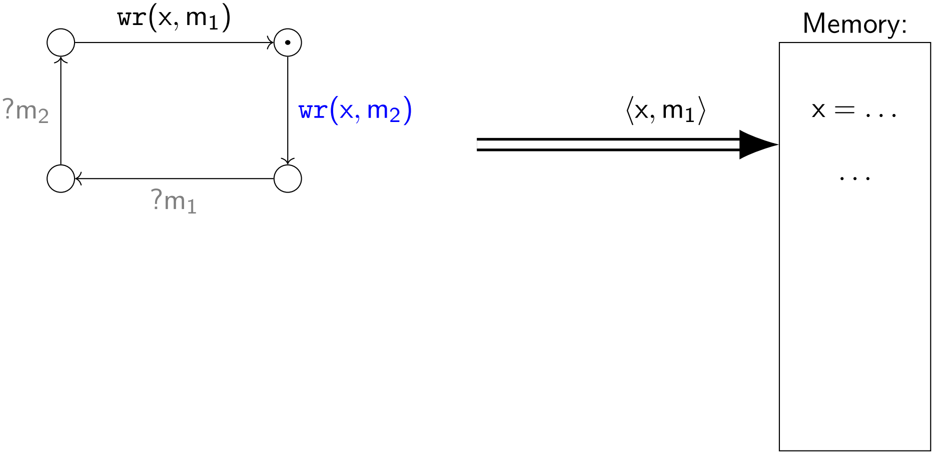

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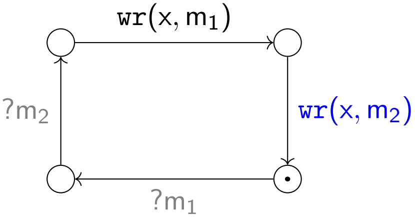
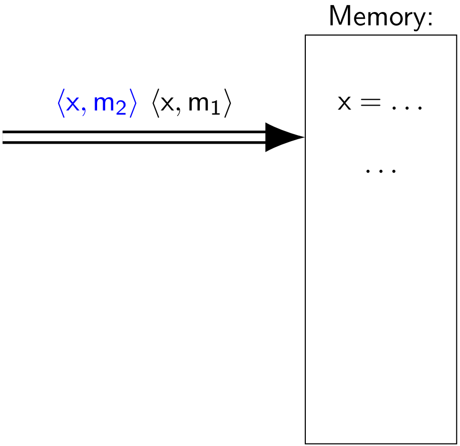




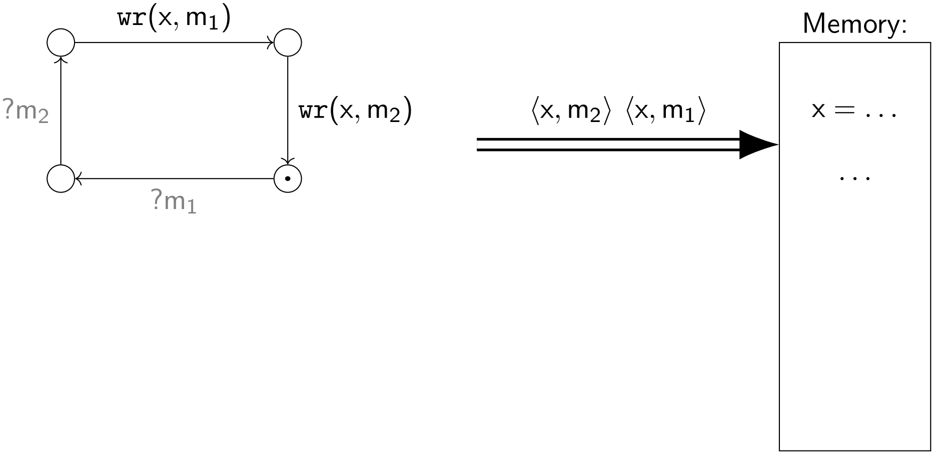
$\langle x, m_2 \rangle$ $\langle x, m_1 \rangle$

Memory:

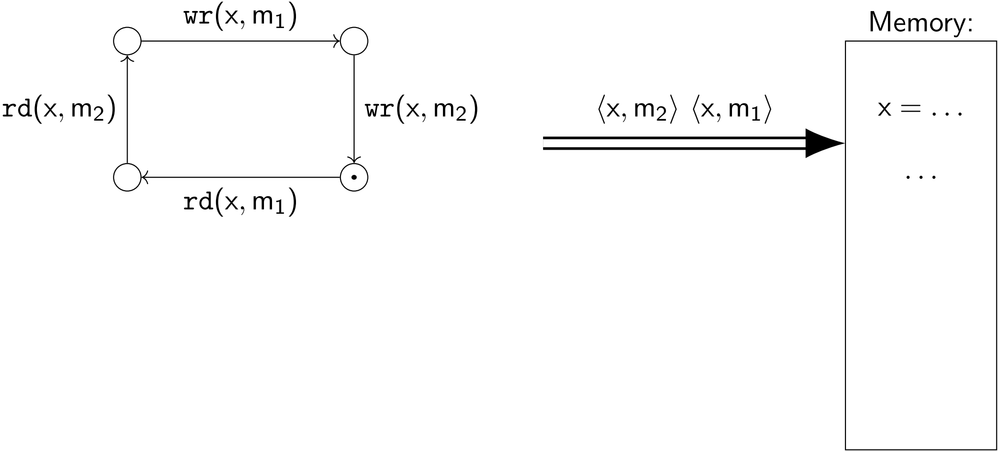
$x = \ldots$

$\ldots$

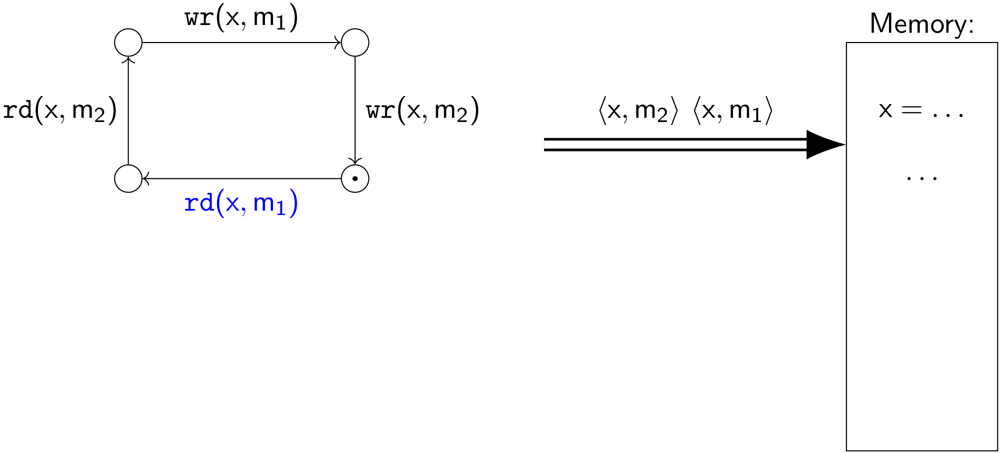
Update Fairness - PCS Reduction



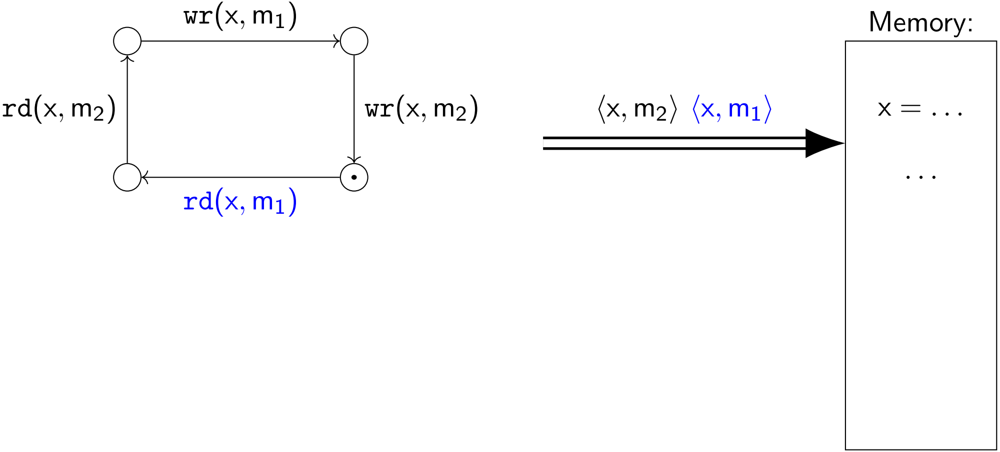
Update Fairness - PCS Reduction



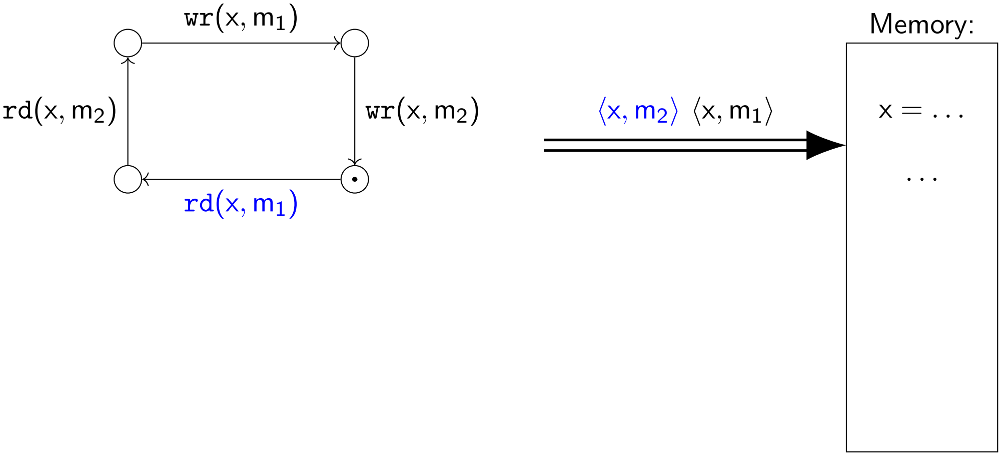
Update Fairness - PCS Reduction



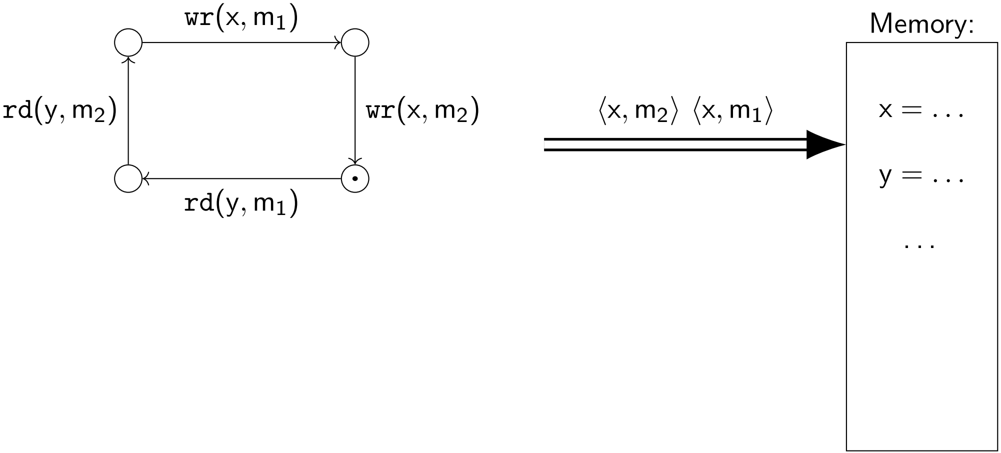
Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



$\mathtt{wr}(x, m_1)$

$\mathtt{rd}(y, m_2)$

$\mathtt{wr}(x, m_2)$

$\mathtt{rd}(y, m_1)$

$\langle x, m_2 \rangle \ \langle x, m_1 \rangle$

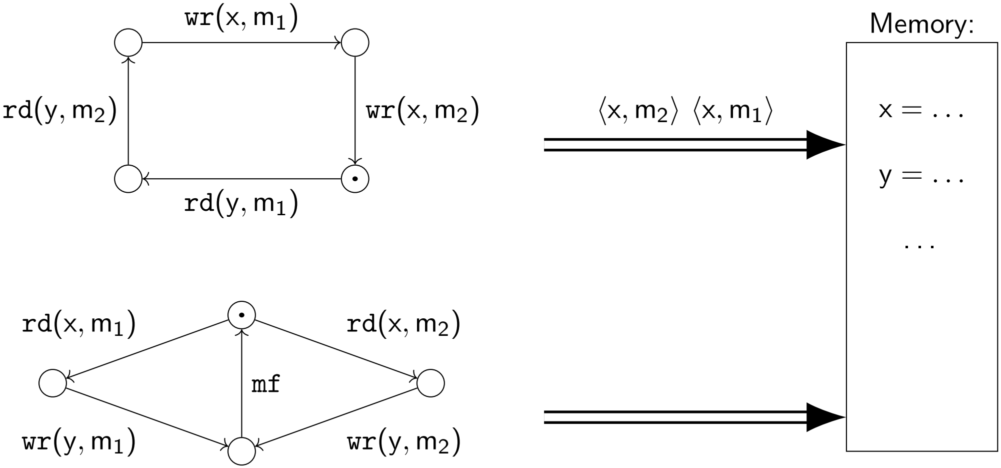
Memory:

$x = \ldots$

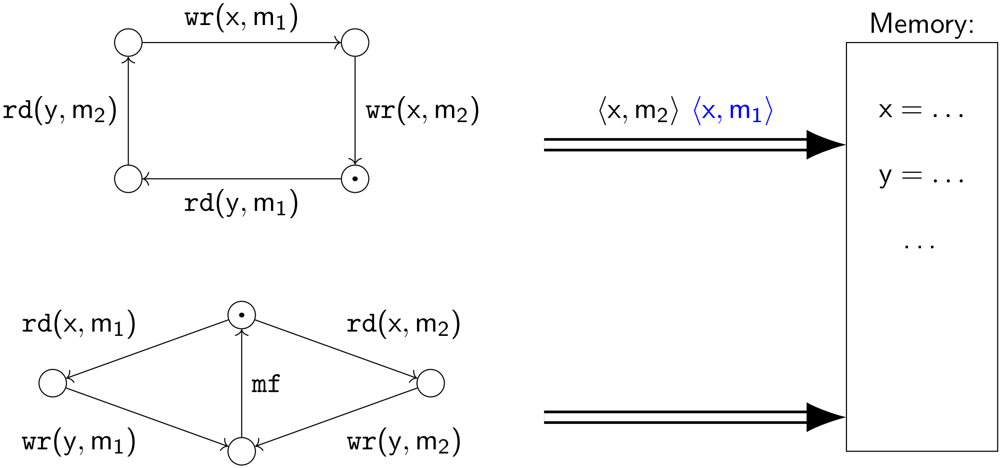
$y = \ldots$

$\ldots$

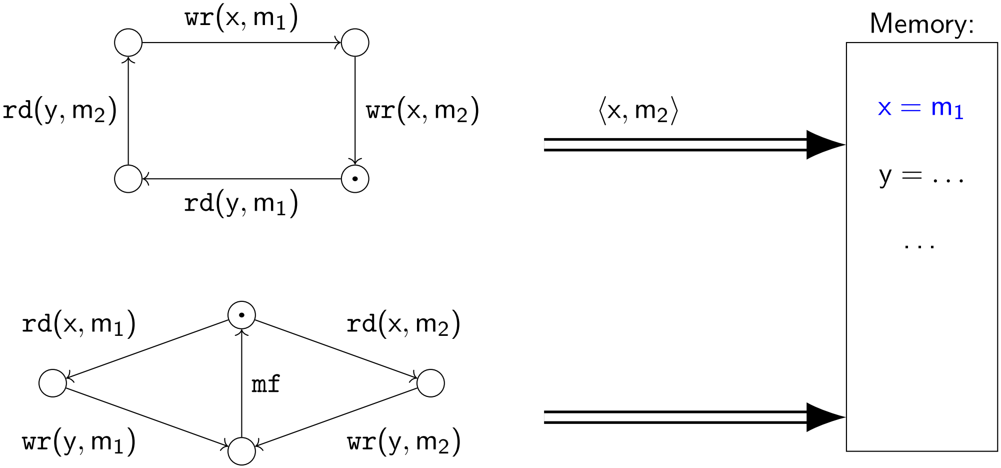
Update Fairness - PCS Reduction



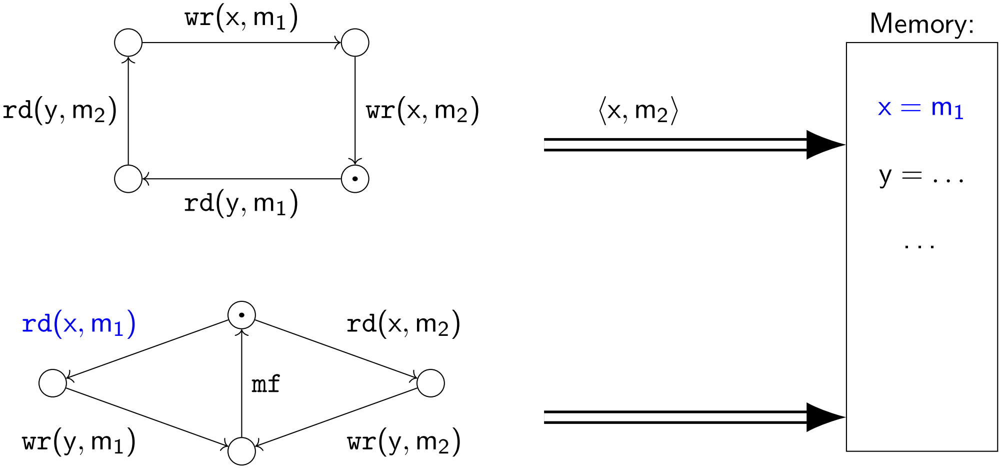
Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



$\mathtt{wr}(x, m_1)$

$\mathtt{rd}(y, m_2)$

$\mathtt{wr}(x, m_2)$

$\mathtt{rd}(y, m_1)$

$\langle x, m_2 \rangle$

Memory:

$x = m_1$

$y = \ldots$

$\ldots$

$\mathtt{rd}(x, m_1)$

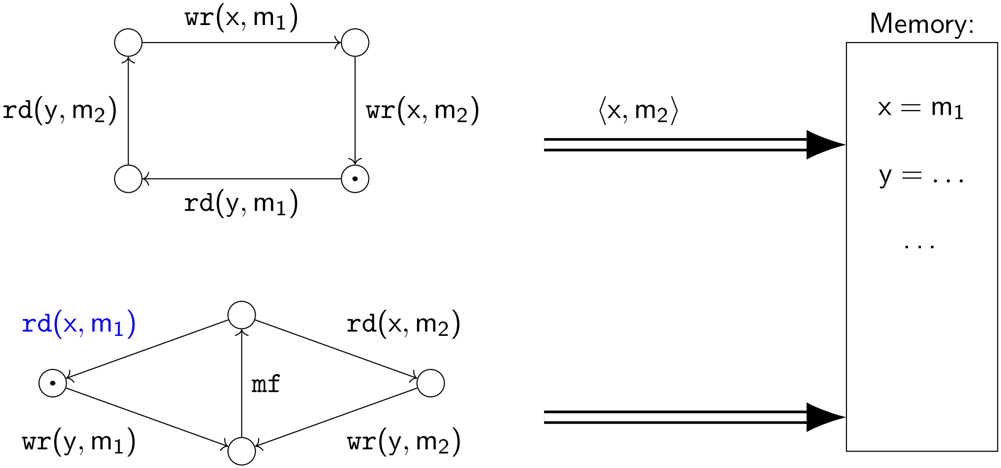
$\mathtt{rd}(x, m_2)$

$\mathtt{mf}$

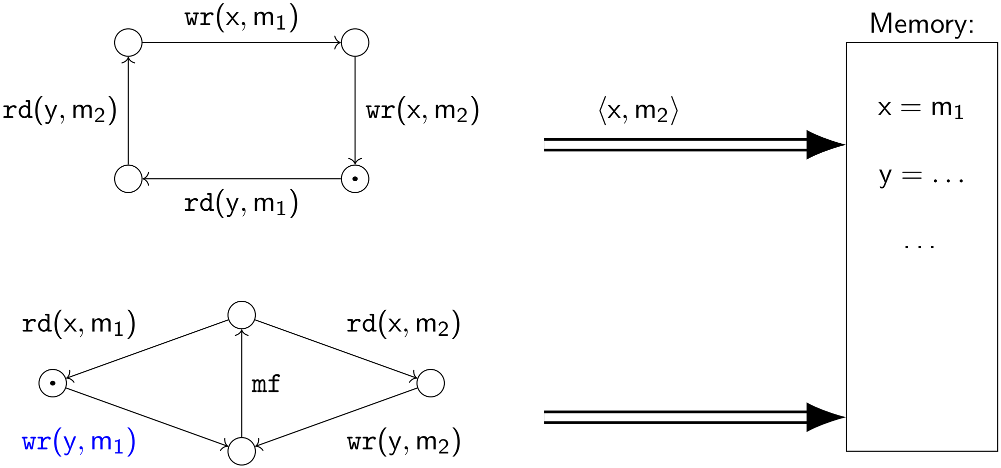
$\mathtt{wr}(y, m_1)$

$\mathtt{wr}(y, m_2)$

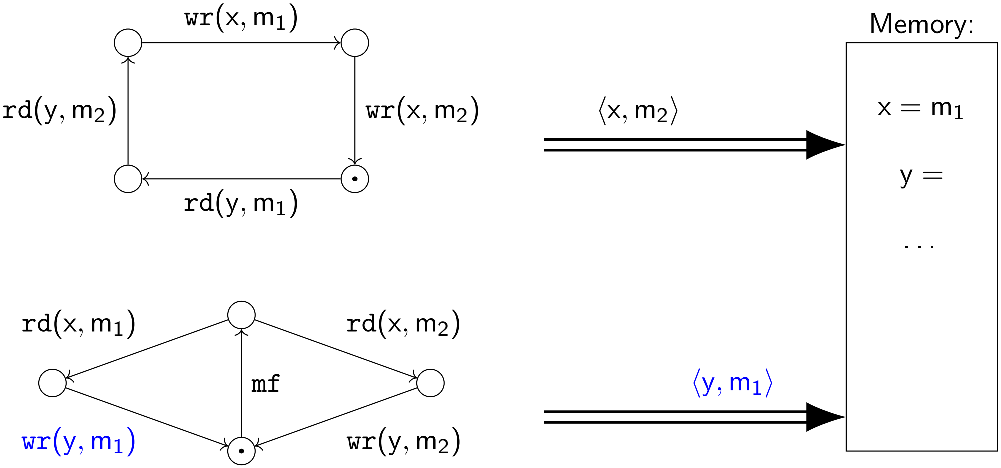
Update Fairness - PCS Reduction



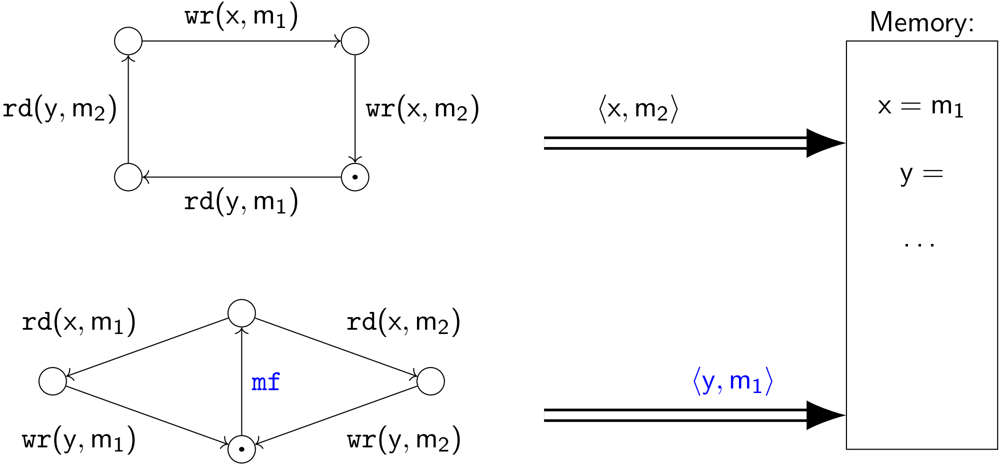
Update Fairness - PCS Reduction



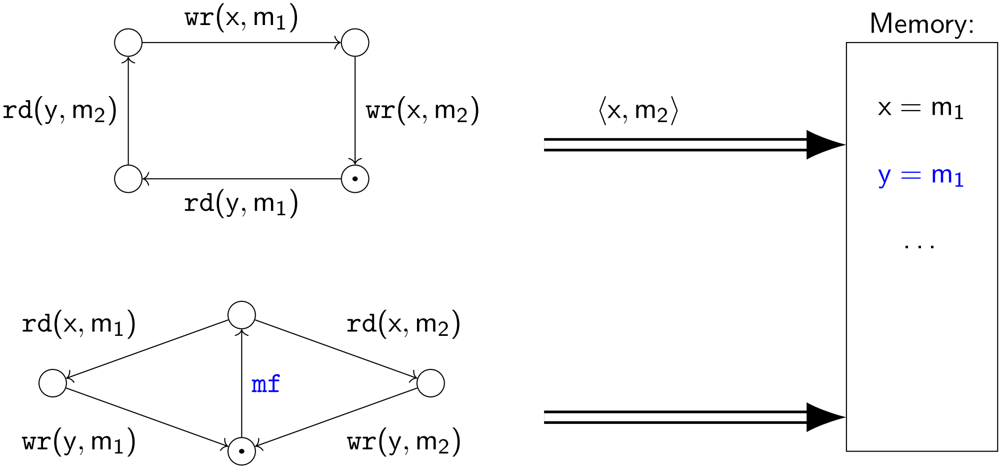
Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



$\mathtt{wr}(x, m_1)$

$\mathtt{rd}(y, m_2)$

$\mathtt{wr}(x, m_2)$

$\mathtt{rd}(y, m_1)$

$\langle x, m_2 \rangle$

Memory:

$x = m_1$

$y = m_1$

...

$\mathtt{rd}(x, m_1)$

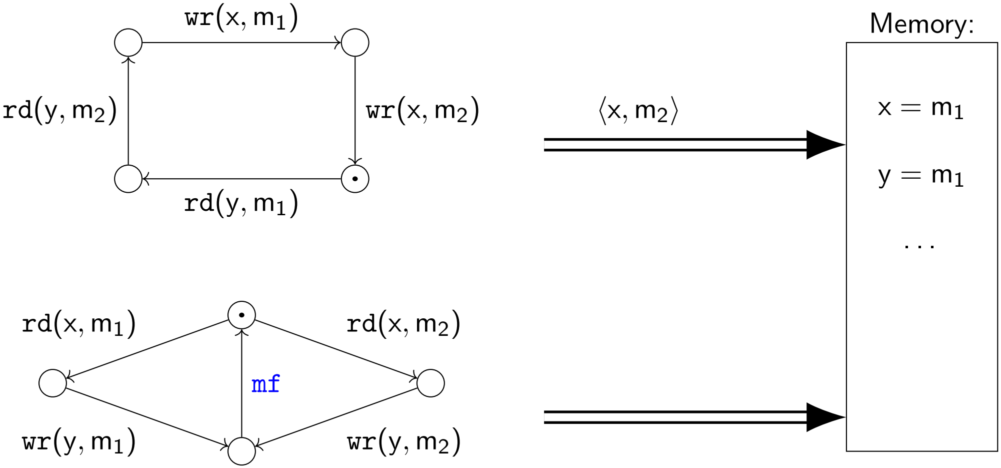
$\mathtt{rd}(x, m_2)$

$\mathtt{mf}$

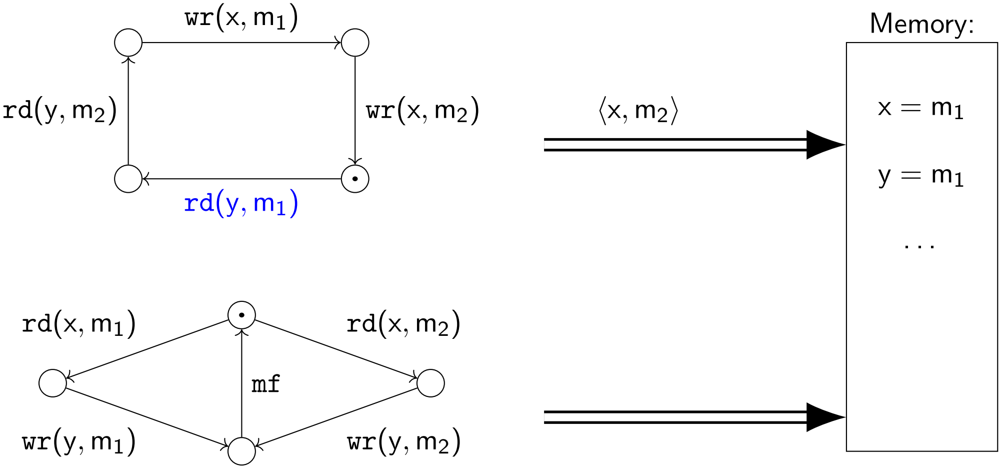
$\mathtt{wr}(y, m_1)$

$\mathtt{wr}(y, m_2)$

Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



$\mathtt{wr}(x, m_1)$

$\mathtt{rd}(y, m_2)$

$\mathtt{wr}(x, m_2)$

$\mathtt{rd}(y, m_1)$

$\langle x, m_2 \rangle$

Memory:

$x = m_1$

$y = m_1$

$\dots$

$\mathtt{rd}(x, m_1)$

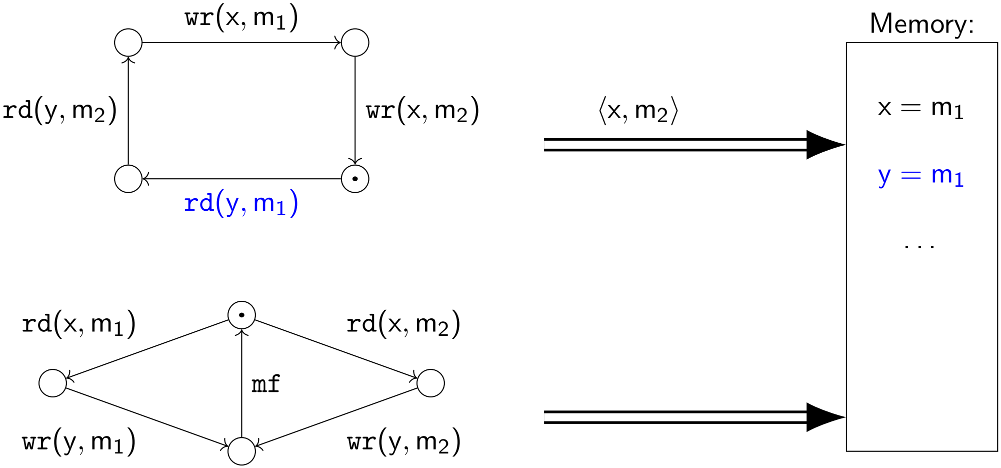
$\mathtt{rd}(x, m_2)$

$\mathtt{mf}$

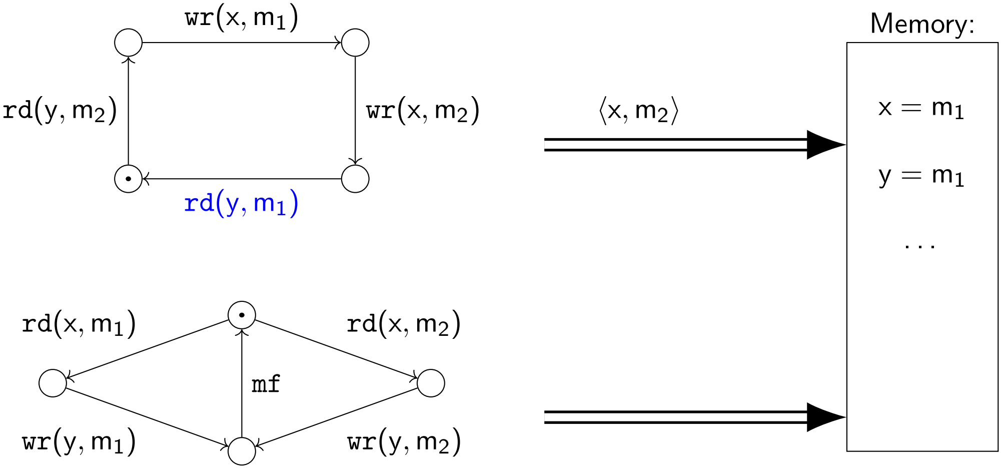
$\mathtt{wr}(y, m_1)$

$\mathtt{wr}(y, m_2)$

Update Fairness - PCS Reduction



Update Fairness - PCS Reduction



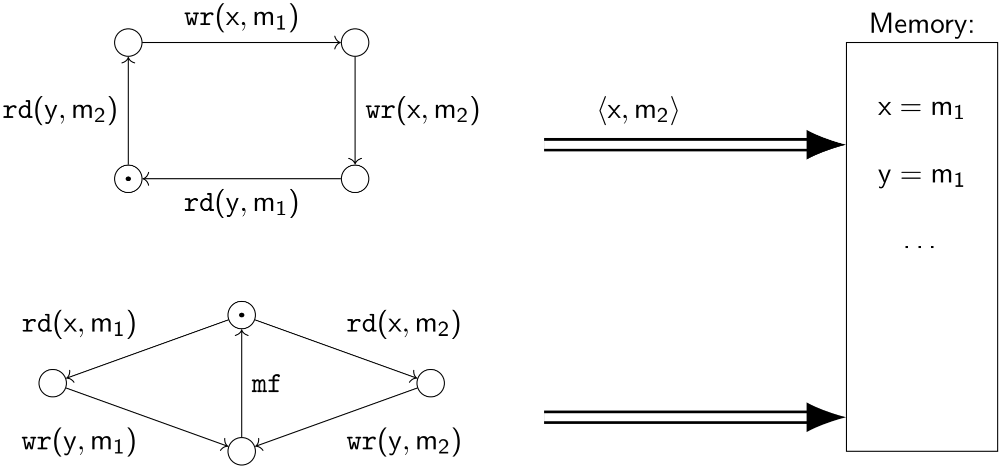
Memory:

$x = m_1$

$y = m_1$

$\dots$

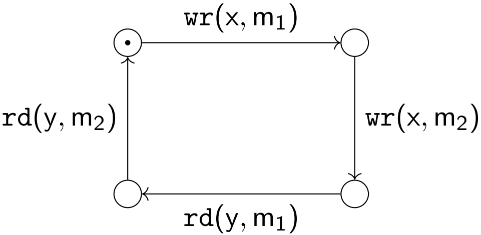
Update Fairness - PCS Reduction



Update Fairness

Eventually, every buffer message must be updated to the memory.

- use TSO buffer to simulate PCS channel
- reduce PCS reachability (undecidable) to TSO reachability game



Theorem

The reachability problem under TSO semantics with update fairness is undecidable.

Process Fairness

Every enabled process must be executed infinitely often.

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Every enabled process must be executed infinitely often.

reachability games?

Process Fairness

Every enabled process must be executed infinitely often.

~~reachability games?~~ $\longrightarrow$ safety games!

Process Fairness

Every enabled process must be executed infinitely often.

~~reachability games?~~ ⟶ safety games!

Idea: update player simulates PCS run, process player is passive

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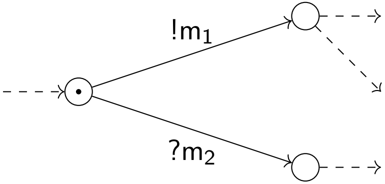
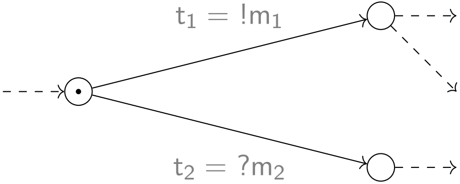
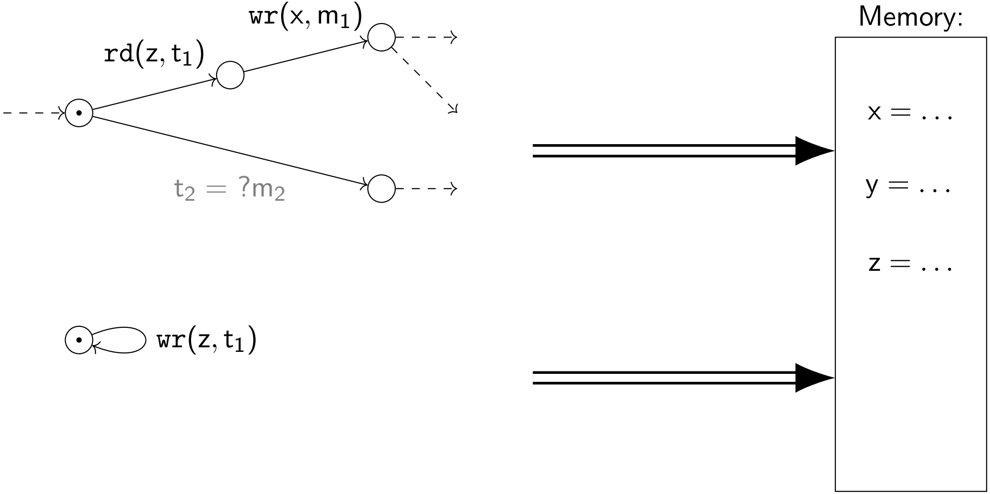


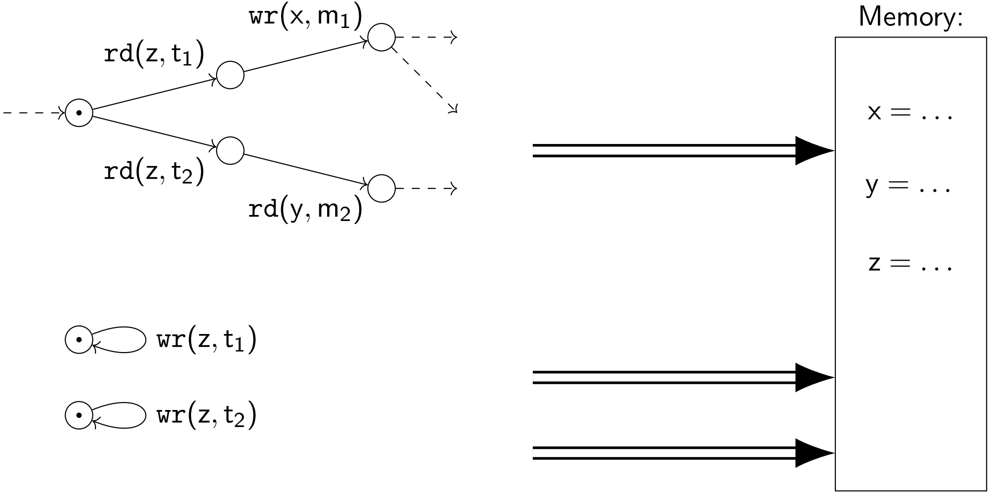
Process Fairness



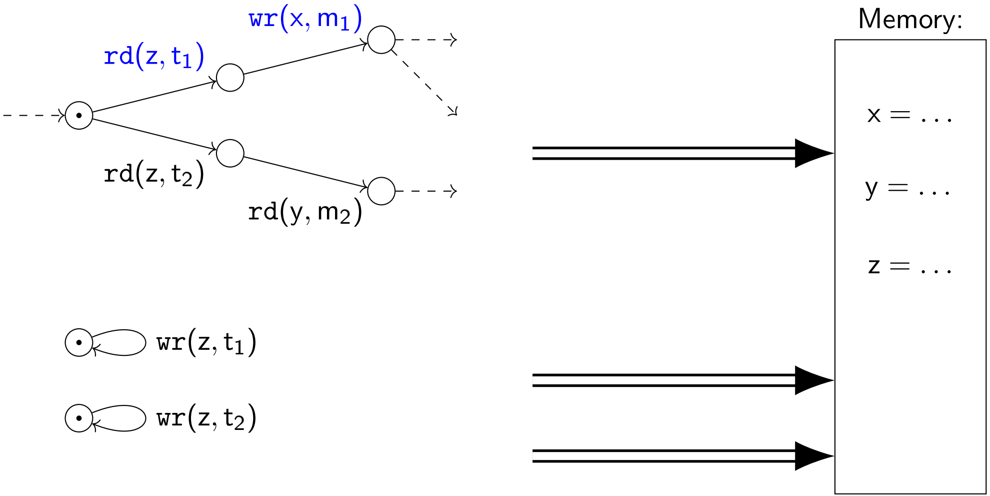
Process Fairness



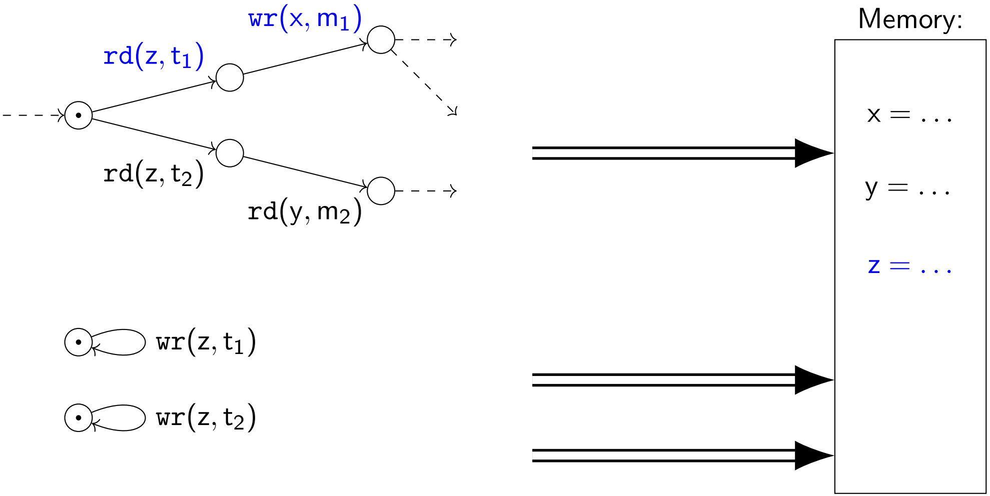
Process Fairness



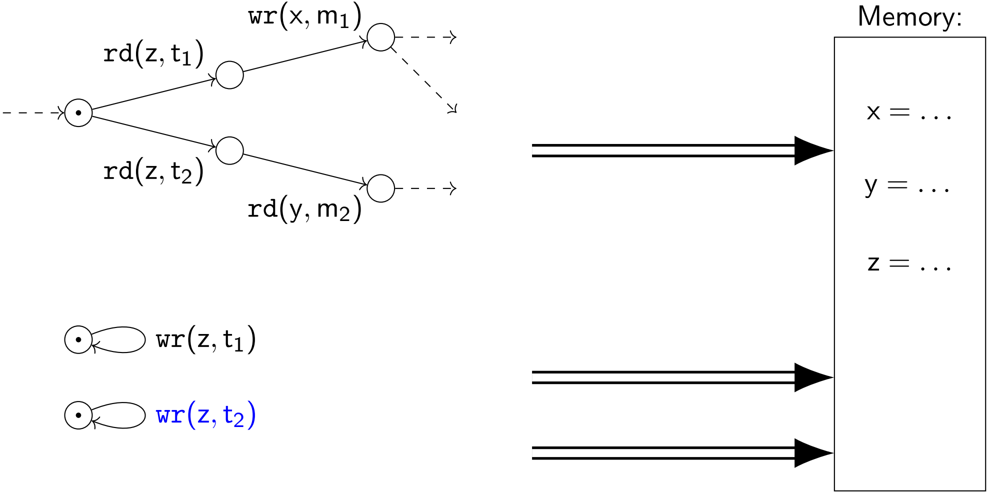
Process Fairness



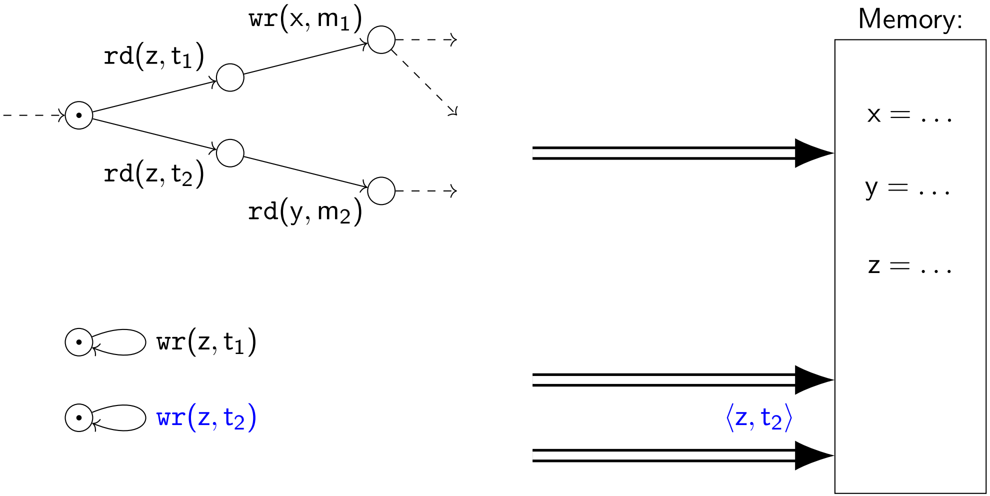
Process Fairness



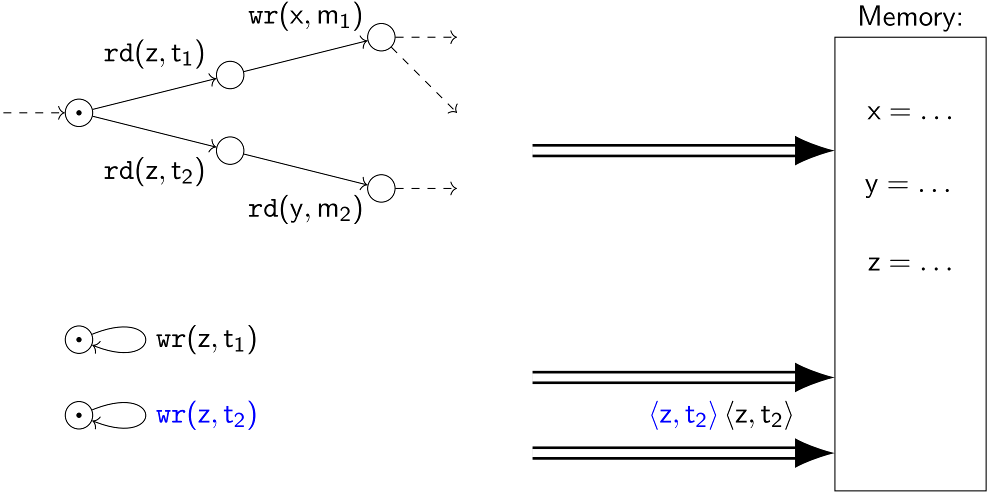
Process Fairness



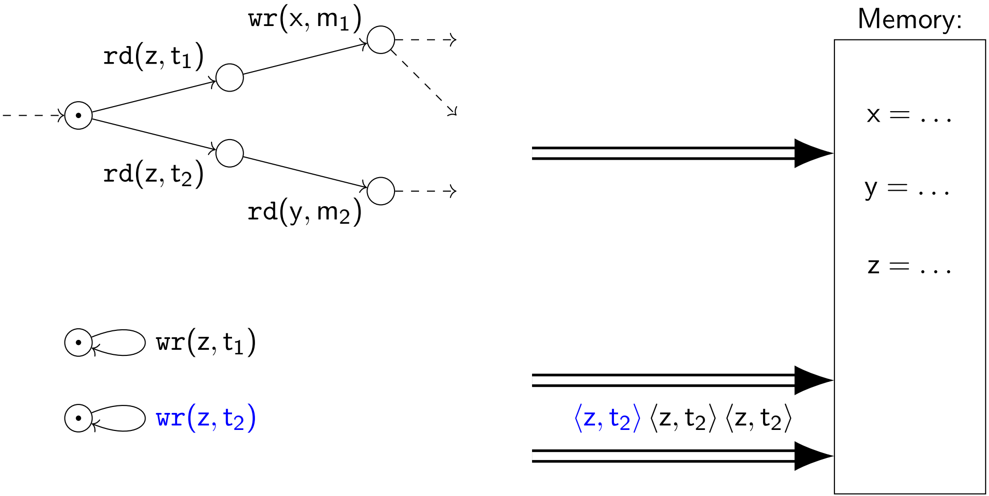
Process Fairness



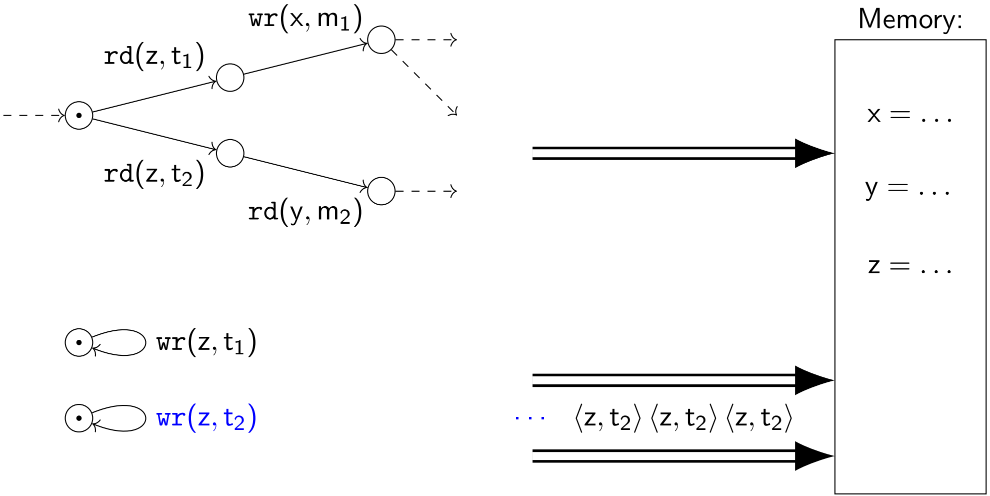
Process Fairness



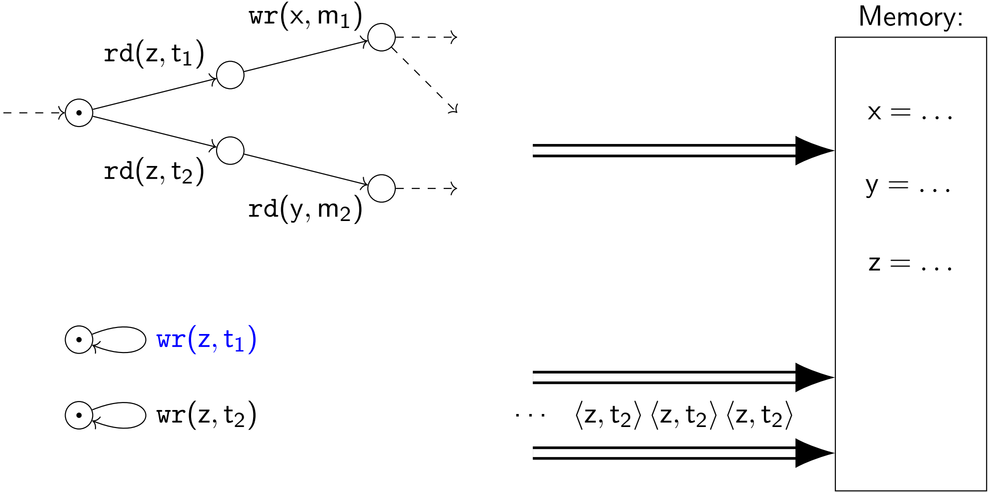
Process Fairness



Process Fairness



Process Fairness



$\mathtt{wr}(x, m_1)$

$\mathtt{rd}(z, t_1)$

$\mathtt{rd}(z, t_2)$

$\mathtt{rd}(y, m_2)$

Memory:

$x = \ldots$

$y = \ldots$

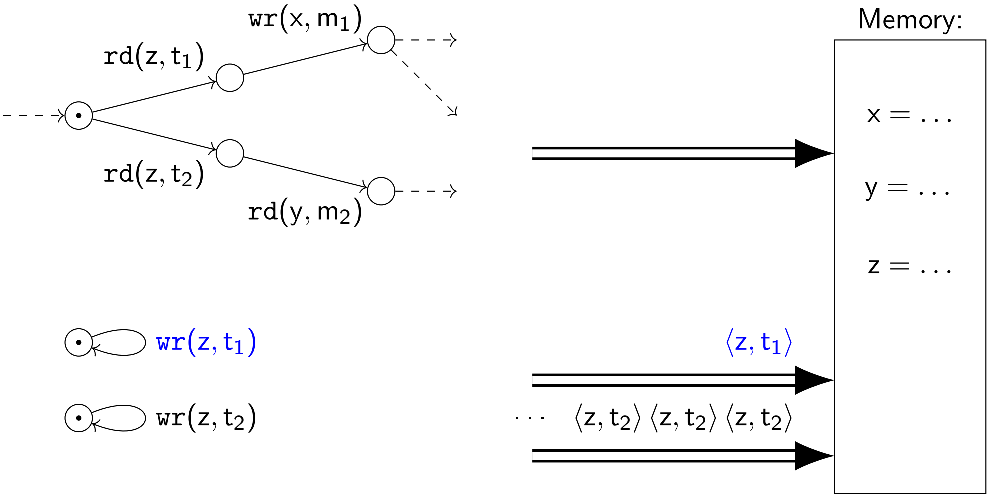
$z = \ldots$

$\mathtt{wr}(z, t_1)$

$\mathtt{wr}(z, t_2)$

$\cdots \quad \langle z, t_2 \rangle \, \langle z, t_2 \rangle \, \langle z, t_2 \rangle$

Process Fairness



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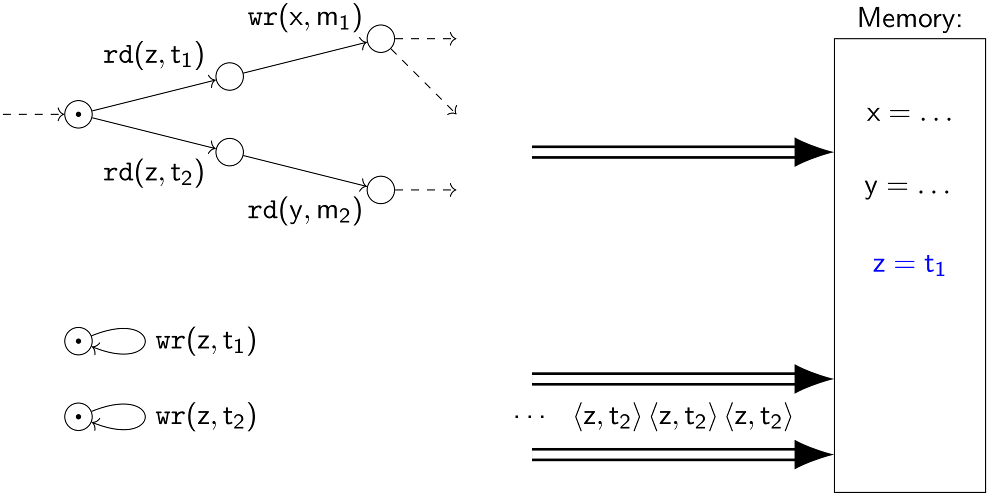
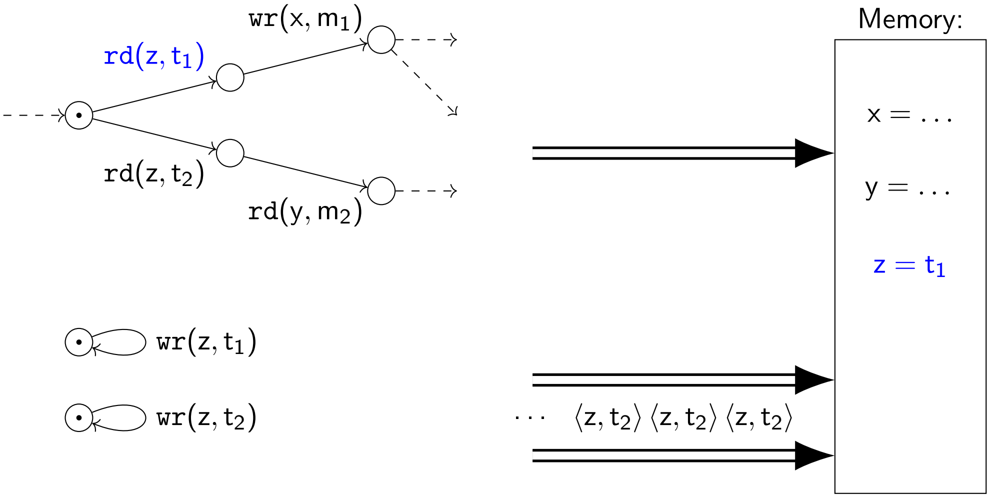
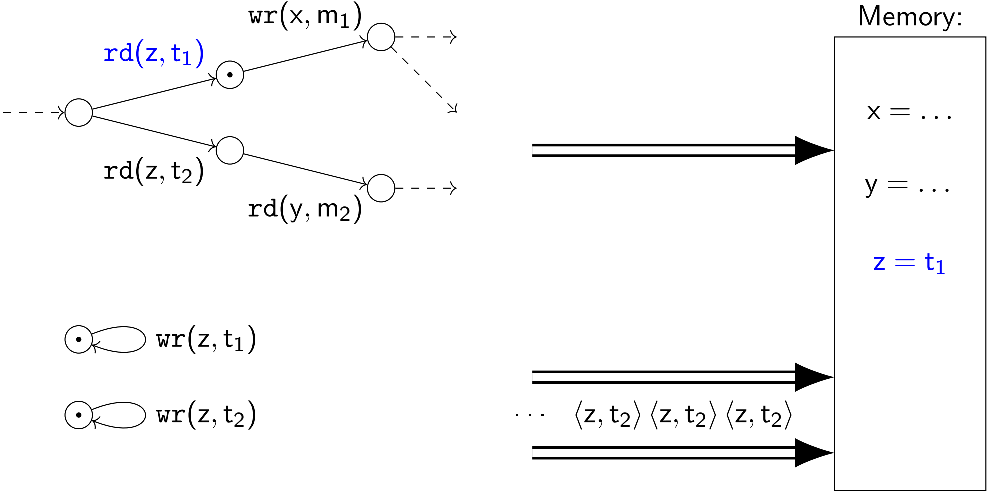


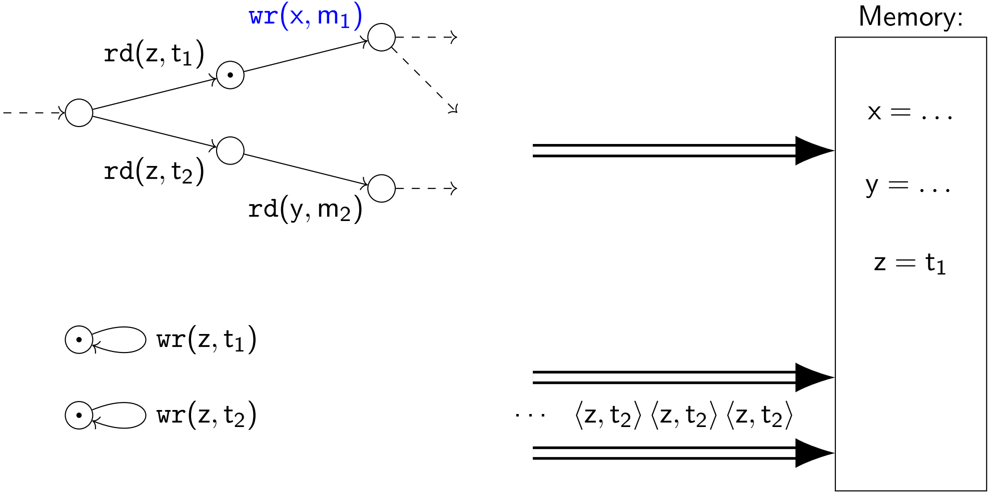
Process Fairness



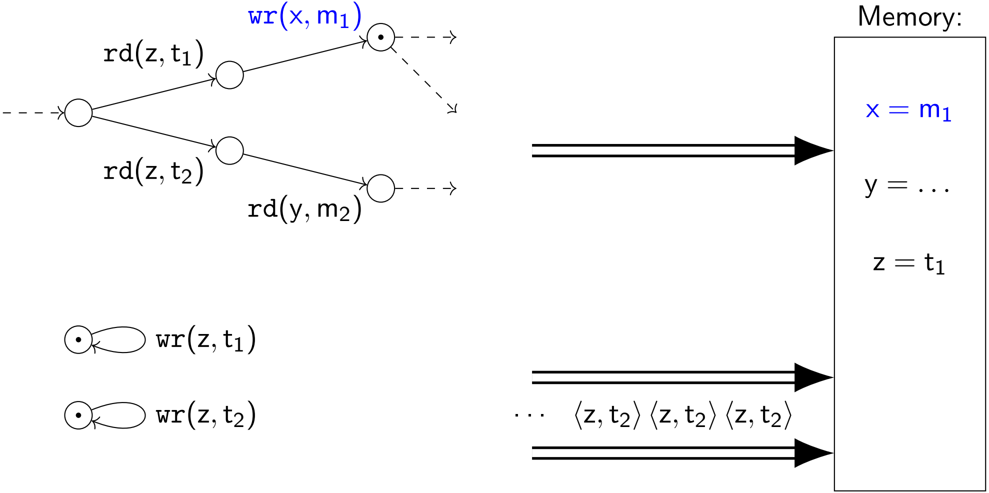
Process Fairness



Process Fairness



Process Fairness



$\mathtt{wr}(x, m_1)$

$\mathtt{rd}(z, t_1)$

$\mathtt{rd}(z, t_2)$

$\mathtt{rd}(y, m_2)$

$\mathtt{wr}(z, t_1)$

$\mathtt{wr}(z, t_2)$

$\cdots \quad \langle z, t_2 \rangle \, \langle z, t_2 \rangle \, \langle z, t_2 \rangle$

Memory:

$x = m_1$

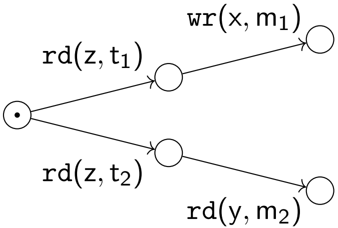
$y = \ldots$

$z = t_1$

Process Fairness

Every enabled process must be executed infinitely often.

- similar to reachability games
- update player simulates PCS run, process player is passive
- reduce PCS reachability (undecidable) to TSO safety game



Theorem

The safety problem under TSO semantics with process fairness is undecidable.

Conclusion

- reachability and safety *without* fairness
  - reduce to single-process programs
  - finite behaviour / PSPACE-complete

Conclusion

- reachability and safety *without* fairness
  - reduce to single-process programs
  - finite behaviour / PSPACE-complete

- reachability with update fairness and safety with process fairness
  - reduction from PCS reachability
  - undecidable

Conclusion

- reachability and safety *without* fairness
  - reduce to single-process programs
  - finite behaviour / PSPACE-complete
- reachability with update fairness and safety with process fairness
  - reduction from PCS reachability
  - undecidable
- further work could consider other
  - winning conditions
  - fairness conditions
  - weak memory models

Reachability and Safety Games under TSO Semantics

SCooL / GandALF 2024 in Reykjavik

Stephan Spengler

Uppsala University, Sweden

20 June 2024