▸ TIM BUTTON, *MOON theory: Mathematical Objects with Ontological Neutrality.*
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The iterative notion of set starts with a simple, coherent story, and yields a paradise of mathematical objects, which "provides a court of final appeal for questions of mathematical existence and proof" ([5, p.26]). But it does not present an attractive mathematical ontology: it seems daft to say that every mathematical object is "really" some (pure) set. My goal, in this paper, is to explain how we can inhabit the set-theorist's paradise of *mathematical objects* whilst remaining *ontologically neutral*.

I start by considering stories with this shape: (1) Gizmos are found in stages; every gizmo is found at some stage. (2) Each gizmo reifies (some fixed number of) relations (or functions) which are defined only over earlier-found gizmos. (3) Every gizmo has (exactly one) colour; same-coloured gizmos reify relations in the same way; same-coloured gizmos are identical iff they reify the same relations.

Such a story can be told about (iterative) sets: they are monochromatic gizmos which reify one-place properties. But we can also tell such stories about gizmos other than sets. By tidying up the general idea of such stories, I arrive at the notion of a MOON theory (for Mathematical Objects with Ontological Neutrality).

With weak assumptions, I obtain a metatheorem: *all MOON theories are synonymous*. Consequently, they are (all) synonymous with a theory which articulates the iterative notion of set (LT_+; see [1]). So: all MOON theories (can) deliver the set-theorist's paradise of mathematical objects. But, since different MOON theories have different (apparent) ontologies, we attain ontological neutrality.

My metatheorem generalizes some of my work on Level Theory ([1], [2], [3]). It also delivers a partial realization of Conway's "Mathematician's Liberation Movement" [4, p.66].

[1] Button, T. Level Theory, Part 1: Axiomatizing the bare idea of a cumulative hierarchy of sets. *Bulletin Of Symbolic Logic*. **27**, 436-60 (2021)

[2] Button, T. Level Theory, Part 2: Axiomatizing the bare idea of a potential hierarchy. *Bulletin Of Symbolic Logic*. **27**, 461-84 (2021)

[3] Button, T. Level Theory, Part 3: A boolean algebra of sets arranged in well-ordered levels. *Bulletin Of Symbolic Logic*. **28**, 1-26 (2022)

[4] Conway, J. On Numbers and Games. (Academic Press, Inc,1976)

[5] Maddy, P. Naturalism in Mathematics. (Oxford University Press,1997)