► KATARZYNA W. KOWALIK, *A non speed-up result for the chain-antichain principle over a weak base theory.*
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The chain-antichain principle (CAC), a well-known consequence of Ramsey's Theorem for pairs and two colours, says that for every partial order on $\mathbb{N}$ there exists an infinite chain or antichain with respect to this order. We study the strength of this principle over the weak base theory RCA_0^*, which is obtained from RCA_0 by replacing the Σ_1^0-induction scheme with Δ_1^0-induction.

It was shown by Patey and Yokoyama in [3] that RT_2^2 is Π_3^0-conservative over RCA_0 and from [4] it follows that RT_2^2 is also Π_3^0-conservative over RCA_0^* (cf. [1]). The conservativity results lead to the question whether RT_2^2 has significantly shorter proofs for Π_3^0-sentences. The answer depends on the choice of the base theory: it was proved in [2] that RT_2^2 can be polynomially simulated by RCA_0 for Π_3^0-sentences but it has non-elementary speed-up over RCA_0^* for Σ_1^0-sentences.

The speed-up result was obtained by the use of the exponential lower bound for the finite version of RT_2^2. However, it follows from Dilworth's theorem that the upper bound for the finite version of CAC is polynomial. This suggests that CAC, despite being a relatively strong consequence of RT_2^2, might not have an analogous speed-up over RCA_0^*. We confirm this conjecture by constructing a two-step forcing interpretation of RCA_0^*+CAC in RCA_0^*.

[1] Leszek A. Kołodziejczyk, Katarzyna W. Kowalik, Keita Yokoyama, *How strong is Ramsey's theorem if infinity can be weak?* Submitted. Available at arXiv:2011.02550.

[2] Leszek A. Kołodziejczyk, Tin Lok Wong, Keita Yokoyama, *Ramsey's theorem for pairs, collection, and proof size.* Submitted. Available at arXiv:2005.06854.

[3] Ludovic Patey, Keita Yokoyama, *The proof-theoretic strength of Ramsey's theorem for pairs and two colors*, ***Advances in Mathematics***, vol. 330 (2018), pp. 1034–1070.

[4] Keita Yokoyama, *On the strength of Ramsey's theorem without Σ_1-induction*, ***Mathematical Logic Quarterly***, vol. 59 (2013), no. 1-2, pp. 108–111.