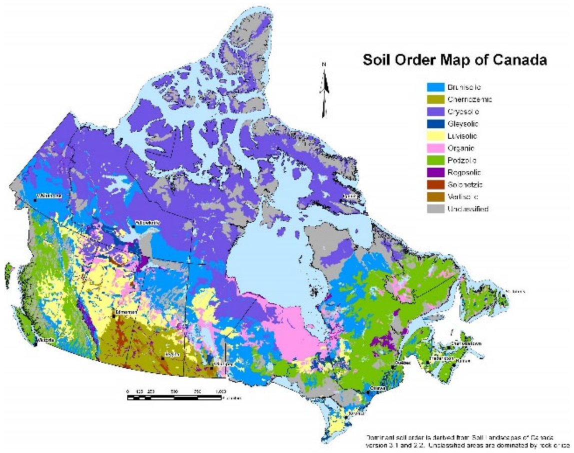
Infiltration pond http://www.estesdesign.com/EDI_awards.html

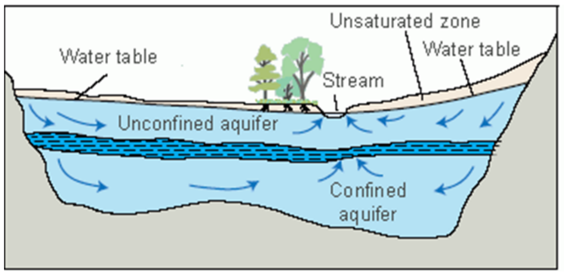
Civ E 321: Principles of Environmental Modelling and Risk

WEEK 4: INFILTRATION

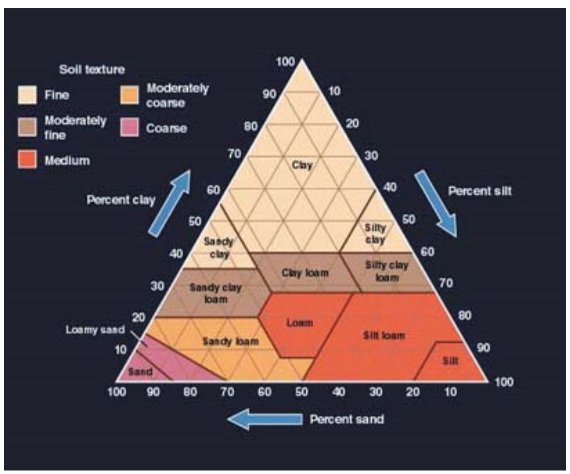
Most information in Week 4 is from pages 122-137 *of* ***Bedient et al. (2019)***



http://www.soilsofcanada.ca/



http://ga.water.usgs.gov/edu/watercycleinfiltration.html



http://www.thisland.illinois.edu/50ways/50ways_21.html

Section overview

- Introduction
- Soil properties and water flow
- Green-Ampt model
- Horton and Φ-index models

1. INFILTRATION

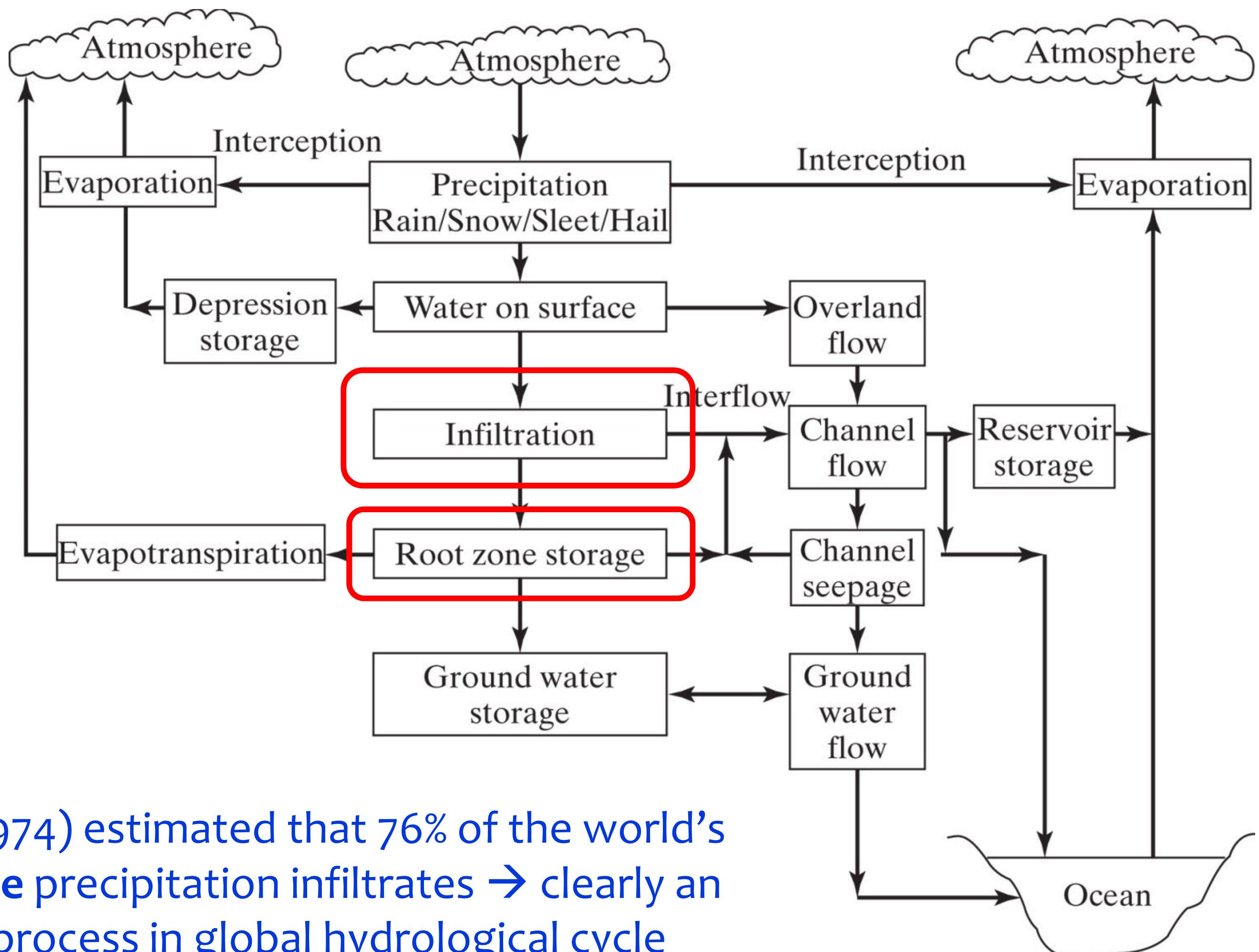
"Infiltration is the process by which **precipitation** is abstracted by *seeping into the soil* below the land surface.

"Once below the surface, the abstracted water moves either **laterally**, as *interflow*, into streams, lakes and rivers, or **vertically**, by *percolation*, into aquifers.

"*Infiltration is a complex process*. It is described by either an instantaneous infiltration or an average infiltration rate, both measured in millimeters per hour" Ponce (1989: 34).

Pervious pavement:
http://www.estesdesign.com/EDI_awards.html, and see https://youtu.be/LWiqoNbJmaw

Hydrological cycle



L'vovich (1974) estimated that 76% of the world's **land surface** precipitation infiltrates → clearly an important process in global hydrological cycle (Dingman 2002)

Bedient et al. (2019)

Importance of infiltration

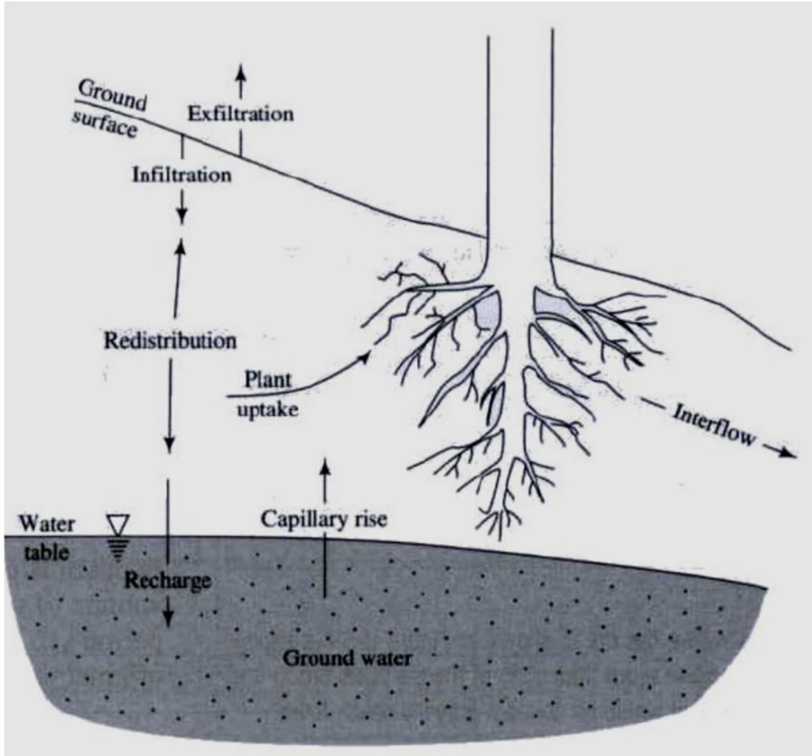
"Infiltration provides all the water used by natural and cultivated plants, and almost all the water that enters groundwater reservoirs"

- Understanding infiltration and redistribution is essential for
  1. Predicting hydrological response to rain or snowmelt events
  2. Estimating the timing and amounts of groundwater recharge
  3. Developing plans for crop irrigation
  4. Understanding chemical processes in soils, including natural weathering and movement of natural nutrients, fertilizers, and contaminants to ground and surface waters

Dingman (2002: 220)

Infiltration processes

- **Infiltration** is movement of water from soil surface into the soil
- **Redistribution** is subsequent movement of infiltrated water in unsaturated zone of soil. Redistribution can involve,
  - **Exfiltration:** evaporation from upper soil layers
  - **Capillary rise:** movement from saturated zone upward to unsaturated zone through surface tension
  - **Recharge:** movement of percolating water from unsaturated to saturated zone
  - **Interflow:** flow in soil that moves downslope
  - **Percolation:** general term for downward flow in unsaturated zone

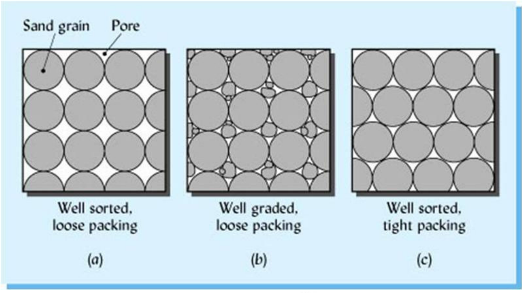


Dingman (2002), Chapter 6

Infiltration rates

- Infiltration rates vary widely, based on,
  1. Condition of land surface (**crust**)
  2. Type, extent and density of **vegetative cover**
  3. Physical properties of the **soil**, including grain size and gradation
  4. **Storm character** (intensity, depth, duration)
  5. Water temperature
  6. Water quality including chemical constituents

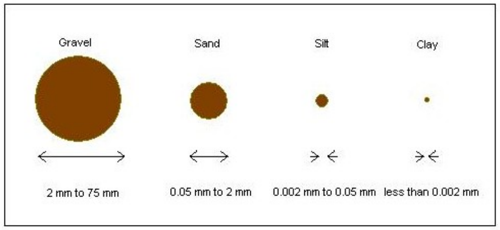
http://faculty.yc.edu/ycfaculty/ags105/week05/soil_physical_properties/soil_physical_properties_print.html



http://www.extension.iastate.edu/CropNews/2009/0430alkaisihanna.htm

http://salixrw.com/project/calverton-tip-drainage-channels

http://www.unibas.it/desertnet/dis4me/indicator_descriptions/vegetation_cover.htm



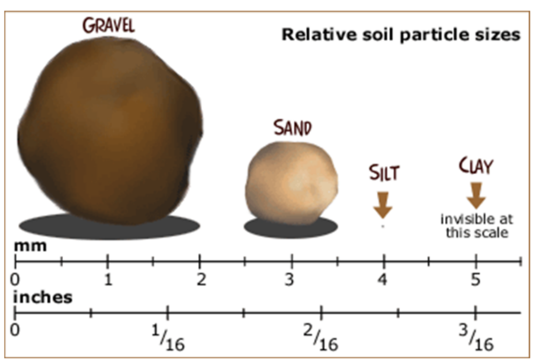
http://techalive.mtu.edu/meec/module06/SoilClassification.htm

Ponce (1989)

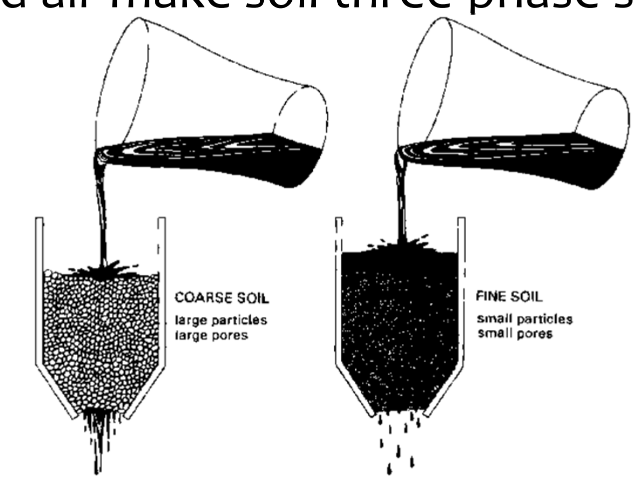
2.1 SOIL PROPERTIES

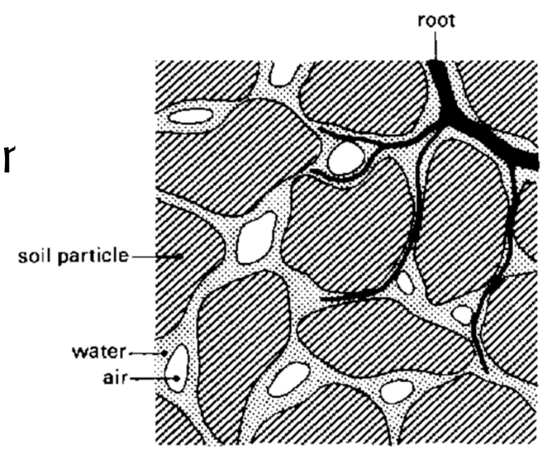
- To understand and apply physical principles to modelling infiltration and redistribution, need to understand
  - Material and hydraulic properties of soils
  - Water conditions of natural soils

- Imagine soil as *matrix of individual solid grains* (mineral or organic; *we will not separately consider the organic component*)
  - Between particles: interconnected **pore spaces**
  - These pore spaces contain **water** and/or **air**
  - Grains, water and air make soil three-phase syster



http://school.discoveryeducation.com/schooladventures/soil/images/particle_sizes1.gif



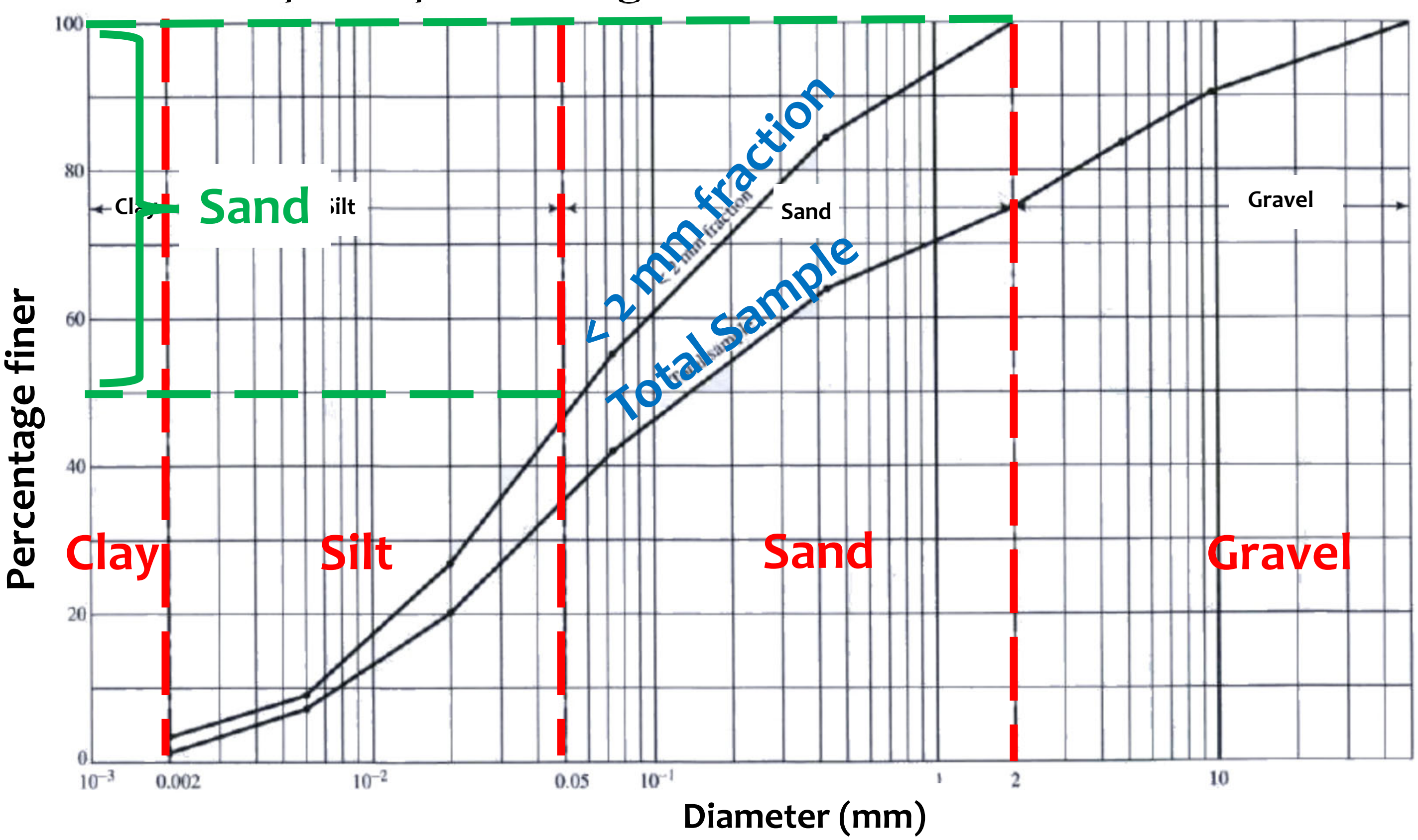


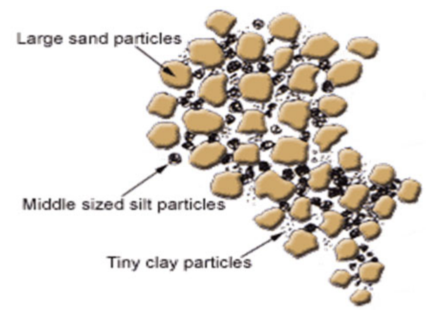
Brouwer et al. (1985), *FAO*:
http://www.fao.org/docrep/r4082e/r4082e03.htm

Dingman (2002)

Soil properties

- Pore sizes (*smaller, but*) approx. equal to grain size, with pore-size distribution determined mainly by grain-size distribution
- Most soils a mix of grain sizes:
  - Distribution can be shown as cumulative frequency plot of *grain diameter vs. weight fraction* of grains with smaller diameter
  - *Steeper slope* means grains-size distribution more uniform





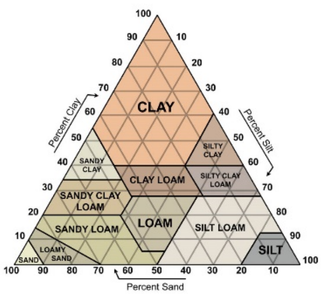
http://toolboxes.flexiblelearning.net.au/demosites/series6/605/html/resources/depot/hort_file/soil_struct/soil_struct.html

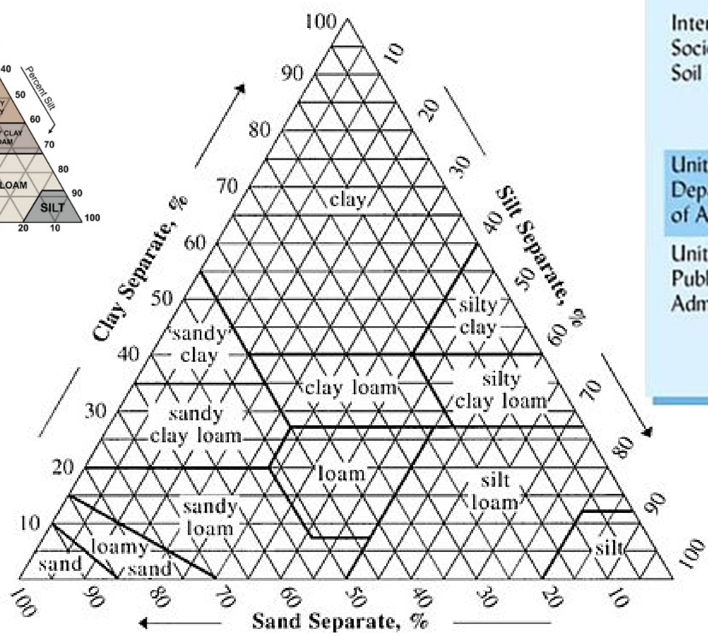
https://twitter.com/EnginePassion/status/1242860978787753990

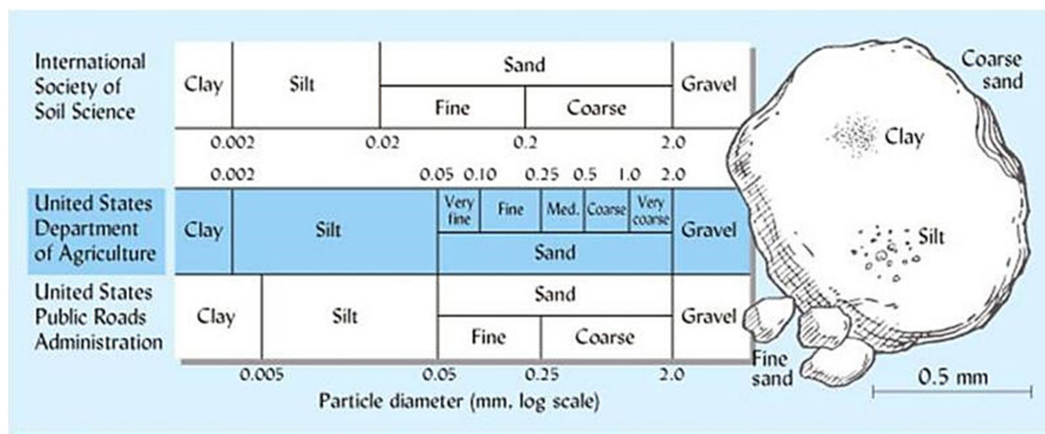
Dingman (2002)

Soil texture

- For many purposes, the term "soil texture" is used to characterize the particle-size distribution
  - Based on proportions by weight of clay (< 0.002 mm), silt (0.002 mm to 0.05 mm) and sand (0.05 mm to 2 mm)
    - *See video on soil types at* https://youtu.be/dsfJRwZXaVk (AgriLife Starr, 2015)
  - Texture defined as in figure, after particles *larger than sand* removed
  - If significant proportion of soil (> 15%) larger than sand, is gravel or larger, soil-texture term includes "*gravelly*" or "*stony*"







http://faculty.yc.edu/ycfaculty/ags105/week05/soil_physical_properties/soil_physical_properties_print.html

Dingman (2002), Chapter 6

Ex 1: Soil analysis

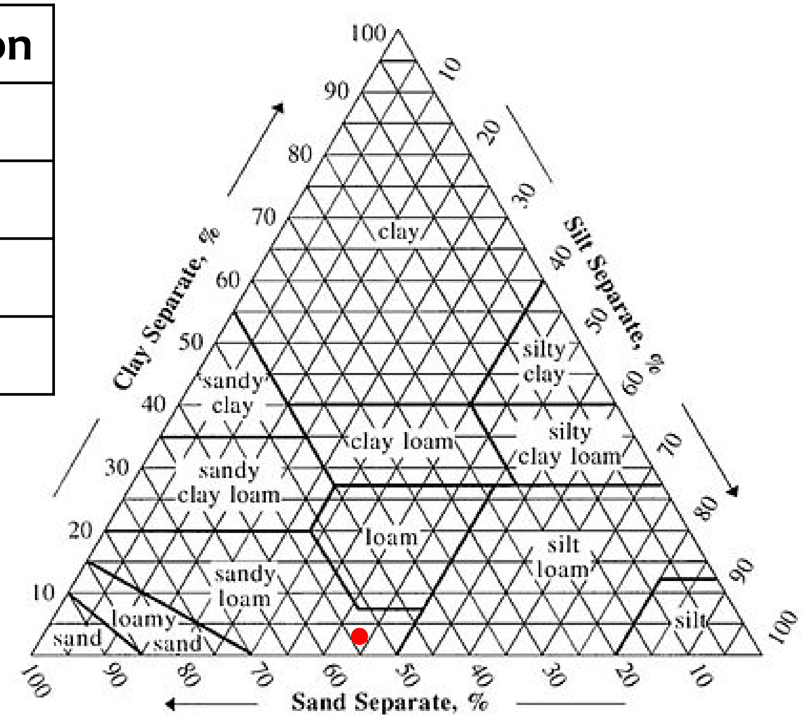
A soil formed on glacial till in SW New Hampshire has the grain size distribution shown in the table below. What is the soil texture?

Diameter (mm)	50	19	9.5	4.76	2.0	0.42	0.074	0.02	0.005	0.002
% finer	100	95	90	84	75	64	42	20	7	2

Solution:

	Total soil	**< 2 mm fraction**
% > 2 mm	25	–
% Sand (0.05-2)	40	53
% Silt (0.002-0.05)	33	44
% Clay (< 0.002)	2	3

The soil is a "sandy loam", and since > 15% is in the gravel range, the soil is a "gravelly sandy loam" (see grain size distribution, slide 9!)



Example 6-1 in Dingman (2002)

Soil property definitions

- **Particle density,** ρ_m, is weighted average density of mineral grains making up the soil:

$$\rho_m = \frac{M_m}{V_m}$$

where M_m is mass, and V_m is volume of the mineral grains

  - Value for a given soil not usually measured; instead, estimated based on mineral composition. Value of 2650 kg m^{-3} usually assumed

- **Bulk density,** ρ_b, is the dry density of the soil

$$\rho_b = \frac{M_m}{V_s} = \frac{M_m}{V_a + V_w + V_m}$$

where M_m is mass, V_s is total volume of the soil sample, made up of the volumes of the air (V_a), water (V_w), and mineral (V_m) components

  - Bulk density commonly increases with depth through compaction

Dingman (2002)

Soil property definitions

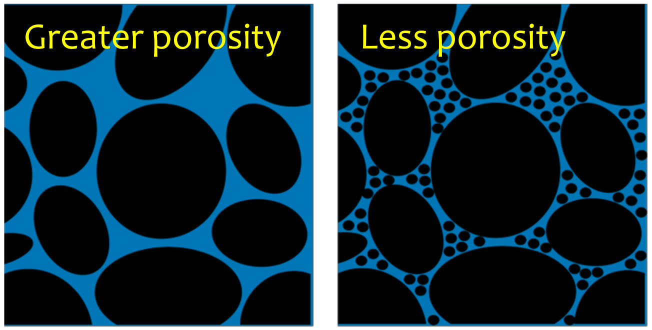
- **Porosity**, ϕ, is the proportion of pore spaces in a volume of soil:

$$\phi = \frac{V_a + V_w}{V_s}$$

  - Like bulk density, porosity is constant over the time periods considered in most hydrological analyses
  - Porosity decreases with depth in most soils because of compaction, and increases near the surface through biological activities near surface that create **macropores**

- Porosity usually determined by measuring ρ_b and assuming value for ρ_m. Then,

$$\phi = 1 - \frac{\rho_b}{\rho_m}$$



Dingman (2002)

Soil-water storage

- *The **water-holding capabilities** of soil are also important*

- **Volumetric water storage**, or **water content**, θ, is ratio of water volume to soil volume,

$$\theta = \frac{V_w}{V_s}$$

  - Water content can vary in both time and space
  - Theoretically, $\mathbf{0 \leq \theta \leq \phi}$ (i.e. from **completely dry** to **saturation**). However, range for natural soils is less than this
  - In the lab, θ is determined by 1) weighing a soil sample of known volume, 2) over-drying it at 105°C, 3) reweighing it and finding,

$$\theta = \frac{M_{s\,wet} - M_{s\,dry}}{\rho_w \cdot V_s}$$

    - Where $M_{s\,wet}$ and $M_{s\,dry}$ are the weights before and after drying

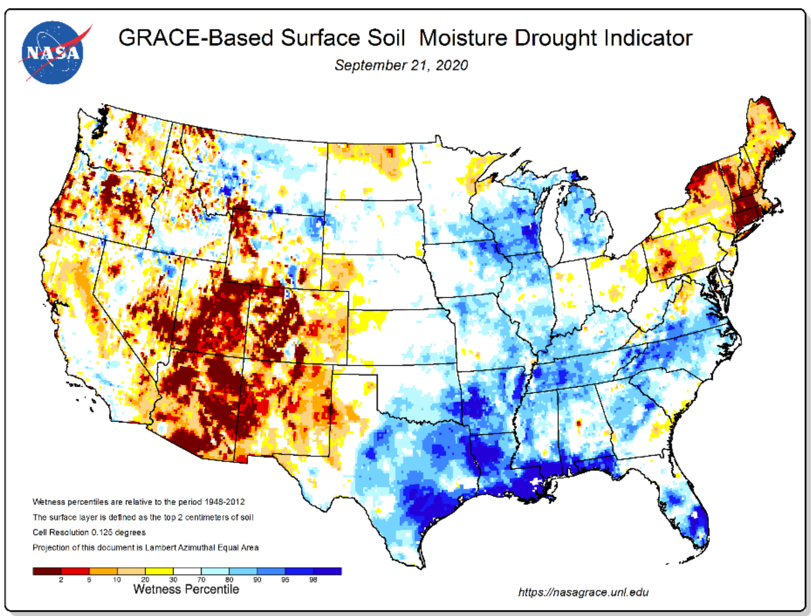
Dingman (2002)

Soil-water storage

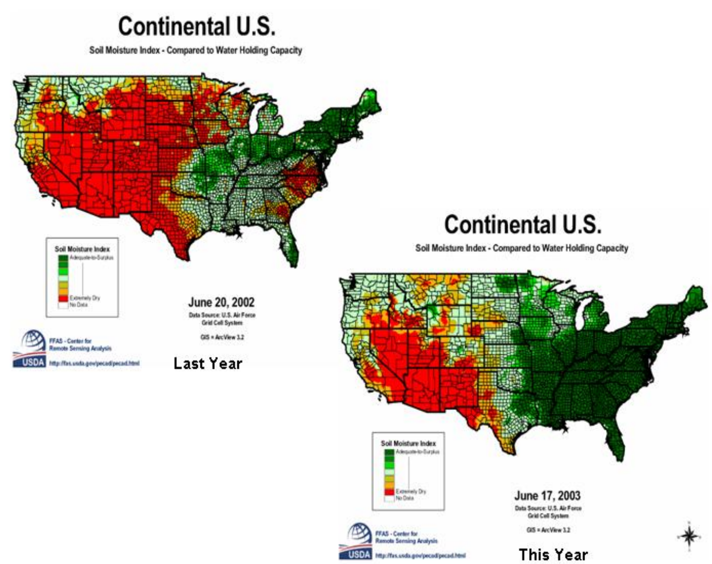
- **Degree of saturation**, or **wetness**, S, is the proportion of pores that contain water,

$$S = \frac{V_w}{V_a + V_w} = \frac{\theta}{\phi}$$

  - Wetness is not directly measured; instead it is calculated using the equation above



https://nasagrace.unl.edu/



http://www.fas.usda.gov/pecad2/highlights/2003/06/Friday%20Briefs/030620/Slide6.JPG

Dingman (2002)

Ex 2: Soil properties

A 10-cm long sample of the soil described in the previous example is taken with a cylindrical sampling tube that has a 5-cm diameter. On removal from the cylinder, the sample weighs 331.8 g. After oven drying at 105°C, the sample weighs 302.4 g.

Compute the bulk density, porosity, water content, and wetness.

Example 6-2 in Dingman (2002)

2.2 SOIL-WATER FLOW

- **Gravity** and **soil-water pressure** gradient drive flow:
  - Conventional to measure pressure relative to atmospheric pressure → *gauge pressure*
  - In **saturated flows,** $p > 0$
  - In **unsaturated flows,** $p < 0$
    - Negative pressure often called **tension** or **suction**

Water table is surface where $p = 0$

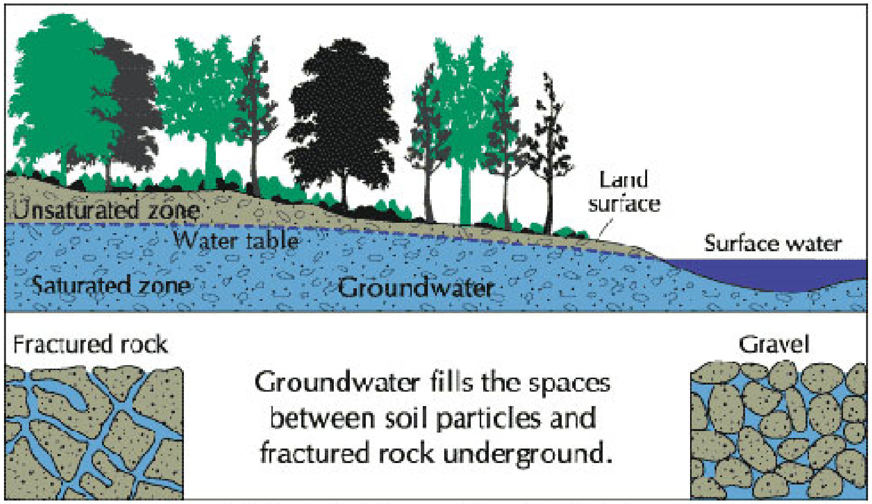


Image compliments of US Geological Survey, adapted by The Groundwater Foundation.

Dingman (2002)

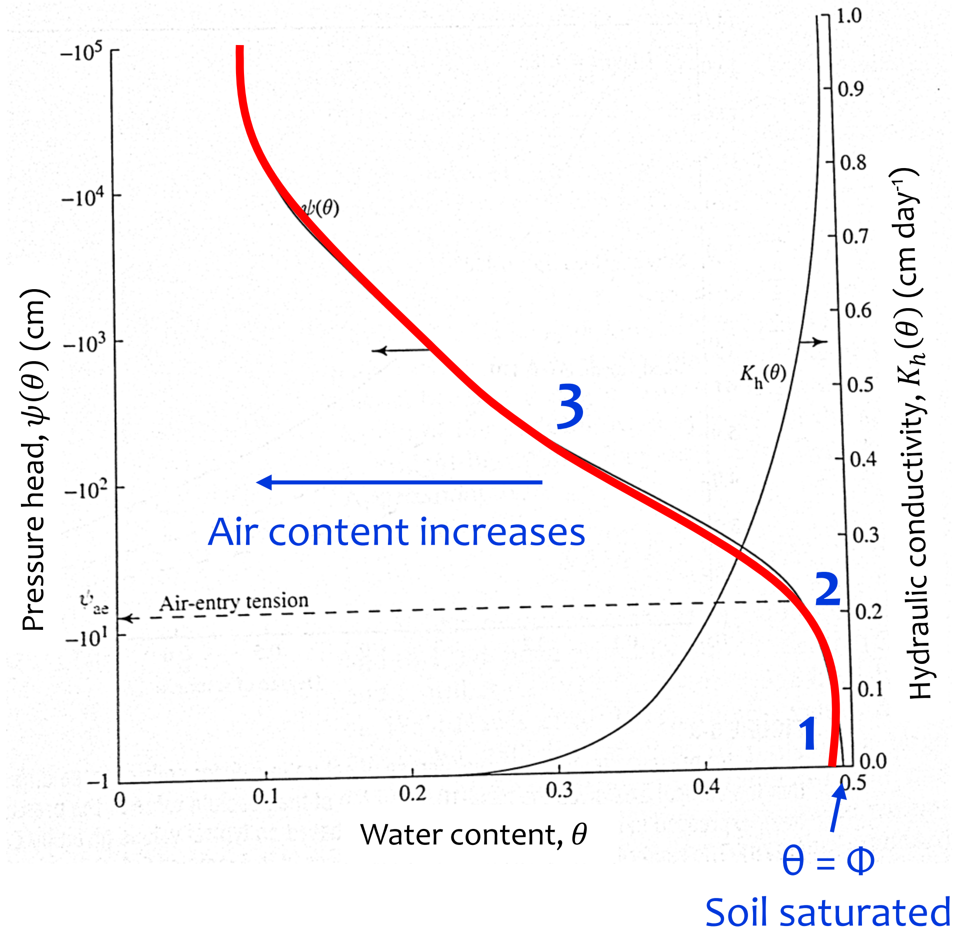
Soil-water flow

- **Pressure head**, ψ, can be defined as $\psi = p/\gamma_w$, where $\gamma_w = \rho_w g$, the specific weight of water
  - *Recall pressure head, velocity head, elevation head of Bernoulli's equation → same idea here*
    - *Think about units:* $p/\rho_w g$ → [N/m²]/[N/m³] = [**m**]
  - Pressure head, ψ, also called **tension head, matric potential, matric suction** or **capillary suction**, when $p < 0$
  - *Like pressure,* $\psi < 0$ *in the unsaturated zone*

Dingman (2002)

Soil-water flow

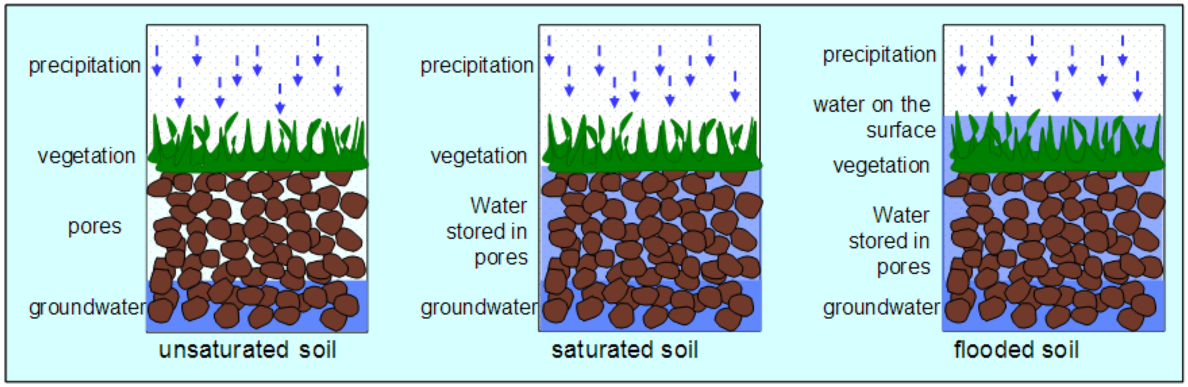
- The **moisture-characteristic curve** shows the relationship between pressure head, ψ, and water content, θ
- Notes:
    1. Pressure head = 0 when water content = porosity (i.e. $\theta = \phi$; here, ϕ = 0.5)
    2. Water content changes little as tension increases to *inflection point* called ψ_{ae} → the "**air-entry tension**"
    3. As tension increases beyond air-entry value, *water content decreases rapidly*, then more gradually
    - *Note: curve shape slightly different for each soil type*



Dingman (2002)

Soil-water flow

- **Hydraulic conductivity,** K_h, is *rate* (volume per unit time per unit area) at which water moves through a porous medium under a unit potential-energy gradient
  - *Or, put more simply, "how easily fluid (water) can move through pore spaces or fractures…"*
  - Rate determined mainly by size (cross-sectional area) of *pathways available for water movement*
  - **Saturated conditions:** *pathway size* determined by soil-grain size
  - **Unsaturated flows:** rate depends on grain size, degree of saturation

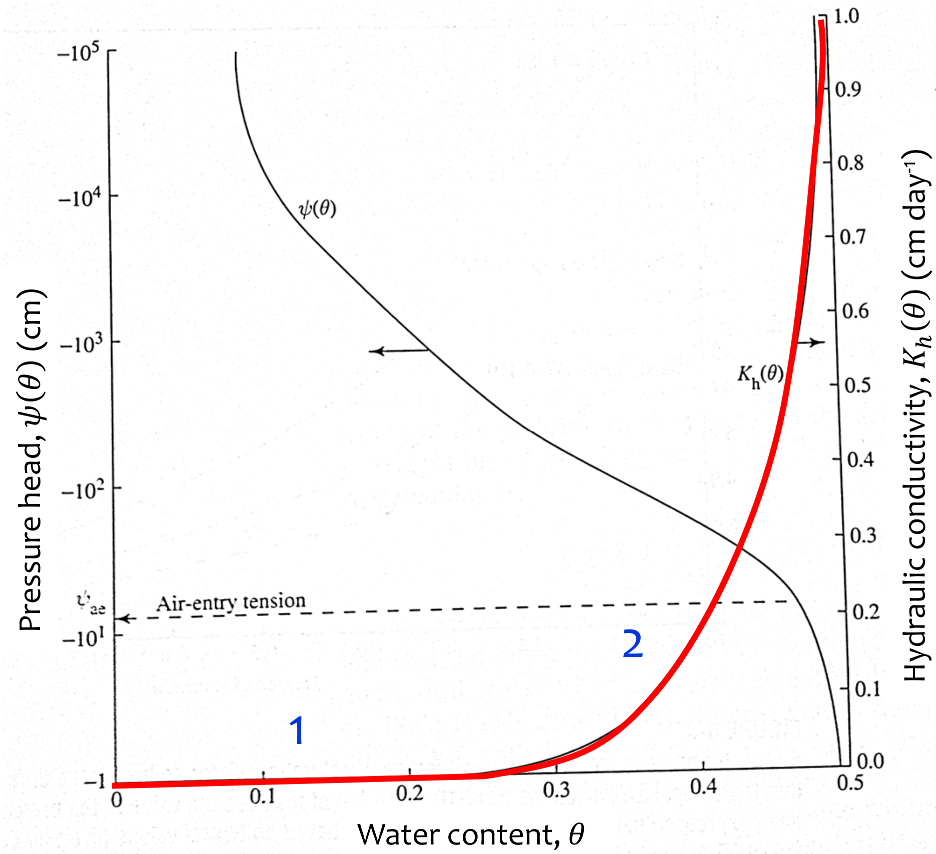


http://www.floodsite.net/juniorfloodsite/html/en/student/thingstoknow/hydrology/waterstorage2.html

Dingman (2002)

Soil-water flow

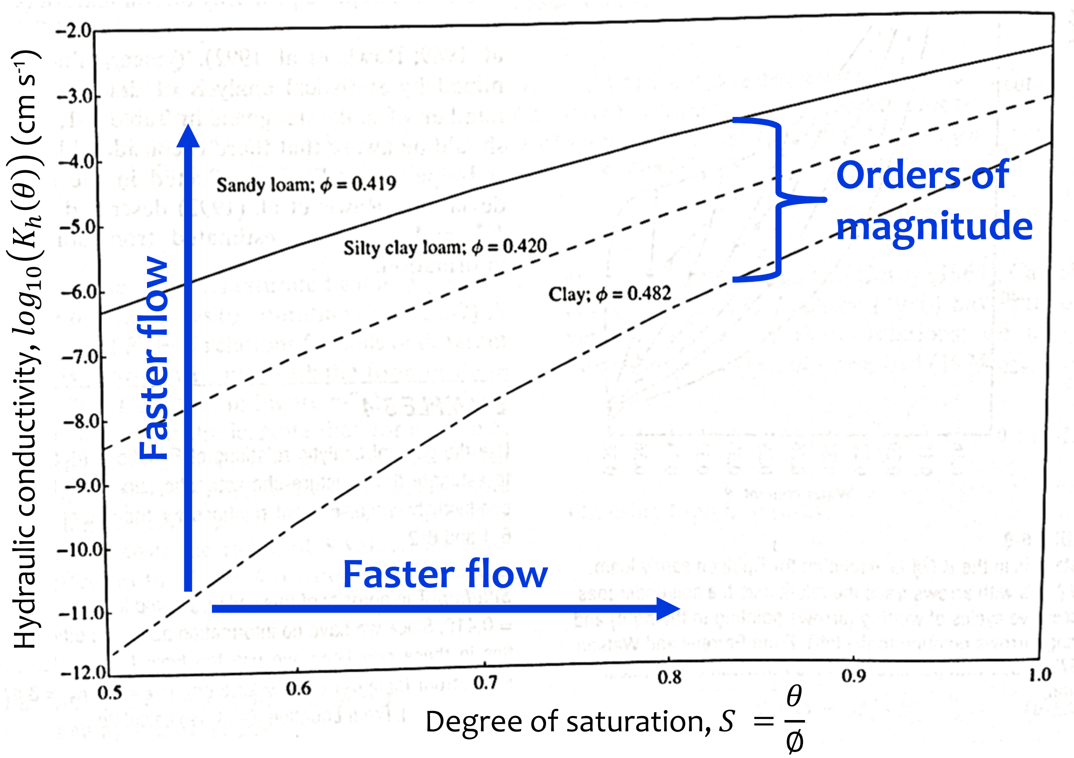
- Hydraulic conductivity, K_h:
  1. For given soil, K_h is **very low** at low to moderate water contents → water just coats soil grains
     - *Unsaturated conditions*
  2. Then, K_h increases **nonlinearly** to saturated value, K_h^*, as pore spaces fill and water content increases to **saturation**



Dingman (2002)

Soil-water flow

- Hydraulic conductivity depends on both **soil grain size** and **degree of saturation:**
  - Figure shows comparison of relationship between K_h and S



Dingman (2002)

Water conditions in natural soils

- Imagine a soil is saturated, and then allowed to drain (*i.e. without evaporation, plant uptake, capillary rise*)
  - Its **water content** will decrease indefinitely in a quasi-exponential fashion
  - The **drainage rate** also declines exponentially and becomes negligible after a few days
    - Define "**field capacity**", θ_{fc}, as the water content below which drainage becomes basically negligible
    - Field capacity is then an "index" of *water content that can be held against force of gravity*
    - θ_{fc} values for sands > 0.1, and for clays may be > 0.3

  - *An experiment you can try at home:* https://youtu.be/z_HYyVCyt9k (ThinkTac, 2019), or https://youtu.be/CBmDlbFOZKU (IzaakWaltonLeague, 2019)

Dingman (2002)

Water conditions in natural soils

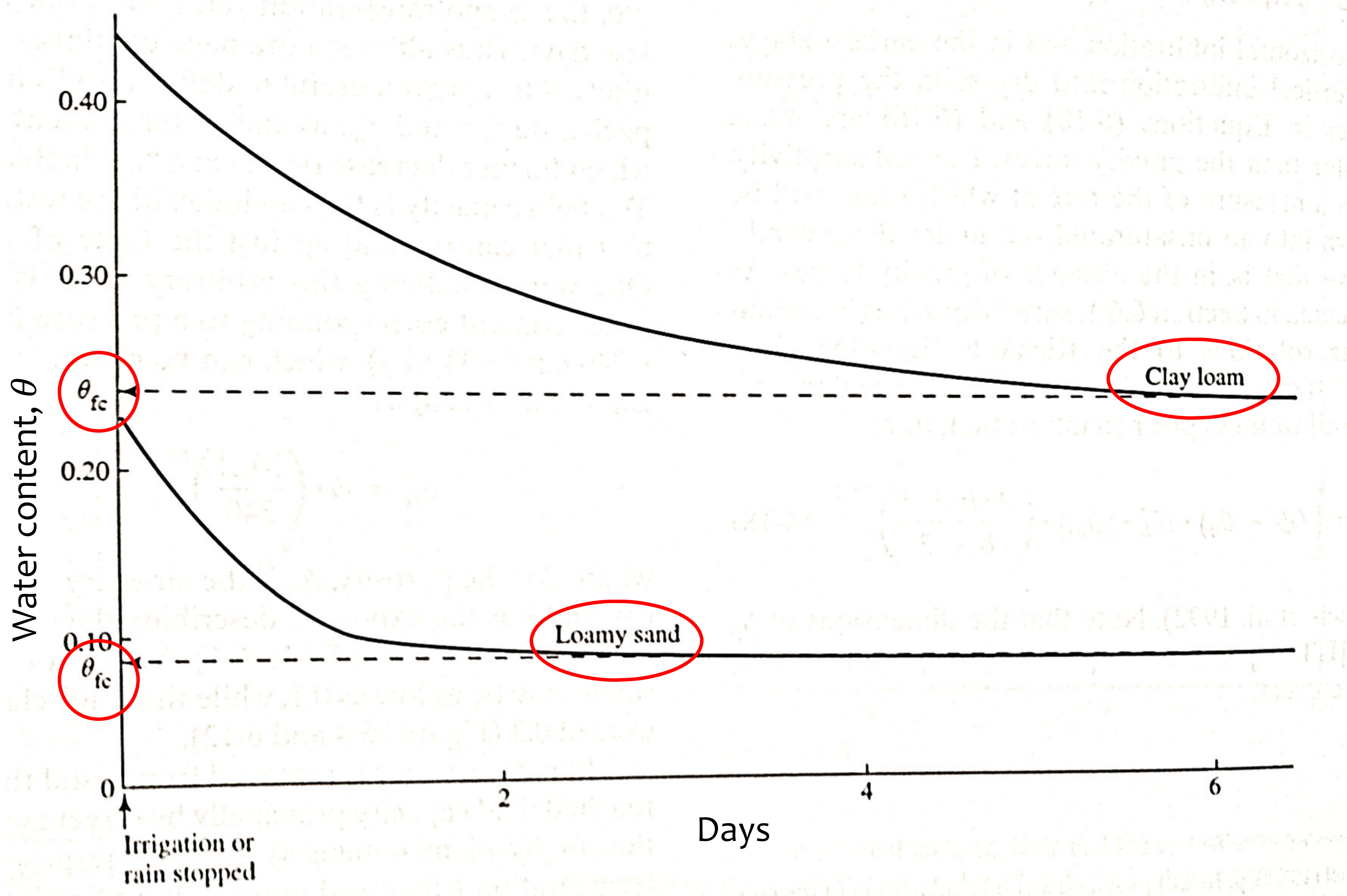


Fig. 6-12 in Dingman (2002)

Water conditions in natural soils

Further, *“**In nature,** water is removed from a soil that has reached field capacity principally by direct **evaporation** or by plant uptake as part of the process of **transpiration**.*

*“However, plants cannot exert suctions stronger than [a certain level] and, when the water content is reduced [to that point], transpiration ceases and plants **wilt**… ”*

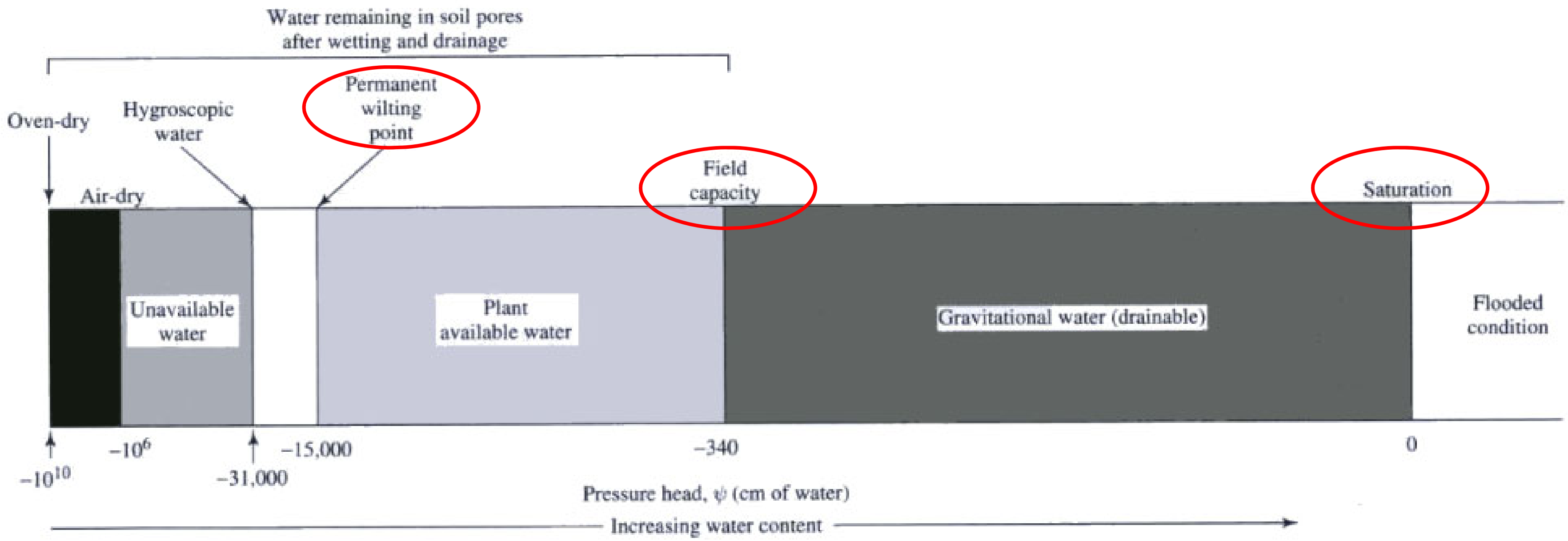
- This point is called the **permanent wilting point,** θ_{pwp}
- Water available to plants is called **available water content,** θ_a

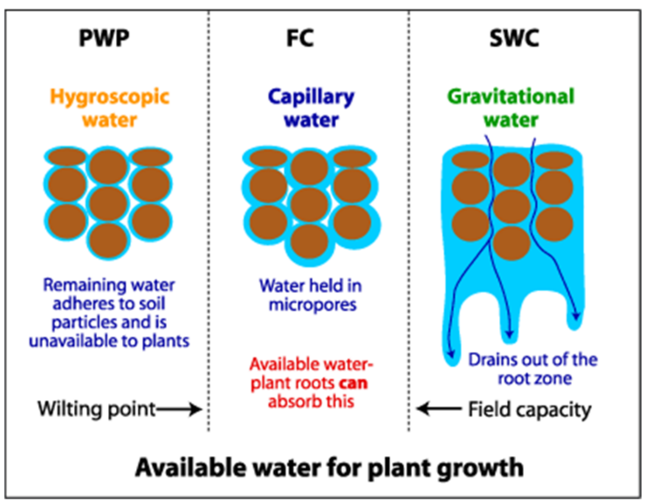
$$\theta_a = \theta_{fc} - \theta_{pwp}$$

- *Illustration of the definitions (using sponges!) is at* https://youtu.be/KvwdVEnv68s (New Mexico State U, 2016)
- *Can also see experiments showing how water moves through soil at* https://youtu.be/ego2FkuQwxc (University of Liège, 2014)

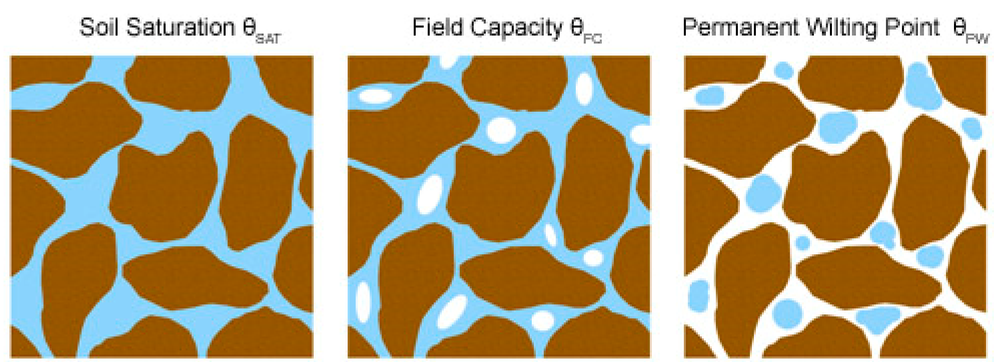
Dingman (2002: 236)

Water conditions in natural soils





http://toolboxes.flexiblelearning.net.au/demosites/series6/605/html/resources/depot/seeya/managing.htm



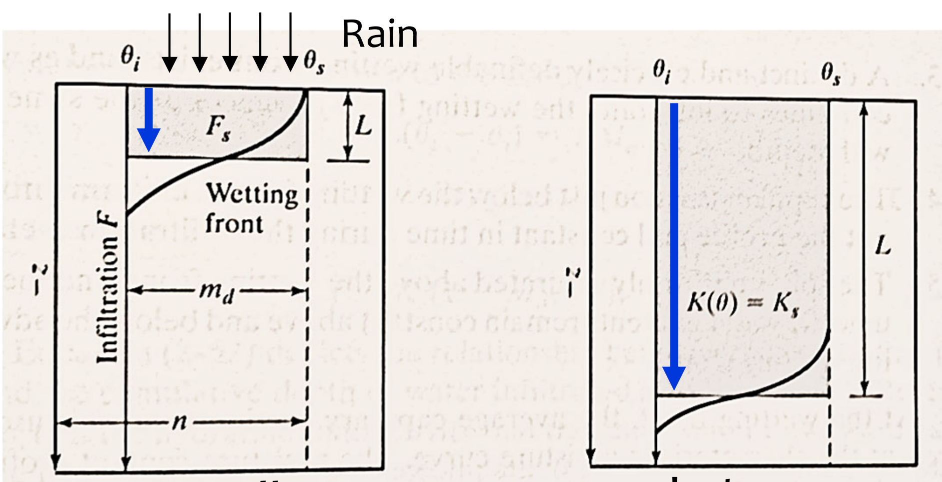
http://www.stevenswater.com/articles/irrigationscheduling.aspx

Fig. 6-13 in Dingman (2002)

3. Infiltration Models

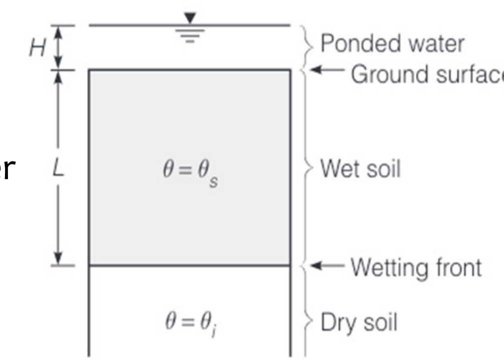
- Infiltration modelled as unsaturated flow
  - Both water and air are present
  - Suction (pressure) force, gravity, and friction are present
  - Soil (usually) starts as unsaturated at beginning of rainfall
    - Infiltration decreases as soil saturates → *implications for runoff*

- **Green-Ampt infiltration model** (1911) represents relationship between moisture content and soil depth as "sharp wetting-front" separating saturated soil at top of soil column from initial moisture content at column base



Initially Later

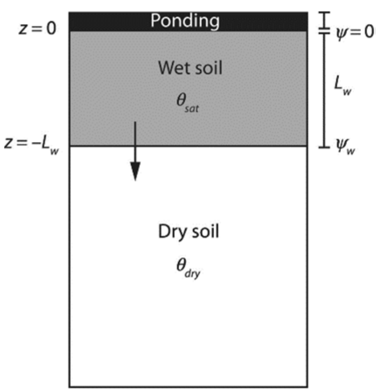
Clearer view:



Bedient et al. (2019)

3.1 Green-Ampt model

https://global.oup.com/us/companion.websites/9780195311259/resources/chapter4/figure4_10/



Bonan (2019)
https://doi.org/10.1017/9781107339217.010

- Green-Ampt model has **five key assumptions:**
  1. Soil under consideration homogeneous and stable (i.e. no macropores, no preferential flow pathways)
  2. Supply of ponded water at surface unlimited
  3. Distinct, precisely definable wetting front exists, wetting front advances at same rate with depth
  4. Capillary suction (pressure head) below wetting front is uniform, constant in time during infiltration event
  5. Soil uniformly saturated above wetting front, volumetric water contents remain constant above and below advancing wetting front

Bedient et al. (2019)

Green-Ampt model

- The **volume of infiltration** down to the wetting front is given as F, and can be calculated as,

$$F = (\theta_s - \theta_i)L = M_d L$$

where θ_i and θ_s are the initial and saturated water contents, L is the **depth of the wetting front**, and M_d is the **moisture deficit**, $\theta_s - \theta_i$

- Volume of infiltration is amount of "new" water added to soil
  - **Δθ·depth**·*unit surface area of soil column* = **Volume**

- Then, based on Darcy's law (*covered in Topic 10…*), which is,

$$q = -K(\theta)\frac{dh}{dz}$$

where q is the Darcy velocity (cm/s)

z is the depth below surface (cm)

h is head = $z + \psi$ (i.e. *gravity and pressure*; cm)

[ψ is **suction head**, or **pressure head** (negative cm)]

$K(\theta)$ is unsaturated hydraulic conductivity (cm/s)

θ is water content

Bedient et al. (2019)

Green-Ampt model

- Darcy's law is applied to the saturated zone and simplified to,

  Δ head / Δ depth

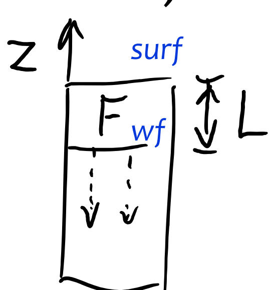
  $$q = -f \cong -K_s\left(h_{surf} - h_{wf}\right)/\left(z_{surf} - z_{wf}\right)$$

  Where "*surf*" and "*wf*" are *surface* and *wetting front*, respectively,

  – Darcy velocity, q, assumed to equal *downward* **infiltration rate**, $-f$,

  – **Saturated hydraulic conductivity**, K_s, used to represent conditions in the saturated zone (i.e. above the wetting front)

- Substitutions: Distance to the wetting front, with z positive upward is $-L$, so that $h_{wf} = z + \psi \cong -L + \psi$, and $h_{surf} = 0$, then:



$$-f = -K_s[0 - (-L + \psi)]/[0 - (-L)]$$

$$f = K_s\left(1 - {}^{\psi}/_{L}\right)$$

Bedient et al. (2019)

Green-Ampt model

- Finally, recalling that $F = L(\theta_s - \theta_i) = LM_d$, we can rearrange to get the **Green-Ampt equation,**

$$\boxed{f = K_s(1 - M_d\psi/F)}$$

$L = {}^{F}/_{M_d}$

Bedient et al. (2019)

Green-Ampt model

- The **Green-Ampt equation** is,

$$\boxed{f = K_s(1 - M_d\psi/F)}$$

- Terms in the equation are,
  - The infiltration rate, f, in cm/h (*or other time units*)
  - Volume of infiltration to the wetting front, F, in cm
  - The saturated hydraulic conductivity, K_s, in cm/s
  - The moisture deficit, M_d, is the difference between the final and initial water contents, $\theta_s - \theta_i$
  - The pressure head, ψ, in cm

Bedient et al. (2019)

Green-Ampt model

- **Interpretation:** Green-Ampt equation represents relationship between infiltration rate (f) and cumulative depth of the water infiltrated (F)
  - Since the pressure head, ψ, is negative, *infiltration rate is greater than saturated hydraulic conductivity*
  - This situation continues as long as there is sufficient water at the surface for infiltration (ponding = puddles)
  - Infiltration rate, f, decreases as cumulative infiltration, F, increases

i.e. f = f(*F*), and F↑ *causes f*↓

Bedient et al. (2019)

Green-Ampt model

- Solution requires estimation of three parameters: K_s, M_d, and ψ
  - K_s is easiest to find and is the limiting downward rate of water transmission (i.e. as $F \rightarrow \infty$)
  - M_d depends on "effective porosity" and initial saturation (effective porosity is space available for water flow, where $\theta_e = \emptyset_e < \emptyset$)
  - ψ is difference between atmospheric (*gauge*) and hydrostatic pressures

Bedient et al. (2019)

Green-Ampt equation use

- Major advantage of Green-Ampt: parameters K_s, ψ, and M_d can be determined from physical measurements of soil
  - Lots of data are available on **tension** and **moisture content**
  - In practice, Green-Ampt parameters typically calibrated, especially when the equation is used in simulation models
- Values tabulated below for variety of soils
  - Effective porosity accounts for trapped air and residual water; more reasonable value to use in computations: $M_d = \eta - \theta_i$ or $= \theta_e - \theta_i$

Table 2–4 Green–Ampt Infiltration Parameters for Various Soil Texture Classes

Soil Class	Porosity η or $\emptyset$	Effective Porosity θ_e or $\emptyset_e$	Wetting Front Suction Head ψ (-cm)	Hydraulic Conductivity K (cm/hr)
Sand	0.437	0.417	4.95	11.78
	0.374–0.500	0.354–0.480	0.97–25.36	
Loamy sand	0.437	0.401	6.13	2.99
	0.363–0.506	0.329–0.473	1.35–27.94	
Sandy loam	0.453	0.412	11.01	1.09
	0.351–0.555	0.283–0.541	2.67–45.47	
Loam	0.463	0.434	8.89	0.34
	0.375–0.551	0.334–0.534	1.33–59.38	
Silt loam	0.501	0.486	16.68	0.65
	0.420–0.582	0.394–0.578	2.92–95.39	
Sandy clay loam	0.398	0.330	21.85	0.15
	0.332–0.464	0.235–0.425	4.42–108.0	

Bedient et al. (2019)

Green-Ampt parameters

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Sandy clay loam	0.398	0.330	21.85	0.15
	0.332–0.464	0.235–0.425	4.42–108.0	
Clay loam	0.464	0.309	20.88	0.10
	0.409–0.519	0.279–0.501	4.79–91.10	
Silty clay loam	0.471	0.432	27.30	0.10
	0.418–0.524	0.347–0.517	5.67–131.50	
Sandy clay	0.430	0.321	23.90	0.06
	0.370–0.490	0.207–0.435	4.08–140.2	
Silty clay	0.479	0.423	29.22	0.05
	0.425–0.533	0.334–0.512	6.13–139.4	
Clay	0.475	0.385	31.63	0.03
	0.427–0.523	0.269–0.501	6.39–156.5	

Finer particles

$-\psi$ increases

K_s decreases

Source: Rawls et al., 1983.

Bedient et al. (2019)

Green-Ampt model variants

- Green-Ampt equation can also be used to determine cumulative infiltration, F, as a function of f
  - **One application:** derive volume of infiltration, F_s, occurring at the *moment of surface saturation*, where $f = i$:

$$F_s = M_d \psi / (1 - i/K_s)$$

Here, $i > K_s$ and recall that capillary suction, ψ, is negative

i.e. $F_s = \frac{[-ve]}{[-ve]} = [+ve]$

Note: "moment of surface saturation" → time when *ponding begins*, so $t = t_p$

Bedient et al. (2019)

Green-Ampt model variants

- Finally, Green-Ampt equation predicts f as a **function of $\boldsymbol{F}$**, ***not time***
  - However, since $f = dF/dt$, can convert the equation into a form that can be solved **iteratively** for $F(t)$

- Two useful equations that include the **cumulative infiltration** and **time** are,
  - A determination of the "time to saturation", also called the "time to ponding" (where $t_s = t_p$), which uses the F_s (also called F_p) equation given earlier to find, $t_s = F_s/i$
  - Chow et al. (1988) developed an equation connecting the "*cumulative infiltration to ponding*" (F_s or F_p) to "*time to ponding*",

$$K_s(t - t_p) = F - F_p + \psi M_d ln\left[\frac{M_d\psi - F}{M_d\psi - F_p}\right]$$

Bedient et al. (2019)

Green-Ampt model interpretation

- What happens when it rains? Infiltration or runoff?
- *Infiltration governed either by **rainfall intensity**, i, of the storm, or by the Green-Ampt equation*

- Three rainfall scenarios for i are possible:

Light rainfall

1. $i < K_s$: Rainfall intensity less than maximum downward hydraulic conductivity → *runoff will not occur*, because all rainfall infiltrates regardless of rainfall duration

Complicated

2. $K_s < i < f$: Rainfall intensity greater than saturated hydraulic conductivity, but less than infiltration rate → *time to ponding* varies for different rainfall intensities

Heavy rainfall

3. $i > f$: Rainfall intensity greater than infiltration rate, and *runoff can occur*

Bedient et al. (2019)

Green-Ampt model interpretation

- Showing these three possible cases graphically, based on various **rainfall intensities, *i*,** over time:

(A) 1. If $i \leq K_s$, then $f = i$

(B) 2. If $i > K_s$, then $f = i$ until ponding occurs, when $F = it_s = F_s$

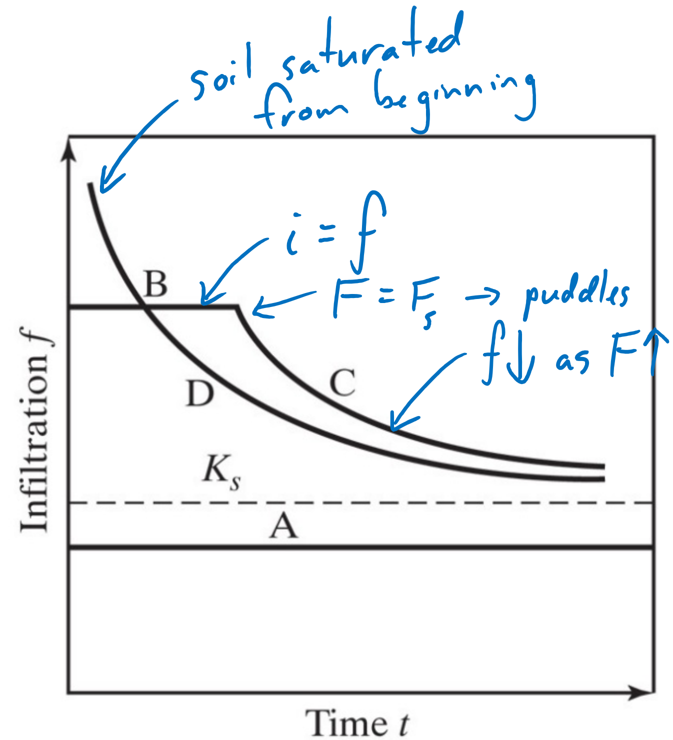
3. Following surface saturation,

(C) and **(D)** $f = K_s\left(1 - \frac{M_d\psi}{F}\right)$ for $i > K_S$,

OR $f = i$ for $i \leq K_s$

Time	Infiltration rate	Cumulative infiltration	Process
$t < t_p$	$f = i$	$F(t) = i \cdot t$	No ponding
$t = t_p$	$f = f_p = i$		Ponding begins
$t > t_p$	$f = f_p$	Chow et al. (1988)	Ponding continues

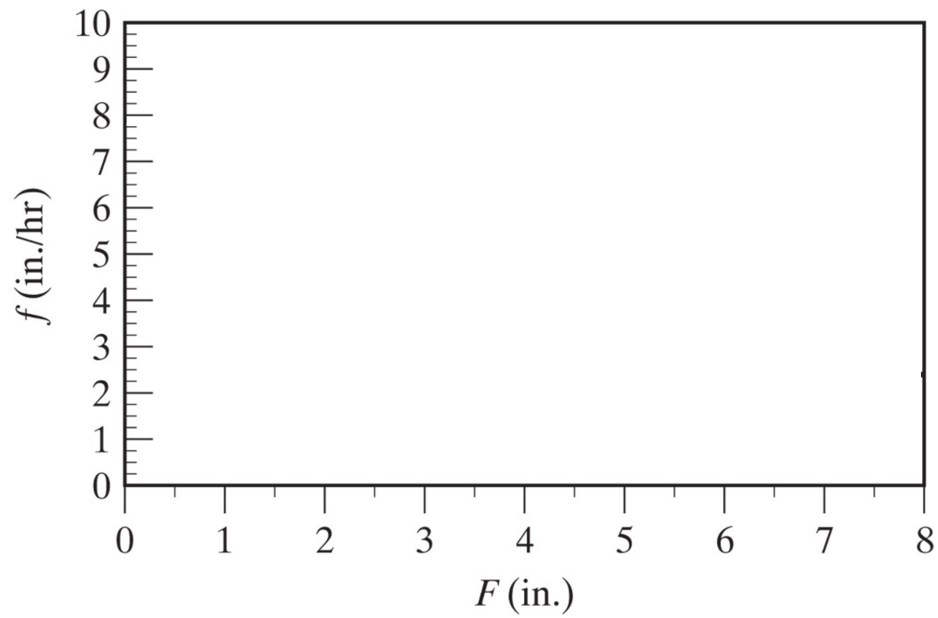
Note: where ponding occurs for $t > 0$, then t_p is the "time to ponding"
f_p is infiltration capacity when ponding occurs, i.e. when $t = t_p$



Bedient et al. (2019)

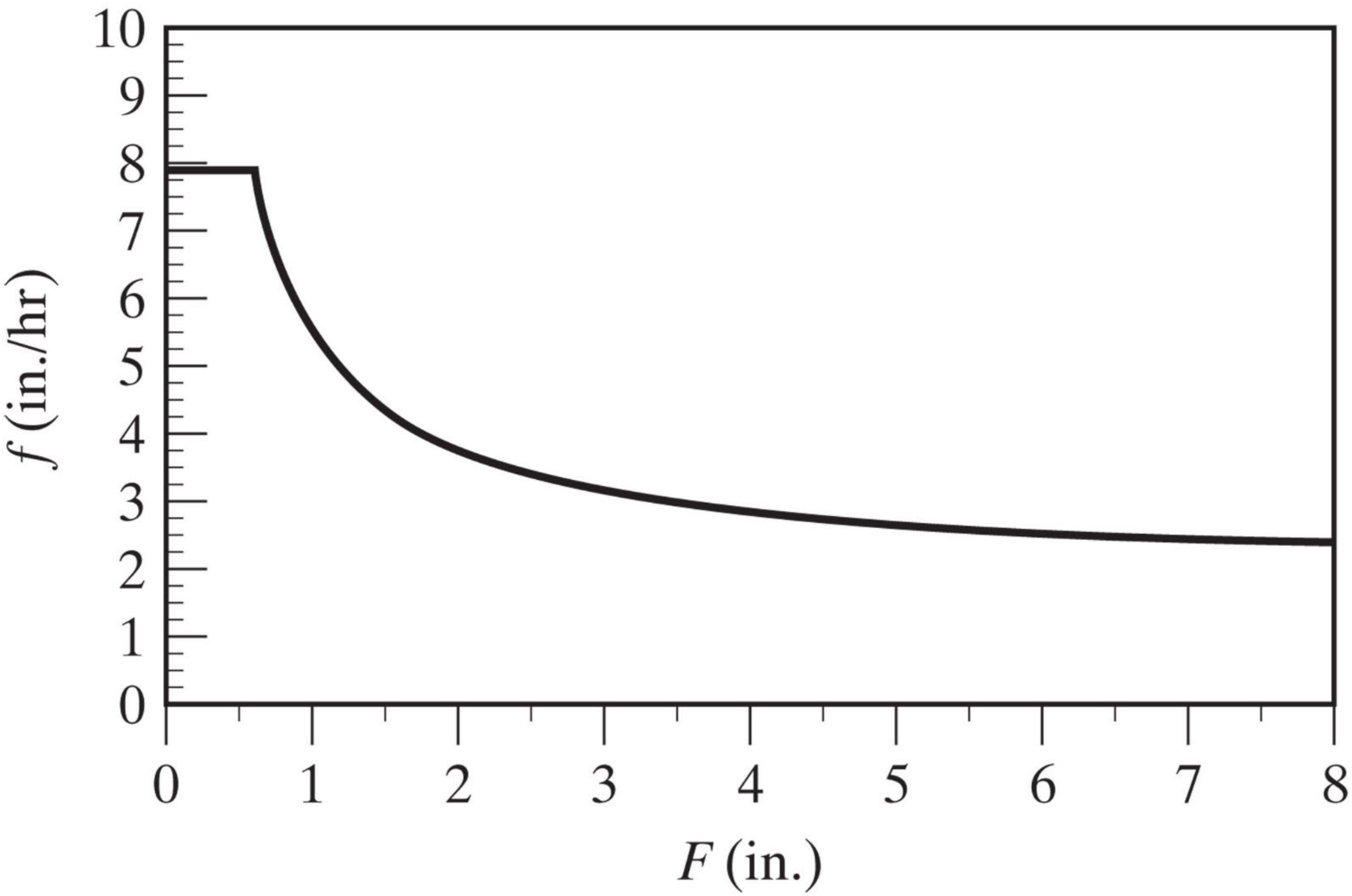
Ex 3: Green-Ampt equation

For the following soil properties, develop a plot of infiltration rate f vs. infiltration volume F using the Green-Ampt equation: K_s = 1.97 in/h, θ_s = 0.518, θ_i = 0.318, ψ = -9.37 in, i = 7.88 in/h



Example 2-12 in Bedient et al. (2019)

Solution



Ex 4: Green-Ampt equation

Silt loam has the following soil properties for use in the Green-Ampt equation: K_s = 1.81 x 10^{-4} cm/s, θ_s = 0.523, ψ = -17 cm

For an initial moisture content of θ_i = 0.3, what is the ponding depth (F_s) and ponding time (t_s) based on a rainfall intensity of $i = 6K_s$ for 10 minutes? What will be the infiltration depth 20 minutes after the beginning of the rainfall? (Assume the rainfall had a constant intensity.)

Example 2-12 in Bedient et al. (2019)

Solution

Find V of infiltration to the moment of surface saturation, F_s

GREEN–AMPT TIME TO SURFACE SATURATION **EXAMPLE 2–13**

SOLUTION

$F_s = \frac{M_d \psi}{(1 - i/K_s)}$ (Slide 35)

i.e. $f = i$ until $F = F_s$ (Slide 39)

Then $f < i$ (ponding)

$\theta_s - \theta_i$

The initial moisture deficit $M_d = 0.523 - 0.300 = 0.223$. In order to obtain the ponding time, we compute the ponding depth required to produce saturation from Equation (2–53):

$$F_s = (-17.0 \text{ cm})(0.223)/(1 - 6K_s/K_s) = \underline{0.76 \text{ cm}}.$$

The ponding time (t_s) is

$i = 6K_s$

$$t_s = F_s/i = (0.76 \text{ cm})/[(6 \times 1.81 \times 10^{-4} \text{ cm/s})(60 \text{ s/min})] = \underline{11.7 \text{ min}}$$

It shows that it takes rainfall 11.7 minutes to saturate the soil. However, after this point (11.7 min < 20 min), the infiltration depth is not going to be equal to the product of the rainfall intensity (i) and time but increase exponentially according to the equation as follows (Chow et al., 1988):

$$* \quad K_s(t - t_p) = F - F_p + \psi M_d \ln\left[\frac{M_d\psi - F}{M_d\psi - F_p}\right]$$

$F_p = F_s, \quad t_p = t_s$

Solution (2)

We're finding F for $t = 20\,\text{min}$

GREEN–AMPT TIME TO SURFACE SATURATION **EXAMPLE 2–13**

Apparently, infiltration depth (F) varies with time (t) in this equation. There is only one unknown variable (F) in the equation at any time after ponding. We have known $t_s = 11.7$ min, $M_d = 0.223$, $\psi = -17.0$ cm, $F_s = 0.76$ cm, and $t = 20$ min. By Goal Seek and rearranging the above equation in Excel spreadsheet to make it equal to zero, the infiltration depth for 20-minute rainfall can be calculated as 1.20 cm. It indicates that 0.44 cm (1.2 cm − 0.76 cm) of rainfall has been infiltrated in 8.3 minutes (20 min − 11.7 min) after the ponding.

$$0 = F - 0.85 - 3.791 \ln\left[\frac{-3.791 - F}{-4.551}\right]$$

Key: Chow et al. (1988) equation is ***implicit****: if solving for F, it can be found on both the right-hand and left-hand side of the equation after rearranging – i.e. $F = f(F)$. Can use Goal Seek in Excel to solve, as Bedient et al. suggest, or another approach if you prefer.*

3.2 Horton infiltration equation

- Horton (1940) suggested the following infiltration equation, where rainfall intensity $i > f$ at all times,

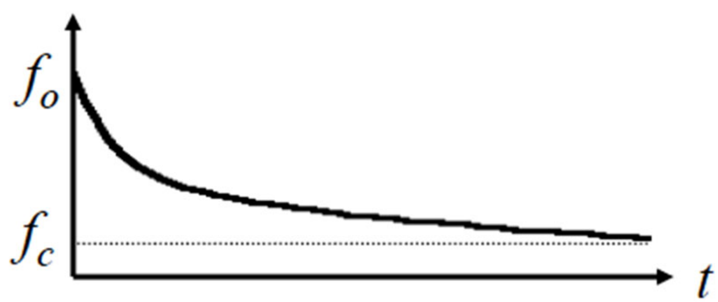
$$f = f_c + (f_0 - f_c)e^{-kt}$$

where f = infiltration capacity (cm/h)

f_0 = initial infiltration capacity (cm/h)

f_c = final infiltration capacity (cm/h)

k = empirical constant (h^{-1})



Bedient et al. (2019)

Horton infiltration equation

- Specific curves can be predicted using Horton's equation, given rainfall intensity, initial soil conditions, and a set of unsaturated characteristic curves for the soil
  - f_c numerically equivalent to *saturated hydraulic conductivity* of soil

- **A limitation of approach**: Horton's equation models infiltration capacity as *decreasing over time regardless* of actual amount of water available for infiltration
  - In other words, equation assumes **ponding on surface** and reduction of infiltration capacity even if i does not exceed f_0 (a problem particularly in sandy soils, where f_0 could be as much as 58 cm/h)
  - Instead, **infiltration capacity**, f, should be reduced in proportion to cumulative infiltration volume, F, not **time**

Bedient et al. (2019)

Horton infiltration equation

- **Cumulative infiltration volume** comes from integrating Horton's equation,

$$F(t) = f_c t + \left[\frac{f_0 - f_c}{k}\right]\left(1 - e^{-kt}\right)$$

- Finally, the table below shows typical parameter values for Horton's equation

Table 2–3 Typical Values of the Parameters of f_0, fc, and k of the Horton Model

Soil Type	f_c (in./hr)	f_0 (in./hr)	k (hr^{-1})
Alphalpha loamy sand	1.40	19.00	38.29
Carnegie sandy loam	1.77	14.77	19.64
Dothan loamy sand	2.63	3.47	1.40
Fuquay pebbly loamy sand	2.42	6.24	4.70
Leefield loamy sand	1.73	11.34	7.70
Tooup sand	1.80	23.01	32.71

After Rawls et al., 1983.

Bedient et al. (2019)

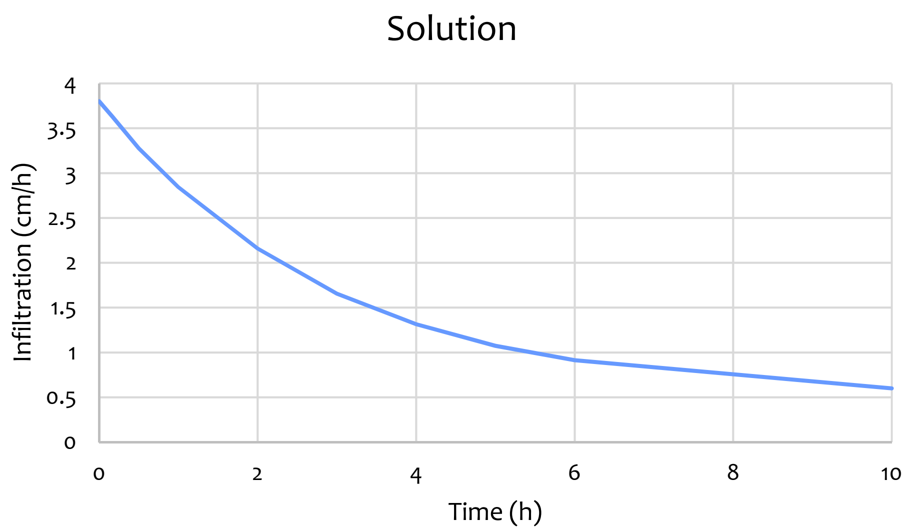
Ex 5: Horton infiltration

The initial infiltration capacity, f_0, of a watershed is estimated as 3.8 cm/h and the time constant is taken to be 0.35 h^{-1}. The equilibrium capacity, f_c, is 0.5 cm/h.

Use Horton's equation to find (a) the values of f at t = 10 min, 30 min, 1 h, 2 h, and 6 h, and (b) the total volume of infiltration over the 6-h period.

t (h)	f (cm/h)
1/6	
1/2	3.28
1	2.84
2	2.16
3	1.66
4	1.31
5	1.07
6	0.91

Adapted from example 2-10 in Bedient et al. (2019)

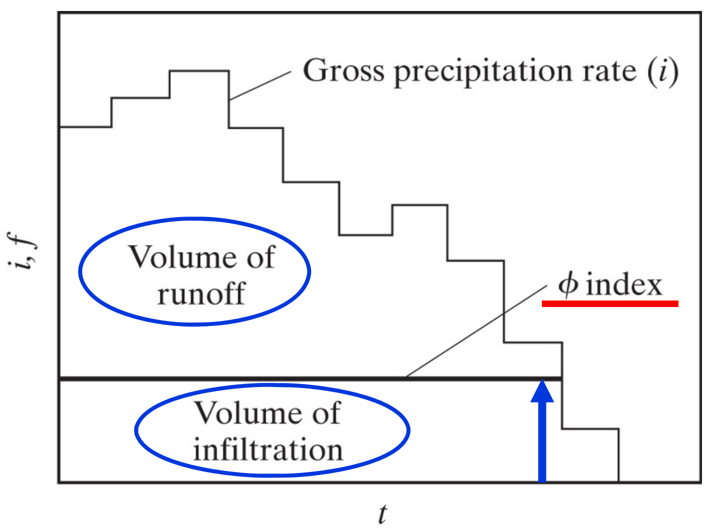


Full set of values:

t (h)	f (cm/h)	F (cm)
1/6	3.61	0.62
1/2	3.28	1.76
1	2.84	3.28
2	2.16	5.75
3	1.66	7.63
4	1.31	9.10
5	1.07	10.29
6	0.91	11.27

3.3 ϕ-INDEX METHOD

- ϕ index is simplest infiltration method
- Calculated by finding the *loss difference* between gross precipitation and surface runoff measured as hydrograph
- Assumption is that the loss (*infiltration*) is uniformly distributed across rainfall pattern
  - Shaded area under the ϕ index line sums to volume of infiltration
  - Area above ϕ index line is the direct runoff



We find the value of Ø such that $\sum_i (P_i - I) = R \;\rightarrow\; \sum_i (P_i - \emptyset) = R$, where P_i is the rainfall at time *i*, R is the total runoff, and I is the infiltration, set to an *unknown constant* value of Ø

Bedient et al. (2019)

Example: ϕ index method

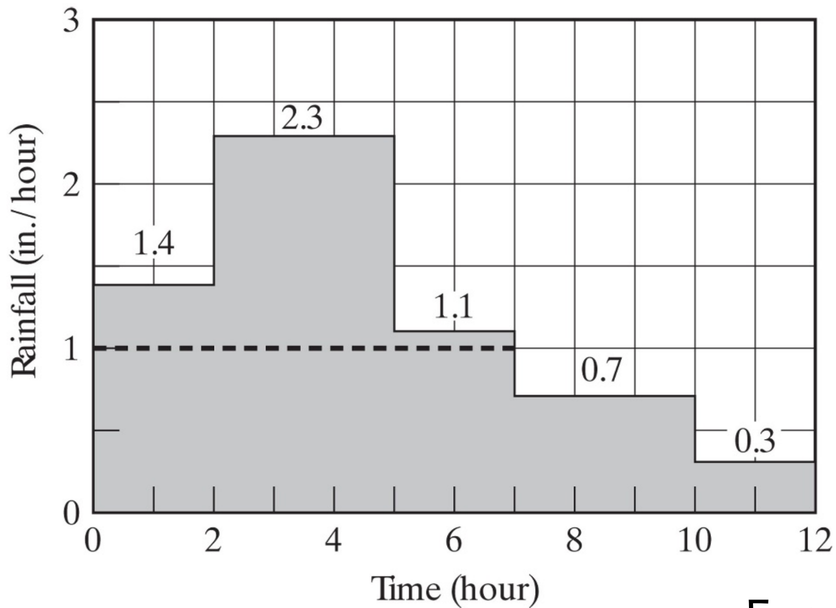
PHI INDEX METHOD FOR INFILTRATION **EXAMPLE 2–11**

Use the rainfall data below to determine the ϕ index for a watershed that is 0.875 square miles, where the runoff volume is 228.7 ac-ft.

Time (hr)	Rainfall (in./hr)
0–2	1.4
2–5	2.3
5–7	1.1
7–10	0.7
10–12	0.3

Note: We will learn about runoff and hydrographs next week

Solution:



Example 2-11 in Bedient et al. (2019)

Solution

PHI INDEX METHOD FOR INFILTRATION **EXAMPLE 2–11**

SOLUTION

The first step involves graphing the given data, as in Figure E2–10. To approach the problem, we must first change the area of the watershed into acres:

$$\text{area(ac)} = 0.875 \text{ sq mi } (640 \text{ acres/sq mi}),$$
$$\text{area} = 560 \text{ acres},$$

We develop an equation for rainfall minus infiltration for each of the time intervals as given:

$$2(1.4 - \phi) + 3(2.3 - \phi) + 2(1.1 - \phi) + 3(0.7 - \phi) + 2(0.3 - \phi) = 4.9$$

Note that if ϕ is greater than the net rainfall for a specific time period, no negative rainfall is added into the runoff calculation.

The rate of infiltration can be found only by trial and error:

Assume $\phi = 1.5$ in./hr. The runoff is the volume of water above the line at which $y = 1.5$ on the graph in Figure E2–11. This ϕ index would then account for $3(2.3 - 1.5) = 2.4$ in. of runoff (neglecting negative components), which is less than 4.9 in. Try again.

Example 2-11 in Bedient et al. (2019)

Solution (2)

PHI INDEX METHOD FOR INFILTRATION **EXAMPLE 2–11**

Assume $\phi = 0.5$ in./hr. This ϕ would account for $2(1.4 - 0.5) + 3(2.3 - 0.5) + 2(1.1 - 0.5) + 3(0.7 - 0.5) = 9.0$ in. of runoff, which is greater than 4.9 in. Try again.

Assume $\phi = 1.0$ in./hr, and the solution is found where the runoff is equal to $2(1.4 - 1.0) + 3(2.3 - 1.0) + 2(1.1 - 1.0) = 4.9$ in., the required amount.

From the calculations, one can see that below the dotted line at which $\phi = 1.0$ in./hr, the rainfall infiltrates into the ground and the rainfall above this line (a total of 4.9 in. in 12 hr) runs off, as required.

Example 2-11 in Bedient et al. (2019)

Runoff and surface flows

- Recall that **rainfall** is the key input to the surface component of hydrological cycle: *the **water budget***,
$$P - R - G - E - T = \Delta S$$
- Have now addressed **other components** of the water budget: the **losses** or "**abstractions**"
  - *Evaporation, transpiration and infiltration*

- *Next topic is the "remainder" after losses:* **runoff**
$$P - R - G - E - T = \Delta S$$