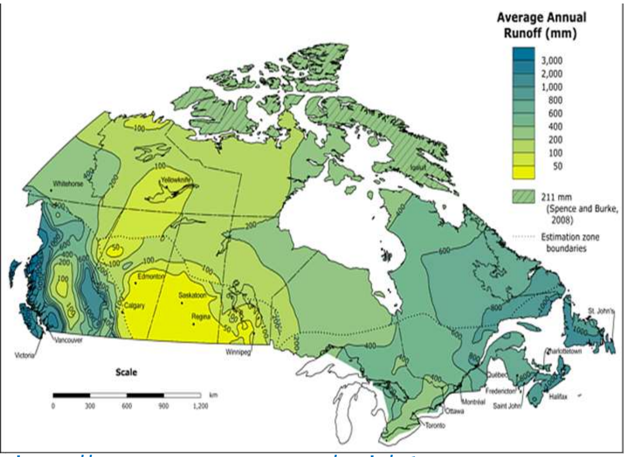
Spring runoff (MS Windows wallpaper)

Civ E 321: Principles of Environmental Modelling and Risk

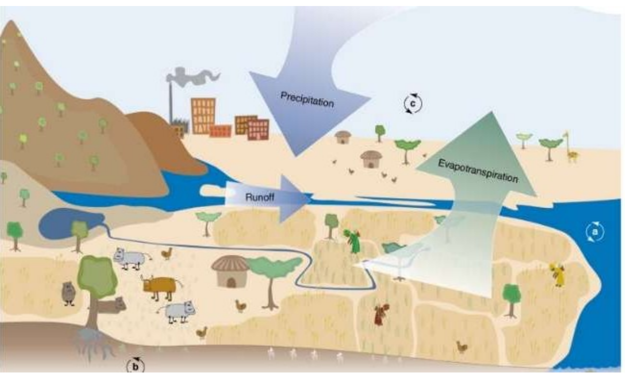
WEEK 6: RUNOFF

Most information in Topic 6 is from pages 42-55 and 75-108 *of* ***Bedient et al. (2019)***

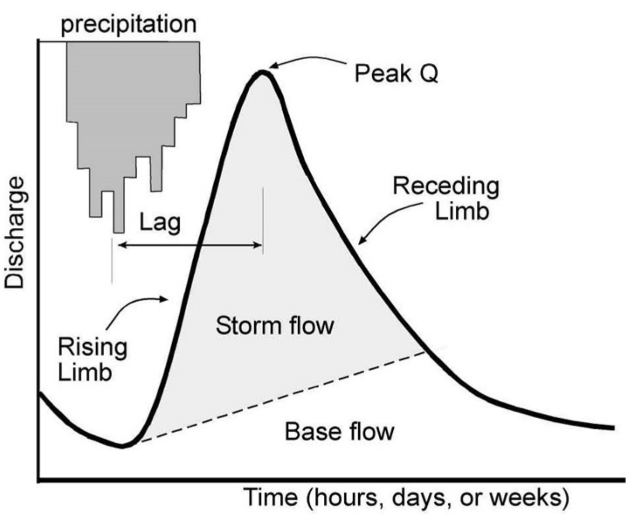
Section overview



http://www.statcan.gc.ca/pub/16-002-x/2009002/map-carte/map-carte001-eng.htm



Gordon et al. (2007), *Trends in Ecol and Evol.*



http://www.geogonline.org.uk/storm_hydrograph.htm

- Introduction to runoff
- Hydrograph theory
- Unit hydrographs (UH)
- UH convolution
- S-curves
- Synthetic unit hydrographs

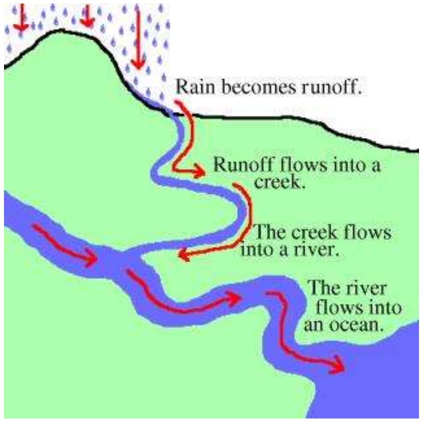
INTRODUCTION TO RUNOFF

"Surface runoff, or simply '**runoff**', refers to all the waters flowing on the [Earth's surface], either by **overland sheet flow** or by **channel flow** in rills, gullies, streams, or rivers.

"Runoff is a **continuous process** by which water is constantly flowing from higher to lower elevations through [gravitational forces]. Small streams combine to form larger streams which eventually grow into rivers.

"In time, rivers carry their flow into the ocean, *completing the hydrological cycle*" Ponce (1989: 62).

http://water.me.vccs.edu/concepts/runoff.htm



http://en.wikipedia.org/wiki/Surface_runoff

http://islandnature.ca/2012/11/rosewall-creek-ramble/

http://visibleearth.nasa.gov/view_rec.php?id=4927

Examples

1. Heavy rainfall on impervious surfaces
T. Struckmeier (2015), https://youtu.be/P1mB2FVQcgs

2. Overland flow → channel flow
Internet Geography (2017), https://youtu.be/JVRna7Awndo

3. Flash flood in Texas (rain occurred upstream)
J. Garza (2015), https://youtu.be/b-Aa45zeUdw

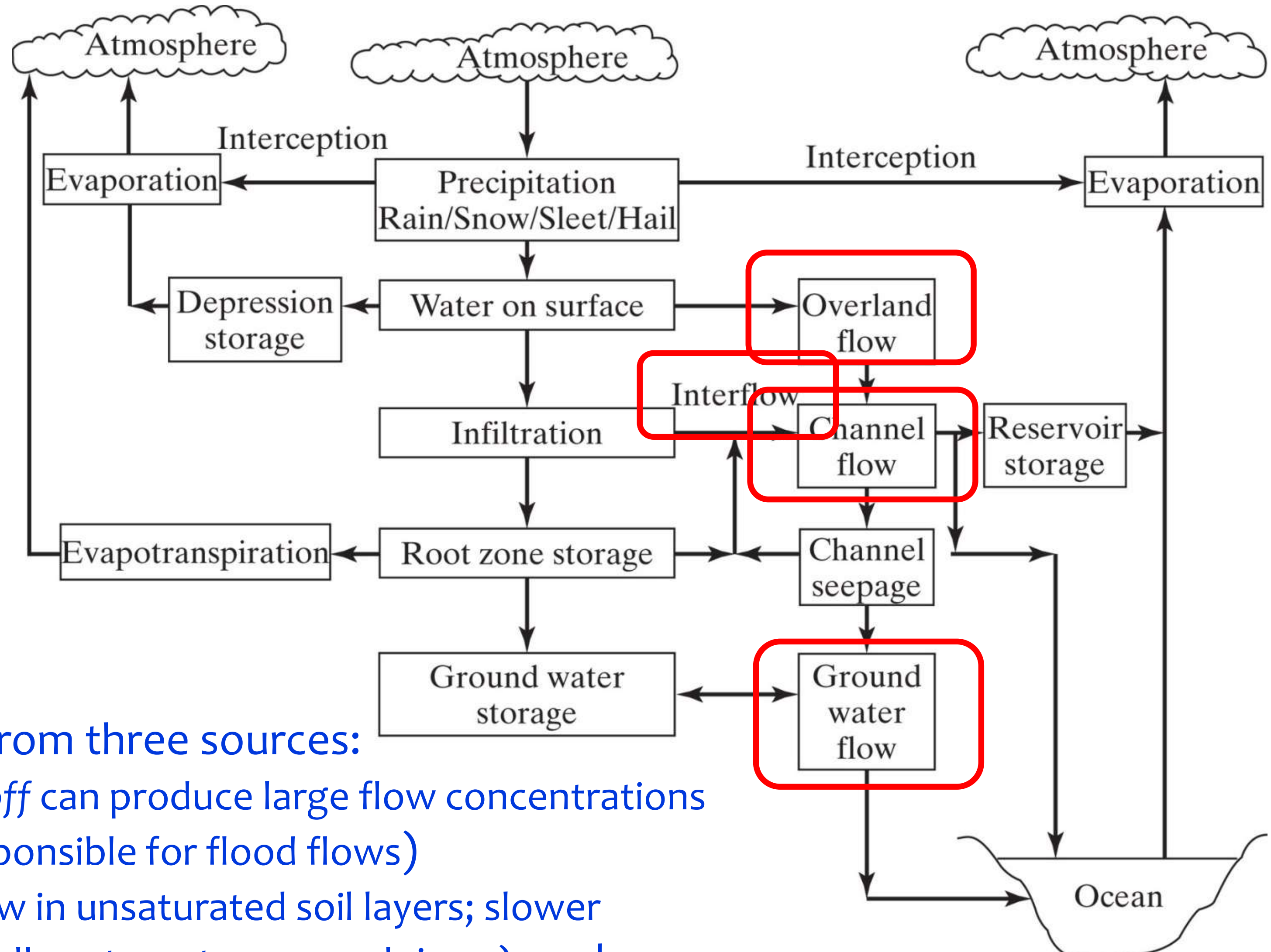
4. The Athabasca River at Jasper, E. Davies (2020)

Importance of runoff

*"Hydrologists are concerned with the **amount of surface runoff generated in a watershed** for a given rainfall pattern, and attempts have been made to analyze historical rainfall, infiltration, evaporation and streamflow data to develop predictive relationships"* [i.e. *models*] (Bedient et al. 2019: 42)

- "There are **three principal scientific and practical motivations** for studying stream response to water-input events:
  1. **Water supply**: precipitation excess moving through stream network constitutes water resources available for human use and management
  2. **Flood prediction and forecasting**: *Flood predictions* are basis for design of bridges, dams, levees, and formulation of floodplain land-use plans and regulations. *Flood forecasts* are estimates of the streamflow response to an actual event that is occurring or is forecast to occur. These are used to guide operation of reservoir systems and provide flood warnings.
  3. **Water quality**: Strongly influenced by chemical and biological reactions that occur as water moves over and through land surface toward streams" (Dingman 2002: 389).

Hydrological cycle and runoff



"Runoff consists of water from three sources:

(1) Surface flow (*direct runoff* can produce large flow concentrations in short period, largely responsible for flood flows)

(2) Interflow (subsurface flow in unsaturated soil layers; slower process, but water eventually enters streams and rivers), and

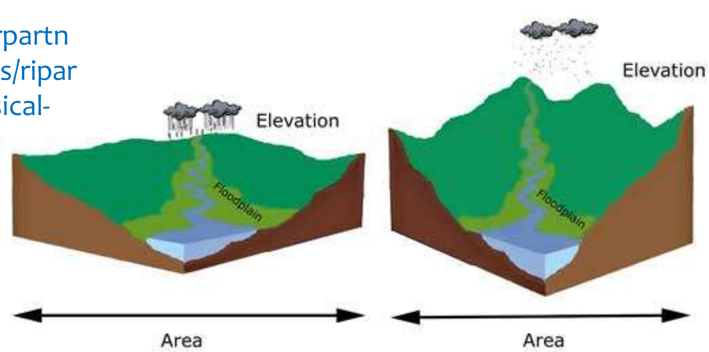
(3) Groundwater flow (saturated flow through alluvial deposits, portion of infiltrated water that has reached water table; slow process, with water eventually reaching oceans) (Ponce, 1989: 64)

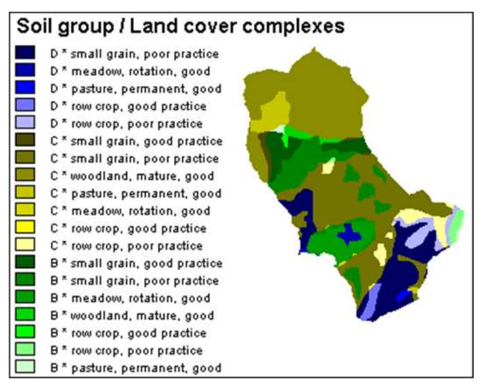
Bedient et al. (2019)

Important determinants of runoff

- Main watershed characteristics that affect hydrological response: **size, shape, slope, soil type**, and **storage** within watershed area
  - **Week 1** discusses watershed delineation, size and shape, length measures, slope
    - Larger watershed can be divided into smaller areas
    - **Drainage area** reflects volume of water that can be generated from rainfall
    - **Slope** reflects rate of change of elevation with distance along main channel; used in performing unit hydrograph, flood routing, and time-of-travel calculations
  - **Week 2** discusses precipitation: rainfall intensity, duration, areal averages
  - **Week 4** discusses soil types and infiltration rates
  - **Land use and land cover** (e.g. *pavement vs. soil*) can have profound effects on watershed response

http://www.riverpartners.org/resources/riparian-ecology/physical-river-processes/





http://www.itc.nl/ilwis/applications/application11.asp

Bedient et al. (2019)

Runoff generation

1. **Precipitation** initially falls on land surface
2. May distribute from there to fill depressions (make puddles), infiltrate to become soil moisture and shallow ground water, or travel as interflow to receiving stream
   – Depression storage capacity usually filled early in storm passage, followed by infiltration capacity into the soil
3. Overland flow and surface runoff commence after soil storage and depression storage satisfied
   – Overland flow quickly moves downhill toward nearest rivulet or channel, which flows into next stream, and eventually reaches main stream channel as "open-channel flow"

- *Recall that evaporation is small component of storm events*
- *Interflow* and *groundwater flow* continue outside storm events

Chapter 1 in Bedient et al. (2019: 43)

Streamflow generation

- Classic concept of streamflow generation by **overland flow** comes from Horton (1933)
  - Horton proposed that overland flow is common and areally widespread. *However,*
  - Later investigators incorporated heterogeneity across watersheds, developed *partial-area contribution* concept
    - "Only certain portions of a watershed regularly contribute overland flow to streams, no more than 10% of a watershed contributes overland flow"
    - Further, significant runoff occurs only after soil saturation
- River channels may also contain **baseflow** from groundwater and soil contributions, even absent rainfall

M. Dean (2015), "River Lune burst its banks", https://youtu.be/vop2u4KsbTM

Bedient et al. (2019: 44)

Overland Flow

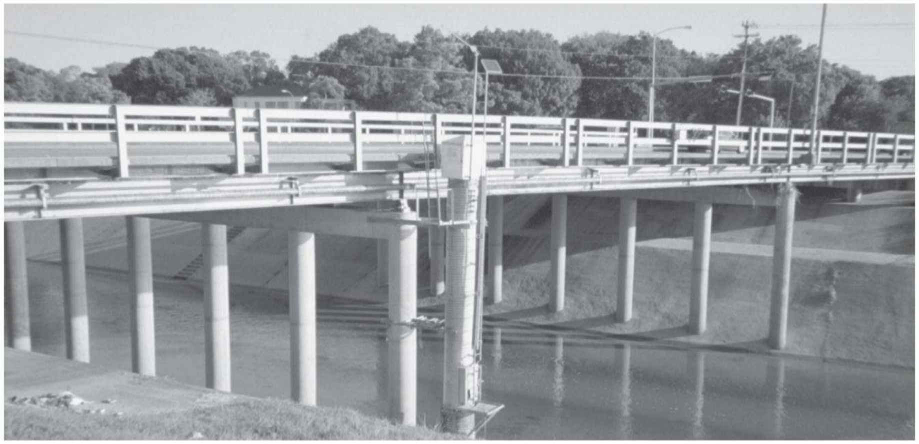
- Overland flow is the product of "rainfall excess"
  - Rainfall excess is the *rainfall minus abstractions* (e.g., infiltration, depression storages, and ET)
  - Rainfall excess also called "effective rainfall"
- It can produce a large amount of flow in a fairly short time → largely responsible for floods
- Two generation mechanisms for overland flow:

	"Infiltration Excess"	**"Saturation Excess"**
Definition	Occurs when rainfall intensity exceeds the soil's infiltration capacity ($i > f_p$).	Occurs when the soil becomes fully saturated and cannot absorb any more water.
Causes	• High rainfall intensity • Low infiltration capacity (soil type, compaction, or surface crust)	• Prolonged rainfall • Existing soil moisture fills all pore spaces → no room for additional water
Context	Common in arid and semi-arid regions, or during intense rainstorms	More common in humid regions, or during prolonged periods of rain

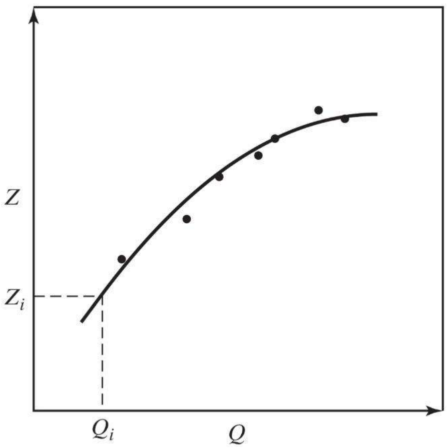
Based on Dingman (2002), after Horton (1933, 1940) and Dunne and Black (1970)

2. Discharge measurement

- Key difference between input (rain) and output (flow):
  - **Rainfall** can be measured in relatively simple way
  - However, **runoff** measurements require elaborate stream-gauging procedure
    - See Water Survey of Canada, https://wateroffice.ec.gc.ca/contactus/faq_e.html, for a brief explanation
    - See Bedient et al. (2019), pages 58-60 for more information on streamflow measurement. In particular: A "rating curve" relates "stage" (depth) to "discharge" (flow)



USGS stream gauge station with telemetry, Fig. 1-32 in Bedient et al. (2019)



Rating curve, Figure 1-31 in Bedient et al. (2019)

"A rating curve is obtained for a particular cross section by finding the total **Q** at a particular stage **z**. The other points are obtained by finding different velocities to obtain Q at different stages. These can change as watersheds change due to land use and channel types" (pg. 60)

Ponce (1989)

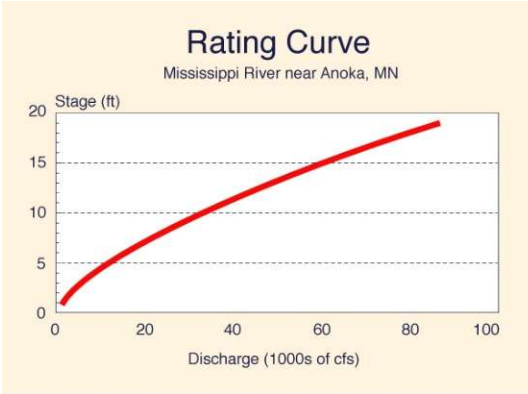
Rainfall-runoff relations

- The difference between "input" and "output" has led to **rainfall data** being more widely available than **runoff data**
  - Typical watershed has more rain gauges than stream-gauging stations
  - Rainfall records likely longer than streamflow records
- Thus, runoff typically calculated using rainfall data

Ponce (1989)

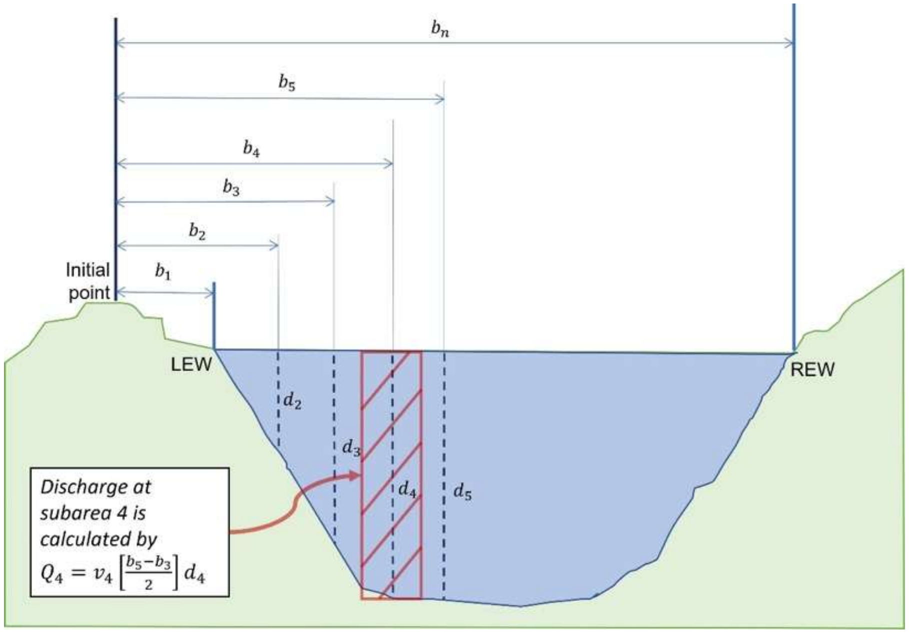
Discharge measurement

- A **rating curve** comes from measurements of stage and discharge multiple times per year
  - Measurements used to "fit" the rating curve
  - Adjustments are made for changes in riverbed, vegetation, flow conditions



Ponce (2014), Fig. 3-7

- Measurement approach:



https://images.app.goo.gl/JSfFG9TDc3H7WVME9

For each rectangle, the discharge is,

$$Q_i = v \cdot A$$

- Q_i = discharge of the ith section [m³/s]
- v = mean velocity [m/s]
- A = cross-sectional area [m²]

Total discharge = $\sum Q_i$ values [m³/s]

Stage measurement

- Water levels are measured by **water level sensors:** https://youtu.be/bHxEXlIHSHY?si=7143XyVPh38RovuV
- Using **telemetry,** water levels can be reported back/online in hours

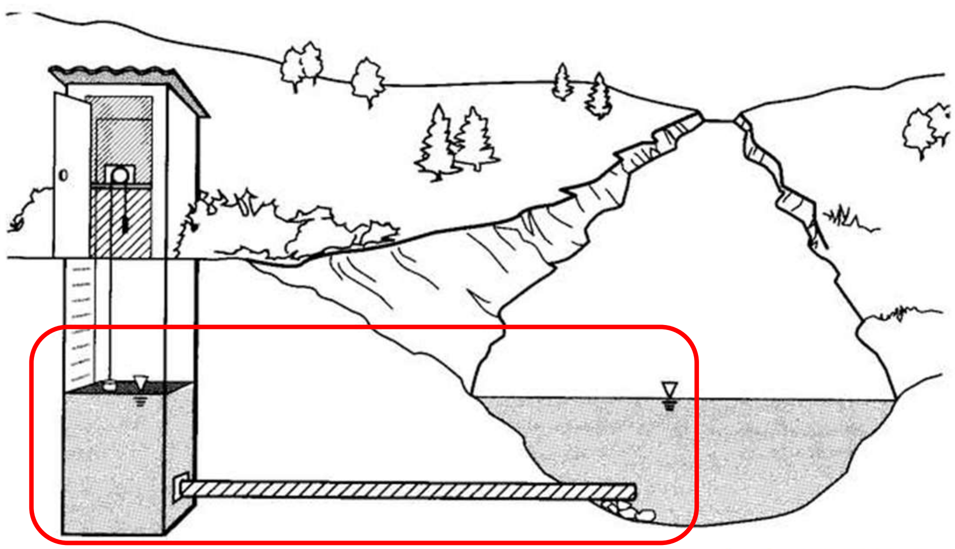
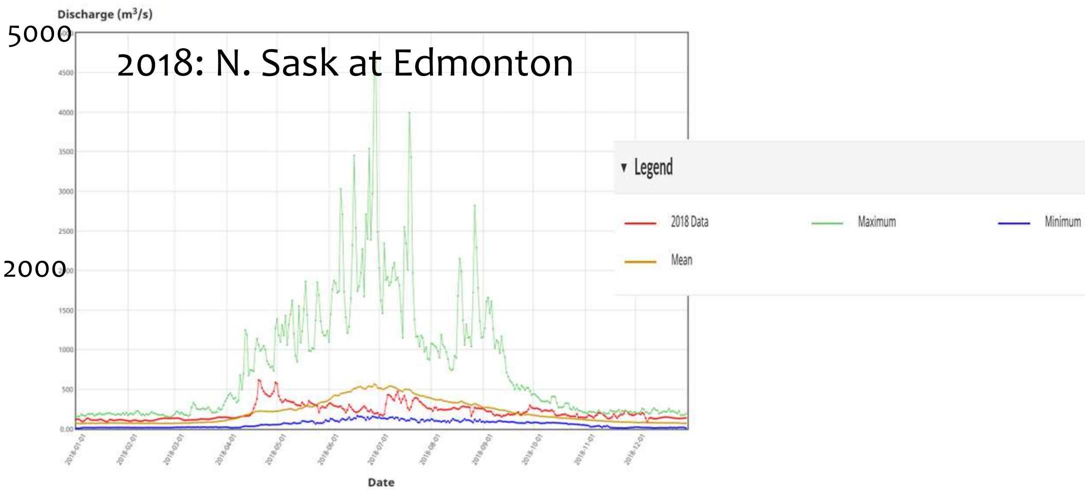


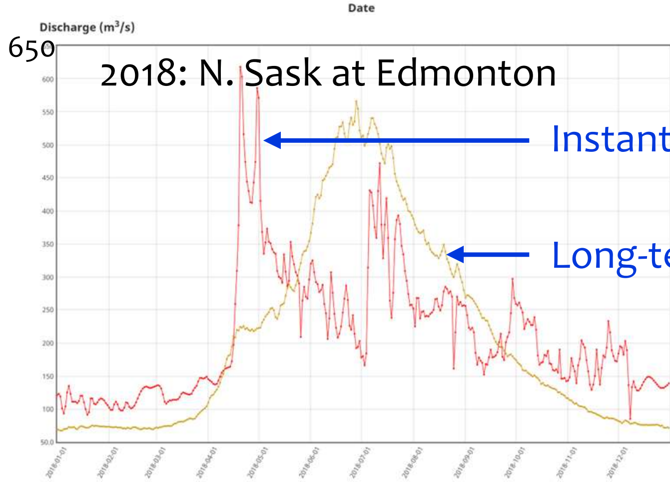
Figure 3.9: Schematic of stream water level monitoring station (after Fig. 3.17, Sanders, 1998). The configuration shown is known as a ``Stilling well'', most stations simply have a PVC tube in place of the well.

https://www.usgs.gov/media/images/usgs-streamgage-station-13295000-valley-creek-stanley-idaho

Runoff measurements

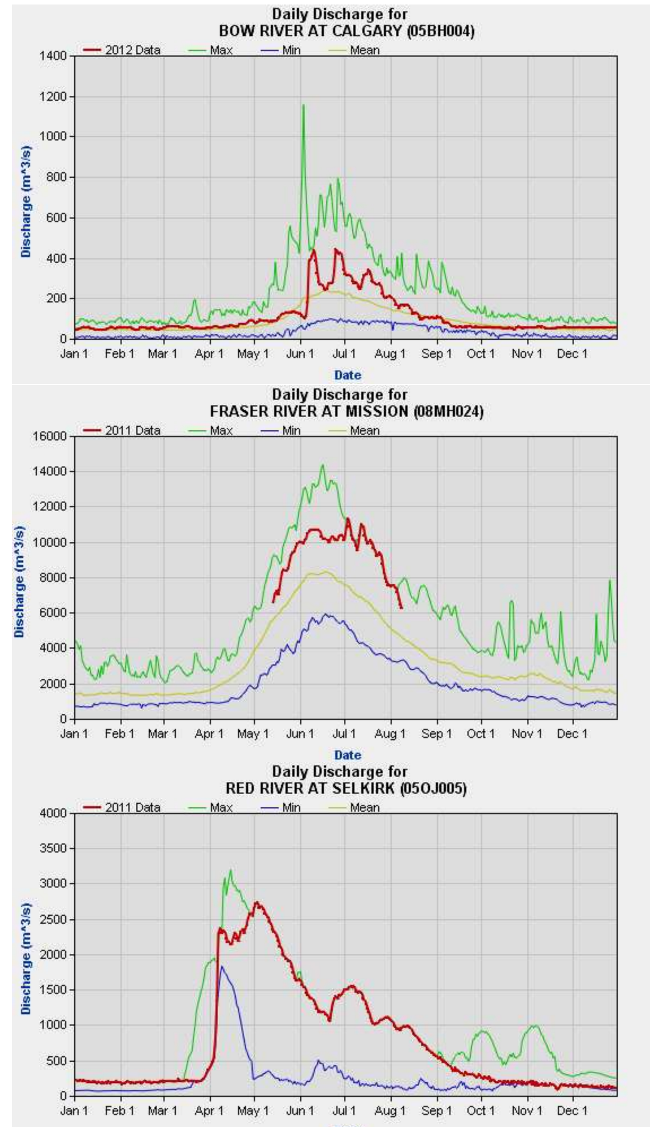
- Runoff expressed in terms of volume or flow rate
  - Runoff volume: m^3 or ft^3
  - Flow rate: m^3/s (cms) or ft^3/sec (cfs)





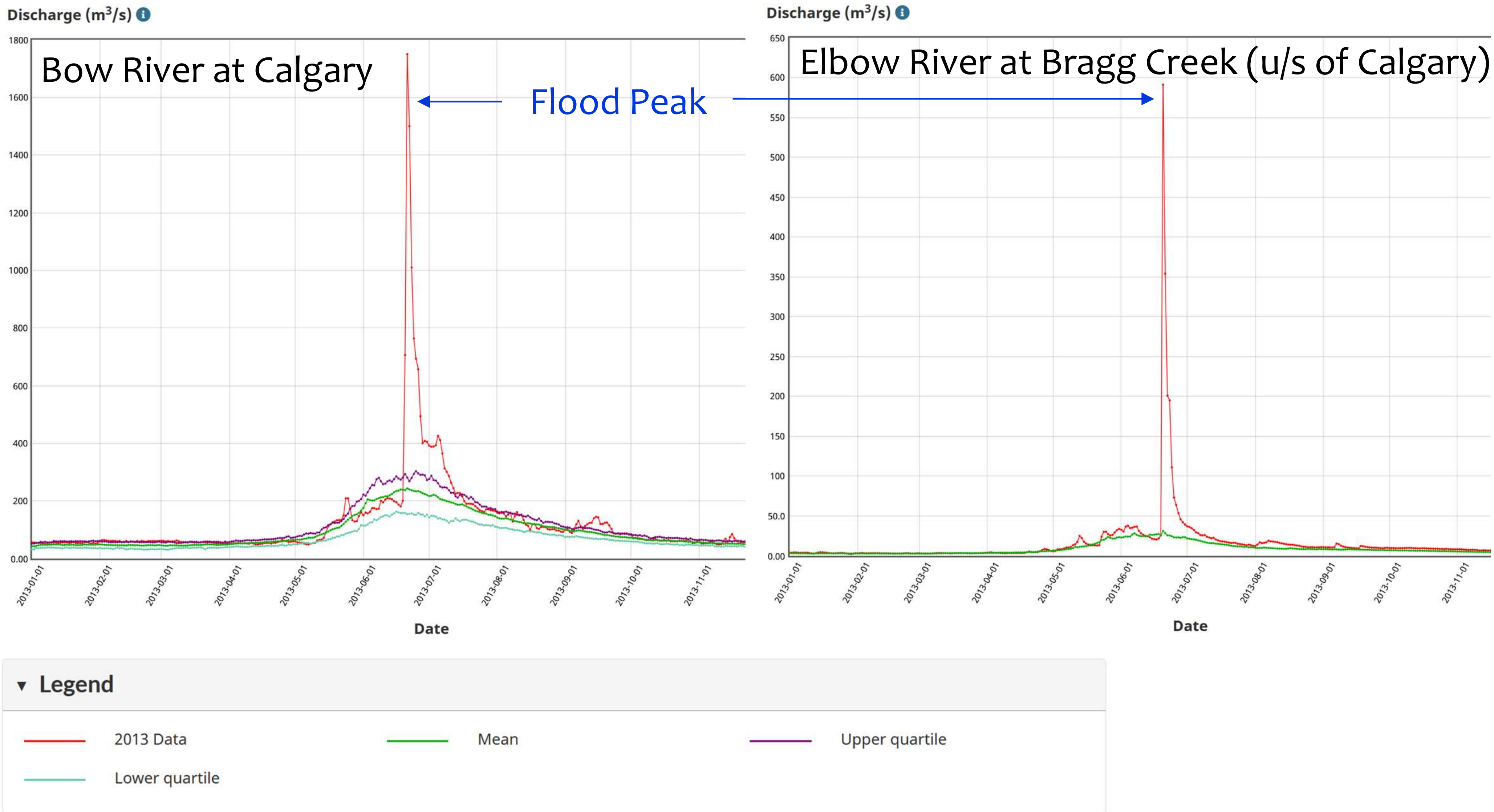
Instantaneous value (2018)

Long-term average value



Water Survey of Canada,
http://www.wsc.ec.gc.ca/applications/H2O/index-eng.cfm

The Calgary Flood, 2013

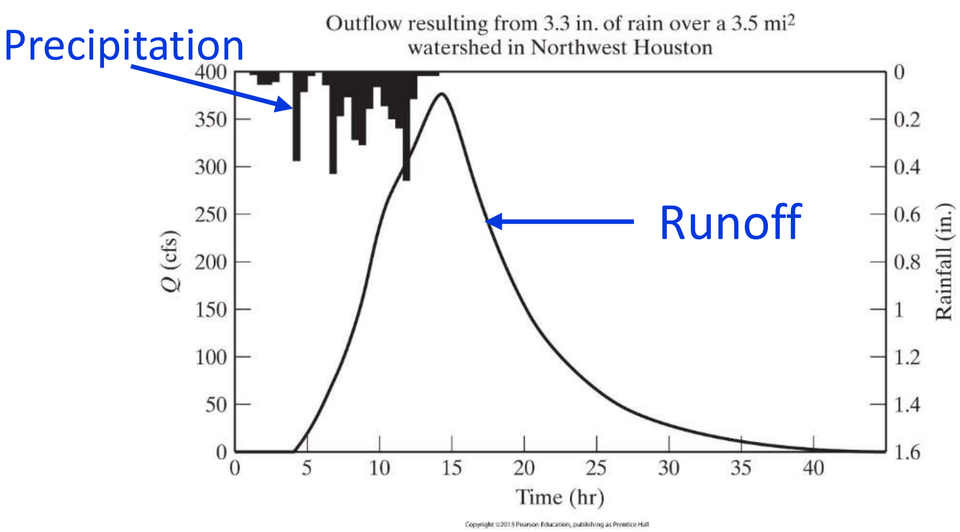


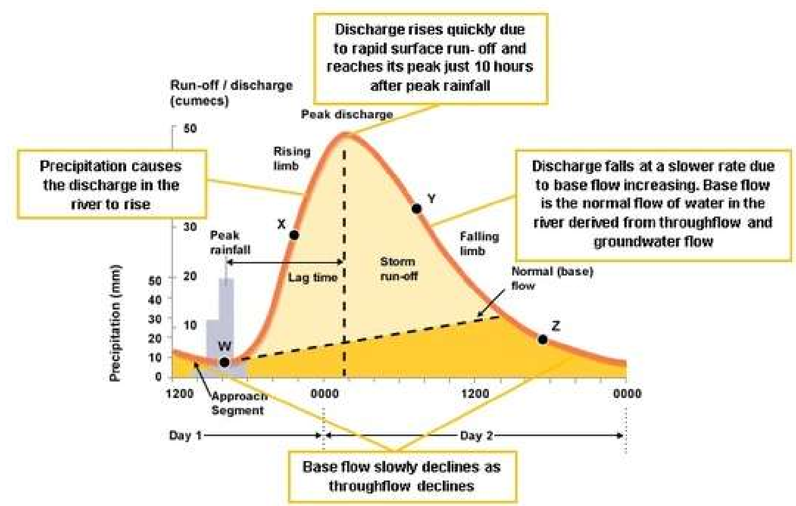
You can access these data from the *Water Survey of Canada*:
https://wateroffice.ec.gc.ca/search/historical_e.html → Search by "Station" (city)
Try "poking around" to see what you can find. Also try https://rivers.alberta.ca

3. The Hydrograph

"The hydrograph is a plot of flow rate vs. time for a given location within a stream and represents the main hydrological response function" (Bedient et al. 2019: 74)

- Discharge from rainfall excess, after infiltration losses subtracted, is the **direct-runoff hydrograph (DRO)**
- **Total storm hydrograph** includes both direct runoff and **baseflow**

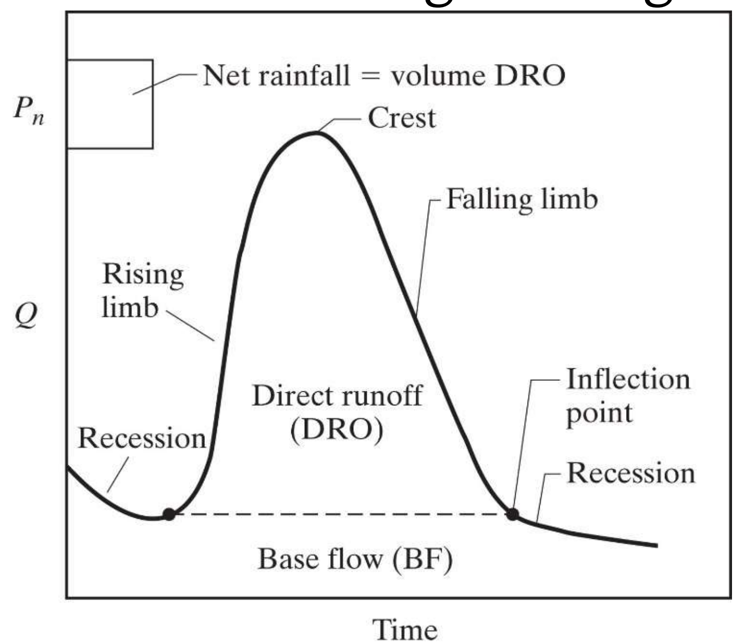




http://thebritishgeographer.weebly.com/hydrographs-recurrence-intervals-and-drainage-basin-responses.html

Hydrograph terminology

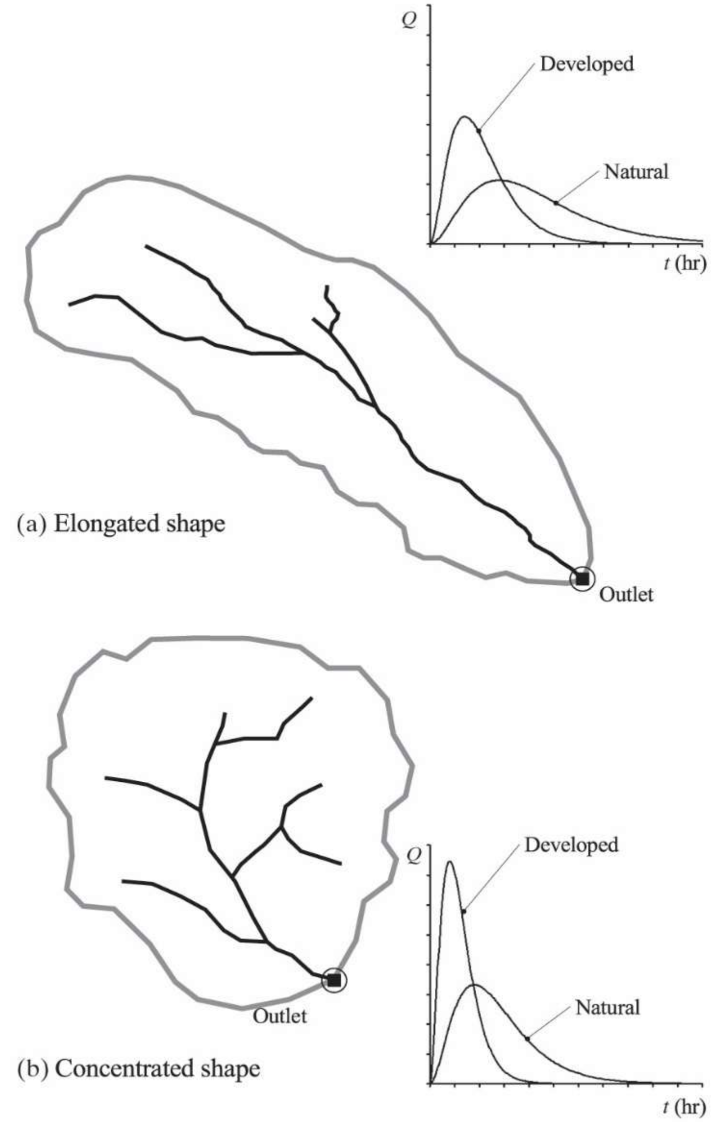
- Typical hydrograph has (1) **rising limb**, (2) **crest segment**, and (3) **recession curve**
  - **Inflection point** on falling limb often assumed to be point at which direct runoff ends
- **Rainfall excess**, P_n, obtained by subtracting infiltration losses from total storm rainfall
  - **DRO** represents hydrograph response of watershed to rainfall excess, P_n = *gross precipitation minus infiltration*
  - Shape and timing of DRO hydrograph related to duration and intensity of rainfall and factors governing watershed area



Bedient et al. (2019)

Hydrograph shapes

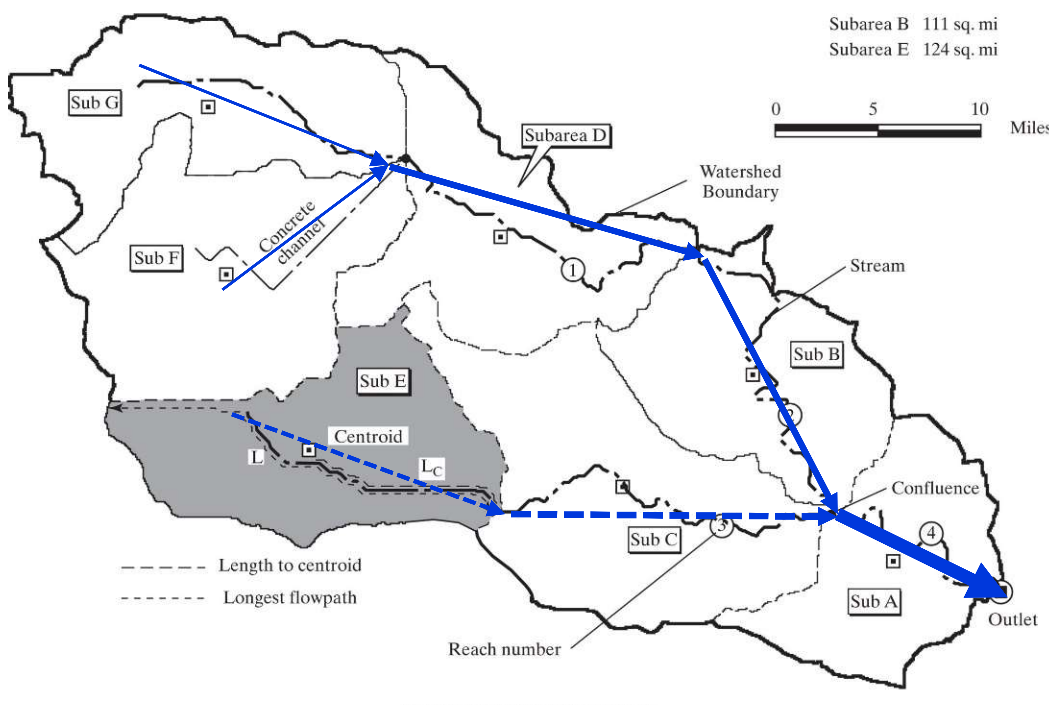
- For a given rainfall in watersheds of similar size, *hydrograph will vary based on*
  - **Watershed shape** and
  - **Land use**
- As watershed urbanizes, response increases in peak flow, decreases in time to peak
  - Result of increased impervious surfaces, greater channel density
- “Hydrology aims to understand and provide tools to predict these relationships…”



Bedient et al. (2019: 45)

Runoff timing

- A typical watershed area (below) receives rainfall input
  - Runoff makes its way from **west to east**, and flows out at the outlet
  - Note that meteorological-, watershed physical-, and human factors (land use) all contribute to response
  - Subareas G and F flow out through area D, then through B, and finally through A, while E flows through C to the confluence at A
- *Contributions from each subarea reach outlet at different times!*

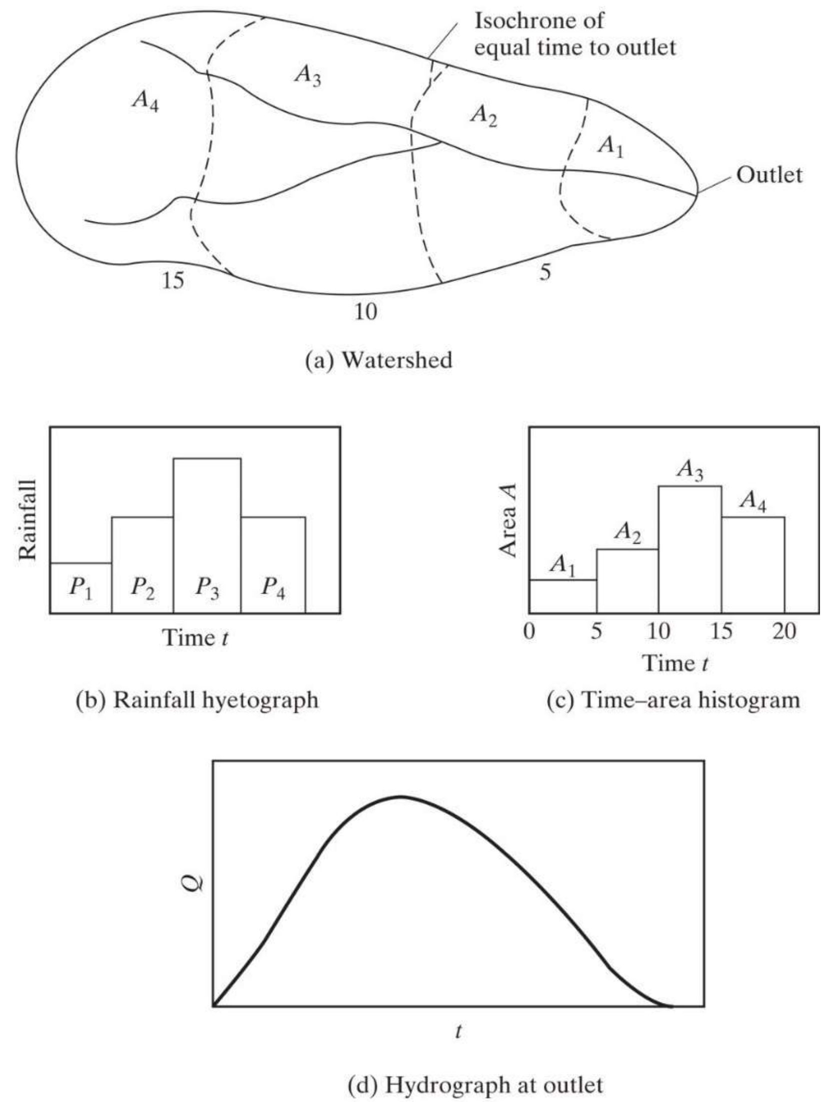


Bedient et al. (2019)

Time-area histogram

- Hydrograph can be built from *contributions from different subareas* of a watershed
  - Subareas chosen that represent equal travel times to the outlet → ***isochrones***
  - Complex rainfall events can then be analyzed by computing products of rainfalls, P_i, and areas, A_i
    - Runoff arrives at outlet first from A_1, then A_2, A_3...
    - Further, rainfall from period P_1 falling on A_2 *arrives at same time* as rainfall from P_2 falling on A_1, which together produce outflow Q_2

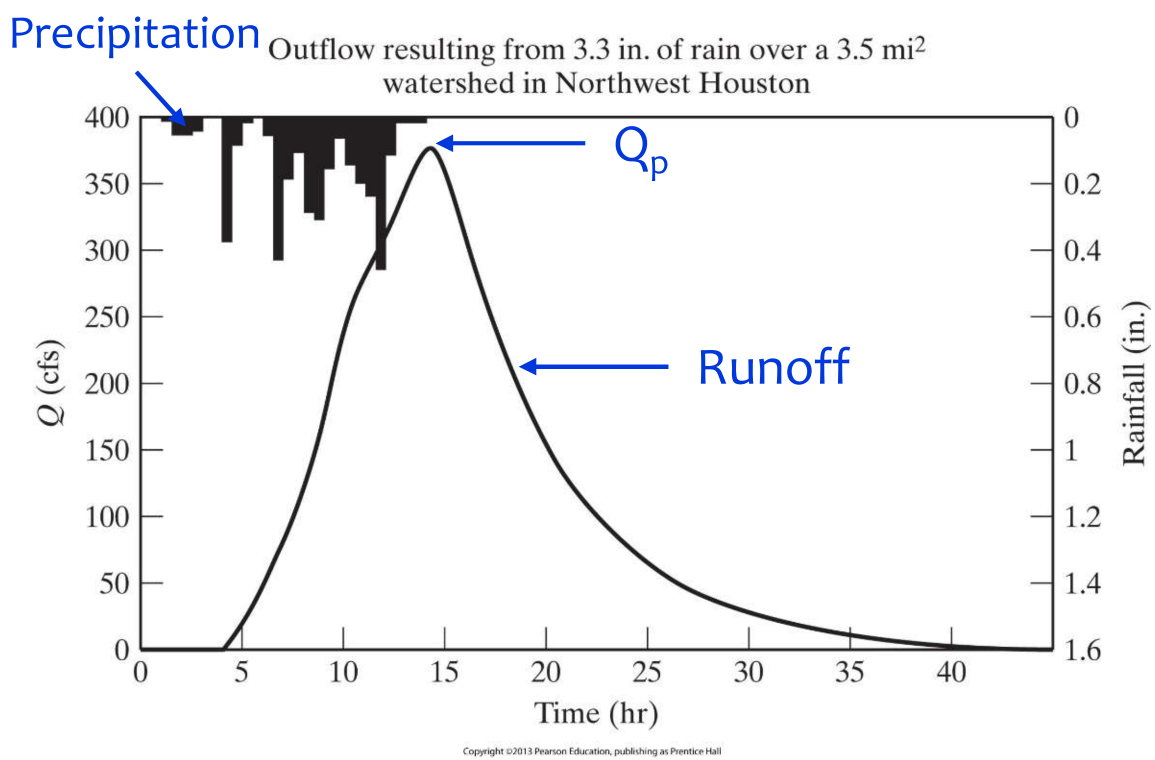
Key Concept!



Bedient et al. (2019)

Precipitation and runoff timing

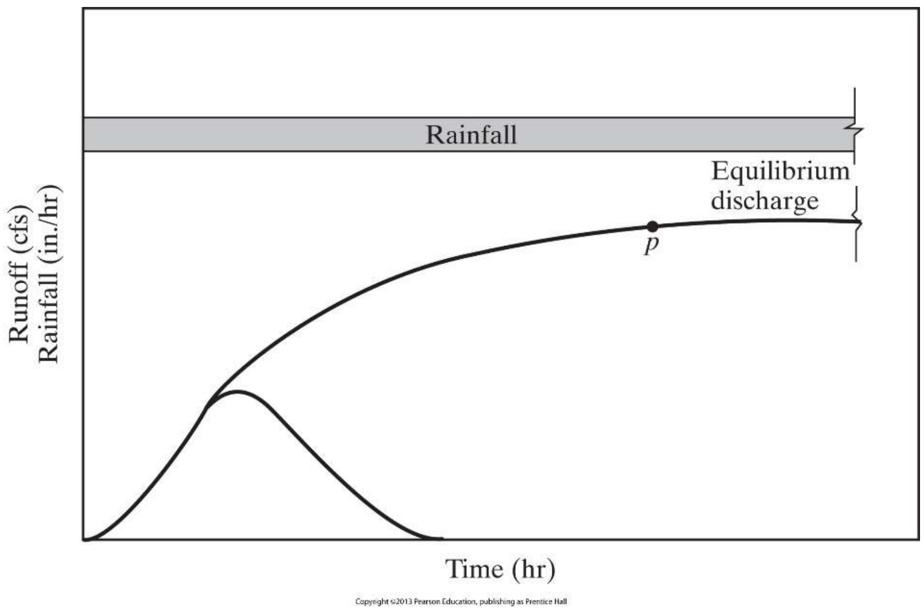
- ***Duration of rainfall** is usually much shorter than time-base of hydrograph*
  - Example below shows **rainfall** and **runoff** for Little Cypress Creek, Houston
  - Rainfall event lasted ~15 hours, *but…*
  - Hydrograph rises to peak flow in ~15 h, and recedes to zero flow after 40 hours
  - Early part of rainfall event infiltrated into soil, so that ~50% of rainfall became direct runoff



Bedient et al. (2019)

Equilibrium hydrographs

- Also possible to have uniform rainfall over long period falling over small watershed
  - Rainfall continuing at constant intensity for very long period fills storage
  - Then **equilibrium discharge** can be reached: *inflow = outflow*
  - Rarely occurs in nature, except for very small basins or parking lots
  - *Will see this concept again with* ***unit hydrographs***

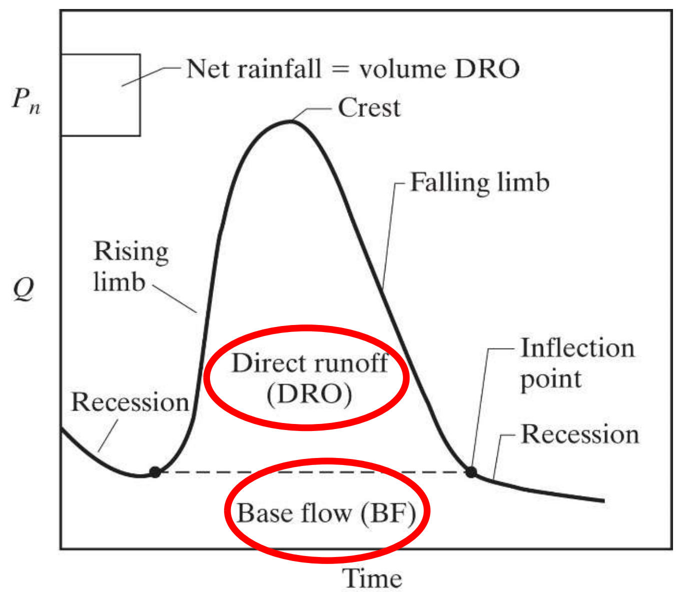


Note also **time of concentration:** time at which entire discharge area contributes to flow (labelled ***p***)

Bedient et al. (2019)

Recession & baseflow separation

- Customary to consider hydrograph as *divided into two parts*: **direct runoff** (DRO) and **baseflow** (BF)
  - DRO may contain some interflow
  - BF considered to be from groundwater (fraction from BF a few percent for urban streams, but can be significant for larger rivers)



Bedient et al. (2019)

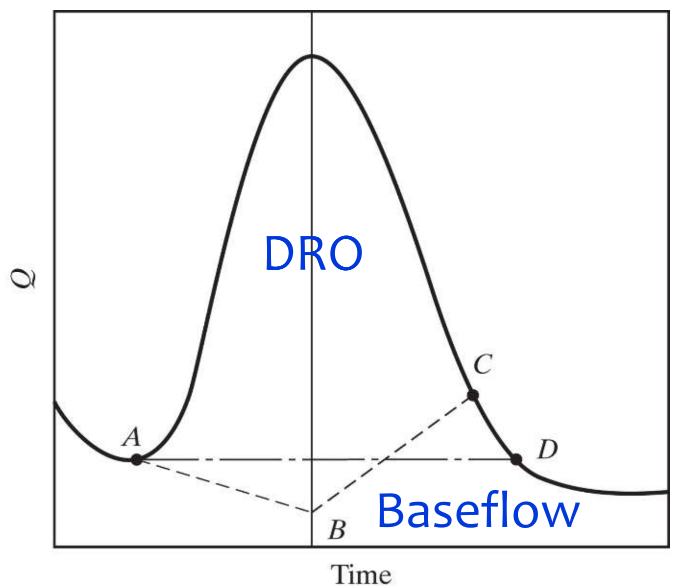
Recession & baseflow separation

- Several techniques available for separating DRO from baseflow, based on analysis of groundwater recession curves
  - One simple form of **recession curve** is an **exponential:** $q_t = q_0 e^{-k}$, where q_0 is the specified initial discharge, q_t is discharge at later time, t, and k is the recession constant
    - Equations of this form often used in engineering to describe first-order decay or depletion → will plot as straight-line depletion on semi-logarithmic paper
    - *Difference between this curve and the total hydrograph plotted on same paper represents the DRO*
  - Several storms should be used to develop this recession curve

Bedient et al. (2019)

Recession & baseflow separation

- Simpler methods also available:
  - Base flow recession can be extended under peak of hydrograph to a point on recession limb (see **line AD**)
  - Concave method extends baseflow under peak of hydrograph and then to inflection point on recession curve (see **line ABC**)
  - All methods are arbitrary and somewhat inaccurate



Bedient et al. (2019)

Storm hydrograph appearance

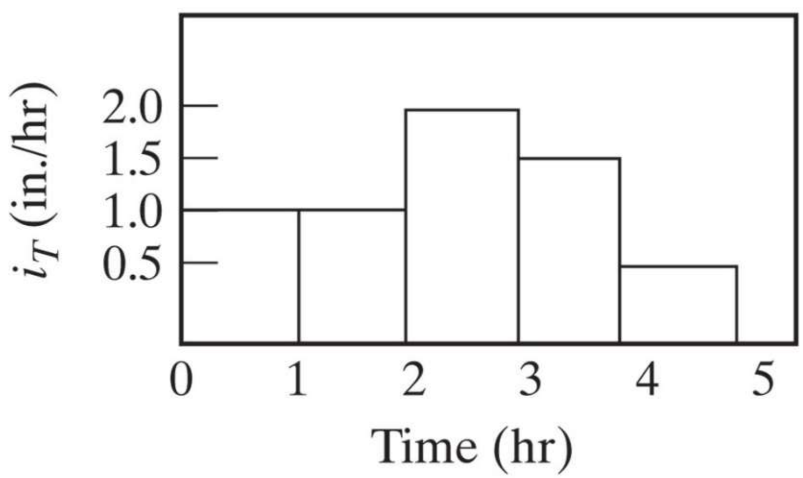
S. Gough (2008), "*Complete storm hydrograph*", https://youtu.be/hJftAYYXpVw

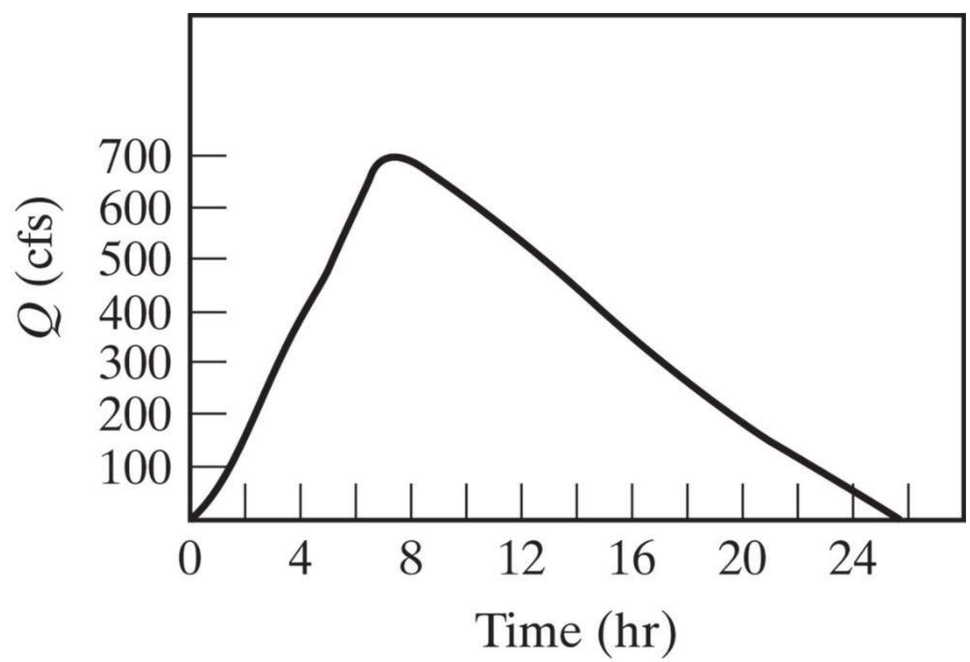
Ex 1: Net rainfall, volume, and area

Rain falls as shown in the rainfall hyetograph below. The ϕ index for the storm is 0.5 in/h and is constant over 5 hours.

(i) Plot the net rainfall on the hydrograph.

(ii) Determine the total volume of runoff and (iii) the watershed area

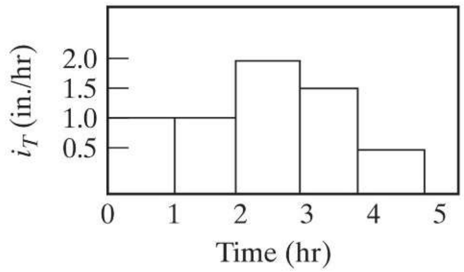


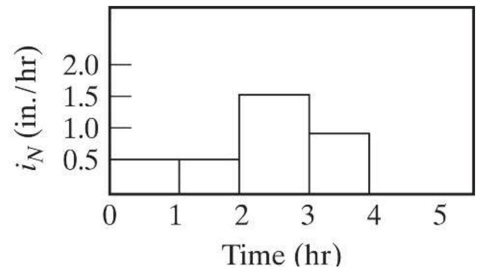


Example 1-6 in Bedient et al. (2019)

1. Develop the net rainfall hyetograph: rainfall – infiltration

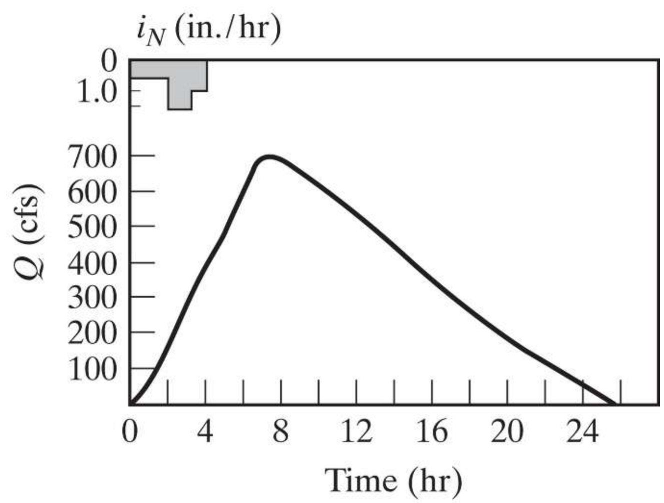
Don’t forget to find P_n!





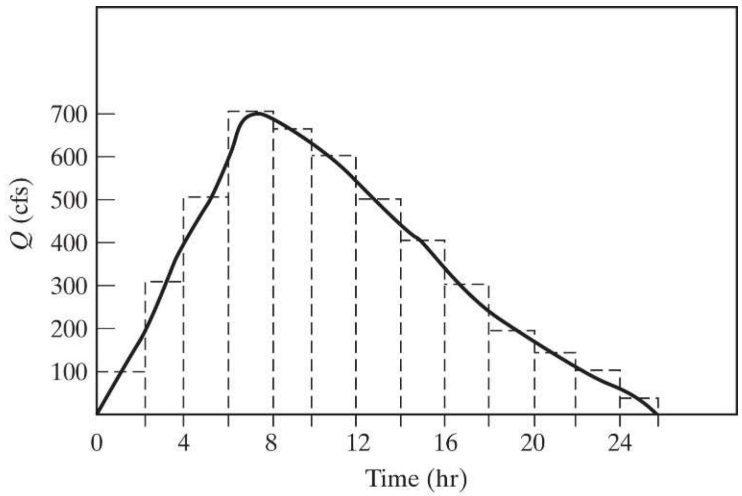
Φ = 0.5 in/h

2. Add graph this to hydrograph plot in *upper-left corner* → **rainfall excess** becomes 3.5 inches total with duration of 4 hours



Bedient et al. (2019)

Volume of runoff is equal to area under hydrograph.



To determine volume of runoff, use $\sum \bar{Q}\,dt$, where $\bar{Q}$ is average flow in cfs (ft^3/s) over time dt (hours)

Imperial units: cfs·hr = V/t·t

Time (hr)	$\bar{Q}$ (cfs)	Volume (cfs-hr)
0–2	100	200
2–4	300	600
4–6	500	1000
6–8	700	1400
8–10	650	1300
10–12	600	1200
12–14	500	1000
14–16	400	800
16–18	300	600
18–20	200	400
20–22	150	300
22–24	100	200
24–26	50	100

(ii) Volume = $\sum \bar{Q}\,dt = 9100$ cfs-hr
$\sum \bar{Q}\,dt$ = 9025 ac-in of runoff

(iii) Since P_n = 3.5 in and $V = AP_n$, the area, A = 2580 acres

4. Unit Hydrographs (UH)

A unit hydrograph is the ***"basin outflow** resulting from 1 mm (1 inch)* [*i.e. one UNIT*] *of **direct runoff** generated uniformly over the drainage area at a uniform rainfall rate during a specified period of rainfall duration"* (Bedient et al. 2019: 77)

- Concept developed by Sherman (1932)
- Important points:
  - UH is composed of 1 mm of direct runoff (DRO), equivalent to 1 mm of net rainfall (the *rainfall excess*)
  - All infiltration losses subtracted before computations

Unit hydrograph assumptions

- Assumptions of the UH approach tend to limit its application for a given watershed:
  1. *Rainfall excesses of equal duration* are assumed to produce hydrographs with equivalent time bases, *regardless of the rainfall intensity*
  2. Direct runoff flows (*ordinate or y-axis*) for a storm of given duration are assumed directly proportional to rainfall excess volumes – i.e. 2 x *rainfall* → 2 x *hydrograph ordinates*
  3. The time distribution of direct runoff is assumed independent of *antecedent precipitation*
  4. *Rainfall distribution* is assumed to be the same for all storms of equal duration, both spatially and temporally

Bedient et al. (2019)

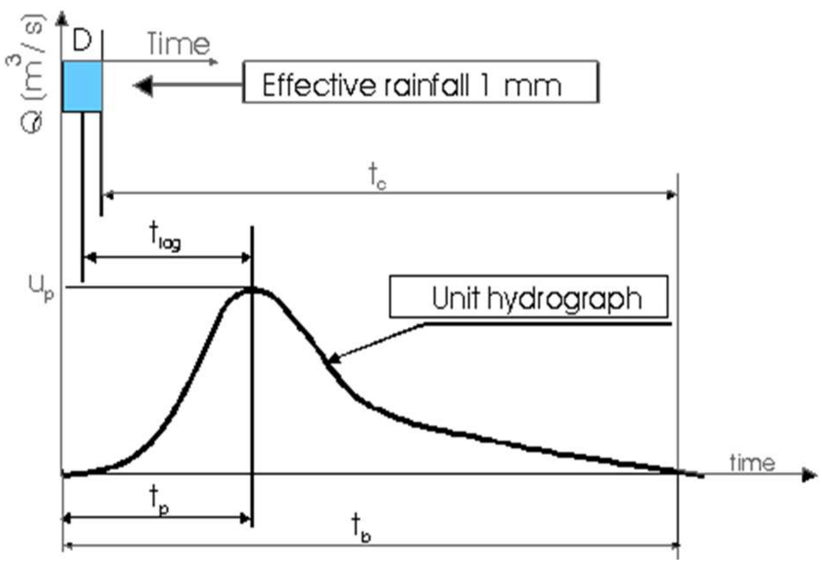
Unit hydrograph assumptions

- Unit hydrograph theory assumes the hydrological system is linear and time-invariant
  - **Linear:** complex storm hydrographs can be produced by adding up individual unit hydrographs, adjusted for rainfall volumes, and lagged appropriately in time
    - E.g. **2-mm rainfall** in x hours will produce double the response of **1 mm** in x hours
  - **Time-invariant:** climate predictable and unchanging, as is the watershed itself
  - *So, the theory is wrong, strictly-speaking, but is useful in many cases*

Bedient et al. (2019)

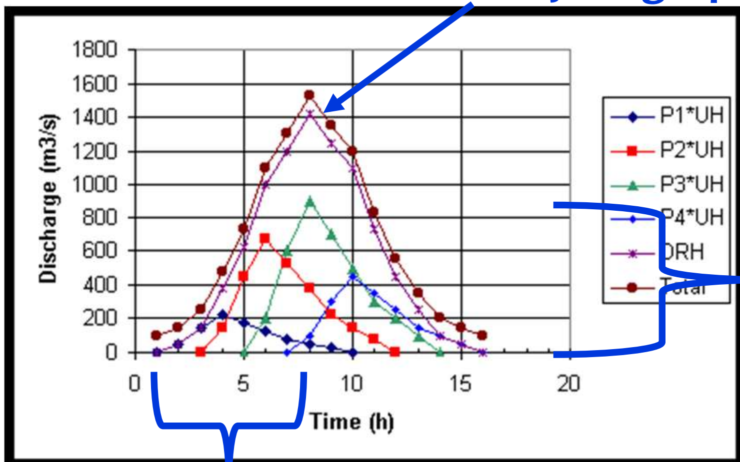
Unit hydrograph use

- UH's useful for a **number of hydrological calculations.**
- Can be developed for almost any area,
  - Where UH methods have been developed, or
  - Where general methods such as SCS curves can be applied.
- For a single basin, the UH can be used with a given storm event to find a **storm hydrograph** through an **'add and lag' procedure**



http://echo2.epfl.ch/VICAIRE/mod_1b/chapt_4/main.htm

Storm hydrograph!



Add all these smaller hydrographs to get…

Notice different start times → "lag"

http://www.engr.colostate.edu/~ramirez/ce_old/classes/cive322-Ramirez/CE322_Web/Example_UnitHydrographs.htm

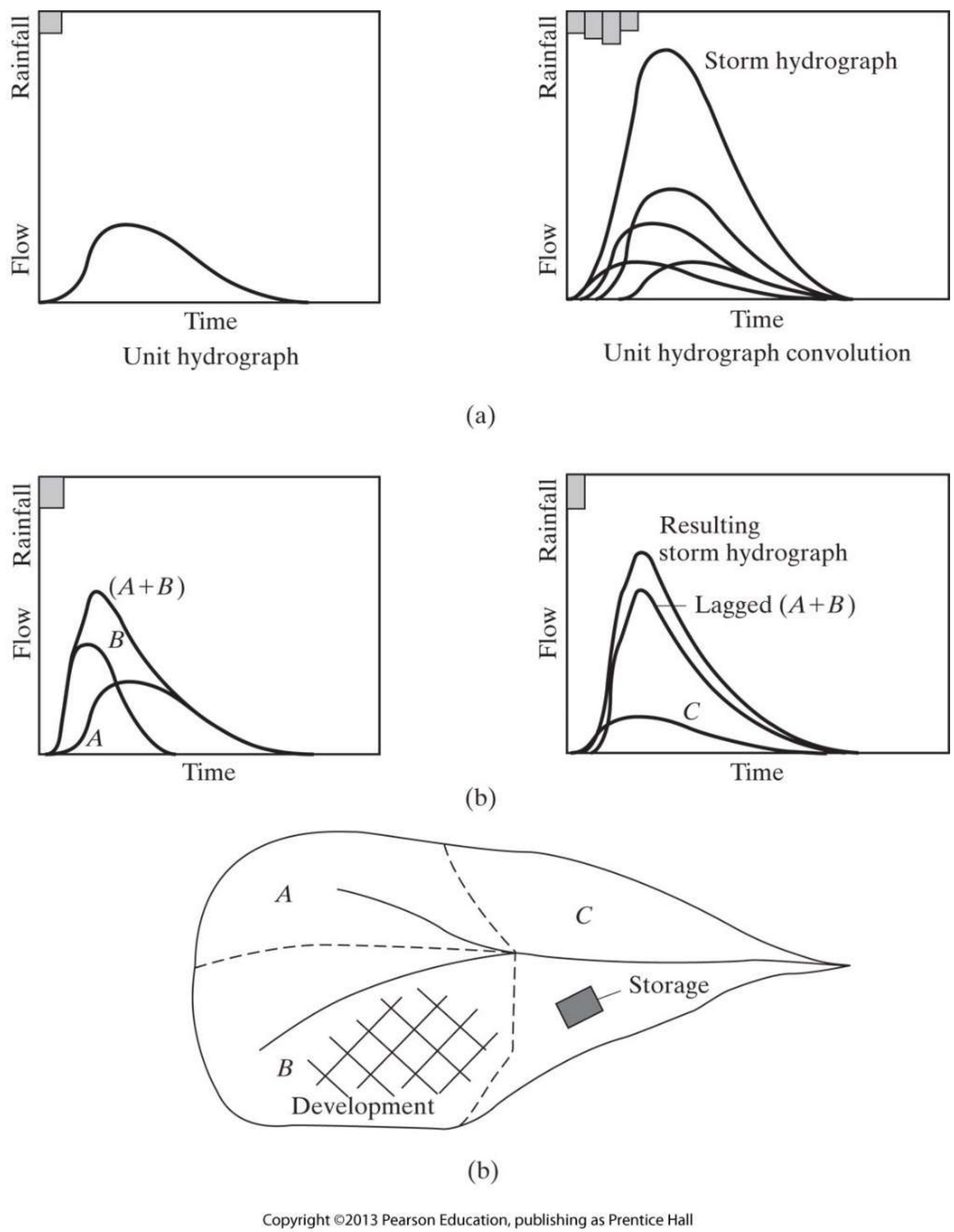
Bedient et al. (2019: 102)

Unit hydrograph use

- In particular,
  1. "Design storm hydrographs for selected recurrence-interval storms (10-yr, 25-yr, 100-yr) can be developed through *convolution procedures* (adding and lagging) for a given watershed area
  2. Effects of land use changes, channel modifications, storage additions, and other variables can be tested to determine changes in the UH
  3. Storm hydrographs for each subbasin can be simulated by adding, lagging, and routing the flows (*routing not covered in Civ E 321*) produced by the unit hydrographs through channel reaches. Effects of various rainfall patterns and land use distributions can be tested on overall hydrological response of the large watershed
  4. Storage routing methods (*not covered in Civ E 321*) can be used to translate inflow UHs through a reservoir or detention basin (e.g. stormwater pond) of particular size to attenuate peak flow or lag time to the peak of the hydrograph. Used for flood analysis."
  5. Note: One-hour UHs are useful for learning and are commonly used in practice. For urban hydrology or small watersheds, can also use 10-min or 30-min UHs, etc.

Bedient et al. (2013: 103)

Unit hydrograph use



Applications:

(a) Development of design storm hydrograph

(b) Development of watershed hydrograph

Convolution → 'add and lag':
In mathematics, convolution is a mathematical operation on two functions (f and g) that produces a third function ($f * g$), expressing how the shape of one is modified by the other.

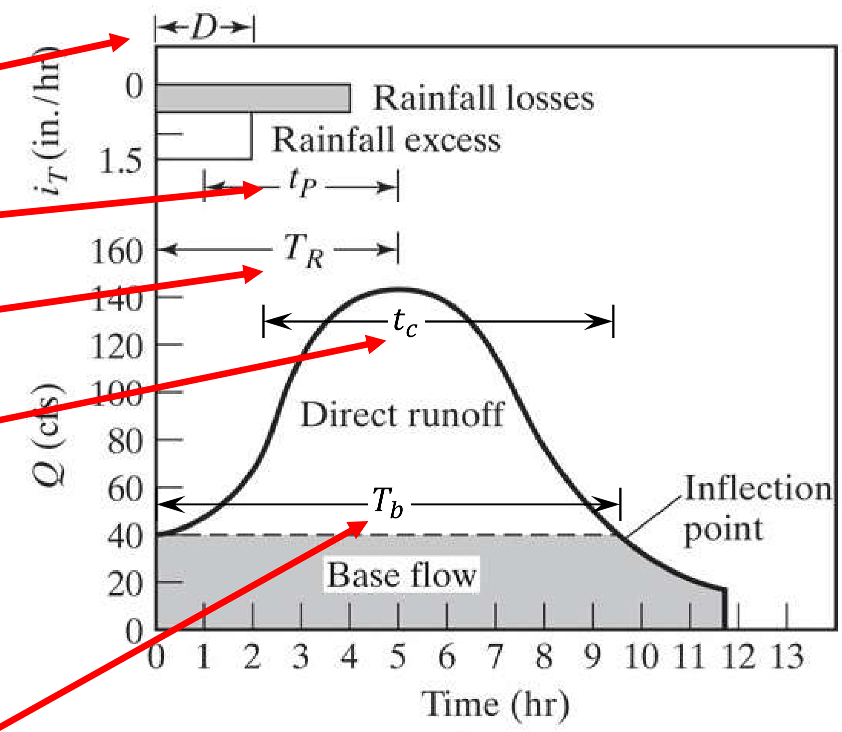
The term *convolution* refers to both the result function and to the process of computing it.

https://en.wikipedia.org/wiki/Convolution
(Yes, I know... Wikipedia…)

Figure 2-9 in Bedient et al. (2019)

Unit hydrograph characteristics

- Figure below shows typical storm hydrograph for a watershed
  - Total storm hydrograph is simple plot of flow vs. time (cfs or cms)
  - Key characteristics include
    - Rising limb, crest segment, recession curve
  - Timing aspects include,
    - **Duration** of rainfall excess: time from start to finish of rainfall excess
    - **Lag time** (t_p): time from centre of mass of rainfall excess to peak of hydrograph
    - **Time of rise** (T_R): time from start of rainfall excess to peak of hydrograph
    - **Time of concentration** (t_c): time for wave of water to propagate from most distant point in watershed to outlet. Can estimate as time from end of net rainfall to inflection point of hydrograph
    - **Time base** (T_b): total duration of the DRO hydrograph



Bedient et al. (2019)

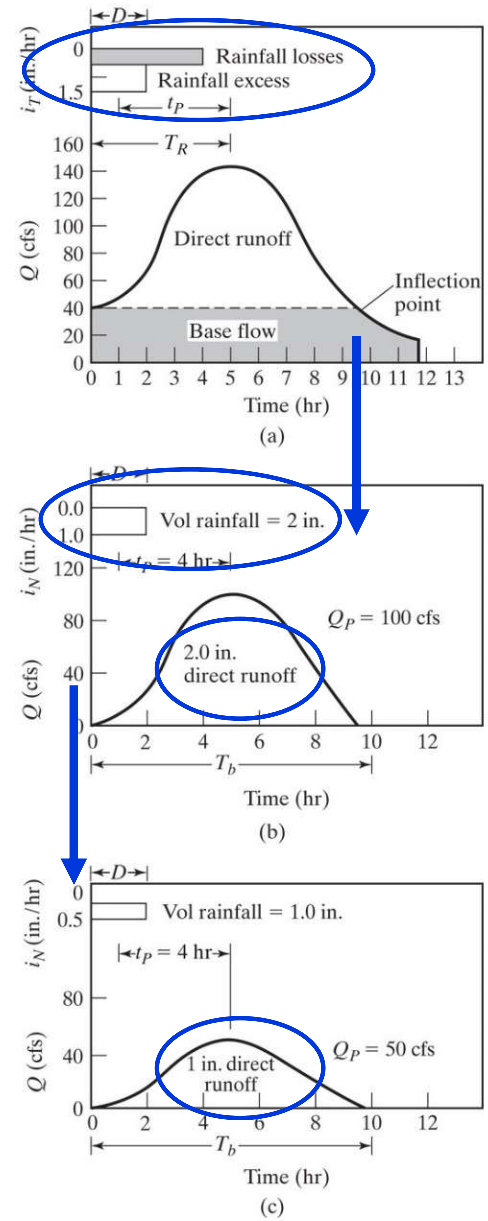
Deriving unit hydrographs

- The following **general rules** are for developing unit hydrographs in gauged watersheds:
  1. Select simple storms – uniform spatial and temporal rainfall distributions
  2. Watershed size between 1 mi^2 and 100 mi^2
  3. Direct runoff between 0.5 to 2 in
  4. Duration of rainfall excess ~25% to 30% of lag time, t_p
  5. Average unit hydrograph for particular duration should be developed from a number of storms of similar duration
  6. Repeat step 5 for several rainfalls of different duration

Bedient et al. (2019)

Deriving unit hydrograph

- To develop a specific unit hydrograph from a single storm hydrograph,
  1. Analyze storm hydrograph and **separate baseflow**
  2. Measure **total volume of DRO** under the hydrograph and convert to mm (inches) over the watershed
  3. Convert total rainfall to **rainfall excess** through infiltration methods. Then rainfall excess = DRO. Determine **duration of rainfall excess** that produced DRO hydrograph
  4. **Divide ordinates of DRO hydrograph** by volume in mm (inches), and plot results as UH for the watershed. Time base, T_b, assumed constant for storms of equal duration and so does not change
  5. Check volume of unit hydrograph to make sure it is 1.0 mm (1.0 inches), and graphically adjust ordinates as required



Bedient et al. (2019)

A few notes of caution…

- Linearity assumption in unit hydrographs means care must be used in applying UHs under conditions that tend to violate linearity. For example,
  - Large variations in intensity over long-duration storm
  - Significant storage effects in watershed
- If nonlinearity exists (from differences in interflow, channel flow times, storage effects…),
  - Derived UHs should be used only for generating similar-size events
  - *Caution should be used in extrapolating to extreme events*
- Larger watersheds (100 mi^2-500 mi^2 or 260 km^2-1280 km^2) should be divided into smaller subareas with their own UHs
  - Can add, or "convolute", and lag subarea UHs together
  - *Unit hydrographs should typically not exceed areas of 12-25 km^2*

Bedient et al. (2019)

4.1 UH Convolution

- **UHs** help in development of **storm hydrographs** from actual rainfall event over a watershed → this is its real value
  - Approach: UH ordinates, U_j, multiplied by rainfall excess, P_n, are added and lagged in a sequence
    - Calculations typically done in hydrological models, or "by hand" in Excel
  - Baseflow values can be added to produce storm hydrograph, if BF values available for the watershed
  - Time increments of rainfall excess need to correspond to duration of UH – e.g. 1-hr increments used with a 1-hr UH

Bedient et al. (2019)

Unit hydrograph convolution

- The **governing equation** is,

$$Q_n = \sum_{i=1}^{n} P_i U_{n-i+1}$$

Or, $Q_n = P_n U_1 + P_{n-1} U_2 + P_{n-2} U_3 + \cdots + P_1 U_j$

where Q_n is the storm hydrograph ordinate (*note, need n of these to make a hydrograph!),* P_i is rainfall excess, U_j is UH ordinate ($j = n - i + 1$)

- It's important to understand the subscripts here. Bedient uses the i and n subscripts to mean the *end* of a time interval.
  - P_1 means the rain that fell from t > 0 to t = 1, or t = (0,1].
  - U_1 means the flow over the interval t > 0 to t = 1, or t = (0,1].

Bedient et al. (2019)

Unit Hydrograph Convolution

- This is the form of the equation for a 4-hour storm and a 7-hour UH, starting with P_1 and U_1:

$$
\begin{aligned}
Q_1 &= P_1U_1,\\
Q_2 &= P_2U_1 + P_1U_2,\\
Q_3 &= P_3U_1 + P_2U_2 + P_1U_3,\\
Q_4 &= P_4U_1 + P_3U_2 + P_2U_3 + P_1U_4,\\
Q_5 &= P_4U_2 + P_3U_3 + P_2U_4 + P_1U_5,\\
Q_6 &= P_4U_3 + P_3U_4 + P_2U_5 + P_1U_6,\\
Q_7 &= P_4U_4 + P_3U_5 + P_2U_6 + P_1U_7,\\
Q_8 &= P_4U_5 + P_3U_6 + P_2U_7,\\
Q_9 &= P_4U_6 + P_3U_7,\\
Q_{10} &= P_4U_7.
\end{aligned}
$$

- *Note that Q_0 = 0, so we start with P_1 and U_1, and don't worry about U_0, since U_0 = 0 anyway…*

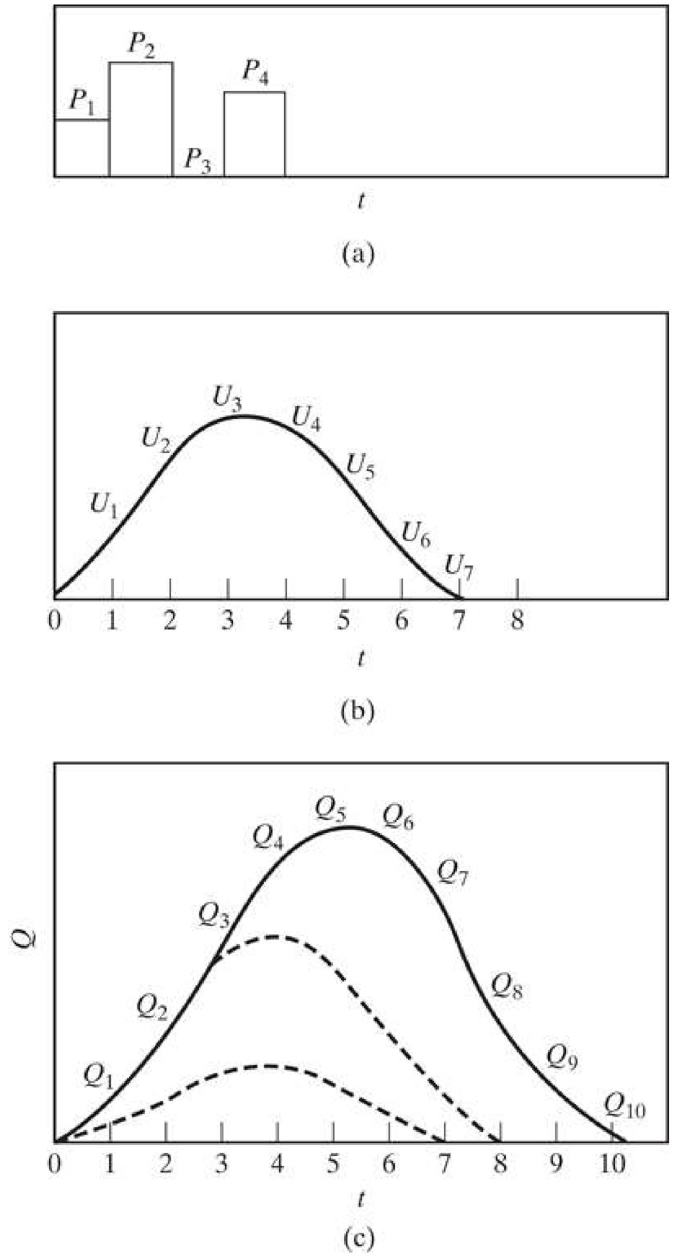


Figure 2-4

Ex 2: Storm hydrograph from UH

Given a rainfall excess hyetograph of P_n = [0.5, 1.0, 1.5, 0.0, 0.5] in, and a 1-hr UH of U_n = [0, 100, 320, 450, 370, 250, 160, 90, 40, 0] cfs, derive the storm hydrograph for the watershed using hydrograph convolution. Compute the resulting storm hydrograph and assume no losses to infiltration or evapotranspiration.

$P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5$

<u>Solution:</u> P_n = [0.5, 1.0, 1.5, 0.0, 0.5] in,

U_n = [0, 100, 320, 450, 370, 250, 160, 90, 40, 0] cfs

$U_0 \quad U_1 \quad U_2 \quad U_3 \quad U_4 \quad U_5 \ldots$

Using $Q_n = P_n U_1 + P_{n-1} U_2 + P_{n-2} U_3 + \cdots + P_1 U_j$,

$Q_0 =$

$Q_1 =$

$Q_2 =$

$Q_3 =$

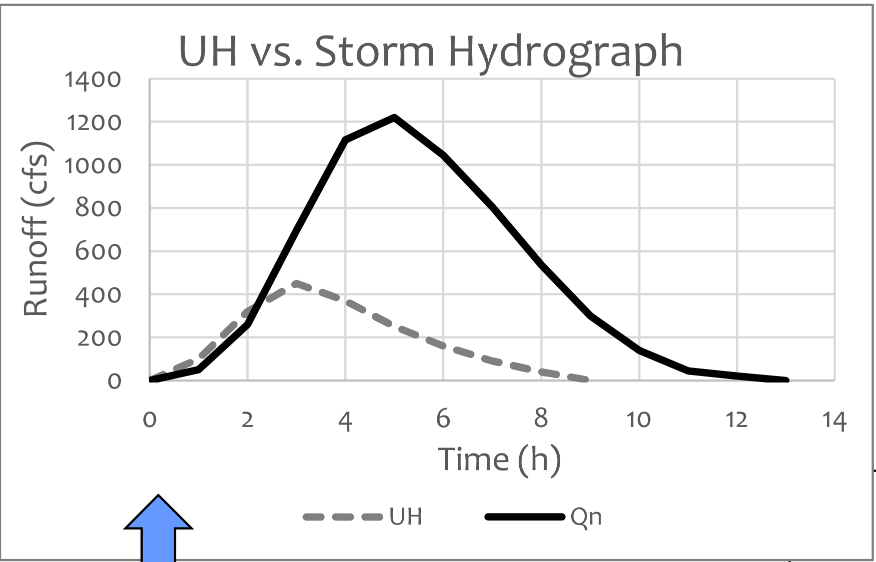
These values can be collected into a **table**:

Table E2–2

Time (hr)	P_1U_n	P_2U_n	P_3U_n	P_4U_n	P_5U_n	Q_n
0	0					**0**
1	50	0				**50**
2	160	100	0			**260**
3	225	320	150	0		**695**
4	185	450	480	0	0	**1115**
5	125	370	675	0	50	**1220**
6	80	250	555	0	160	**1045**
7	45	160	375	0	225	**805**
8	20	90	240	0	185	**535**
9	0	40	135	0	125	**300**
10		0	60	0	80	**140**
11			0	0	45	**45**
12				0	20	**20**
13					0	**0**

Example 2-2 in Bedient et al. (2019)

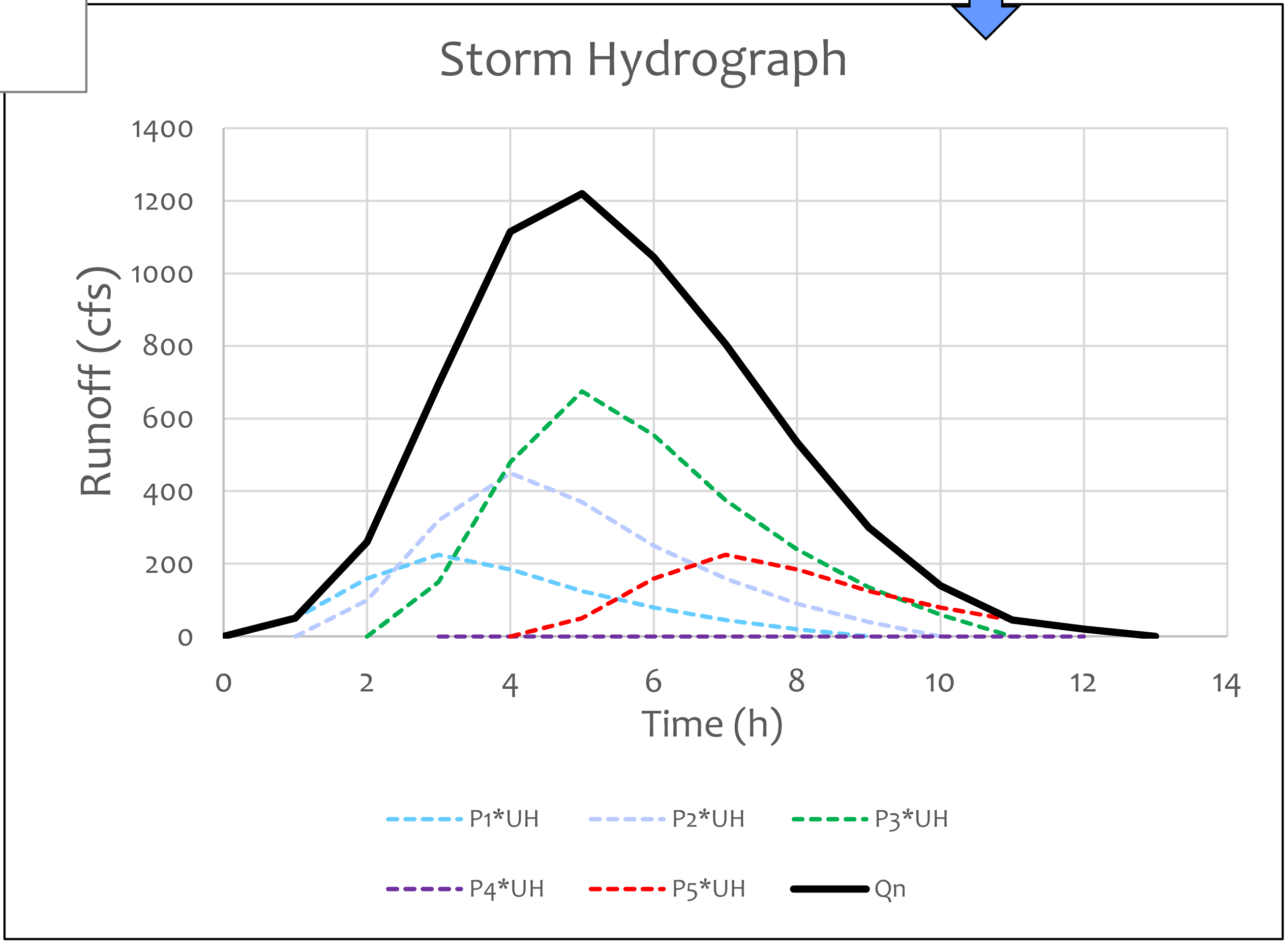
Visualizing the results…



1. Comparison of the UH and the storm hydrograph.
Note that the storm hydrograph is much higher.

- *The UH is the response to 1 inch of rainfall excess over 1 hour.*
- *The storm hydrograph is the response to 3.5 inches of rainfall excess over 5 hours*

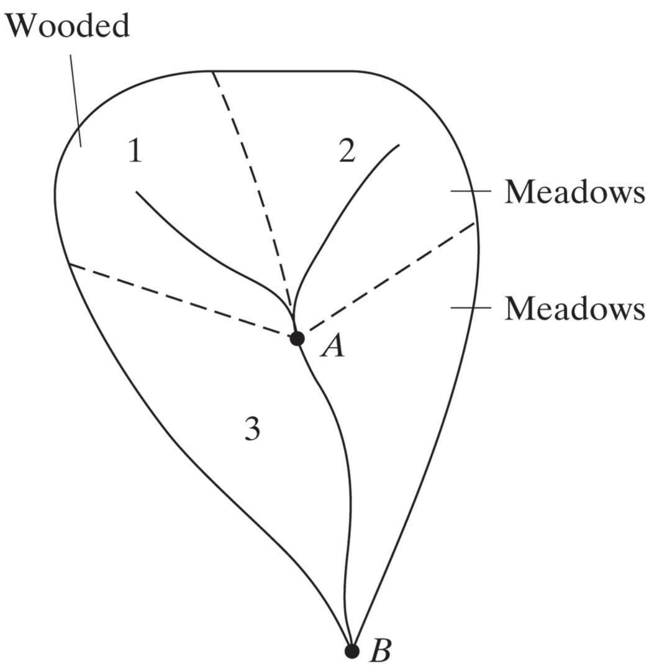
2. Effect of the convolution of all "scaled" Unit Hydrographs (by rainfall depth) to create the full storm hydrograph.
*Note how each scaled hydrograph (P_x*UH) starts slightly after the previous one → this is the "lagging" effect. The storm occurred over five hours!*
The "adding" effect comes from adding up the values of the coloured curves at each hour to get the value of the black line. Values are from the table on the previous slide



Ex 4: Storm hydrograph from UH

Problem 2.18 in Bedient et al. (2019)

Develop storm hydrographs from UH's of subareas 1 and 2 shown in the figure below for the given rainfall and infiltration.



Data provided:

t (hr)	i (in/hr)	f (in/hr)
1	0.5	0.4
2	1.1	0.3
3	3	0.2
4	0.9	0.1

t (hr)	0	1	2	3	4	5	6	7	8	9
UH_1 (cfs)	0	200	450	650	450	300	150	0		
UH_2 (cfs)	0	150	300	500	350	250	125	100	50	0

t (hr)	i (in/hr)	f (in/hr)	
1	0.5	0.4	
2	1.1	0.3	
3	3	0.2	
4	0.9	0.1	

Try graphing the UH's vs. the storm hydrographs!

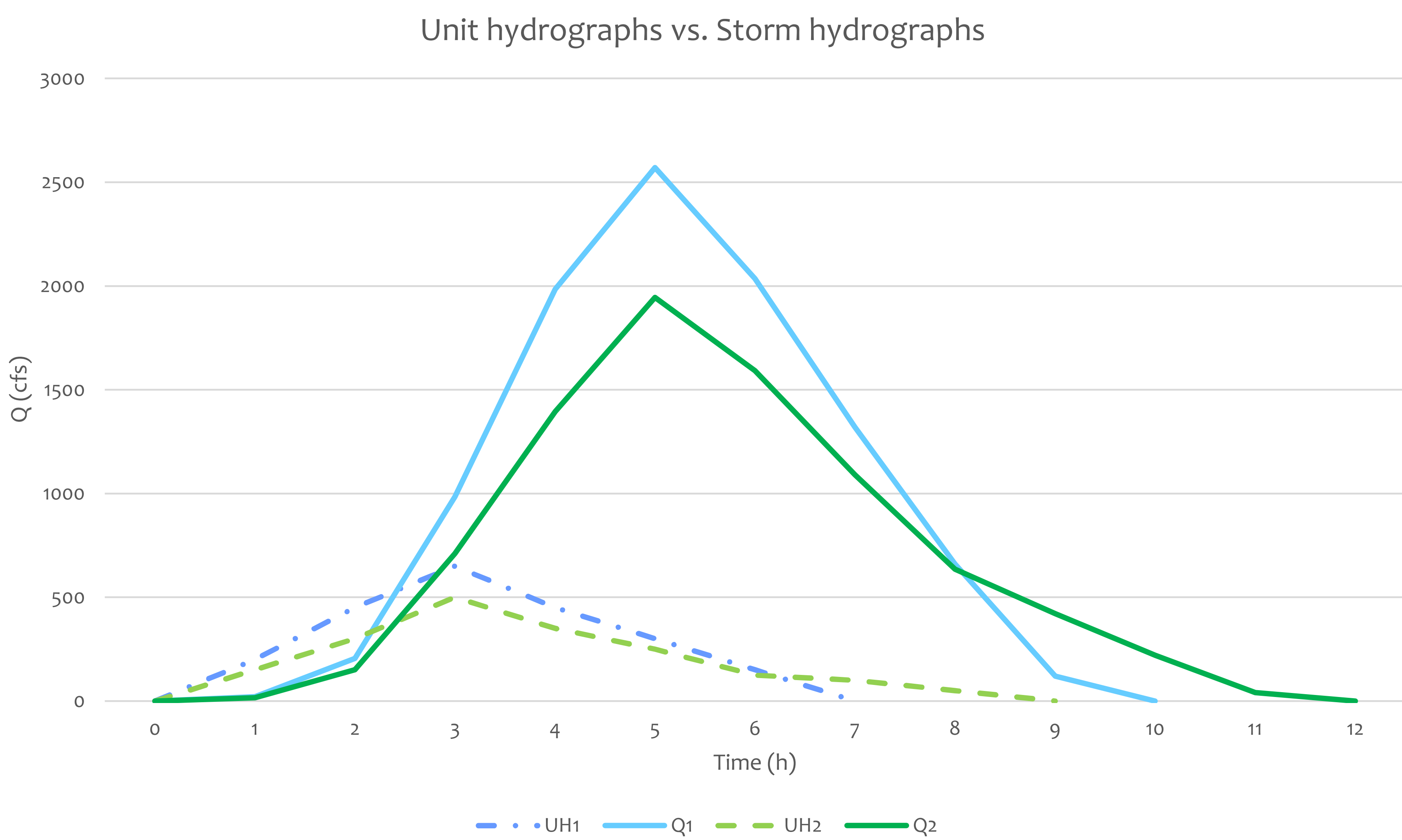
Subarea 1:

Time (h)	UH_1 (cfs)					Q_1 (cfs)
0	0					
1	200					
2	450					
3	650					
4	450					
5	300					
6	150					
7	0					
8	0					
9	0					
10	0					
11						
12						

Subarea 2:

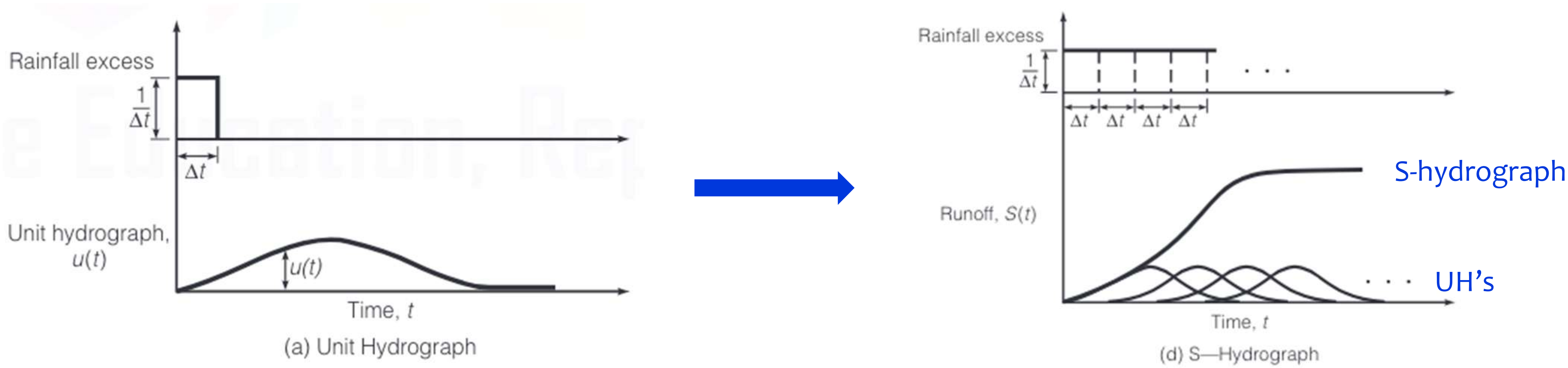
Time (h)	UH_2 (cfs)					Q_2 (cfs)
0	0					
1	150					
2	300					
3	500					
4	350					
5	250					
6	125					
7	100					
8	50					
9	0					
10	0					
11	0					
12	0					

Solution: Hydrograph comparison



4.2 S-HYDROGRAPH

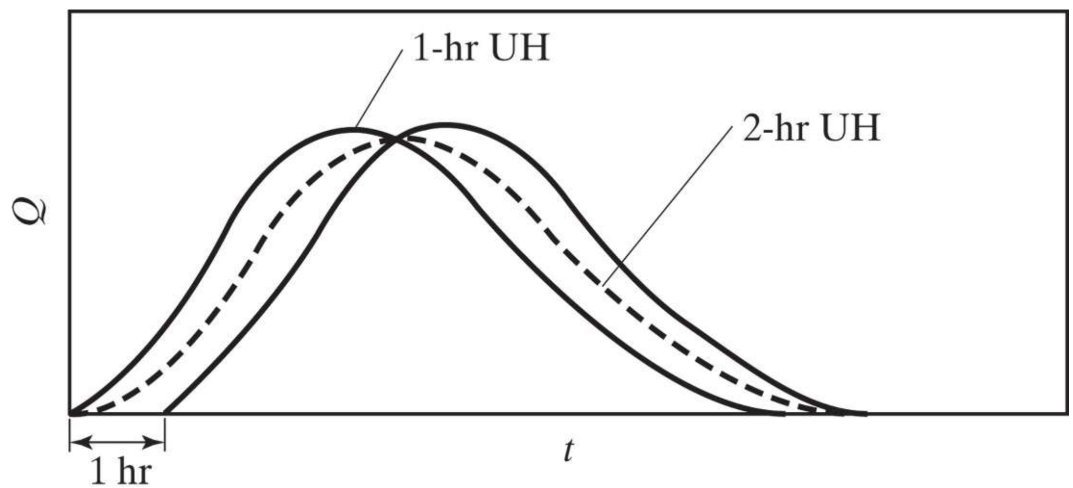
- S-hydrograph is defined as response to storm of infinite **duration**, $D \rightarrow \infty$, and **intensity**, $i = 1/D$
- Develop S-hydrograph by adding lagged UHs for a rainfall excess intensity of $1/D$ mm/h
  - Note that Slide 23 shows an **equilibrium hydrograph** → the response to continuous long-duration rainfall intensity falling until inflow = outflow
  - *S-hydrograph* is often called an *S-curve*



Chin et al. (2014), Fig. 10-4

S-hydrograph use

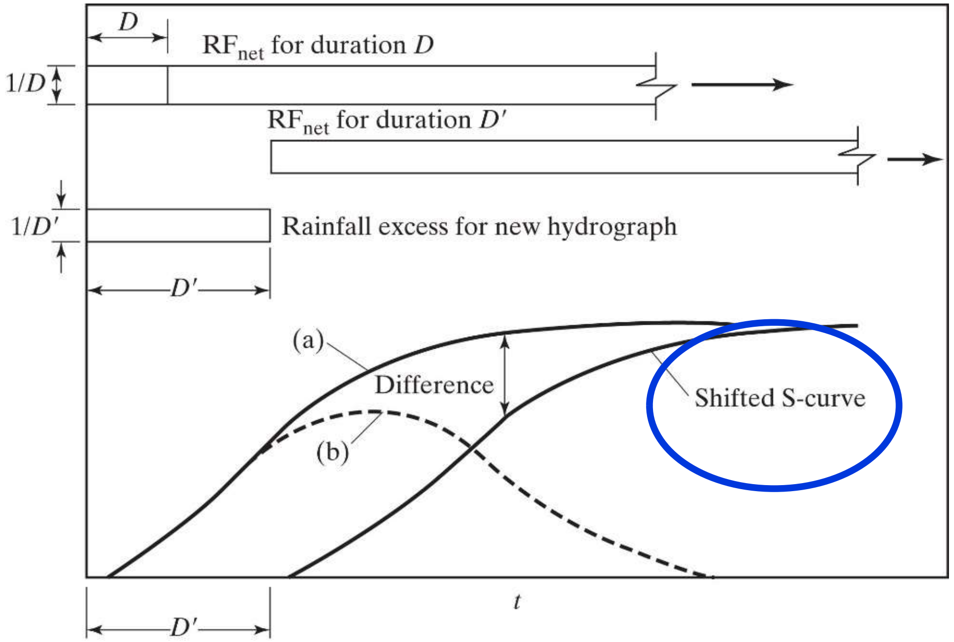
- A unit hydrograph for a watershed is defined for a specific duration of rainfall excess (30 min, 1 hour, 2 hour etc.)
- *However, using the linear property of the UH, a **different UH of shorter or longer duration** can be generated*
  - As an example, a 1-hr UH for a watershed could be transformed to a 2-hr UH by,
    1. Adding two 1-hr UHs, with the second lagged by 1 hr
    2. Adding together the ordinates (cfs or cms values)
    3. Dividing the result by two
  - In this way, the 1 mm of rainfall in 1 hr has been distributed uniformly over 2 hours
  - *A more general approach to conversion uses **S-curves**!*



Bedient et al. (2019)

S-curve method

- S-curve method for converting UH time-base:
  - Assume UH of duration D is known and that new UH for the same watershed with new duration D' is to be generated
- Approach:
  - Add series of UHs of duration D, each successively lagged by D hours
    - Each UH then corresponds to the runoff hydrograph resulting from a continuous rainfall excess of unit depth applied uniformly over duration D (i.e. $i = 1/D$ mm/hr) → results in an S-curve
  - This S-curve is then shifted in time by D' hours, and the ordinates of the two S-curves are subtracted
  - Finally, the resulting hydrograph ordinates are multiplied by D/D'.
    (Subtraction produced runoff corresponding to D'/D units of depth. Scaling ensures the result corresponds to one unit depth)
  - Result: A new UH of duration D'



Ex 3: S-curve application

Use the S-curve method to calculate a 40-min UH based on a 30-min UH for a 1.7 km² urban catchment. The 30-min unit hydrograph is,

Time (min)	0	30	60	90	120	150	180
Runoff (m^3/s)	0	1.2	2.8	1.7	1.4	1.2	1.1

Method: 1) find the S-curve for the 30-min UH; 2) create the new UH, using S-curve method to convert from the 30-min time base to 40-min time base

Solution:

1. Convert the 30-min UH into an S-curve:

Equation for S-curve: $$S(t) = \sum_{k=0}^{\infty} u_{\Delta t}(t - k\Delta t)$$

- $S(t)$ is the set of ordinates for the S-hydrograph or S-curve
- $u_{\Delta t}$ is a unit hydrograph corresponding to a unit rainfall excess applied over duration Δt
- Δt is the duration of the rainfall excess
- $t - k\Delta t$ is the elapsed time since the previous rainfall block

In tabular form, like the other examples, the S-curve is:

Time (min)	$u\Delta t_0$	$u\Delta t_1$	$u\Delta t_2$	$u\Delta t_3$	$u\Delta t_4$	$u\Delta t_5$	$u\Delta t_6$	$u\Delta t_7$	$u\Delta t_8$	$u\Delta t_9$	$u\Delta t_{10}$	**S-curve**
0	0	--	--	--	--	--	--	--	--	--	--	**0**
30	1.2	0	--	--	--	--	--	--	--	--	--	**1.2**
60	2.8	1.2	0	--	--	--	--	--	--	--	--	**4**
90	1.7	2.8	1.2	0	--	--	--	--	--	--	--	**5.7**
120	1.4	1.7	2.8	1.2	0	--	--	--	--	--	--	**7.1**
150	1.2	1.4	1.7	2.8	1.2	0	--	--	--	--	--	**8.3**
180	1.1	1.2	1.4	1.7	2.8	1.2	0	--	--	--	--	**9.4**
210	0	1.1	1.2	1.4	1.7	2.8	1.2	0	--	--	--	**9.4**
240	0	0	1.1	1.2	1.4	1.7	2.8	1.2	0	--	--	**9.4**
270	0	0	0	1.1	1.2	1.4	1.7	2.8	1.2	0	--	**9.4**
300	0	0	0	0	1.1	1.2	1.4	1.7	2.8	1.2	0	**9.4**

2. Develop the new 40-min UH from the given 30-min UH:

From slide 53, the key equation is,

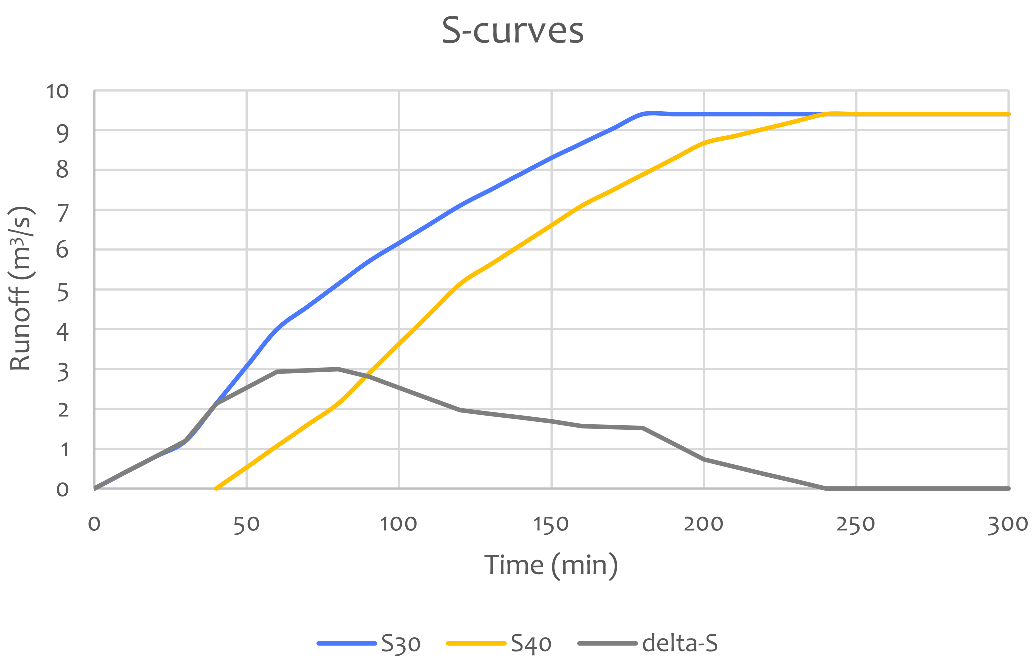
$$UH_{D'}(t) = [S(t) - S(t - D')] \times \frac{D}{D'}$$

Note: the time base for the solution is **40-min**, but values were given at **30-min** increments for the S-curve.

Therefore, we must convert from 30-min increments to get a 40-min UH. We must **interpolate** the S(t) values to 40-min increments!

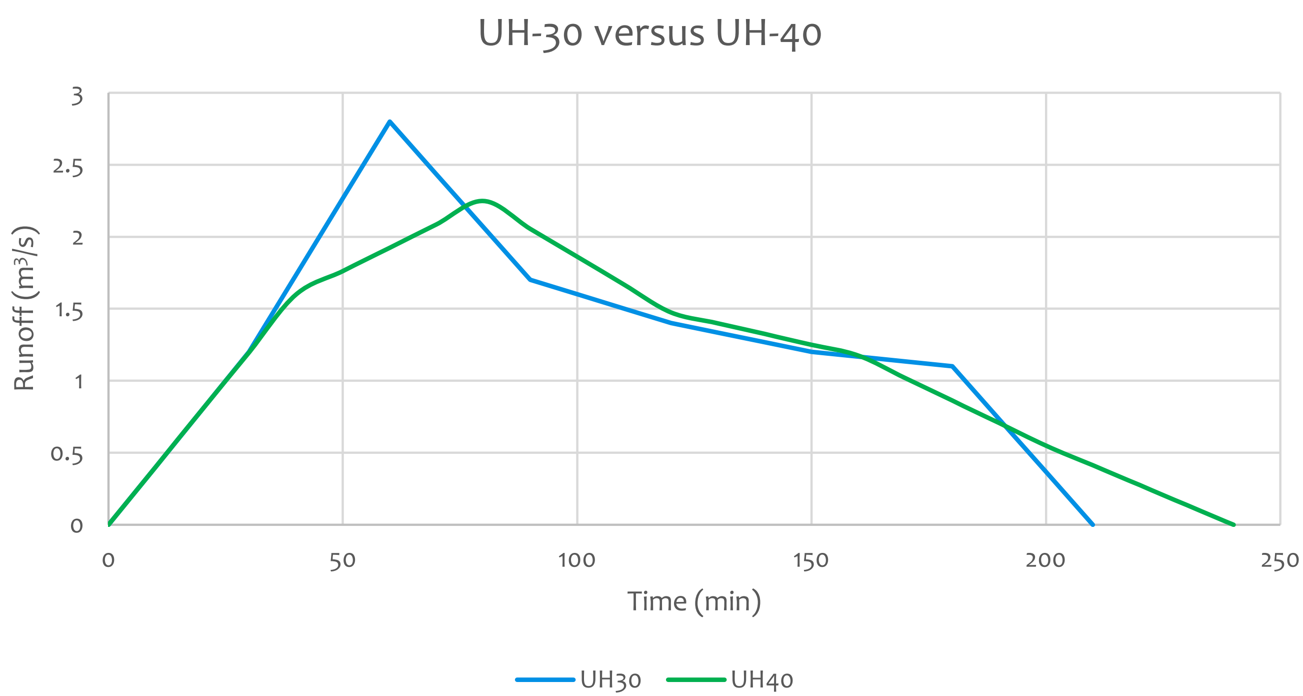
t (min)	$S(t)$ (m³/s)	$S(t-40)$ (m³/s)	$\Delta S = S(t) - S(t-40)$	$UH_{40}(t) = 0.75\Delta S$ (m³/s)
0	0			0
40		0		
80				
120	7.100			
160	8.667	7.100	1.567	1.175
200	9.400	8.667	0.733	0.550
240	9.400	9.400	0	0
280	9.400	9.400	0	0

Solution:



Note: the S-curves should be smoother. I used linear interpolation to graph them, so many segments are linear that should be curved. Delta-S is "jerky" because it's graphed at 10-min intervals.

Note also the difference in vertical scales in the two graphs. UH-30 is more compact and UH-40 is spread out.

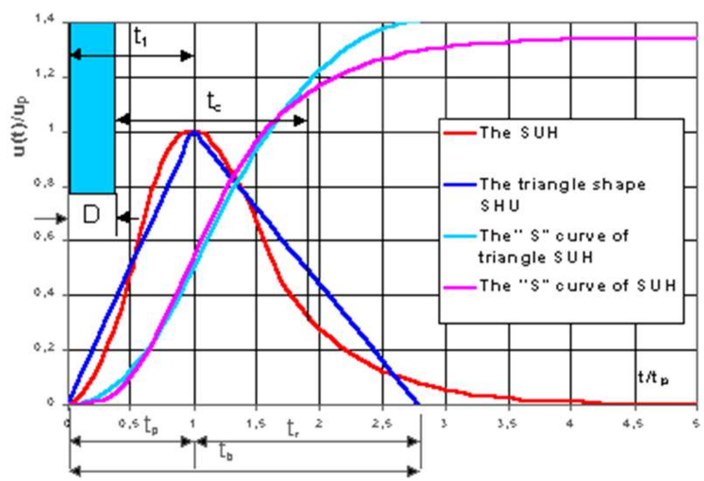


4.3 Synthetic Unit Hydrographs

- The unit hydrographs developed to this point use **gauged data.** *However, such data are not always available* → watersheds without these data are called “ungauged”

“Most [synthetic] UH methods were developed in the period from 1932 to 1970, and they still provide one of the most useful and accurate approaches for hydrological prediction for a given rainfall event”

  – Once developed, synthetic UHs can be used with historical or design rainfalls to produce hydrographs at outlet of watershed
  – *Should update UH occasionally to represent land use change and channel alterations*



http://echo2.epfl.ch/VICAIRE/mod_1b/chapt_4/main.htm

Synthetic unit hydrographs

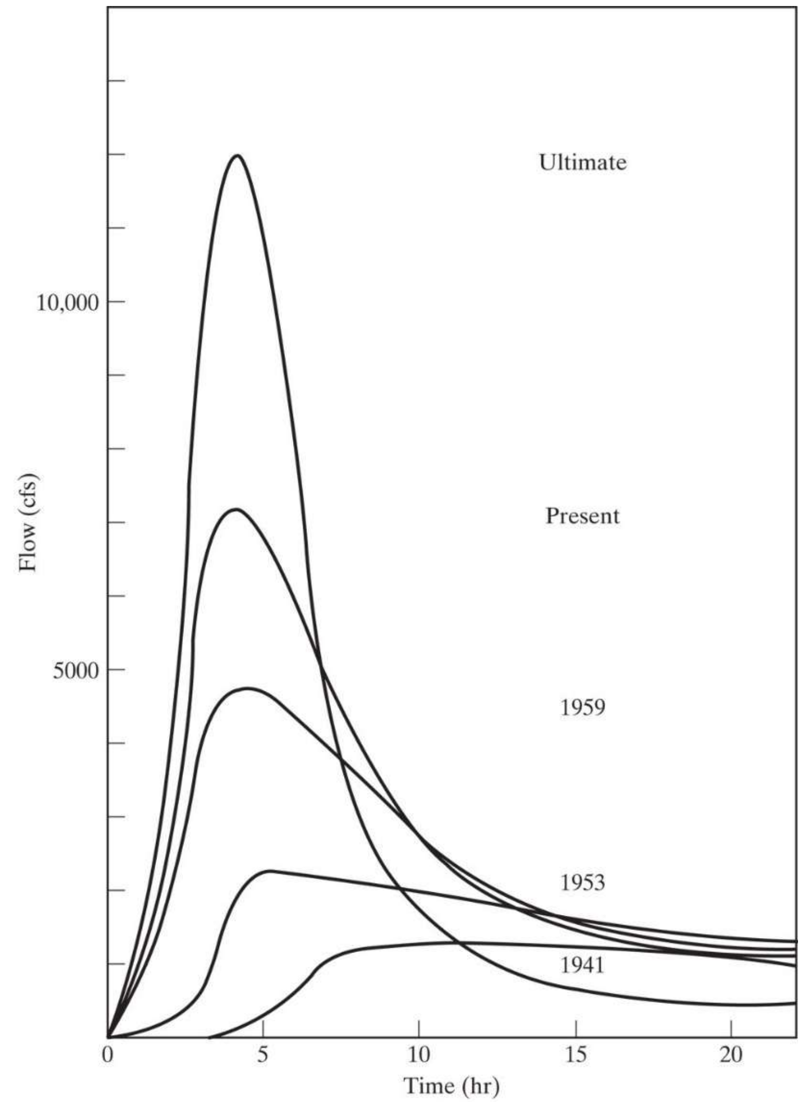
- A **number of equations** have been developed over the years to predict many of the UH parameters as functions of **measureable watershed characteristics**
  - Methods for **ungauged** watersheds come from **theoretical or empirical formulas** relating hydrograph peak flow and timing to integrated effect of size (watershed area, length of main channel), slope, shape and storage characteristics of watersheds

- Most methods for synthetic UHs relate lag time, t_p, or time of rise, T_R, of hydrograph to measures of length of main channel and shape of basin, or to inverse of slope (longer, lower slope basins have higher T_R)
- A second relation is usually presented between peak flow, Q_p, and area of basin, and between Q_p and inverse of t_p or T_R of hydrograph → larger areas produce higher Q_p, and higher Q_p must have smaller t_p to keep volume of UH constant with 1 mm DRO

Bedient et al. (2019: 87)

Modifying factors

- **Original synthetic hydrograph** methods did not consider urbanization effects
  - Figure shows changes in shape of Brays Bayou unit hydrograph as result of land use change
- *More modern empirical formulas account for urban channels, percent impervious area, or percent storm-sewered area*



Bedient et al. (2019)

Snyder's method

- Snyder (1938) was first to develop synthetic UH – for basins from 10 mi^2 to 10000 mi^2 – based on study of Appalachian watersheds
- Snyder's relations define points for a UH produced by rainfall excess of duration $D = t_p/5.5$, and consist of the following three equations,

$$t_p = C_t(LL_c)^{0.3}$$

where t_p = basin lag (h)

L = length of main stream from outlet to divide (mi)

L_c = length along main stream to point nearest watershed's centroid (mi)

C_t = coefficient usually ranging from 1.8-2.2

$$Q_p = 640\, C_p A/t_p$$

where Q_p = peak discharge of unit hydrograph (cfs)

A = drainage area (mi^2)

C_p = storage coefficient ranging from 0.4 to 0.8 (larger values of C_p assoc. with smaller values of C_t), and

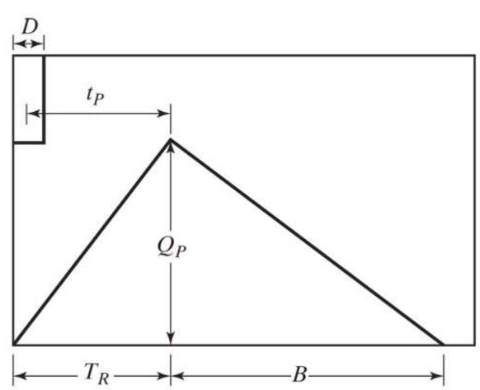
$$T_b = 3 + t_p/8$$

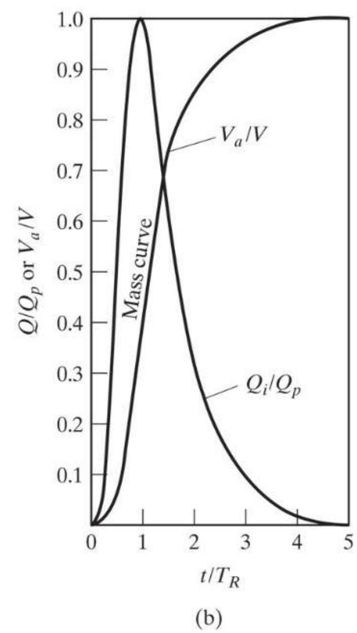
Bedient et al. (2019)

SCS methods

- Soil Conservation Service (SCS) – now called *Natural resources Conservation Service (NRCS)* – developed an approach for synthetic UH based on a **dimensionless hydrograph**
  - Used large number of UHs from gauged watersheds of various sizes and geographic locations
  - Earliest method assumed a hydrograph shaped as simple **triangle**, with rainfall duration D (h), time of rise T_R (h), time of fall B (h), and peak flow Q_p (cfs)

(1) Triangle





(2) Dimensionless Hydrograph

Bedient et al. (2019)

SCS methods

- Triangular UH equations:
  - Volume of direct runoff is,

$$\boxed{Vol = \frac{Q_p T_R}{2} + \frac{Q_p B}{2}}$$

Or,

$$Q_p = \frac{2 \cdot Vol}{T_R + B}$$

  - Then, since a review of a large number of UHs revealed that $B = 1.67 T_R$, the equation simplifies with unit conversions to,

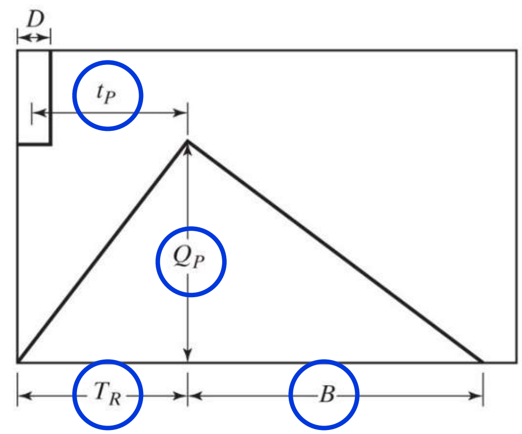
$$\boxed{Q_p = \frac{484A}{T_R}}$$

where A = watershed area (mi²)

T_R = time of rise (h)

The factor of 484 can differ by watershed (range of 300-600)

Solve for these values



Bedient et al. (2019)

SCS methods

- Lag time is estimated from one of several empirical equations; one often used is,

$$\boxed{t_p = \frac{L^{0.8}(S+1)^{0.7}}{1900\sqrt{y}}}$$

where t_p = lag time (h)

L = length to divide (ft)

y = average watershed slope (%)

S = "potential abstraction" = ${}^{1000}/_{CN} - 10$ (in)

CN is a "curve number" for various soils and land uses, as explained below

- Time of rise is very simple (see figure, previous page): $\boxed{T_R = \frac{D}{2} + t_p}$

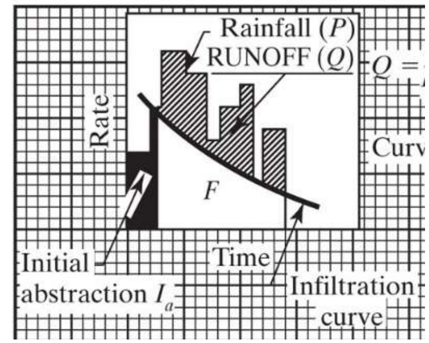
- ***Make sure you use the units as given above for the SCS method***
- *SCS dimensionless UH can be used to develop 1) a **triangular UH**, or 2) a **curved UH**, using the same t_p and Q_p as for the triangular UH*

Bedient et al. (2019)

SCS methods

- SCS runoff estimates assume relationship between total storm rainfall P, runoff Q, and potential abstraction S,

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S}$$



- This is derived from assumptions that,
  - $F/S = Q/P_e$, where $F = P_e - Q$ is the infiltration occurring after runoff begins, and $P_e = P - I_a$ is effective rainfall
  - And that initial abstraction I_a occurs before runoff
    - From SCS watershed studies, $I_a = 0.2S$ → "antecedent moisture condition II" → *there are other assumptions for conditions I and III*

- Equation above is basis for the figure on next slide

Bedient et al. (2019)

SCS curve numbers

- The runoff curve number, CN, is related to potential abstraction by $CN = 1000/(S + 10)$
- Curve numbers vary from 0 to 100:

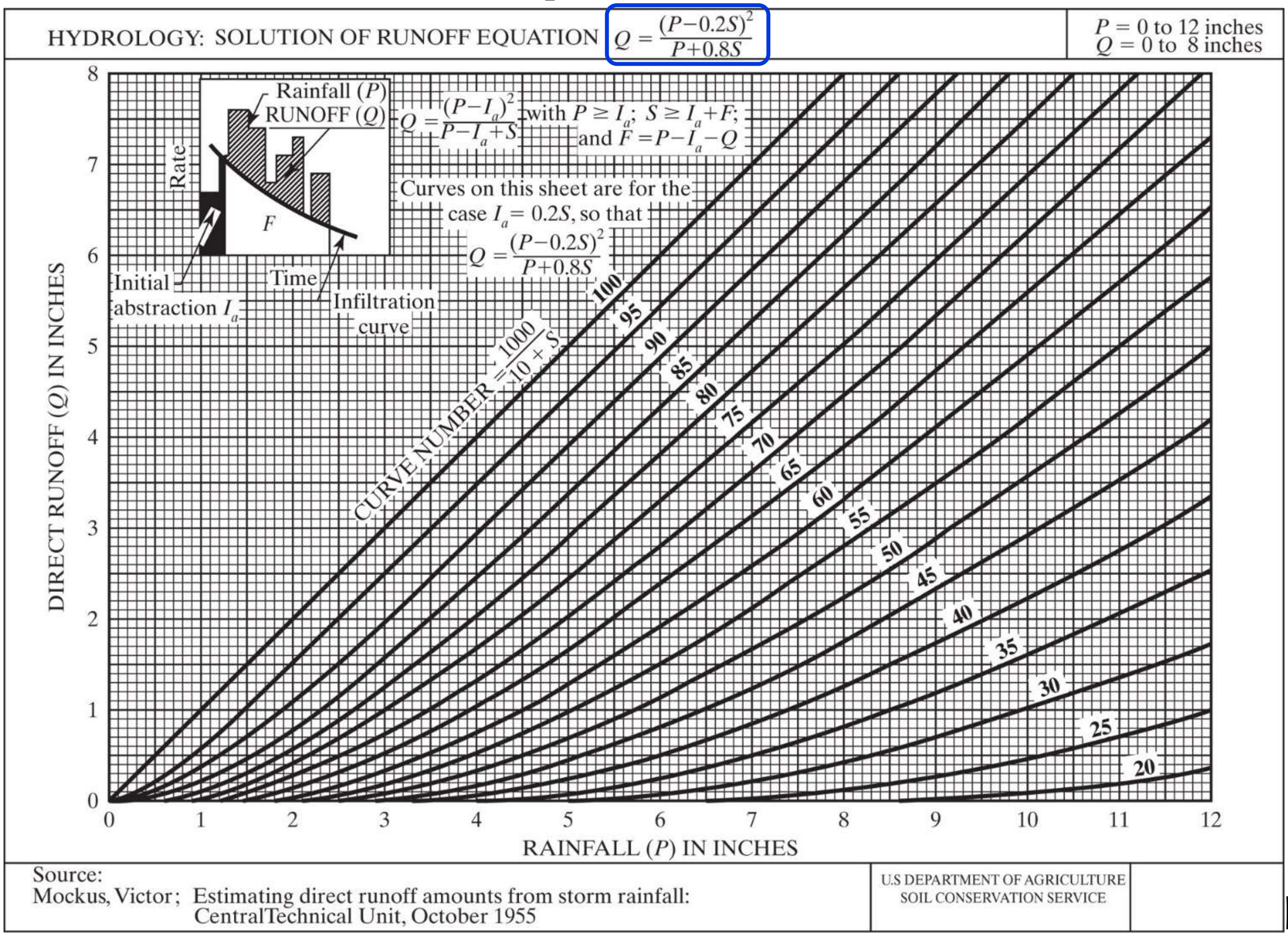


Fig. 2-8 in Bedient: Assumes *antecedent moisture condition II*

Other antecedent moisture conditions and effects of urbanization can be developed using SCS reports

Bedient et al. (2019)

SCS curve numbers

- *Curve numbers tabulated for different land uses and soil types*
  - **Soil group A** is sandy and well-drained, **group B** is sandy loam, **group C** is clay loam or shallow sandy loam, and **group D** is a poorly-drained, plastic clay that swells when wet → A has highest infiltration capacity and D has the lowest
  - *Composite curve numbers can be calculated for watersheds made of several soil types and land uses*

Table 2–1 Runoff Curve Numbers for Selected Agricultural, Suburban, and Urban Land Use (Antecedent Moisture Condition II; $I_a = 0.2S$)

	Hydrologic Soil Group			
Land Use Description	**A**	**B**	**C**	**D**
Cultivated land[1]				
Without conservation treatment	72	81	88	91
With conservation treatment	62	71	78	81
Pasture or range land				
Poor condition	68	79	86	89
Good condition	39	61	74	80

Bedient et al. (2019)

SCS curve number table

Table 2–1 Runoff Curve Numbers for Selected Agricultural, Suburban, and Urban Land Use (Antecedent Moisture Condition II; $I_a = 0.2S$)

Land Use Description		Hydrologic Soil Group A	B	C	D
Cultivated land[1]					
Without conservation treatment		72	81	88	91
With conservation treatment		62	71	78	81
Pasture or range land					
Poor condition		68	79	86	89
Good condition		39	61	74	80
Meadow					
Good condition		30	58	71	78
Wood or forest land					
Thin stand, poor cover, no mulch		45	66	77	83
Good cover[2]		25	55	70	77
Open spaces, lawns, parks, golf courses, cemeteries, etc.					
Good condition: grass cover on 75% or more of the area		39	61	74	80
Fair condition: grass cover on 50%–75% of the area		49	69	79	84
Commercial and business areas (85% impervious)		89	92	94	95
Industrial districts (72% impervious)		81	88	91	93
Residential[3]					
Average lot size	Average % impervious[4]				
1/8 ac or less	65	77	85	90	92
1/4 ac	38	61	75	83	87
1/3 ac	30	57	72	81	86
1/2 ac	25	54	70	80	85
1 ac	20	51	68	79	84
Paved parking lots, roofs, driveways, etc.[5]		98	98	98	98
Streets and roads					
Paved with curbs and storm sewers[5]		98	98	98	98
Gravel		76	85	89	91
Dirt		72	82	87	89

Bedient et al. (2019)

Ex 4: Composite CN

EXAMPLE 2–5 **SCS CURVE-NUMBER METHOD**

A watershed is 40% wooded (good condition) and 60% residential (1/4-ac lots). The watershed has 50% soil group B and 50% soil group C. Determine the runoff volume if the rainfall is 7 in. Assume antecedent moisture condition number II (Table 2–1).

SOLUTION

Land Use	Soil Group	Fraction of Area	CN
Wooded	B	0.4(0.5) = 0.2	55
	C	0.4(0.5) = 0.2	70
Residential	B	0.6(0.5) = 0.3	75
	C	0.6(0.5) = 0.3	83

The weighted CN is

$$\text{CN} = 0.2(55) + 0.2(70) + 0.3(75) + 0.3(83) \text{ or}$$

$$\text{CN} = 11 + 14 + 22.5 + 24.9 = 72.4$$

or, using CN = 72, runoff volume is 3.9 in for the given rainfall (Fig. 2–8).

Example 2-5 in Bedient et al. (2019)

Ex 5: Triangular UH

SCS TRIANGULAR UNIT HYDROGRAPH **EXAMPLE 2–6**

Use the SCS method to develop a UH for the area of 10 mi^2 described below. Use rainfall duration of $D = 2.0$ hr. Sketch the approximate shape of the triangular UH.

$$L = 5 \text{ miles}$$
$$L_c = 2 \text{ miles}$$

The watershed consists of meadows in good condition with soil group D. The average slope in the watershed is 100 ft/mi. Sketch the resulting SCS triangular hydrograph.

SOLUTION

Equation (2–18) gives the following relationship for t_p:

$$t_p = \frac{L^{0.8}(S + 1)^{0.7}}{1900\sqrt{y}}$$

From Table 2–1, the SCS curve number is found to be 78. Therefore,

$$\begin{aligned} S &= 1000/\text{CN} - 10 \\ &= 1000/78 - 10, \\ S &= 2.82 \text{ in.} \end{aligned}$$

Converting $L = 5$ mi, or

$$L = (5 \text{ mi})(5280 \text{ ft/mi}) = 26{,}400 \text{ ft}$$

Example 2-6 in Bedient et al. (2019)

The slope is 100 ft/mi, so convert to percent for the equation

$$\begin{aligned} y &= (100 \text{ ft/mi})(1 \text{ mi}/5280 \text{ ft})(100) \\ &= 1.9 \end{aligned}$$

and

$$\begin{aligned} t_p &= \left[\frac{(26{,}400)^{0.8}(2.82 + 1)^{0.7}}{1900\sqrt{1.9}}\right] \\ &= 3.36 \text{ hr.} \end{aligned}$$

From Equation (2–17) and with rainfall duration $D = 2.0$ hr,

$$\begin{aligned} T_R &= D/2 + t_p \\ &= (2/2) + 3.36 \text{ hr}, \\ T_R &= 4.36 \text{ hr, the rise time of the hydrograph,} \end{aligned}$$

and Equation (2–16) gives for $A = 10 \text{ mi}^2$

$$\begin{aligned} Q_p &= \frac{484\,A}{T_R} \\ &= \frac{484(10)}{4.36} \text{ cfs}, \\ [Q_p &= 1{,}110 \text{ cfs.}] \end{aligned}$$

To complete the graph, it is also necessary to know the time of fall B. The volume is known to be 1 in. of direct runoff over the watershed, so

$$\text{Vol} = (10 \text{ mi}^2)\left(\frac{5280 \text{ ft}}{\text{mi}}\right)^2\left(\frac{\text{ac}}{43{,}560 \text{ ft}^2}\right)(1 \text{ in.}) = 6400 \text{ ac-in.}$$

From Equation (2–12),

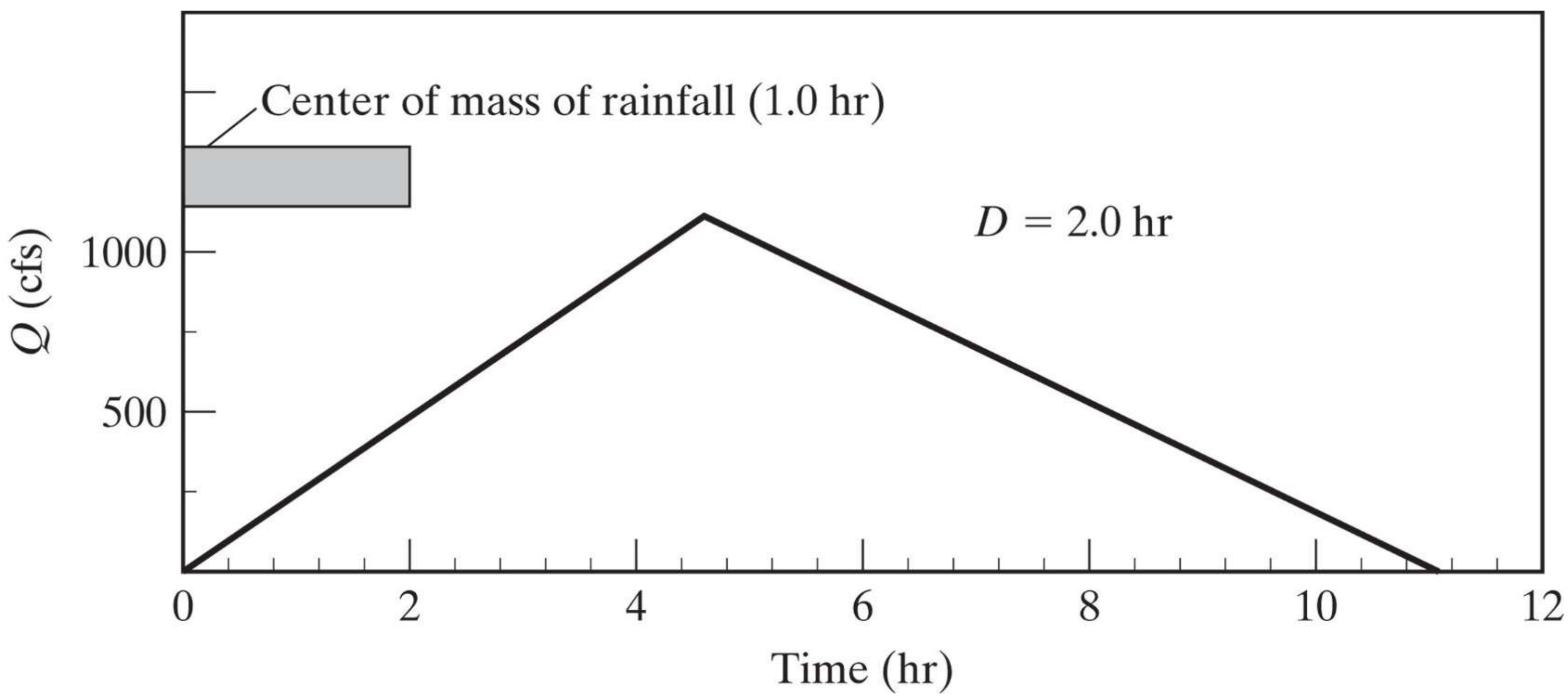
$$\text{Vol} = \frac{Q_p T_R}{2} + \frac{Q_p B}{2} = 6400 \text{ ac-in} = 6400 \text{ cfs-hr},$$

$$6400 \text{ cfs-hr} = \frac{(1110 \text{ cfs} \times 4.36 \text{ hr})}{2} + \frac{(1110 \text{ cfs})(B \text{ hr})}{2},$$

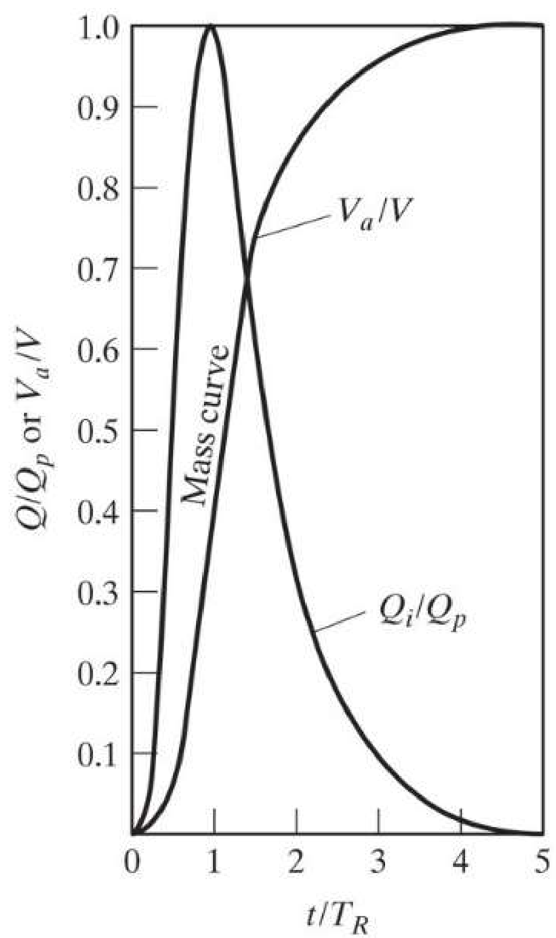
so

$$B = 7.17 \text{ hr}.$$

The triangular unit hydrograph is shown in Figure E2–6; note the time base of 11.5 hr. The next example demonstrates the use of the dimensionless SCS UH; for this example. Table E2–6 lists the resulting shaped SCS hydrograph.



And to convert values to SCS hydrograph:



(b)

Table E2–6 $T_R = 4.36$ hrs and $Q_p = 1110$ cfs

t/T_R	Q_i/Q_p	V_a/V	t (hr)	Q_i (cfs)
0	0	0	0	0
0.2	0.1	0.006	0.87	111
0.3	0.19	0.012	1.3	211
0.4	0.31	0.035	1.74	344
0.5	0.47	0.065	2.18	522
0.6	0.66	0.107	2.62	733
0.7	0.82	0.163	3.05	910
0.8	0.93	0.228	3.49	1032
0.9	0.99	0.3	3.92	1099
1.0	1.0	0.375	4.36	1110
1.2	0.93	0.522	5.23	1032
1.4	0.78	0.65	6.10	866
1.6	0.56	0.75	6.98	622
1.8	0.39	0.822	7.85	433
2.0	0.28	0.871	8.72	311
2.2	0.207	0.908	9.59	230
2.4	0.147	0.934	10.46	163
2.6	0.107	0.953	11.36	119
2.8	0.077	0.967	12.21	85
3.0	0.055	0.977	13.1	61
3.4	0.029	0.989	14.82	32
4.0	0.011	0.997	17.44	12
5.0	0	1.0	21.8	0